Project B: Determining the Relationship of Insulin to Diabetes

- Orientation
- Project Synopsis
- Phase I: Diabetes
  - Phase II: Tools for Monitoring Diabetes
    Activity 1: Fuel
    Activity 2: Controlling Diabetes
    Activity 3: Modelling Two Competing Processes
- Phase III: Solving a Regimen Problem
Project B: Determining the Relationship of Insulin to Diabetes

Orientation

Time:

Phase I: Diabetes
Activity 1: Diabetes Research .................................................. 1 to 2 class periods

Phase II: Tools for Monitoring Diabetes
Activity 1: Fuel ................................................................. 1 to 2 class periods
Activity 2: Controlling Diabetes ................................................. 2 class periods
Activity 3: Modelling Two Competing Processes .................. 1 to 2 class periods

Phase III: Solving a Regimen Problem ................................. 1 class period

Instructional Strategies:
• Direct Instruction
• Case Study
• Group Work
• Issues Inquiry

Real-World Applications:
Some real-world applications of the skills learned in this project include:
• representing and interpreting health data in the form of graphs
• making better decisions about health based on the interpretation of health data
• determining the optimal price for concert ticket sold by scalpers

Project Overview:
Figure 1 on the following page is a schematic to help you conceptualize Project B: Determining the Relationship of Insulin to Diabetes.
Project B: Determining the Relationship of Insulin to Diabetes

**Figure 1: Project B: Determining the Relationship of Insulin to Diabetes**

**Task:** To design a regimen of insulin in order to overcome abnormally low levels of insulin in a person with diabetes

**Modelling a real situation**

**Developing mathematical language**

**Activity 1: Fuel**
- Reinforcing the sense of rational numbers
- Applying the Laws of Exponents

**Activity 2: Controlling Diabetes**
- Making sense of the coefficients in a polynomial (linear) expression
- Modelling a situation with a formula involving rational expressions
- Identifying linear patterns
- Determining the effects of changing the coefficients of a polynomial model

**Activity 3: Modelling Two Competing Processes**
- Developing a sense of two intersecting linear models

**Generalizing**

To model a situation involving two competitive processes
Project Synopsis

The objective of this project is to reinforce student skills in pattern recognition and in performing calculations with rational numbers and powers while developing their consciousness about diabetes.

Students identify and graphically represent patterns. These patterns are then applied to make predictions and draw conclusions.

Students analyse two competitive processes (input and output) with linear models in order to develop their understanding of polynomial coefficients and their significance.

Students are evaluated on their skills and abilities in:
• statistics, that is, establishing the correlation between two variables (Activity 1: Diabetes Research)
• number concepts and number operations, that is using rational numbers and powers in a real context by applying the laws of exponents (Activity 1: Fuel)
• patterns, variables and equations that is, model a real situation with polynomials and solving first-degree equations (Activity 2: Controlling Diabetes, and Activity 3: Modelling Two Competing Processes).

This project has three phases.

In Phase I: Diabetes, students are introduced to the topic of diabetes and assigned the task of collecting more information about the disease.

Phase II: Tools for Monitoring Diabetes has three major activities, Activity I: Fuel; Activity 2: Controlling Diabetes; and Activity 3: Modelling Two Competing Processes. In Activity 1: Fuel, students identify the variables that are involved in diabetes development. Students relate the causes, the diagnosis and the treatment of diabetes. In Activity 2: Controlling Diabetes, students recognize the patterns representing the change in the variables (sugar and insulin) over time. Students also have an opportunity to design regimens. Finally, in Activity 3: Modelling Two Competing Processes, students learn about the biochemical properties of insulin, and its role in the metabolism of sugar and in regulating the blood glucose level by injection. Students are assigned the tasks of designing other situations where two competing processes are involved and of modelling the result.

Phase III: Solving a Regimen Problem is the evaluation component of this project. In this phase, the mathematical and scientific concepts are evaluated from reports prepared by groups of students. These reports include: a description of the preparation of a regimen of insulin injections from a table of blood sugar readings; answers to questions about the impact of changes in the diet; and the social and economic implications of diabetes.
Activity 1: Diabetes Research

Objective:
The objective of this activity is for students to become familiar with the nature of diabetes, the vocabulary used, and the physiological effects of the illness. Students collect and represent data related to the occurrence of diabetes and analyse the results in terms of two variables. You may also choose to do Phase I as part of the science course or in collaboration with the science teacher.

Note: Before you begin this project, check with your school administrator to determine whether there is any school, district or Ministry of Education policy governing the use of body fluids for testing purposes. Some activities require authentic data from humans. For sources of human physiological data, consult local health officials and/or diabetes organizations.

Materials:
• information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centres
• access to Internet
• prepared microscope slide of a pancreas cell(s)
• Student Activity: Diabetes Research
• access to a nurse, physician, dietitian or student with diabetes

Procedure:
Diabetes is a condition in which the body cannot properly use the energy from food eaten. When food is ingested, some of it is broken down by the digestive system into forms of sugar, one of which is called glucose. Glucose is the main fuel of the body. It enters the bloodstream and goes to the cells where it is used as energy or where it is stored in the liver for future use.

Insulin is a chemical made and stored in specific cells of an organ called the pancreas. Refer to Figure 2: Where Insulin is Made. Insulin is needed to help sugar leave the bloodstream and enter cells of the body: liver cells following digestion, and all body cells when human activity levels rise. Refer to Figure 3: Microscopic Sections of Pancreas and to Figure 4: Intake of Glucose Into an Islet Cell.
Diabetes develops when the body cannot:
• make any insulin;
• make enough insulin; or
• properly use the insulin it makes.

