Mathematics 9

A RESOURCE FOR TEACHERS

Building Confidence Through Doing

British Columbia
Ministry of Education

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Acknowledgements

Mathematics 9: A Resource for Teachers is a response to a need expressed by many teachers faced with the challenges of implementing new curriculum and expanding their pedagogy.

The project has benefitted from the collective vision and talents of many individuals over the past two years including:

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Dave Chowdhury	Centre for Applied Academics

In addition, several BC teachers reviewed or piloted some of the materials.

The Ministry of Education is pleased to acknowledge the efforts of each individual as well as the collective synergy of everyone involved in the project.

M A T H E M A T I C S 9

Table of Contents

About Mathematics 9: A Resource for Teachers

- What is this resource?
- Why was this resource developed?
- How is this resource organized?
- How can I use this resource to help me organize my program?
- Planning My Program
- Instructional Strategies
- Assessment
- Feedback, Please

About Mathematics 9: A Resource for Teachers

What is this resource?

Mathematics 9: A Resource for Teachers complements and augments the Grade 9 Mathematics curriculum. It provides hands-on, activity-based learning experiences that support classroom instruction and appeal to a broad range of students. The opportunities for communicating, connecting, applying mathematics reasoning within an understandable context, and using technology help give students the skills, knowledge, and the attitude they need to use mathematics at home and in their future careers.

Mathematics 9: A Resource for Teachers allows for new material to be added as you experiment with different activities. The resource includes a variety of projects that you can sample and modify to meet the needs of your students. Assessment techniques appropriate for a broad range of students have also been referenced.

Finally, the material included in *Mathematics 9: A Resource for Teachers* has been piloted in Grade 9 classrooms like yours across British Columbia.

Why was this resource developed?

Mathematics 9: A Resource for Teachers was developed to support teachers in their planning for teaching and learning the Mathematics 9 curriculum. This resource helps teachers expand their repertoire of teaching strategies, so that they can better meet the wide-ranging learning outcomes found in the Grade 9 Mathematics curriculum. In particular, it helps with those learning outcomes that address the applied focus, student decision making, critical thinking and problem solving.

Mathematics 9: A Resource for Teachers has three major sections. This first section, About Mathematics 9: A Resource for Teachers, provides an orientation to this resource and its use.

The second section, Introduction to Mathematics 9, has been developed to support teachers as they make the shift to a more applied approach. This section includes:

- an overview with information about mathematics
- a brief description of the process used in teaching mathematics
- a brief description of the principles of learning mathematics

- brief descriptions of the key players involved in teaching and learning Mathematics 9 - students, teachers, parents and community.

The remaining section focuses on examples of real-world problems. Background information has been provided to ensure a context for the problems. These problems are organized as projects that include activities. By working through each phase of a project, students:

- are introduced to the problem or determine the problem for themselves by teasing it out of the situation
- propose solutions to the problem
- determine what additional skills and concepts they need in order to propose solutions and evaluate them
- acquire new skills and ways of thinking
- apply new skills, concepts and ways of thinking to solve the original problem.

The projects presented in this Resource provide opportunities for students to develop the attitudes, knowledge, skills, and levels of thinking needed to identify the mathematics within problems, and to ask mathematical questions.

Many students find it helpful for the teacher to describe the problem-based learning process in which they are engaged. That is:

- posing problems
- posing questions
- generating and identifying alternatives
- making choices
- researching and planning
- generating and constructing products
- evaluating products
- assessing choices and reflecting on them
- reflecting on the effectiveness of the products.

At the end of each project, students should realize that they have learned new mathematical concepts, skills and processes.

For students, teacher and parents, one of the most important aspects of problem-based learning is how it relates to the developmental stage of students. In Grade 9, students are examining their own lives and how they ‘fit’ into the universe. The Mathematics 9 curriculum encourages learning in which students use their academic understandings and abilities in a variety of in-school and out-of-school settings to solve real-world or simulated problems both alone and as part of a group. By examining real-world applications of mathematics, students learn that there is order, predictability and structure expressed through patterns, both in the mathematical world and their ‘real world’, reassuring them that all is not random or chaotic.

How is this resource organized?

Mathematics 9: A Resource for Teachers is organized in the following way:

- About *Mathematics 9: A Resource for Teachers* (section 1)
- Introduction to Mathematics (section 2)
- Sample Project: Counting Kokanee
- Project A: Predicting Satellite Collisions
- Project B: Determining the Relationship of Insulin to Diabetes
- Project C: Planning for a Teen Recreation Centre
- Project D: Drawing a Scale Map for a Teen Recreation Centre
- Project E: Building a Scale Model for a Teen Recreation Centre
- Appendix A: Correlation Between Projects and Mathematics 9 Learning Outcomes
- Appendix B: Blackline Masters

The first two sections of the Resource effectively set the stage for embarking on the applied mathematics journey.

Whether you are a teacher who has been on this journey for several years or one just about to begin, we suggest that you spend some time going through these first two sections. The first section provides the information you need to know about the boat on which you are sailing while the second section, provides you with the information about the journey, the destination, and the other people on the journey with you.

The third section includes six projects. Each of these projects is organized in a similar way, by phases.

Phase I sets the stage for the project by describing the problem to be solved or the project to be undertaken. It also identifies the skills and mathematical concepts needed to find solutions.

Phase II are the activities designed for students to acquire the specific knowledge, skills and mathematical concepts important to solving the problem or carrying out the project.

Phase III brings closure to the project as students develop, evaluate, and reflect upon one or more products that are solutions for the problem or project described in the first phase.

The following are synopses of the projects found in the third section of this resource.

Sample Project: Counting Kokanee

Many Grade 9 students may have wondered how the communications media know how many youth attended an outdoor concert, or how many salmon have returned to a local river to spawn. In this sample project, students learn sampling and estimation skills.

The objective of the Sample Project is to give you and your students an opportunity to become familiar with the approach used in this resource. The mathematics utilized in the Sample Project is based on the Grade 8 Mathematics curriculum.

The project contains a discussion on mathematical modelling that is designed to help you explain to your students how real-world systems can be described in terms of mathematical models. A simple heuristic is provided as well as a concrete example.

Project A: Predicting Satellite Collisions

By Grade 9, many students have their first experience buying stereo speakers or a new mountain bike. The skills they learn in this project can help them make better choices as consumers.

The objective of this project is to develop student awareness about the risk of collision between satellites, and between satellites and fragments of satellites or meteorites. The risk of collision is greater because of the dramatic increase in the number of satellites placed into specific orbits.

In this project, students review rational numbers. They learn to model situations with formulae involving powers, and to apply the Laws of Exponents.

Project B: Determining the Relationship of Insulin to Diabetes

Most students are keenly interested in their physical appearance and health. In this project, they learn to make better decisions with respect to food and exercise, and to gain a better understanding of what it means to stay healthy.

The objective of this project is to reinforce student skills in pattern recognition and in performing calculations with rational numbers and powers. Students learn more about diabetes and the role insulin plays in its prevention.

Students learn to identify and graphically represent patterns, and to apply them. They analyse two competing processes using linear models, and gain an understanding of the significance of coefficient of polynomial models.

Project C: Planning for a Teen Recreation Centre

By Grade 9, most students are increasingly independent in terms of their out-of-school activities. In this project, they investigate the need for a teen recreation centre. They learn about surveys, ways to construct questions, and skills that help them understand what survey results actually mean.

The objective of this project is for students to research the community-based recreational needs of youth.

Students learn about designing and implementing surveys, and explore the use of scatterplots and lines of best fit.

Project D: Drawing a Scale Map for a Teen Recreation Centre

Most students have had the opportunity to use a scale map or plan. They might have used these for hiking and orienteering, sewing and quilting, designing decorations for a school dance, or even rearranging their rooms.

The objective of this project is for students to learn some basic surveying techniques.

Students learn about triangulation, tangent ratio, sine and cosine, similarity, and congruence. Once they learn these concepts and skills, they prepare a scale map.

Project E: Building a Scale Model for a Teen Recreation Centre

Building a scale model is often an essential step toward building a final product. For Grade 9 students who are designing the sets for a school play, knowing how to build a scale model is an important skill. For model railway enthusiasts and doll house builders, the building of a scale model is the final product.

The objective of this project is to construct a scale model.

Students learn about solving problems using trigonometry as well as geometry in preparation for constructing their scale models.

You will note that each of Projects C, D and E focusses on a different aspect of the development of a Teen Recreation Centre. You may choose to do one, two or three of these projects. Together, they comprise a much larger investigation.

Appendices

The Appendices include additional materials to support teachers. When these materials are referred to within the text of each project, they are either recommended or required.

Appendix A contains a table correlating the five main projects with the Mathematics 9 learning outcomes. It can also be used to indicate areas where new activities may be developed.

Appendix B contains a few blackline masters that may be used in support of student investigations.

How can I use this resource to help me organize my program?

Mathematics 9: A Resource for Teachers is a tool for supporting teachers engaged in teaching mathematics. It is not a traditional mathematics text to be rigidly followed, or a prescription for creating the context for mathematics.

As the teacher, you choose the number and order of the projects to help build your mathematics program. This selection should be based on the motivation, skills, interests, needs, and abilities of your students. We have included the approximate number of class periods each project requires, but you may wish to expand or contract the scope or amount of time you spend on the projects.

Basically, there are three options for using this Resource:
1. just do it - adopt the resource as a complete program
2. pick and choose - augment an existing approach or course organization with selected activities and projects
3. create own vision - create a unique program based on large-scale problems and projects patterned after those in this resource.

Some of you have always taught mathematics in a practical and applied manner. For you, these projects may simply provide some new ideas and points of departure as you develop your own projects.

Some of you may choose to teach the projects as you became more comfortable using this mathematics approach.

Planning My Program

In order to plan your Mathematics 9 program, you will need:
- an assessment of the motivation, skills, interests, needs, and knowledge of your students
- an assessment of your own expertise, interests, and experience as a teacher of applied mathematics
- the Grade 9 Mathematics curriculum
- the *Mathematics 9: A Resource for Teachers* (this Resource)

As a starting point, refer to the Grade 9 mathematics curriculum and to the table, Correlation Between Curriculum Organizers and Projects, to familiarize yourself with the curriculum organizers and their relationship to the individual projects in this Resource. In addition, for your convenience, the learning outcomes have been reproduced and correlated with projects where the learning outcomes are addressed in a significant way. This can be found in Appendix A.