Refer to Figure 5: What Happens if Insulin is Not Produced or if Insulin Does Not Work to see the impact on a cell of the above conditions. When diabetes develops, glucose stays in the blood, is not being stored in the liver and is excreted in the urine. Body cells do not get the glucose they need to produce smaller energy packets. Instead the body starts burning stored fat to make energy. When fat is broken down by the digestive system, poisons form called ketones. The body tries to rid itself of the ketones by sending them out in the urine. A buildup of ketones in the body can be dangerous and lead to a problem called ketoacidosis.

Invite a student with diabetes, a nurse, or dietitian to make a brief presentation that includes a videotaped demonstration of how a blood glucose reading is taken using a meter or a strip.

Provide a microscope preparation of the pancreas showing the scattered Islets of Langerhans. Have students draw the Islets and their cells, and estimate the dimensions of the pancreas and the Islets.

Have students do Student Activity: Diabetes Research to guide their investigation. Then ask them to present their findings in a short report.

Divide students into small groups and have each group select two variables. Students can represent the relation between these two variables as a graph or a diagram. Have them describe the type of relationship between the two variables by determining if there is a strong correlation and if there is a direct or inverse relationship.

**Figure 2: Where Insulin is Made**
Figure 3: Microscopic Sections of Pancreas

Note: the boundary of this ‘thin’ section region defines the Islet.
Figure 4: Intake of Glucose Into an Islet Cell

Note: this diagram shows insulin promoting the intake of glucose into an Islet cell.

Figure 5: What Happens if Insulin is Not Produced or if Insulin Does Not Work

Glucose entry denied when:
- receptor does not accept insulin
- insulin is not present
- receptor sites are damaged
Student Activity: Diabetes Research

Student Name: _________________________ Date: _________________

1. Research: What is diabetes?
   • What is diabetes?
   • What are the causes of diabetes?
   • What are some of the characteristics of people who have diabetes, for factors such as age, gender and geographic location? Are there some groups of people who have a higher than average incidence of the disease? If so, why might this be?
   • What is the number of Canadians with diabetes?
   • What is the percentage of Canadians with diabetes?
   • How many people have Type I diabetes and how many have Type II?

   Complete Table 1: Comparing Type I and Type II Incidence by Age Group. Complete Figure 6: Type I and Type II Incidence by Age Group.

Table 1

Comparing Type I and Type II Incidence by Age Group

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Type I Diabetes</th>
<th>Type II Diabetes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>
Figure 6: Type I and Type II Incidence by Age Group

Percentage of Canadians

Age Groups
What are the symptoms of diabetes?

What are the consequences to the individual of having diabetes?

What are the financial costs to the individual, the province and the nation?

What are the costs involved in the treatment of diabetes? How do these costs compare to the treatment costs 50 years ago?

Is there a cure for diabetes? What are some recent developments from medical research?

What are some of the ways that people cope with diabetes?

What is the significance of 1-800-BANTING, used as the toll-free number for the Canadian Diabetes Association.
Comment on the following terms related to diabetes:

- **Pancreas:**
  - Description:
  - Dimensions:
  - Diagram:

- **Islet of Langerhans:**
  - Description:
  - Dimensions:
  - Diagram:

- **Beta cell:**
  - Description:
  - Dimensions:
  - Diagram:
• Describe the role that Banting and Best played in the history of diabetes research.

• Insulin:
  Definition:

  Chemical formula:

• Glucose:
  Definition:

  Chemical formula:

• Sugars:
  Definition:

  Chemical formula generalized:

• Ketones:
  Definition:

  Chemical formula generalized:
Carbohydrates:
Definition:

Chemical formula generalized:

Type I Diabetes:
Definition:

Type II Diabetes:
Definition:

2. What are the complications of diabetes?

Low blood sugar (Hypoglycemia)
Causes:

Signs of mild hypoglycemia:

Ways to treat low blood sugar:

Ways to avoid low blood sugar:
Student Activity: Diabetes Research

- High blood sugar (Hyperglycemia)
  Causes:

  Signs of hyperglycemia:

  Ways to treat high blood sugar:

  Ways to avoid high blood sugar:

- Ketoacidosis
  Causes:

  Signs of mild ketoacidosis:

  Ways to avoid ketoacidosis:

  Ways to treat ketoacidosis:

Description of long term complications:
3. Statistics

Name and describe any two variables related to diabetes.

Describe the kind of relationship you predict between the two variables for a given set of conditions. Explain.

Represent their relationship to each other in a diagram.

Complete Table 2: Values of the Two Variables, by recording the values of the two variables for example, over time. Properly identify the units of each of the variables.

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values of the Two Variables</td>
</tr>
<tr>
<td>Variable #1</td>
</tr>
<tr>
<td>__________</td>
</tr>
<tr>
<td>__________</td>
</tr>
<tr>
<td>__________</td>
</tr>
<tr>
<td>__________</td>
</tr>
<tr>
<td>__________</td>
</tr>
<tr>
<td>__________</td>
</tr>
</tbody>
</table>
Create a scatter graph in the space below for the two variables and analyse the relationship. Were your predictions correct?
Phase II: Tools for Monitoring Diabetes

Activity 1: Fuel

Objective:
Students perform operations with rational numbers and powers. They develop simple linear models representing two competing processes in order to develop their understanding of the coefficients of a polynomial model representing a real situation.

Materials:
• information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centres
• Student Activity: Fuel
• graphing calculator or mathematics software for example Mathcad and Mathematica
• access to a nurse, physician, dietitian, or to a student with diabetes
• copies of diabetes diaries
• Good Health Eating Guide Poster or equivalent

Procedure:
Students learn about controlling the glucose level in the bloodstream by identifying three factors involved in a relationship when fuel is burned:
1. diet
2. activity
3. insulin.
See Figure 7: Balancing Act below.

Figure 7: Balancing Act
These factors contribute to the fluctuations in the blood glucose level. People with Type I diabetes need to adjust the glucose level by manipulating these three factors. To do this they must understand the effect and contribution of each of the factors.

Have students use the 24-hour timeline (see Figure 12: 24-Hour Timeline in Student Activity: Fuel) to show what they expect the glucose level to be in the blood of a normal person under normal conditions with the proper amount of insulin. This is called a qualitative estimation, since students can only estimate whether the level is higher or lower.

Ask students to include meals, sleep time, exercise, and stress on a timeline and to estimate the effect these factors have on the glucose level.

For example, in Figure 8: Typical Eating Pattern for Individuals in the Human Population below, it is apparent that there is a regular interval of food intake. In Figure 9, Blood Glucose (mmol/L) Normal Pattern, the curve represents a typical pattern for blood glucose. Comment upon the fluctuations during the day.

**Figure 8: Typical Eating Pattern for Individuals in the Human Population**

<table>
<thead>
<tr>
<th>Breakfast</th>
<th>Lunch</th>
<th>Supper</th>
<th>Snack</th>
</tr>
</thead>
</table>

Tell students that the blood glucose level is expressed in mmol/L. This unit is used by doctors and nurses when referring to diabetes and concentrations of some blood constituents. (One mmol, millimole, is 0.001 of one mole.)

Tell students that one mole of glucose weighs 180 g (if applicable, refer to the Activity 2: Controlling Diabetes for the chemical properties of glucose and for the concept of mole). Tell students that chemists express concentrations using a unit called the mole (mol) instead of grams.

Indicate that the mole of a substance is the number of elementary particles \(6.023 \times 10^{23}\) of that particular substance. Have students perform some calculations using the concept of moles. For instance, what is the number of moles of one marble when one marble weighs 10 g? What is the number of moles in a rice grain when one grain of rice weighs 1 g? What is the number of moles in a grain of sand when one grain of sand weighs 0.1 g?

Before eating, the normal level of blood sugar (glucose) is 4-6 mmol/L. Eating raises the level (1 or 2 hours after a meal, the blood sugar can go up to 8mmol/L); exercising lowers the level because it utilizes glucose (energy). After a meal, the blood glucose level increases. When this happens, insulin goes from the pancreas to the blood to help the extra glucose enter the liver’s cells. Figure 10: Insulin Rate (in U/h) Normal Response, shows how the body sends extra insulin into the blood to look after the sugar that has been absorbed into the bloodstream after a meal.
Project B: Determining the Relationship of Insulin to Diabetes

Ask students to determine the amount of glucose (in g) normally present in one litre and in 10 litres of blood. (Ten litres is the average volume of blood in adults.)

- 180 g is the weight of 1 mole of glucose
- 0.18 g is the weight of 1 mmol of glucose
- 0.18 x 4-6 mmol/L = 0.72 g to 1.08 g of glucose in 1 litre of blood
- or 7.2 g to 10.8 g of glucose in the entire circulatory system.

Provide students with the following problem about burning calories.

When one mole of glucose (that is, 180 g) is burned by oxygen in cells, the process releases 2820 kJ (kilojoules) of heat energy in the form of chemical bonds of ATP.

If your glucose reading rises to 8 mmol/L within the next hour after breakfast and comes down to 4 mmol/L after 3 hours, how many calories did you store in your liver as glucose?

Students should represent the situation on a graph to make sure they understand the situation and then make the unit conversions.

From the above example, 0.18 g of glucose is the weight of 1 mmol so,

\[ 0.18 \text{ g:mmol} \times 8 \text{ mmol/L} = 1.44 \text{ g/L}. \]

In 10 L, one has \( 1.44 \text{ g/L} \times 10 \text{ L} = 14.4 \text{ g} \) of glucose present in the entire system after 1 hour.

After 3 hours, half of the glucose remains, that is, 7.2 g and, therefore, 7.2 g of glucose has been stored. How much energy does this represent?
Note that: 1 Joule = 0.23901 calories
    1 kJ = 239.01 calories
    1 calorie = 1/0.23901 J
    = 4.184 J

As indicated earlier 180 g releases 2820 kJ or
2820 x 239.01 cal = 674 008.2 calories or
674 kcal, therefore,
7.2 g releases 674 kcal x 7.2 g/180 g = 27 kcal of energy when burned.

Discuss the results with your students in relation to the daily amount of
energy required by an average adult (1500 kcal at rest in a warm room to
3000 kcal for a person doing average work).

Ask students:
• “Why are the results so low?”
• “Does the result make sense?”

You can provide them with some clues to the answer:
• the process is continuous
• the glucose is only one source of energy; other carbohydrates, fats and
  proteins also produce energy while they are metabolized
• glucose is at the end of the chain of decomposition of carbohydrates;
  intermediate steps already produced energy.

These values are approximately the same as the amount of energy
consumed by a 100-watt light bulb operating for a 24-hr period.

Table 3, below, shows the fuel value of some common foods.