Table 1

Correlation Between Curriculum Organizers and Projects

Curriculum Organizer	Proj A	Proj B	Proj C	Proj D	Proj E
Problem Solving	•	•	•	•	•
Number (Number Concepts)	•	•			
Number (Number Operations)	•	•			
Patterns and Relations (Patterns)	•	•			•
Patterns and Relations (Variables and Equations)	•	•		•	
Shape and Space (Measurement)				•	•
Shape and Space (3-D Objects and 2-D Shapes)				•	•
Shape and Space (Transformations)	•				•
Statistics and Probability (Data Analysis)	•	•	•		
Statistics and Probability (Chance and Uncertainty)	•				

Your next step is to develop your program in conjunction with the following fundamental ideas:
- provide learning opportunities for students to develop an understanding of Grade 9 mathematics, and to integrate that knowledge into their lives so that it becomes a resource for further education and life-long learning
- actively engage students in their own learning
- provide context for all learning
- encourage students to see relevance in their learning by allowing them to focus on personal passions
- promote the use of technology that encourages the exploration of important mathematical questions
- apply context, co-operative learning, constructivist learning and communication (4 C's) as the foundation for an applied approach
- provide opportunities and support for students to take responsibility for planning, exploration, projects and exercises
- encourage students in their development of a positive attitude towards life-long learning and understanding the role of mathematics in their lives

We recommend that you:
- read through the sample project, Counting Kokanee, to get a flavour of how the other projects in this *Mathematics 9: A Resource for Teachers*, are organized
- examine the Project Planning Template, Figure 1, and review it for the sample project
- read through each project completely
- assess each project in relation to the motivation, interests, skills, knowledge and readiness of your students

- select the most appropriate project or projects
- decide whether to do all or part of each project
- determine which projects you wish to modify, if any, and how
- decide upon the order you will do the projects

Prior to implementing your program, you will need to have the following elements in place:
- a set of activities in support of the projects
- a set of learning resources for both the teacher and students; this Resource forms only part of the teacher and student materials that could be used
- a repertoire of instructional strategies that have been chosen on the basis of particular learning activities
- a range of assessment techniques and a particular assessment strategy overall
- an assessment and evaluation plan

The six projects included in this resource provide a broad range of learning experiences for your students in terms of content, process and product.

As a starting point, consider learning in the context of large-scale projects such as the building of a teen recreation centre.

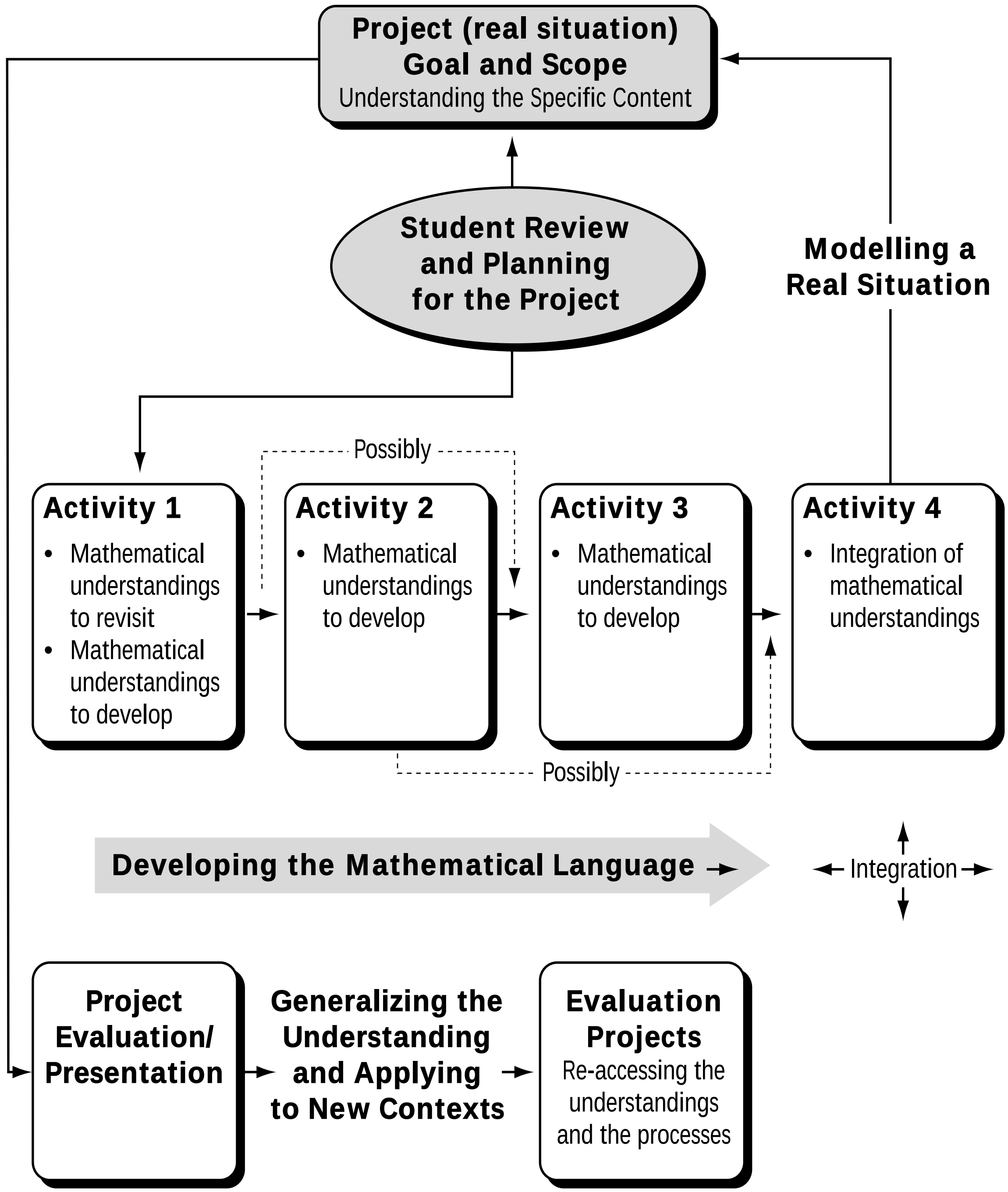
Each project provides the context for the learning of:
- mathematics concepts
- mathematical skills
- mathematical processes
- mathematical modelling
- attitudes towards mathematics
- ways to see and understand the world through mathematics

We have also included instructional strategies that may be used in each phase of a project. Using a variety of instructional strategies helps ensure that the learning styles of a broad range of learners are addressed.

Instructional Strategies

Teachers have found a number of instructional strategies to be particularly useful in applied mathematics programs. We hope that you find the following information helpful in your planning.

Figure 1: Project Planning Template



A teacher's role as facilitator, instructor, or mentor is both dynamic and flexible. Depending on the particular situation, the balance of responsibility for initiating and completing learning tasks shifts in favour of the teacher or the student. By focussing on an individual student's particular learning style and rate, you can select and apply appropriate strategies to increase the likelihood of success.

The selection of instructional strategies should be based upon how well they foster critical thinking and independent learning. Students will be more engaged in the learning process if you provide them with opportunities to articulate their own problems, apply appropriate strategies, and reach defensible solutions.

Student learning can be deepened and extended by combining instructional strategies. The way strategies are applied depends on several factors including:
- nature of the required activities
- nature of the class
- available resources
- available time
- experience of the teacher
- school and community support
- purpose of instruction
- student motivation
- student readiness

Several instructional strategies that may be used effectively in applied mathematics are briefly described below. It is generally highly appropriate to use a variety of strategies, allowing them to overlap or be used in combination. These instructional strategies include:
- Issues Inquiry
- Case Study
- Learning Centres
- Direct Instruction
- Field Study
- Math Labs
- Group Work or Co-operative Learning
- Simulation or Role Play
- Independent or Self-Directed Learning

Issues Inquiry

What is the Issues Inquiry Strategy?

An Issues Inquiry approach is based on identifying challenging inquiries that lead students to further questions about a problem or an issue. Students create their own questions and then ask themselves, "What do I need to know in order to answer these questions?" In mathematics, indeed in all problem-based activities, issues inquiry is a key instructional strategy.

Why use the Issues Inquiry Strategy?

Issues Inquiry requires and develops critical thinking, creative thinking, problem solving, and communication skills. These help students deal confidently and effectively with the ever-increasing quantity of information available through the news media and the Internet.

Issues Inquiry hooks students into exploring questions that are self-motivating. When exploring issues and answering their own questions, students have an opportunity to experience mathematics as a critical tool that helps them view the world in richer ways.

How can the Issues Inquiry Strategy be used?

Issues Inquiry can be initiated by either the teacher or a student with, for example, discussion of a current news story, the presentation of an engaging conundrum, or the introduction of an intriguing topic.

Issues Inquiry is facilitated when students ask questions that stimulate different aspects of thinking:

- analyzing
- assessing
- classifying
- comparing
- defining
- describing
- evaluating
- hypothesizing
- inferring
- interpreting
- observing
- predicting
- recalling
- recognizing
- reflecting
- synthesizing

The process typically involves the following without regard to a particular order:

- describing the issue or problem
- determining criteria and methods for making decisions
- deciding on constraints of the inquiry, for example, the time frame
- researching, choosing and organizing information
- analysing and synthesizing information to identify alternatives
- selecting alternatives using decision-making criteria
- presenting selected alternatives and a rationale for selection

What is the teacher's role in the Issues Inquiry Strategy?

The teacher's role involves:

- preparing students for self-directed inquiry
- ensuring that issues are of importance to the students engaged in the inquiry
- organizing and structuring appropriate groupings of students

- encouraging students to explore alternatives rather than thinking in terms of a right answer
- focussing questioning to ensure students explore appropriate pathways
- helping students work towards higher levels of questioning
- guiding students to develop assessment criteria, for example:
  - In what ways was your research appropriate and effective?
  - What are some limitations of your data?
  - What are some limitations of your analysis?
  - What are some ways you would do your analysis differently and why?
  - What are some ways you would do your research differently if you were to do it again and why?

What is the student's role in the Issues Inquiry Strategy?

The student's role involves:
- identifying and taking ownership of an interesting topic
- identifying and questioning assumptions
- identifying criteria for decision making
- gathering and assessing information
- analysing data
- identifying alternatives
- selecting and defending an alternative

Case Study

What is the Case Study Strategy?

A Case Study is a written narrative (e.g., story, drama, news article) based upon a set of events that presents a specific real-life dilemma (e.g., a resource management situation). Broad issues and several points of view are examined through a set of focussed study questions.

Why use the Case Study Strategy?

A Case Study focusses on decision-making skills, problem solving and critical thinking. Students are encouraged to explore real-world problems from their own personal perspective.

It is easier for students to understand and remember a particular example that is embedded in a meaningful context than general principles presented in isolation. A Case Study provides students with an opportunity to see the value of mathematics in understanding an issue or problem, and provides them with knowledge and opportunities to apply mathematical concepts and skills. A Case Study based in the local community, provides students with opportunities to access additional learning resources beyond those available at home or school.

What is the teacher's role in the Case Study Strategy?

The teacher's role involves:
- preparing an appropriate study that:
  - is specific and local

- is developed by students and teacher
- includes graphs, maps, photographs, text, and data
- encourages students to gather additional material
- focusses on a specific problem or situation that develops around big ideas
- develops conceptual understanding
- moves student thinking from specific to general; from abstract to concrete and back
- establishes the context for the study
- monitors depth and direction of small group discussion
- asks questions that encourage students to extend their thinking, for example, open-ended questions that cannot be answered with a simple 'yes' or 'no'
- facilitates whole-class discussion and presentations

What is the student's role in the Case Study Strategy?
The student's role involves:

- being motivated to investigate and learn from a local real-world problem
- using mathematics to think critically about issues identified in the Case Study
- developing a personal opinion based on a mathematical analysis
- discussing, communicating, and presenting personal opinion
- acknowledging other perspectives and reaching a consensus based on mathematics

Learning Centres

What is the Learning Centre Strategy?
Learning Centres are self-contained learning stations. These can be located in the classroom, around the school, or in the community. They can be focussed on the use of specific media (for example, software, website information, video, or print), on specific skill development, or on the integration of knowledge and skills. Small groups of three to five students cycle through the Learning Centres, then debrief in order to share ideas and learning.