<table>
<thead>
<tr>
<th>Food</th>
<th>Protein (grams)</th>
<th>Fat (%)</th>
<th>Carbohydrate (%)</th>
<th>Fuel Value (kcal/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apples</td>
<td>0.4</td>
<td>0.5</td>
<td>13</td>
<td>0.64</td>
</tr>
<tr>
<td>Beer</td>
<td>1.0</td>
<td>0</td>
<td>4.0</td>
<td>4.8</td>
</tr>
<tr>
<td>Bread (white, enriched)</td>
<td>9.0</td>
<td>3.0</td>
<td>32.0</td>
<td>2.8</td>
</tr>
<tr>
<td>Cheese (cheddar)</td>
<td>28.0</td>
<td>37.0</td>
<td>4.0</td>
<td>4.7</td>
</tr>
<tr>
<td>Eggs</td>
<td>13.0</td>
<td>10.0</td>
<td>0.7</td>
<td>1.6</td>
</tr>
<tr>
<td>Fudge</td>
<td>2.0</td>
<td>11.0</td>
<td>81.0</td>
<td>4.1</td>
</tr>
<tr>
<td>Green beans</td>
<td>1.0</td>
<td>0</td>
<td>5.3</td>
<td>2.2</td>
</tr>
<tr>
<td>Hamburger</td>
<td>22.0</td>
<td>30.0</td>
<td>0</td>
<td>3.6</td>
</tr>
<tr>
<td>Milk</td>
<td>3.3</td>
<td>4.0</td>
<td>5.0</td>
<td>0.74</td>
</tr>
<tr>
<td>Peanuts</td>
<td>26.0</td>
<td>39.0</td>
<td>22.0</td>
<td>5.6</td>
</tr>
<tr>
<td><strong>Average Fuel Energy (kcal/g)</strong></td>
<td><strong>4.1</strong></td>
<td><strong>9.3</strong></td>
<td><strong>3.8</strong></td>
<td></td>
</tr>
</tbody>
</table>
The fuel energy value of carbohydrates is approximately 4 kcal/g of carbohydrate (3.8 in Table 3 above) and the fuel energy value of fats is approximately 10 kcal/g (9.3 in Table 3 above).

To maintain body weight, a normal active, healthy adult must take in about 130 kJ of food energy per kilogram of body weight per day. Have students prepare a daily diet model (Student Activity: Modelling the Blood Sugar Level) and compare the data in Table 3: Fuel Value of Some Common Foods, above with the required number of calories to maintain their body weight. Make sure that students first convert the energy from kJ to kcal.

Tell students that although exercise is great for improving tone and firming up sagging tissues, it is not an effective way to lose weight. A one hour brisk walk over average terrain uses only about 700 kJ of stored energy. This is about one quarter of the energy contained in a quarter-pound hamburger in a bun.

Note: the popular term calorie is actually a kilocalorie and is sometimes written calorie (1000 cal = 1 Cal).

Have students complete the Figure 11: Blood Sugar Level Over Time, in Student Activity: Modelling the Blood Sugar Level in a more rigorous fashion from the data students have provided through their own research. Insist that only carbohydrates are considered in glucose production and that the lower part of the graph showing the input of insulin (time and quantity) be included. Tell students that insulin is measured in Units of insulin per hour (U/h). As a rule, a person needs about 0.3 U of insulin per kilogram of weight per day. For example, a 50 kg person would need 15 Units a day.

Note: if students are unable to locate blood glucose data, then make it available to them. They should be encouraged, however, to research plausible sources.
Figure 11: Blood Sugar Level Over Time

How many grams of glucose are there in the blood (10 litres in all the body) when the reading of the blood sugar indicates 5.5 mmol/L?

1 mol of glucose weighs ________ g
1 mmol of glucose weighs ________ g
In one litre, 5.5 mmol of glucose weighs ________ g
In 10 litres, there are ________ mmol glucose and ________ g glucose

How many grams of glucose have been burned and transformed into energy between each of the two readings in Table 4: Amount of Glucose Burned below? Again assume a 10 litre volume of blood in the body.
Project B: Determining the Relationship of Insulin to Diabetes

Student Activity: Fuel

Table 4

Amount of Glucose Burned

<table>
<thead>
<tr>
<th>#</th>
<th>Reading 1</th>
<th>Reading 2</th>
<th>Grams of Glucose Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.4 mmol/L</td>
<td>4.8 mmol/L</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>4.8 mmol/L</td>
<td>6.0 g</td>
</tr>
<tr>
<td>3</td>
<td>10.4 mmol/L</td>
<td></td>
<td>8.4 g</td>
</tr>
</tbody>
</table>

For each of the three situations above, estimate the amount of energy produced and express this in kJ and kcal in Table 5: Amount of Energy Produced.

Table 5

Amount of Energy Produced

<table>
<thead>
<tr>
<th>#</th>
<th>Grams of Glucose Burned</th>
<th>Energy (kJ)</th>
<th>Energy (kcal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the Good Health Eating Guide Poster or equivalent to plan a daily diet. Use the above data to prepare a 24-hour timeline indicating only the blood sugar level contribution of each of the sources of carbohydrates expressed in mmol/L. You will use this graph later to prepare an insulin regimen.

First, estimate the weight of blood sugar produced by 10 g of carbohydrate, and then, convert this into the number of mmol/L. When you have completed the conversions, complete Table 6: Amounts of Blood Sugar Produced.
### Table 6

**Amounts of Blood Sugar Produced**

<table>
<thead>
<tr>
<th>Time</th>
<th>Starch</th>
<th>Fruits</th>
<th>Milk</th>
<th>Sugars</th>
<th>Total mmol/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 12: 24-Hour Timeline

Time Line

- 7 a.m.
- Noon
- 7 p.m.
- Midnight
- 7 a.m.
Activity 2: Controlling Diabetes

Objective:
The students develop simple linear models representing two competing processes in order to develop their understanding of the coefficients of a polynomial model representing a real situation.
The students solve problems related to insulin deficiency by using their knowledge of patterns and relations. They propose insulin regimens and solve problems related to insulin adjustment in cases of low and high readings, illness, exercise and special circumstances.

Materials:
- information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centres
- Student Activity: Modelling the Blood Sugar Level
- Student Activity: Planning the Day
- graphing calculator or mathematics software for example Mathcad and Mathematica
- access to a nurse, physician, dietitian, or to a student with diabetes
- copies of diabetes diaries
- copy of Good Health Eating Guide poster or equivalent

Procedure:
Ask students why people with diabetes regularly monitor their blood glucose (before meals and at bedtime, 1 to 4 times a day).
Diabetics generally use a diary to record their blood glucose levels along with insulin doses and comments. Comments might include information about delayed meals, extra snacks or having the flu.
Provide students with anonymous diary information from a diabetic source where blood glucose levels have been monitored during a normal day. They must record the measurements on the timeline in Student Activity: Modelling the Blood Sugar Level. They also need to identify and record times of meals, snacks, and exercise on the timeline.

Modelling Blood Sugar Level
Provide students with examples in Student Activity: Modelling the Blood Sugar Level and have them interpret each of the curves. These examples represent different patterns of variation in blood sugar at different times throughout the day. Students are asked to find the equation for each curve.
Project B: Determining the Relationship of Insulin to Diabetes

For example, in Example 1 the blood glucose level at 7 a.m. is 5 mmol/L. Blood glucose is identified below by the letter g. The function represented by an equation would read:

\[ g = 5 \text{ (for } t = 7) \]

In Example 2, the blood sugar level increases from 5 mmol/L at 7 a.m. to 7.5 mmol/L at 12 p.m., a period of 5 hours. Have students determine that the equation is:

\[ g = 0.5t + 5 \text{ (for } t > 7 \text{ and } t \leq 12) \text{ or } (7 < t \leq 12) \]

In Example 3, the level decreases from 7.5 mmol/L to 5.5 mmol/L over a period of 4 hours. The equation should read:

\[ g = -0.5t + 7.5 \text{ (for } t > 12 \text{ and } t \leq 4) \text{ or } (12 < t \leq 4) \]

In Example 4, the level increases from 5.5 mmol/L to 8.5 mmol/L within an hour. The equation should read:

\[ g = 3t + 5.5 \text{ (for } t > 4 \text{ and } t \leq 5) \text{ or } (4 > t \leq 5) \]

Ensure that students understand the significance of the coefficients of the independent variable; that is, negative means decrease, positive means an increase in glucose level; the larger the coefficient is, the bigger the increase (input).

Have students compare available readings to these observations and then comment on the differences in terms of meals, snacks and exercise.

Have students solve the problems in the Student Activity: Modelling the Blood Sugar Level. In these problems they must add or subtract two competing processes, that is input and output.

Modelling Insulin Release

Have students set a target range for blood glucose control from their 24-hour timeline glucose level established in the previous activity. They should assume that the body is not releasing any insulin into the bloodstream. The recommended target is 4 to 7 mmol/L of glucose.

Distribute some insulin manufacturer's pamphlets describing how insulin works. There are several types of insulin having different types of action. See Figure 13: Action Times of Insulin.

Rapid acting insulin (Humalog or Lispro) begins to work 5 to 15 minutes after injection, peaks in one hour and has a duration of 3 to 4 hours.

Short acting insulin (Regular or R or Toronto insulin) begins to work one-half hour after injection, peaks in 2 to 4 hours and has a duration of 6 to 8 hours.

Intermediate acting insulin (also N or NPH or L-insulin) begins to work 1 to 3 hours after injection, peaks in 6 to 12 hours and has a duration of 18 to 24 hours.

Long acting insulin (or U-insulin) begins to work 4 to 6 hours after injection, and has a duration of 24 to 28 hours.
Have students determine equations representing each of the four types of functions for insulin. To do this they need to simplify the model to a linear one by doing the following:

- time 0 to the time insulin begins to work: constant level zone
- time insulin begins to work to peak: increasing zone (linear)
- time insulin stops increasing to time insulin stops working: decreasing zone (linear)

For example, where the release of insulin is denoted by \( I \), the rapid acting insulin could be modelled by the equations shown below.

\[
I = 0 \quad \text{(for } t \leq 0 \text{ and } t \leq 0.4) \quad \text{or} \quad (0 \leq t \leq 0.4)
\]

\[
I = \frac{40}{3} t \quad \text{(for } t \geq 0.4 \text{ and } t \leq 1) \quad \text{or} \quad (0.4 \leq t \leq 1)
\]

\[
I = -\frac{10}{3} t \quad \text{(for } t \geq 1 \text{ and } t \leq 4) \quad \text{or} \quad (1 \leq t \leq 4)
\]

40/3 represents a rate of increase in the insulin level.
-10/3 represents a rate of decrease in the insulin level.

Provide students with some typical insulin regimens like those illustrated in Figures 14, 15, and 16 below and have them comment on the patterns in relation to the increase in blood sugar (input).
**Figure 14: One Injection Regimen**

**One Injection: regular and intermediate acting insulin**
- **Breakfast**: blood glucose test reflects the tail-end action of the intermediate acting insulin given the day before.
- **Lunch**: blood glucose test reflects the action of the regular insulin.
- **Supper**: blood glucose test reflects the action of the intermediate acting insulin.
- **Bedtime**: blood glucose test reflects the action of the intermediate acting insulin.