Why use the Learning Centre Strategy?
A range of student learning styles, strengths, and interests are easily addressed through the use of Learning Centres. These can help students address specific skills through practice.

Students are encouraged to learn concepts and skills, and use them in a personal creative way through active involvement with other students. Students can contribute their own material, for example photos, samples, data, and questions, to a Learning Centre thereby enriching the experience for themselves and others.

How can the Learning Centre Strategy be used?
Learning Centres can be re-used and improved each year. The organization and structure of the centres can remain relatively stable but the content and data may change.

Teachers should select concepts and skills that can be learned and practised relatively independently. Start with three or four Learning Centres. Providing duplicate Learning Centres for large classes helps minimize preparation and class management issues.

Students can also build Centres themselves if they are given a prototype and encouraged to contribute appropriate information. Once Learning Centres have been established, groups rotate and address the questions posed by their peers. Teachers might suggest that students ask themselves, "What are three questions I want answered about the information in my display?"

What is the teacher's role in the Learning Centre Strategy?

The teacher's role involves:

- structuring concepts and skills into learning centres
- guiding and encouraging students in their use of centres
- facilitating individual groups of students by discussing the kinds of ideas and questions students are formulating
- assisting students to reflect on what they have done, clarify the objectives they want to achieve, and extend their mathematical thinking
- working with students to establish assessment criteria

What is the student's role in the Learning Centre Strategy?

The student's role involves:

- attending to specific concepts or skills
- sharing knowledge and skills with the group
- completing and recording individual work
- reflecting on and evaluating own learning

Direct Instruction

What is the Direct Instruction Strategy?

Direct Instruction is traditionally the most commonly used strategy. It can take a variety of forms including lectures, demonstrations, and video presentations. It can also be effectively integrated into other teaching strategies.

Direct Instruction may be appropriate when a list of facts, concepts, or rules needs to be presented as requisite knowledge. Discussion about the relevance or importance of the material is generally limited. Practice to consolidate learning often follows Direct Instruction.

Why use the Direct Instruction Strategy?

This is an effective means to impart information and procedures that students need in order to accomplish a task. Attention to, and retention of the material is improved when the success of an impending task depends upon specific content.

When many students are experiencing similar problems, it is often most effective to present material to the whole class using direct instruction.

Encouraging students to discuss their perspectives and present a more comprehensive view, helps them to refocus their thinking.

How can the Direct Instruction Strategy be used?

Direct instruction is particularly useful in combination with other strategies to summarize material and/or experiences. It can also be used to introduce new activities, structure choice, explain or clarify procedures, provide information in a consistent format, and summarize learning.

What is the teacher's role in the Direct Instruction Strategy?

The teacher's role involves:

- organizing and structuring the material
- relating the material to student tasks
- integrating other strategies
- providing student strategies such as taking notes, summarizing, and webbing
- assessing effectiveness of the strategy

What is the student's role in the Direct Instruction Strategy?

The student's role involves:

- listening attentively
- recording and assimilating information
- recapitulating information
- acting on information
- linking information with prior knowledge

Field Study

What is the Field Study Strategy?

Field Studies are community expeditions and activities that provide students with opportunities to do research, apply their knowledge, and observe the application of concepts to situations outside the classroom setting.

Why use the Field Study Strategy?

Field study makes learning come alive, helping students link the practical and concrete to the theoretical and abstract. First-hand experience is an excellent way to convey many mathematical concepts. Students are more likely to be successful at recalling and applying knowledge and skills in new contexts when they have had opportunities to manipulate materials and develop specific skills within a real-life context.

Field Study provides opportunities for students to develop and practise skills in settings that are different from the traditional classroom. This provides additional opportunities for a larger number of students to succeed.

How can the Field Study Strategy be used?

The immediate school surroundings and community provide a variety of field study opportunities.

Effective Field Study has specific objectives that focus the learning experience and encourage the development of problem-solving skills and higher-level thinking. Field Study should include instructions and activities at pre-determined stations.

A Field Study may be:
- expert-led; teacher-facilitated
- teacher-organized with instructions to be followed by individual students on their own time, and completed within a designated time
- a focussed investigation that includes data gathering or observation within a limited time period or over a continuous period of time
- a travel experience that includes the study of a variety of relevant physical and human manifestations of mathematical concepts

For example, learning outcomes from the Shape and Space curriculum organizer can be brought to life using a Field Study approach. Learning and applying simple surveying techniques provide an opportunity for students to realize the importance of measurement and trigonometry in many real-world situations.

What is the teacher's role in the Field Study Strategy?

The teacher's role involves:
- planning and organizing an appropriate study
- defining tasks and providing focus (including assessment criteria)
- making arrangements for students to utilize community resources
- considering transportation and safety issues
- preparing students for the learning experience
  - discussing data gathering and observation
  - considering interviewing, measuring, and sampling techniques
  - encouraging hypothesizing, analysing, and synthesizing
- debriefing, discussing, and reflecting on the experience

What is the student's role in the Field Study Strategy?

The student's role involves:
- doing the field study
- thinking critically about the objectives and tasks
- assisting in the planning of the study
- observing and collecting data
- hypothesizing and analysing
- discussing, reflecting, and assessing
- evaluating the impact of the study on their mathmatical understanding of the world

Math Labs

What is the Math Lab Strategy?

A Math Lab is a place of experimentation, exploration, and analysis. Math Labs provide students with opportunities to use a wide range of tools including manipulatives, models, maps, photographs, data collection devices, calculators, and computer software to explore mathematical relationships in a controlled environment.

The Math Lab is a place where basic mathematical skills can be developed and where ideas, skills and tools can be applied to make sense of real-world data.

Why use the Math Lab Strategy?

A Math Lab provides hands-on interaction with equipment and physical objects. This interaction helps students construct meaning. They provide opportunities for students to work on specific skills in the context of a larger inquiry.

Math Labs help students to experience mathematics as a living and vital subject that helps them see further and understand more. Math Labs provide an opportunity for students to develop their thinking and a range of skills and processes that include:

- using technology
- visualizing
- reasoning
- problem solving
- estimating
- connecting
- communicating

How can the Math Lab Strategy be used?

Math Labs can be combined with a Field Study to focus on requisite skills, field study techniques, and data analysis. Math Labs can be conducted to model a physical situation. Students can then analyse the results of an empirical study, and discuss differences and similarities between a mathematical model and the real world.

The relationship of mathematics to other subjects, for example visual art, science and history, can be effectively explored in a laboratory setting.

What is the teacher's role in the Math Lab Strategy?

The teacher's role involves:

- selecting appropriate lab equipment
- setting up a lab and organizing access to it
- providing the context for lab activities
- assessing and discussing lab work with students

What is the student's role in the Math Lab Strategy?

The student's role involves:

- posing and testing hypotheses
- observing, measuring, classifying
- predicting, inferring, interpreting
- formulating models, designing experiments, controlling variables
- assessing data and evaluating results
- communicating and presenting lab results

Group Work or Co-operative Learning

What is the Group Work or Co-operative Learning Strategy?

Group Work or Co-operative Learning occurs when two or more students strategize and co-operatively solve problems to accomplish a shared objective.

As members of a group, students present and explain their points of view, summarize the group's position, develop a strategy to address the task, and interact synergistically to achieve an objective.

Why use the Group Work or Co-operative Learning Strategy?

Group Work or Co-operative Learning provides opportunities for students to develop interpersonal and small group communication skills, and to experience individual accountability. For many students, group work or co-operative learning provides an opportunity for positive interdependence and mutual support.

Group Work or Co-operative Learning skills become increasingly important when individuals assume greater responsibility as workers, employers, parents, and citizens, and develop a positive attitude towards the contributions of others.

How can the Group Work or Co-operative Learning Strategy be used:

Group Work or Co-operative Learning can be incorporated into many different learning activities. By assigning roles, for example, reader, checker, observer, or reporter, different skills and learning styles can be addressed.

Group Work or Co-operative Learning can be used to address a large meaningful problem that could not otherwise be considered. Students experience the power of co-operative projects and are given the opportunity to appreciate the diverse skills and perspectives of the students in their group.

What is the teacher's role in the Group Work or Co-operative Learning Strategy?

The teacher's role involves:

- emphasizing the importance of social skills such as communicating, accountability, and negotiating for agreement
- working with students to select groups appropriate for the particular task at hand
- encouraging students to discuss tasks in ways that use higher level thinking
- facilitating and supporting students as they interact in the learning process and support each other's efforts
- providing opportunities for students, both as a group and individually, to reflect on group dynamics and the value of each person's contribution to the success of the group or team
- providing strategies for students to deal with dysfunctional groups
- assessing group work appropriately based on both the process used and the product developed

What is the student's role in the Group Work or Co-operative Learning Strategy?

The student's role involves:

- selecting appropriate team members
- taking responsibility for himself and the group
- reflecting on how the group is functioning
- adjusting personal behaviour as required
- assessing personal work and the group's work

Simulation or Role Play

What is the Simulation or Role Play Strategy?

Simulation or Role Play is a scenario in which students interact within a structured context. Rules of interaction and the initial state of the scenario are presented first. Students become the actors and play out a story using available information and their mathematical understandings. This instructional strategy is a key one in applied mathematics.

The three parts to this strategy are:

1. preparation, where learning takes place about the scenario;
2. the actual simulation where the students play out their roles; and
3. a critical debriefing of the simulation.

Why use the Simulation or Role Play Strategy?

Simulation or Role Play provides students with opportunities to explore and immerse themselves in real-world or imaginary situations that they could not otherwise access. By interacting with each other and within known constraints, students are challenged to analyse their thinking in light of other ideas. Simulation or Role Play helps students see things from different perspectives, understand that there are other ways to view a situation, and that there are many ways to solve a problem.

Critical thinking, decision making, problem solving, and communication skills are emphasized in Simulation or Role Play. The consequences of a sequence of decisions, consequences not necessarily apparent at the start of the simulation, can be explored and discussed.

How can the Simulation or Role Play Strategy be used?

A Simulation or Role Play can be used to summarize or bring closure to an activity. Roles can be assigned and researched based on the interests and styles of individual students.

Students who do not participate directly in the main simulation can be assigned the role of reporter, for example, for Much Music. The student may then choose to report on the event in print, audio, or video, or through a website. The report should focus on the critical issues and arguments that emerged during the simulation activity.

What is the teacher's role in the Simulation or Role Play Strategy?

The teacher's role involves:

- setting the scene:
  - describing the circumstances of the simulation
  - identifying some of the attributes of the main characters
  - setting the objective for each character
  - identifying the rules of the simulation
- ensuring students have sufficient time and information to prepare for the activity
- guiding students as they prepare their roles
- facilitating interaction during the simulation
- helping students debrief their experience

What is the student's role in the Simulation or Role Play Strategy?

The student's role involves:

- preparing for her individual role by gathering information and developing mathematical skills to make sense of information
- engaging in the simulation by interacting and exploring possibilities
- reflecting on the simulation to arrive at meaningful conclusions about the importance of mathematics in real-life scenarios

Independent or Self-Directed Learning

What is the Independent or Self-Directed Learning Strategy?

Independent or Self-Directed Learning consists of a time-limited research or learning project chosen by the student in consultation with the teacher. Self-directed learning should always be an option for students.

Why use the Independent or Self-Directed Learning Strategy?

This strategy provides students with the opportunity to take responsibility for:

- identifying learning objectives
- working independently towards achieving those objectives
- actively assessing their accomplishments in relation to predetermined criteria

This strategy has the added benefits of:

- appealing to many students by helping them focus on a particular passion
- appealing to many students who do not work well in groups
- helping students develop research skills that will benefit them in later years

How can the Independent or Self-Directed Learning Strategy be used?

Independent or Self-Directed Learning requires a contract with which the student feels comfortable before starting the project. Students must develop a vision of their project, identify an objective, and select a specific challenge within that objective.

Students write a contract either developing their own proposal or working from an initial list of challenges provided by the teacher. The contract should state that, "I've chosen this topic/challenge; I'm going to do these particular things and this is how I'm going to do it; I'll be finished by this date and will deliver a product (e.g., report, presentation, video, model) that will demonstrate my learning."

An additional component of the contract with the student could include an assessment of the experience doing the topic. For example: "My experience doing this project has helped me to understand the world of mathematics in the following ways..." If assessment has been incremental, it will be nearly complete when the project is completed.

What is the teacher's role in the Independent or Self-Directed Learning Strategy?

The teacher's role involves:

- guiding the student in developing a plan and a timeline that address the student's learning style, include appropriate milestones, and anticipate obstacles
- facilitating implementation of the plan
- monitoring the learning
- assessing learning incrementally according to agreed-upon criteria

What is the student's role in the Independent or Self-Directed Learning Strategy?

The student's role involves:

- working with the teacher to determine a preferred learning style
- developing a learning plan around a topic of personal interest and including appropriate assessment criteria
- entering into a project contract
- working through the project and providing deliverables on time

A note about using technology...

Technology supports and enhances the instructional strategies described above.

Technology not only engages students but it is also intricately linked to past and present mathematical exploration. Similarly, mathematics is embedded and entwined with all technological development. Using technology can help students understand the fundamental role mathematics plays in our lives. The use of technology helps prepare students for the workplace. Various learning styles can also be addressed by using technology.

One of the expectations of the Mathematics curriculum is that technology will be used as a tool to help students develop mathematical concepts and solve complex problems.

Technology can be used for:

- facilitating interaction (e.g., e-mail, Internet, fax)
- conducting simulations
- practising skills

- manipulating information (e.g., spreadsheets, databases, calculators),
- collecting data (measuring devices)
- modelling and analysing real-life phenomena
- stimulating critical thinking, problem solving, and decision making

The teacher's role involves:
- providing students with access to a wide variety of technology
- identifying available technology
- determining appropriate use of technology
- organizing access to and use of the technology
- providing context for using technology to support specific mathematical activities
- using technology to facilitate the applications of mathematics
- providing alternatives to using a specific technology

The student's role involves:
- attending to the mathematics rather than dwelling on the technology
- selecting and using appropriate technology
- using technology effectively

Assessment

Assessment is the systematic process of gathering information to provide ongoing feedback about critical or significant aspects of a student's learning. Students benefit from assessment when they clearly understand the learning objectives and expectations.

Student performance assessment is based on a wide variety of methods and tools. These range from portfolio assessment to pencil-and-paper tests. Appendix D of the IRP includes a detailed description of assessment and evaluation.

Provincial reference sets can also help you assess the skills that students acquire across curricular areas, in particular:
- Evaluating Problem Solving Across Curriculum (RB 0053)
- Evaluating Group Communication Skills Across Curriculum (RB 0051)
- Evaluating Mathematical Development Across Curriculum (RB 0052)

A series of assessment handbooks that provide guidance for teachers as they explore and expand their assessment repertoires includes:
- Performance Assessment (XX0246)
- Portfolio Assessment (XX0247)
- Student-Centred Conferencing (XX0248)
- Student Self-Assessment (XX0249)

The assessment strategies listed in Column 3 of the IRP suggest several approaches to gathering information about student performance. These assessment strategies have been developed by specialist and generalist teachers to assist their colleagues. Ideas for developing a repertoire of assessment strategies may come from reviewing the earlier section on Instructional Strategies, and reviewing the suggested instructional strategies in each Column 2 of the Grade 9 Mathematics curriculum.

Feedback, Please

We welcome any comments or questions you might have about this resource and encourage you to share the experiences you and your students have had using it, for example, through a listserv or website. Please contact:

Mathematics Coordinator
Ministry of Education
Fax (250) 387-1527

Introduction to Mathematics

- What are the Different Instructional Approaches?
- What is the Process of Teaching Mathematics?
- What are the Principles of Learning Mathematics?
- Who are the Key Players in Mathematics 9?

Introduction to Mathematics

There is a growing sense of excitement and anticipation as teachers, students and parents discover the wonder of mathematics through applications.

Mathematics 9 is a rigorous, academic mathematics curriculum. Developed as a means to make mathematics accessible to all students, this curriculum has renewed a focus on pedagogy and, perhaps most important, the relationship among mathematics, the learner and the real world.

Mathematics 9 is one of a growing number of curricula having a significant applications focus. This curriculum is based on a holistic understanding of the learning process and provides an approach to learning that makes sense to a great many students.

The Mathematics 9 curriculum focuses specifically on the relationship between given skills or aspects of knowledge and the contexts in which people use them. Applications relates the theoretical to the practical and connects the abstract to the concrete. They tie what students already know and their sense of the real world to what they are about to learn. This helps give students a sense of how they might use what they are learning.

What are the different instructional approaches?

The lessons build upon the mathematical knowledge and understanding students already have. It is essential that students see the mathematics lesson as an extension or reconstruction of what they already know. This can lead to some principles, theory and practice of skills to support that theory. Then, *as* mathematical ideas are used by students in decision making and in critical thinking within new contexts, they become integrated as part of the student's knowledge and skill set. This emphasis on actual thinking about the real world makes the mathematics very important to the students, rather than something that they must do.

The most important aspect of the mathematics lesson is that it is engaging. It should genuinely spark curiosity in the student, and inspire a sense of questioning and empowerment—a sense that mathematics is a practical and necessary activity that all humans, in one way or another, do.
To encourage this enthusiasm and interest, an open forum must be created where students can explore ideas and issues without the feeling that there must be a single and, therefore, predictable 'right answer'.

A second important component of an applied lesson is a clear delineation among mathematical concepts, contexts, calculations and critical thinking (4 C's). Students are very willing to tackle even 'drill and practice' if they understand it is an exercise that is useful for accomplishing some activity in which they are interested. In this regard, mathematics students are like those who play basketball, or any other sport that requires drill and practice. The important thing for you the teacher, is to occasionally give the players (i.e., students) a chance to use what they have learned in a game situation.

What is the process of teaching mathematics?

Throughout the process of teaching mathematics, teachers help students make connections with the world they experience around them, and their current and future roles and responsibilities as students, family members, citizens, and workers.

One process for teaching mathematics follows:

- Describe the context of a problem and understand what is involved in doing a project intended to shed light on the problem. At this point, research and key planning questions are addressed.
- Build a workplan and identify important activities. Include the following:
  - why each activity is important
  - what will be achieved by doing each activity
  - what method will be used to assess and evaluate each completed activity.
- Create the product.
- Assess and evaluate the product using both formative and summative methods.
- Assess and evaluate the learning associated with developing the product.

Many students find it very helpful to have the teacher describe the problem-based learning process in which they are engaged. That is:

- posing problems
- posing questions
- generating and identifying alternatives
- making choices
- researching and planning
- generating/constructing products
- evaluating products
- assessing choices/reflecting on decisions
- reflecting on the effectiveness of products

At the end of the process, students should realize that they have learned new mathematical concepts, skills and processes, and they should be able to communicate their understanding with respect to these.

What are the principles of learning mathematics?

A major goal of *Mathematics 9: A Resource for Teachers* is to bring curriculum closer to the issues in students' lives now and in the future. This is done by focussing on the following key principles of learning mathematics:
- developing positive attitudes
- solving problems
- communicating mathematically
- connecting and applying mathematical ideas
- developing mathematical ideas in context
- reasoning mathematically
- using technology
- estimating and doing mental mathematics
- modelling

These key principles are elaborated below:

Developing Positive Attitudes

Research, including provincial assessments, emphasizes the direct relationship between student attitudes and student performance levels. The mathematics classroom should be a learning environment that engages the interest and imagination of all students. When students develop a positive attitude to mathematics, they are more willing to take risks and grow in their tolerance of ambiguity. They are much less likely to communicate, "I can't do math."

Solving Problems

Problem solving should be integrated into all areas of learning. The goal of problem solving is to provide students with opportunities to explore new mathematical concepts by building on previous experience and knowledge. Students must be empowered to:
- read and analyse a problem and identify the significant elements
- select an appropriate strategy that is likely to generate solution(s) to the problem
- draw upon the skills and concepts developed within areas of mathematics as well as in other subject areas
- work individually and in groups
- verify an answer and judge how reasonable it is
- communicate mathematically to different audiences

Communicating Mathematically

Mathematical communication skills are developed when students are actively exploring, investigating, describing, justifying decisions, explaining, and solving problems collaboratively. It is important that students build confidence in expressing their mathematical understanding when communicating and demonstrating what they have learned.

Connecting and Applying Mathematical Ideas

Learning activities should help students understand that mathematics is a changing and evolving domain, one that has benefited from the contributions of many cultural groups. A major instructional focus of mathematics is helping students connect mathematics to their own experiences and future career interests.

Developing Mathematical Ideas in Context

Activities should challenge students to learn mathematical skills and concepts in real-world situations. They should also demonstrate how mathematical ideas are powerful tools by which to understand and influence the world. Students should view mathematics as a vital human activity that evolves through approximation, refinement, and communication.

Reasoning Mathematically

Mathematics instruction should help students develop confidence in their ability to reason and justify their thinking. Students need the freedom to explore, conjecture, validate, and convince others if they are to develop mathematical reasoning skills. It is important that their abilities to reason well and to take risks are valued as much as their ability to find correct answers.

Using Technology

The natural curiosity of students is engaged when a range of technology tools is used such as computers, calculators, and measuring devices. Technology allows students to explore computationally complex situations while concentrating on the underlying mathematical relationships. Students must be able to select and use the most appropriate tools for data collection, analysis, and presentation.