**Figure 15: Two Injection Regimen**

**Two Injections: regular and intermediate acting insulins**
- **Breakfast**: blood glucose test reflects the action of the intermediate acting insulin given the evening before.
- **Lunch**: blood glucose test reflects the action of the morning regular insulin.
- **Supper**: blood glucose test reflects the action of the morning intermediate acting insulin.
- **Bedtime**: blood glucose test reflects the action of the supper time regular insulin.
Figure 16: Four Injection Regimen

Have students prepare (Student Activity: Planning the Day) an insulin regimen from the blood sugar pattern they have prepared in the former activity in order to maintain the glucose level in the 4-7 mmol/L range all day (24 hours). Provide several copies of the Student Activity: Planning the Day to each student so that they can propose different options for insulin regimens.
Determine the mathematical model that represents the evolution of blood levels sugar in the eight activity pages that follow. Use the example below to help determine each model you create.

Example of Modelling the Evolution of Blood Sugar

Data:
Before 7 a.m., the reading is 5.0 mmol/L (INPUT) Breakfast is at 7 a.m.
The reading at 9 a.m. is 8.5 mmol/L (OUTPUT) At 10 a.m., the reading is down to 7.25 mmol/L
At 11 a.m., the reading is 6.0 mmol/L (INPUT) At lunch time (12 p.m.), the reading is 5.0 mmol/L
These data points are plotted below:

Equations:
Before 7 a.m. : \( g = 5 \) mmol/L
Between 7 a.m. and 9 a.m., the level increases by 3.5 mmol/L over a period of 2 hours that is, \( 3.5/2 \) per hour:
\( g = 5 + 3.5/2 \ t = 5 + 1.75 \ t \)
Between 9 a.m. and 11 a.m. :
\( g = 8.5 - 2.5/2 \ t = 8.5 - 1.25 \ t \)
Between 11 a.m. and 12 a.m. :
\( g = 6 - 1 \ t = 6 - \ t \)
Checking:
Use the third equation to calculate the level at 10 a.m.:
\( g = 8.5 - 1.25 \times 1 = 8.5 - 1.25 = 7.25 \) mmol/L
Student Activity: Modelling the Blood Sugar Level

Example Model #1

7 a.m. INPUT

9 a.m. OUTPUT

11 a.m.

12 p.m. INPUT
Example Model #2

7 a.m. INPUT  9 a.m. OUTPUT  11 a.m.  12 p.m. INPUT

Student Activity: Modelling the Blood Sugar Level

Project B: Determining the Relationship of Insulin to Diabetes
Student Activity: Modelling the Blood Sugar Level

Example Model #3

7 a.m. INPUT
9 a.m. OUTPUT
11 a.m.
12 p.m. INPUT
Project B: Determining the Relationship of Insulin to Diabetes

Student Activity: Modelling the Blood Sugar Level

Example Model #4
Student Activity: Modelling the Blood Sugar Level

Example Model #5

7 a.m.  9 a.m.  11 a.m.  12 p.m.
INPUT  OUTPUT  INPUT
Student Activity: Modelling the Blood Sugar Level

Example Model #6

<table>
<thead>
<tr>
<th>7 a.m.</th>
<th>9 a.m.</th>
<th>11 a.m.</th>
<th>12 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPUT</td>
<td>OUTPUT</td>
<td></td>
<td>INPUT</td>
</tr>
</tbody>
</table>
Student Activity: Modelling the Blood Sugar Level

Example Model #7

7 a.m. INPUT

9 a.m. OUTPUT

11 a.m.

12 p.m. INPUT
Student Activity: Modelling the Blood Sugar Level

Example Model #8
Use the pattern of Student Activity: Fuel showing the changes of blood sugar over a 24-hour period and propose a regimen by combining the different types of insulin. Your target is 4 to 7 mmol/L for the entire period. Suggest different options. Submit your proposals to a nurse or a physician for comments.

Obtain a diabetes diary. These are available from most major drug store chains. Compare this diary to the one provided by your teacher.

Insulin Regimen: Planning a Day

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 a.m.</td>
<td>INPUT</td>
</tr>
<tr>
<td>9 a.m.</td>
<td>OUTPUT</td>
</tr>
<tr>
<td>11 a.m.</td>
<td></td>
</tr>
<tr>
<td>12 p.m.</td>
<td>INPUT</td>
</tr>
</tbody>
</table>

Comments:
Follow the guidelines below to adjust the insulin:

If the insulin dose is
- less than 8 units, adjust it by 1 unit at a time
- between 8-20 units, adjust it by 2 units at a time
- greater than 20 units, adjust it no more than 10%. For example, if the dose is 30 units, adjust by 3 units.

**Adjusting for Low Readings**
- A reading of 3.5 mmol/L glucose or less is a low reading.
- Use the chart What is your Blood Sugar Level? A low glucose reading is called an Insulin Reaction. An insulin reaction can be treated by having 10 g of carbohydrate (2 teaspoons of sugar or a half cup of juice).
- A low reading can result from a delayed meal, missed portions of food and/or extra activity.
- When unexpected insulin reactions occur, the insulin dose must be decreased.

Adjust the following situation:
- Mary is taking 5 units of regular and 15 units of intermediate insulin in the morning. The glucose readings have been between 4 and 6 mmol/L.
- One day, an unexplained insulin reaction occurred before lunch. What is the solution?