Estimating and Doing Mental Mathematics

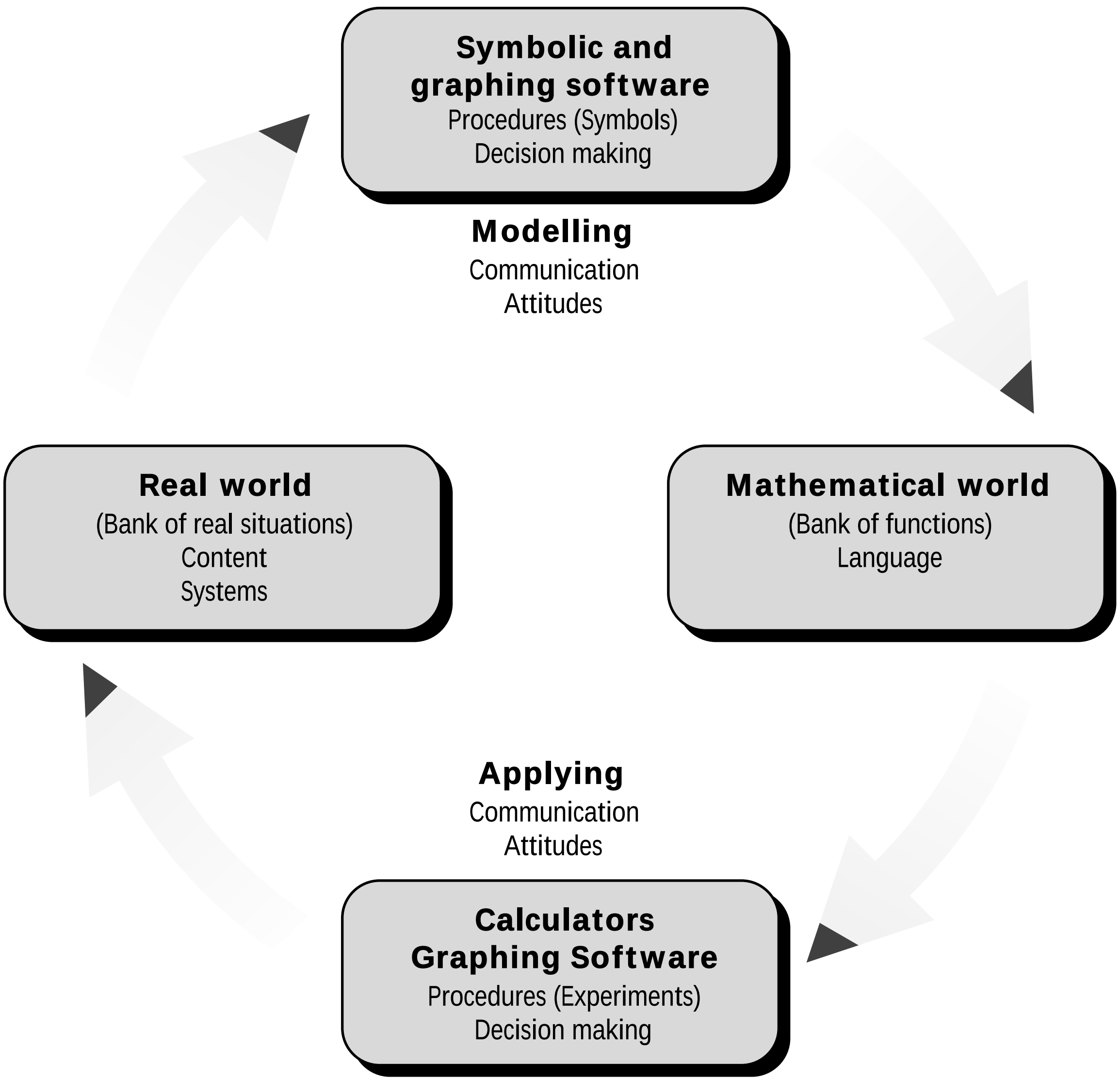
Instruction should emphasize the value of estimation. Students need to use reasoning, judgment, and decision-making skills when estimating. Self-confidence and precision in mathematics increase when students can perform simple mental mathematical operations.

Modelling

An important element of problem-based learning is that students create and use mathematical models of real-world situations. Modelling a situation means translating the situation into mathematical language in order to better understand the real world, for example, modelling the trajectory of a projectile with a quadratic function. See Figure 1: Modelling.

Figure 1: Modelling

Modelling
Modelling a situation means translating the situation into mathematical language in order to better understand the real world, for instance, modelling the trajectory of a projectile with a quadratic function.



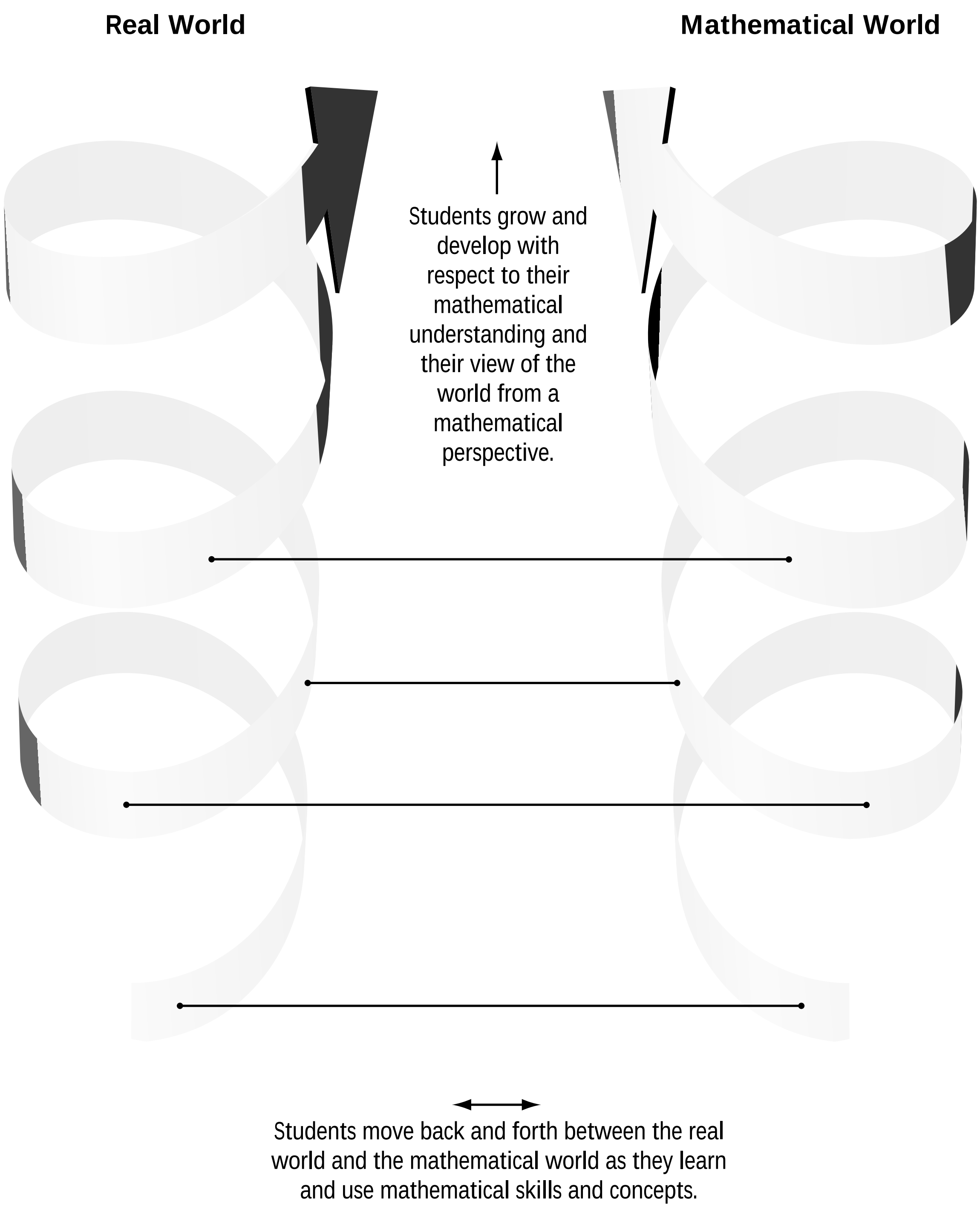
Who are the key players in Mathematics 9?

The Learner

The Mathematics 9 curriculum places an emphasis on students. It is designed to provide opportunities for all students to understand, enjoy and ultimately use mathematics in their present and future lives. It allows students to use intuitive sense to draw valid and useful connections between abstract concepts and concrete situations. With this kind of experience, students are able to explore potential applications in various fields of endeavor, ranging from business and finance to science or engineering. In many cases, a student's interest in this kind of mathematics will come from an interest they have in a particular career. Just as often, it can grow out of personal hobbies and interests, or out of curiosity about the mathematics they have begun to explore in this way.

The mathematics learner comes from a diverse population that includes exceptionally bright students wanting to learn about mathematics from a contextual point of view, in a hands-on way, and those students for whom the algorithmic approach is a struggle, and the mathematics doesn't make much sense. These students need to understand mathematics as it relates to who they are rather than as something external to themselves or their world. The more that they are able to 'construct' the mathematics *for themselves*, the more likely they will view mathematics as *part of themselves*. See Figure 2: Moving Between the Real World and the Mathematical World.

Figure 2: Moving Between the Real World and the Mathematical World



The Teacher

Teachers of a mathematics curriculum make a point of knowing about mathematics in the real world. They tend to look, and to help students look at the whole picture, keeping in mind that mathematics is not just learning the various operations, but also understanding how it can be used to make decisions and construct meaning in the world. They see the teaching of mathematics as a broad and deep responsibility. They realize that mathematics must become richer than pure calculation for most students to develop skills and a true sense of numeracy.

Mathematics teachers begin to teach a concept by placing it in the context of the student's own experience—a context that the student can easily understand.

Mathematics teachers are open to the learning of mathematics, and use the appropriate materials and resources to help them. However, mathematics does not mean that teachers have to transform everything they do. Rather, teachers constantly need to add to their personal repertoire of instructional strategies. Teachers, for example, use more than one approach to stimulate individual learning. When a student says, "I don't get it," teachers try a second and third way of approaching the problem. This constant search for new, innovative and effective ways to teach mathematics helps make mathematics accessible to everyone.

Parents and Community

The excitement that is engendered in the Mathematics classrooms takes students beyond the artificial bounds of the classroom. What children produce is important to their view of the world, so it is necessary to get them into that world, investigating, exploring, and making mathematical sense of it.

The nature of the mathematics curriculum broadens the concept of the mathematics classroom itself. The classroom now extends to the edge of the universe. Parents and the community will find many classrooms looking considerably different from the way they do now.

Teachers find a real-world way to describe applications to parents. Their homework assignments include mathematics application questions that stimulate discussion and debate with siblings and parents.

These parents may be the resources that teachers need to bring the mathematics into the classroom. Many parents have interesting backgrounds in mathematics or have access to people who do.

One of the characteristics of the mathematics curriculum is that it is not fixed or static. It varies with the character of each community, school and classroom. It is no longer a dry and lifeless discipline for the initiates—it is an adventure.

Sample Project: Counting Kokanee

- Orientation
- Project Synopsis
- Phase I: Experiment
- Phase II: Modelling
- Phase III: Verification of Model and Discussion
- Background Information on Mathematical Modelling

Sample Project:
Counting Kokanee

Orientation

Time:

Phase I: Experiment .... 1 class period
Phase II: Modelling .... 1 class period
Phase III: Verification of Model and Discussion.................. 1 class period

Instructional Strategies:
- Case Study
- Direct Instruction
- Group Work
- Simulation

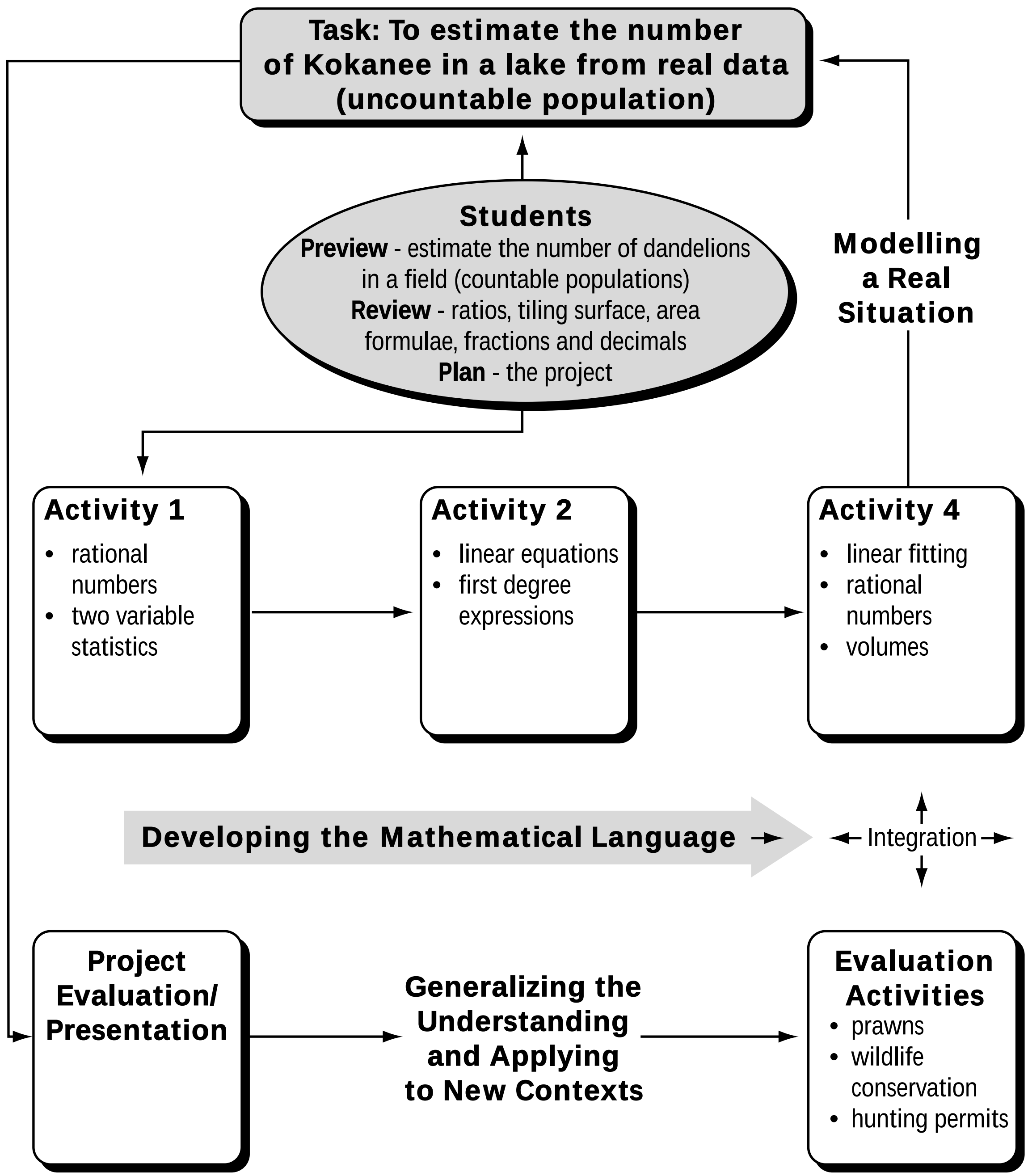
Real-World Applications:

Some real-world applications of the skills learned in this project include:
- estimating the number of youth attending an outdoor rock concert
- estimating the number of salmon that have returned to spawn in a local river
- determining how many units of a trendy clothing item to stock
- doing quality control spot checks in a factory
- polling on a specific political issue
- spotting trends
- developing models to predict outcomes
- sampling populations
- demonstrating information in graph form

Project Overview:

Figure 1:Project Kokanee, is a schematic to help you conceptualize the Sample Project and begin the planning process with your students. Background information on mathematical modelling is provided at the end of this chapter. We recommend you work through that section before attempting to implement the Sample Project.

Figure 1: Project Kokanee —
'How to count everything without counting everything'



Project Synopsis

This two-hour sample project gives you and your students an opportunity to become familiar with the style of activities in *Mathematics 9: A Resource for Teachers*. The mathematics involved in the sample project is at the Grade 8 level and can be used as a review of ratio and proportion.

This project provides an opportunity for students to see how ratios and proportions are used in the real world.

In this project students construct a mathematical model that helps them estimate the number of Kokanee in a lake, and that can be extended to count other uncountable objects.

While working through the project, students should pay particular attention to establishing co-operative working relationships and to observing how a mathematical model is developed.

Background information on mathematical modelling is provided at the end of this project on Kokanee. Having students read this material, with your guidance, prior to beginning the sample project will help ensure that they get a consistent idea of how and why they might want to use a mathematical model. As they work through this sample project, you may find it helpful to discuss Figure 6: Modelling Flowchart now, and again at the end of the project.

In Phase I, students use the Capture-Mark-Recapture technique to estimate the size of a population. In a Math Lab, students represent fish by beads, and estimate the number of beads contained in a bag by applying the technique of Capture-Mark-Recapture. Students then assess the reliability of the technique.

In Phase II, students learn how to ‘count the uncountable’ by constructing a mathematical model. This model helps them estimate the number of Kokanee in a lake.

In Phase III, students verify and discuss the models that they developed.

The project introduces a 6-step process for modelling a real-world situation:
1. Identify a specific problem.
2. Identify variables that are inherent in the problem.
3. Determine possible relationships among the variables.
4. Represent the variables and their relationships numerically, graphically, and symbolically.
5. Verify the model in a constrained situation.
6. Validate the model by testing it in real-world situations.

The following questions can help guide the discussion for this project:
- What are some situations where it is important to estimate population size? For example, people at an outdoor concert, or bird populations.

- What are some smarter, faster and easier ways of determining the size of a population rather than by counting every single individual in the population?
- What are some indirect ways of determining the size of a population when you cannot count them one by one?

Summarize the discussion by explaining that we can:
- model the real world in the laboratory
- test and validate hypotheses
- apply our new knowledge to the real world.

Phase I: Experiment

Objective:

The objective of this phase is to have students determine situations where it is important to estimate populations and to have them learn a technique for doing this.

Materials:

For each pair of students:
- 1 large bag with approximately 400 to 500 white beads (this number is not disclosed to students)
- 1 small bag with between 15 and 40 red beads
- 1 jar or bowl

Procedure:

Have students identify situations where it is important to know or to estimate size of populations.

Have student brainstorm some indirect counting (estimation) procedures. Then have them describe the relevance of using any of these to determine the size of a population of fish.

Organize students into pairs and distribute the materials listed above.

Explain that students will be using a real-world technique commonly used by wildlife personnel. It consists of netting, marking, and releasing X number of fish. The marked fish are then given a period of time to disperse throughout the unmarked population. After a given period of time a new sample of size X is netted and the marked fish are counted.

Explain that the beads represent the population of fish in a lake and have each pair of students estimate the number of beads in the container. Each pair should record their initial estimate, and perhaps, their estimation techniques.

Each student pair should take a different size sample of between 15 and 40 beads from the large bag of beads and count them. These beads represent the fish that have been netted in the first sample. Each pair should record the sample size in their Student Activity table.

This sample is then replaced in the container by the same number of red beads. This process models the tagging of the fish.

Ask students to mix the beads thoroughly to represent the assumed natural mixing process of fish in the lake. Have students discuss some of the effects of inadequate mixing on future sampling.

Without looking in the bag, have each pair of students randomly remove the same number of beads as before.

In all likelihood, some of the beads in the second sample will be the same colour as those in the first sample and some will be red.

Count the number of red beads. These represent the tagged fish that have been netted in the second sample.

Ask pairs of students to compare and discuss the accuracy of their estimates for the first sample. Have each pair of students share their experimental results with the rest of the class.

Phase II: Modelling

Objective:

The objective of this phase is for students to learn and apply the basic principles of developing and testing models.

Materials:

For each pair of students:
- 1 large bag with approximately 400 - 500 white beads
- 1 small quantity of red beads
- 1 jar or bowl

Procedure:

What is the quantity you want to determine?
- the total number of normal colour beads in the bag that we will call N

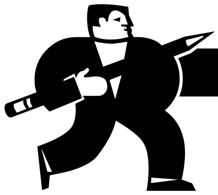
What are the quantities you know or have determined in this experiment?
- the size of the sample, that is the number of beads in the sample that we will call S
- the number of red beads in the second sample, that we will call D

Ask students to formulate the problem in their own words and to use the specific terms. This could be:

- Knowing the number of normal-coloured beads in a sample of *S* beads taken from a bag of beads, I must determine the number of beads contained in the bag.

Point out to students that a larger sample size generally has a larger number of different-coloured beads.

The students may collect all of the data from the other pairs of students and record the results in a table (Student Activity: Table of Mathematical Model). The students may also use their results to plot a graph showing the relationship of the number of different-coloured beads to the sample size (Student Activity: Graph of Mathematical Model).