**Adjusting for High Readings**

Suggest a solution for each of the following three situations:

**Situation #1: Dose:**
- 7 units of regular acting insulin and 18 units of intermediate acting insulin in the morning
- 4 units of regular acting insulin and 8 units of intermediate acting insulin before supper

**Glucose readings in mmol/L are:**

<table>
<thead>
<tr>
<th>Day</th>
<th>7 a.m.</th>
<th>Noon</th>
<th>6 p.m.</th>
<th>10 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>8.0</td>
<td>4.8</td>
<td>14.8</td>
<td>9.8</td>
</tr>
<tr>
<td>Tuesday</td>
<td>6.9</td>
<td>6.2</td>
<td>11.3</td>
<td>9.4</td>
</tr>
<tr>
<td>Wednesday</td>
<td>7.1</td>
<td>7.6</td>
<td>15.9</td>
<td>11.2</td>
</tr>
</tbody>
</table>

**Solution:**
Situation #2: Dose:
- 30 units of intermediate acting insulin in the morning
- 18 units of intermediate acting insulin before supper

Glucose readings in mmol/L are:

<table>
<thead>
<tr>
<th>Day</th>
<th>7 a.m.</th>
<th>Noon</th>
<th>6 p.m.</th>
<th>10 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>10.9</td>
<td>8.9</td>
<td>16.4</td>
<td>12.3</td>
</tr>
<tr>
<td>Tuesday</td>
<td>17.2</td>
<td>12.6</td>
<td>11.1</td>
<td>10.9</td>
</tr>
<tr>
<td>Wednesday</td>
<td>15.8</td>
<td>13.2</td>
<td>12.7</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Solution:

Situation #3: Dose:
- 10 units of rapid acting insulin (Humalog) and 32 of intermediate acting insulin in the morning
- 8 units of rapid acting insulin (Humalog) before supper
- 12 units of intermediate acting insulin at bedtime

Glucose readings in mmol/L are:

<table>
<thead>
<tr>
<th>Day</th>
<th>7 a.m.</th>
<th>Noon</th>
<th>6 p.m.</th>
<th>10 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>4.8</td>
<td>4.8</td>
<td>12.9</td>
<td>4.2</td>
</tr>
<tr>
<td>Tuesday</td>
<td>5.2</td>
<td>8.9</td>
<td>14.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Wednesday</td>
<td>8.6</td>
<td>9.2</td>
<td>16.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Solution:
Activity 3: Modelling Two Competing Processes

Objective:
The objective of this activity is for students to develop simple linear models representing two competing processes. This helps develop their understanding of the coefficients of a polynomial model representing a real situation. The students solve problems related to insulin deficiency by using their knowledge of patterns and relations. They propose insulin regimens and solve problems related to insulin adjustment in situations of low and high readings, illness, exercise and special circumstance. They extend their knowledge by designing a situation where two competing processes are involved.

Materials:
• information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centers

Procedure:
1. Introduction
Assume that \( y \) represents a population and \( t \) represents time. An example of an increasing process is one in which the size of population grows, while a decreasing process is one in which the population decays.

When these two processes (both linear) are combined, the overall result is the sum of the two linear models.

Figure 17: Increasing and Decreasing Processes illustrates two processes:
I: \( y = -t + 15 \) (decreasing at a rate of -1 U/h from 15)
II: \( y = 0.5t + 5 \) (increasing at a rate of 0.5 U/h from 5)

Overall process (Figure 18: Overall Process)
\( Y = -t + 15 + 0.5t + 5 = -0.5t + 20 \) (decreasing at a rate of -0.5 U/h from 20)
Project B: Determining the Relationship of Insulin to Diabetes

At the beginning, the total population is the sum $5 + 15 = 20$ units. The overall population varies from the contribution of the two processes, one increasing, the other decreasing. For example, at point $t = 4$, process I provides 11 units and process II provides 7 units for a total of 18 units.
Typical questions:
• When will the population be zero? (Answer: t = 40)
• What would be the model of a decreasing process such that the population remains constant all the time? (Answer: the coefficient of \( t \) in the decreasing process must be equal and of opposite sign)
• What would be the model for the decreasing process such that the overall population is always increasing? (Answer: anything larger than -0.5)
• What does it mean when a process model has a zero coefficient for \( t \)?

2. Deriving the Equation of a Linear Model

A linear model may illustrate the ratio (proportionality) between two quantities, where one quantity varies at a constant rate with respect to the other. A linear model is the simplest way to model a relation between two quantities and is represented graphically by a straight line. Symbolically, the model is represented by the equation \( y = mx + b \) where \( m \) is the rate of change of \( y \) with respect to \( x \), and \( b \) is the starting point where the situation is analysed.

For example, the length of a spring at rest is 10 cm. Under the action of a force of 5 units, the length is 15 cm. Assuming that a linear model is appropriate for this situation, what is the equation?
Call \( L \) the length of the spring, \( F \) the force and \( R \) the length of the spring at rest. \( L \) is proportional to \( F \). \( L = mF \) where \( m \) is a constant term. If \( F = 0 \) (rest), \( L = R \). Together, the linear model looks like:

\[
L = mF + R
\]

Determining \( m \): for \( F = 5 \), \( L = 15 \). Substitute to obtain \( m = 1 \)

Now \( L = F + 10 \). For example, when the force is 20 units, the length is 30 cm.

**Figure 19: Graphing the Relationship Between Length and Force**

3. Modelling the Action of Insulin Over Time

\( W \) is the time at which the insulin begins to work
\( T \) is the time it takes for the insulin to reach a peak (from the time of injection)
\( N \) is the number of units of insulin at the peak
\( C \) is the duration the insulin remains constant at peak
\( i \) is the level of insulin in units of insulin
\( t \) is the time in hours from the injection of the insulin

The process can be broken down into four phases (see Figure 20: Modelling the Action of Insulin Over Time, Example 1).