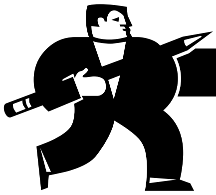
Student Activity: Table of Mathematical Model

Student Name: ______________________________ Date: ____________________

Table 1: Table of Mathematical Model

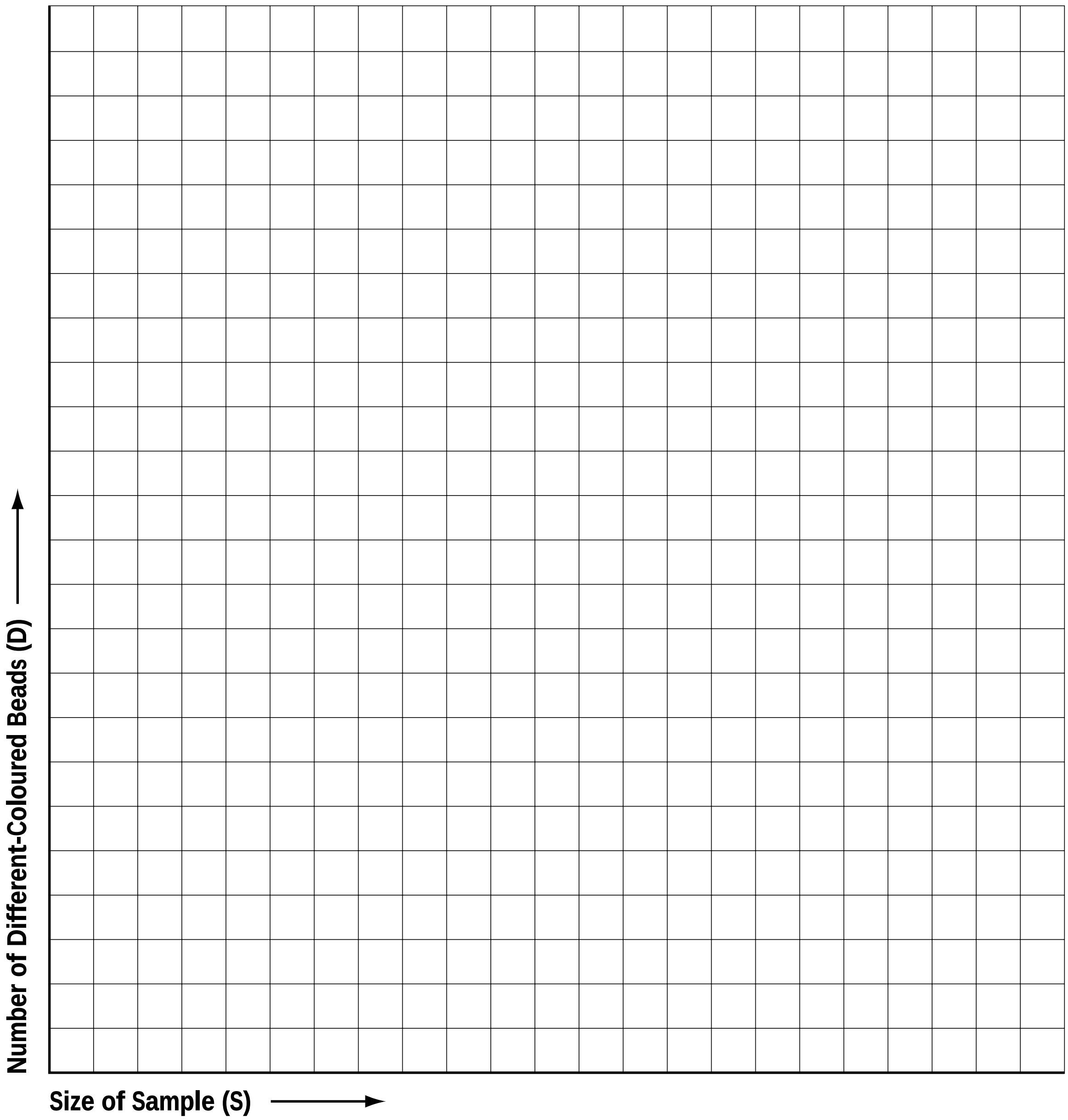
Student Pair	Size of Sample S	Number of Different-Coloured Beads D	Ratio $\frac{D}{S}$

Conclusions:



Student Activity: Graph of Mathematical Model

Student Name: ______________________________ Date: ____________________



Help students interpret the representations of the model from the table and the graph:

- referring to the table where the ratio calculated in the last column is constant, although some variations may be noticed, ask "How is the size of the sample related to the actual number of beads (population)?".
- referring to the graph, ask "Are the data points situated on a straight line? What is the number of beads for a sample size of…)? What would the number be for a large sample?" Invite students to extrapolate the graph to a very large sample size. What does it mean? Do the students realize that a large sample could eventually be the total population?

Have students formulate the model with symbols from the following observation:

the ratio of the number of different coloured beads to the size of the sample is the same irrespective of the size of the sample

or

D divided by S is the same number (a constant)

or

$\frac{D}{S} = k$ (k = a constant)

Assumption

Considering that the beads have been mixed thoroughly, we can assume that the proportion of the number of beads in the second sample is the same as the proportion of all the beads in the total population or

$$\frac{D}{S} = \frac{S}{N}$$

This assumption is based on the results obtained from either the table or from the graph. Whatever the number of beads, the same ratio will be obtained and this is true for the total number, that is, the total population.

Solve the Mathematical Model

Have students substitute the data into the model. Encourage them to first estimate and then use their calculators.

For example,
Size of sample: $S = 20$
Number of different-coloured beads: $D = 2$

Calculate N in the relationship $\frac{D}{S} = \frac{S}{N}$

$\frac{2}{20} = 0.1 = \frac{20}{N}$

$N \times 0.1 = 20$

$N = \frac{20}{0.1}$

$N = 200$

or

Size of sample: $S = 35$
Number of different-coloured beads: $D = 5$

Calculate N in the relationship $\frac{D}{S} = \frac{S}{N}$

$\frac{5}{35} = 0.14 = \frac{35}{N}$

$N \times 0.14 = 35$

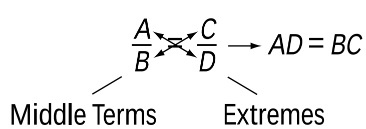
$N = \frac{35}{0.14}$

$N = 250$

Encourage students to determine a way of solving the equation systematically, for example, by using the property of proportionality illustrated below.

If $\frac{A}{B} = \frac{C}{D}$, then $A \times D = C \times B$ or the product of the 'extremes' equals the product of the middle terms:

Figure 2: Property of Proportionality



Ask students to practise the use of the property of proportionality with several results collected in the class. Then have students complete the table in Student Activity: Using and Testing the Model of Capture-Mark-Recapture.

In column one students record sample size. In column two they record the number of different-coloured beads. In column three they use the total population result obtained either in the previous table or graph they completed. In column four they record the total population results obtained from the mathematical model. Finally students estimate the deviation (E) between the actual experimental results and the formal result, and record it in column five.

They may consider expressing this deviation as a percent of the sample size in column six, that is, as a ratio of the deviation to the size of the sample.

Conduct a discussion on the results of Student Activity: Using and Testing the Model of Capture-Mark-Recapture. Have students comment on the size of the sample and its importance in the accuracy of the estimation.

Phase III: Verification of Model and Discussion

Objective:

The objective of this phase is for students to verify and discuss the model they developed.

Materials:

- Background on Mathematical Modelling section at the end of this project
- Figure 6: Modelling Flowchart at the end of this project

Procedure:

As you recall, modelling was presented as a 6-step process:

1. identify a specific problem
2. identify variables that are inherent in the problem
3. determine the relationship among the variables
4. represent the variables and their relationships
   - numerically
   - graphically
   - symbolically
5. verify the model in a constrained situation
6. validate the model by testing in real-world situations

The problem we identified was how to count a population of fish that could not be counted directly, other than by draining the lake! We assumed that we could estimate the population by sampling and then model this with a container of beads.

The variables we identified included:

- sample size *(S)*
- number of marked beads in the second sample *(D)*

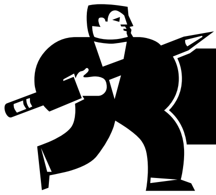
Through experimentation we represented the relationships numerically, graphically, and symbolically $\left(\frac{D}{S} = \frac{S}{N}\right)$

To verify the model, students should count the total number of beads in their containers and determine the accuracy of their experimental estimates.

As a class exercise, all the white beads could be placed in a large container and the procedure applied to the grand total. Does the model facilitate the determination of the total population?

Students should be able to suggest and describe situations where the model can be tested, and be able to design procedures that need to be followed in each case. Ask students to consider and discuss the following questions:

- In what ways does the model make common sense?
- In what ways does it make sense that the ratio must be the same irrespective of the size of the sample?
- What are some of the effects of insufficient mixing?
- What are some systematic ways that the model can be used to determine an unknown quantity?
- Does the model hold up when tested?
- What are some ways you know that the model is precise enough?
- What are some limitations of the model?
- What are some ways that the model can be improved?
- What are some ways that the model might be tested with real-world situations?



Student Activity: Using and Testing the Model of Capture-Mark-Recapture

Student Name: ____________________________ Date: ___________________

Table 2: Testing the Model of Capture-Mark-Recapture

Sample Size S	Number of Different-Coloured Beads D	Size of Population N_E (from Table or Graph)	Size of Population N_M (from Model)	Deviation $E = N_M - N_E$	% Deviation $\frac{E}{S} \times 100$

Conclusions:

Background Information on Mathematical Modelling

A mathematical model represents a system, that is, an assemblage of interdependent components that interact in various ways (e.g., solar system or the global economy).

Examining and analysing models can help students understand:
- how a particular system works
- what causes change in a system
- the sensitivity of the system to certain changes

Models can also help students predict:
- what changes will occur in the system
- when changes will occur in the system

There is a 6-step process for constructing a mathematical model to represent a real-world situation:
1. identify a specific problem
2. identify variables that are inherent in the problem
3. determine the relationship among the variables
4. represent variables and their relationships
   - numerically
   - graphically
   - symbolically
5. verify the model in a constrained situation
6. validate the model by testing it in real-world situations

There are many situations that can be modelled mathematically. For example, suppose an object is dropped from a great height. What are some ways that the fall can be described?
- How long will it take before the world runs out of oil?
- How far can a golfer hit a golf ball?
- What is the required dosage level and the interval of time between doses for a particular drug to be safe and yet effective?
- A chemical has spilled into a lake causing unsafe levels of pollution. Fresh water runs into the lake from a nearby stream, and water runs out of the lake at the same rate. How long will it be before the lake is cleaned to within 10% of the initial pollution level?
- Consider the guideline often given in driver education classes: allow one car length for every 15 km/h of speed under normal driving conditions. How good is this rule? What are some problems with this rule?

Procedure:

1. Identify a Specific Problem

We must be very specific in verbally formulating a problem in order to be successful in translating it into mathematics.

Given the problem of a falling object, there are still many questions we might ask:
- Does a heavier object fall faster than a lighter one?
- Does the object fall at a constant speed?
- If an object's speed changes during the course of the fall, how fast is the object moving when it strikes the earth?
- Does the object reach a constant maximum speed?
- Does the object fall halfway to the earth in half the time of the total fall?
- How long does it take for the object to reach the earth?

In order to successfully model this problem we must simplify the situation even further. For example:
What is the time of fall of an object of given mass dropped from a known height?
We need to be even more precise about the situation. Is it assumed that the object is falling from an aircraft, or simply released from rest? Are there forces that tend to slow the object as it falls? It is important to decide what the scope of the investigation is.
Therefore, let us suppose that:
- the object is released from different levels of a high-rise building
- the effects of the forces slowing the fall are disregarded.