Phase I: time \( W \) needed for the insulin to be working
Equation: \( i = 0, \, 0 \leq t < W \)

Phase II: reaching the peak of \( N \) units at a rate \( m \) over a period \( T \)
Equation: \( i = mt + b, \, W \geq t < T \)

At \( t = W, \, i = 0 \)
At \( t = T, \, i = N \)

\[
m = \frac{N}{T-W} \\
b = -\frac{NW}{T-W} \\
i = \frac{N}{T-W} t \cdot \frac{NW}{T-W}
\]

Phase III: remains constant at \( N \) during a time \( C \)
Equation: \( i = N \)

Phase IV: decreasing at a rate \( M \)
Equation: \( i = Mt + d \)

At \( t = T + C, \, i = N \)
At \( t = 2T + C, \, i = 0 \)

\[
i = \frac{N}{T} t + \frac{N(2T+C)}{T}
\]
Figure 20: Modelling the Action of Insulin Over Time: Example 1

See Figure: 21: Modelling the Action of Insulin Over Time: Example 2.

Example 2:

\[ N = 12 \text{ units} \]
\[ W = 1 \text{ hour} \]
\[ T = 4 \text{ hours} \]
\[ C = 2 \text{ hours} \]

Phase I: \[ i = 0 \]
Phase II: \[ i = 4t - 4 \]
Phase III: \[ i = 12 \]
Phase IV: \[ i = -3t + 30 \]
Figure 21: Modelling the Action of Insulin Over Time: Example 2

![Graph showing the action of insulin over time. The graph is a line plot with the x-axis labeled as 'Time in Hours (t)' ranging from 0 to 35, and the y-axis labeled as 'Insulin Units (i)' ranging from 0 to 20. The graph shows a peak in insulin units at around 10 hours, labeled 'N'.]
4. Types of Insulin

Rapid Acting Insulin (Humalog)

Peak is at 10 units
Time to begin to work is 1/4 hour
Time to reach the peak is 1 hour

Figure 22: Rapid Acting Insulin
Short Acting Insulin (Toronto)
Peak is at 10 units
Time to begin to work is 1/2 hour
Time to reach the peak is 2 to 4 hours

Figure 23: Short Acting Insulin
Intermediate Acting insulin (NPH)

Peak is at 10 units
Time to begin to work is 1 to 3 hours
Time to reach the peak is 6 to 12 hours

In example 1, $N = 10$ units, $W = 3$ hours and $T = 9$ hours
In example 2, $N = 10$ units, $W = 2$ hours and $T = 12$ hours

Figure 24: Intermediate Acting Insulin

Question: At what time are the insulin levels the same?
Long Acting Insulin (U-Insulin)

Peak is at 10 units
Time to begin to work is 4-6 hours
Time to reach the peak is 24-28 hours

Example 1: \( N = 10 \) units, \( W = 4 \) hours, \( T = 28 \)

Example 2: \( N = 10 \) units, \( W = 6 \) hours, \( T = 24 \) hours

**Figure 25: Long Acting Insulin**

Question: At what time are the insulin levels the same?
5. Typical Insulin Problems to Solve

On the following graph determine \( W \) and \( T \), given \( N = 10 \) units.

**Figure 26: Typical Insulin Problem to Solve**

![Graph showing two insulin lines with labeled points](image)

Answer the following:

Which insulin begins to work more quickly?

Which insulin reaches a peak more quickly?

How long does it take for insulin I to reach a peak?
6. Combined Action of Different Insulins

Suppose a patient is injecting a short acting insulin (10 UI) at the same time as 10 UI of long acting insulin.

The short acting insulin begins to work about 0.5 hour and peaks after 3 hours. The level remains the same for 3 hours.

The long acting begins to work after 5 hours and peaks after 12 hours. The level remains the same over a period of 8 hours.

Graph the level of insulin over a 24-hour time period.

Data

Short acting:
N  =
W  =
T  =
C  =

Long acting:
N  =
W  =
C  =

Equations

Short acting Insulin:
Phase I:  \( i = 0 \)
Phase II:  \( i = 4t \)
Phase III:  \( i = \)
Phase IV:  \( i = \)

Long acting Insulin:
Phase I:  \( i = 0 \)
Phase II:  \( i = 10t / 7 \)
Phase III:  \( i = \)
Phase IV:  \( i = \)

During the period between 6 a.m. and 10 a.m., the two insulins are combined. The short acting insulin is in phase IV and long acting insulin is in phase II. Add the two equations to determine the change in insulin levels between 6 a.m. and 10 a.m.
Figure 27: Combined Action of Different Short Acting Insulins

Figure 28: Combined Action of Different Long Acting Insulins
Evaluation Problem

By combining the different types of insulin, design an insulin regimen so that 8 units are released in the blood within 1/2 hour and remain within the limits of 8 to 15 units over a period of 24 hours.

**Figure 29: Designing an Insulin Regimen**
Phase III: Solving a Regimen Problem

Procedure:
In this phase, mathematical and science concepts are evaluated from reports prepared by individual students or groups of students. These reports may include a description of diabetes, preparation of a regimen of insulin injections from a table of blood sugar readings, answers to questions and changes in the diet, and the social and economic implications of diabetes.

Note:
This project has been prepared with the help of Dr. Susanne Voetman, M.D., The Medicine Cabinet, Nanaimo, B.C., and Norma E. Gomerich and Elly Hay, Diabetes Education Centre, Nanaimo Regional General Hospital, Nanaimo, B.C.