2. Identify Variables That are Inherent in the Problem

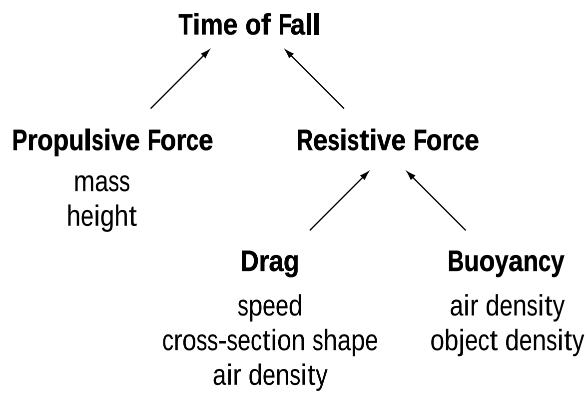
It is not usually possible to capture all of the factors that influence the behaviour under investigation in a usable mathematical model. The task is to simplify the situation by reducing the number of factors, and to determine the relationships among the remaining variables. Again, we can reduce the complexity of the problem by assuming relatively simple relationships (linear or quadratic for example).

Classification of variables: dependent and independent
Variables are the factors that influence the behaviour identified in Step 1. Variables can be classified as dependent or independent. The variables the model seeks to explain are the dependent variables. The remaining ones are the independent variables. Certain variables can be omitted depending upon the questions asked.

Usually before determining relationships among variables, we must simplify the problem. If a problem is too complex, it may be preferable to study sub-models. In this case, one or more of the independent variables are studied separately. Eventually the sub-models are linked together under the assumptions of the main model.

In the 'falling body' problem, as we have described it above, time of fall is a dependent variable and the height of the object above ground is an independent variable. To simplify the problem we assume that the object is moving vertically only. What are the factors that affect the time of the fall or the object's position? Some forces are acting on the object (since it moves). Propulsion forces tend to move the object downward and resistive forces tend to diminish that motion. The resistive forces are caused by drag on the object as it falls and by the buoyancy of the air that supports the object and tends to hold it in space. The time an object takes to complete its fall depends on the propulsion and the resistive forces acting on it. To account for drag and buoyancy, we could create a sub-model that deals with the resistive forces.

Figure 3: Identification of Variables



3. Determine the Relationship Among the Variables

A flowchart of the relationships among the variables can be constructed and assumptions about the way each variable depends on the others can be made.

To further simplify the model you may choose to:
- ignore some of the independent variables. For example, the effect of one variable may be small compared to the others. You may choose to ignore a variable because you want to investigate the situation without the effect of this variable
- simplify relationships among the variables, for example, assuming the propulsion force is a constant multiple of the mass or that the drag equals some constant times the speed of the object. If subsequent tests of the model prove unsatisfactory, you can later refine the model by changing the assumptions.

4. Represent Variables and Their Relationships

- numerically
- graphically
- symbolically

All the sub-models are put together to obtain a numeric, graphic, and algebraic picture of the original situation. One often finds that the model is not complete or is so unwieldy that it cannot be interpreted. In those situations you must return to Step 2 and make additional simplifying assumptions. Sometimes it is even necessary to go back to Step 1 to redefine the problem. The ability to simplify or refine the model is an important part of modelling.

With respect to the 'falling body' problem, under the assumptions made in Step 2 and neglecting all the other variables for now, the algebraic representation is:

$t = kH/m$ where k is the constant of proportionality,
H is the height,
m is the mass of the object, and
t is time.

Note: the model is, of course, wrong since in the real world t is independent of the mass and depends on the square root of H.

5. Verify the Model in a Constrained Situation

Before using a model to reach real-world conclusions, it must be tested.

Several questions should be addressed before designing the tests and collecting data:
- Does the model answer the problem identified in Step 1, or did you stray from the key problem in designing the model?
- Does the model make sense?
- Can the data necessary to test and operate the model be gathered?
- Does the model hold up when tested?

In designing a test or an experiment for the model using actual data, you must include observations made over the same range of values for the independent variables as you would expect to encounter when actually using the model. The assumptions made in Step 2 may be reasonable over a restricted range of the independent variables yet very poor outside of those values.

In the 'falling body' problem, the model helps us determine the time of fall given a specific height. Furthermore, the model makes sense because as the height is increased, the time is longer (that is certainly what is expected). Moreover, it seems reasonable to think that heavier objects would fall faster than lighter objects. All other variables have been disregarded.

What are some ways in which the model can be tested in a constrained situation?

We could conduct some experiments by dropping a steel ball from different heights and measuring the time of the fall. This would take care of the 'height of fall' variable.

Then, we could drop balls of different masses from the same height in order to test the model with respect to the mass.

Finally, we could compare these experimental values against the values predicted by the model.

Graphs are very useful tools to assess models. Experimental results involving dropping a steel ball of a given mass from different heights are given in Table 3.

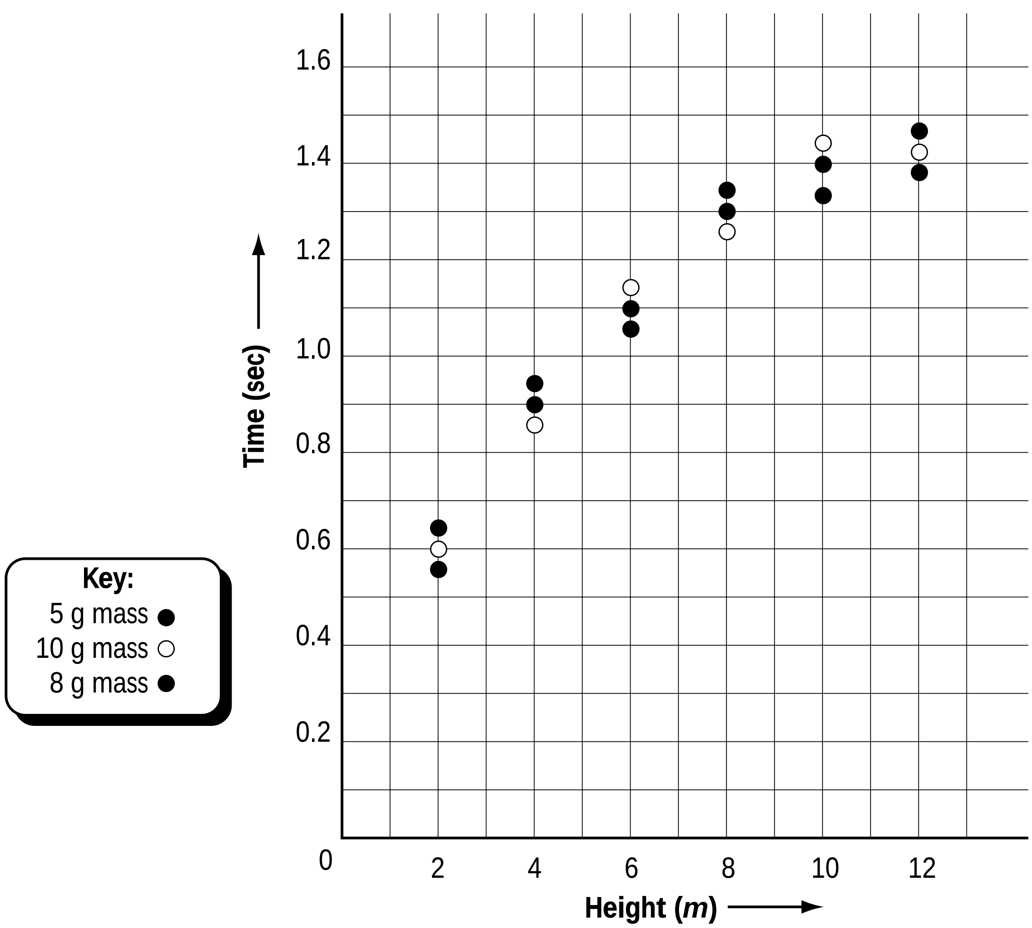
Table 3

Steel Balls Dropped From Different Heights

Height (*m*)	0	2	4	6	8	10	12
Time (sec)	0	0.6	0.9	1.1	1.3	1.4	1.5

Figures below show scatterplots of the data obtained with steel balls of different masses superimposed on the model (straight line).

Figure 4:Testing the Model: Steel Balls Dropped From Different Heights



The data points reveal that experimental results do match the straight line predicted by the model. However the experiments do not support the existence of a relationship between mass and time. The data shows that time is independent of mass.

From this we can conclude that there is something wrong with our assumptions.

Note: Be careful about conclusions drawn from an experiment. Broad generalizations cannot be extrapolated from particular evidence gathered about a model. A model does not become law simply because it is verified repeatedly in specific instances.

Assumptions need to be reviewed and modified in light of the experimental data. Experiments show that rather than being a linear relationship, the relationship between time and height tends to be more quadratic, and that mass has no influence on the time of fall.

A refined model could then be:

$$t^2 = kH$$

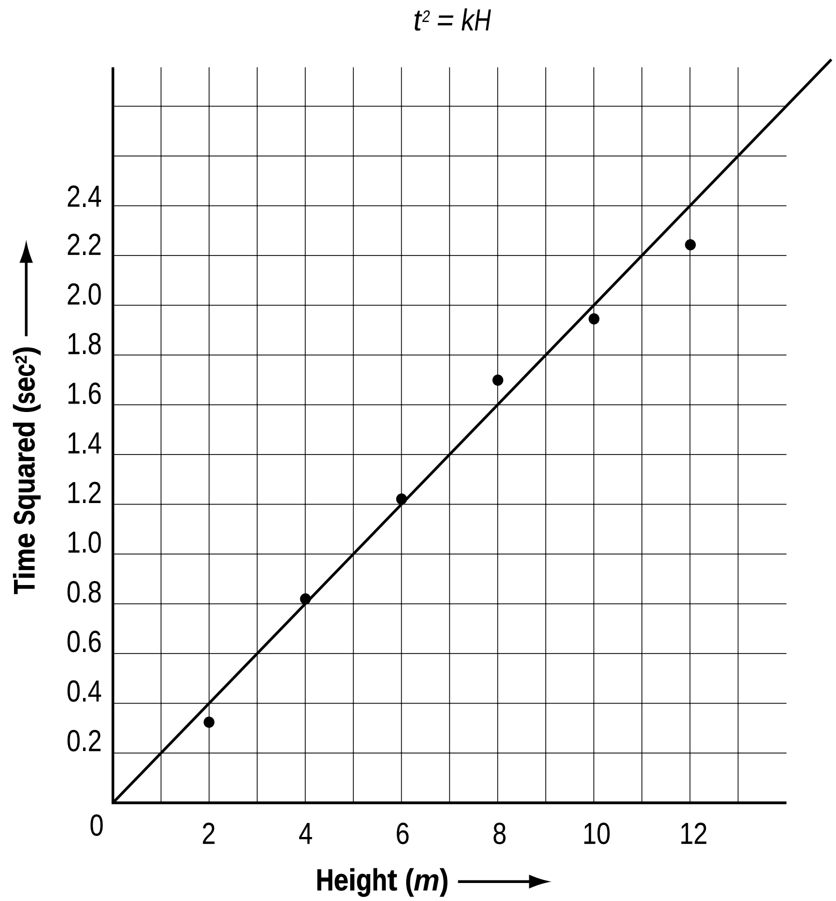
Representing the data in the following table and graphing the square of the time versus the height of fall would give the result shown in Figure 5: Testing the Model Under New Assumptions.

Table 4

Testing the Model Under New Assumptions

Height (*m*)	0	2	4	6	8	10	12
Time (*t*) Squared	0	0.36	0.81	1.21	1.69	1.96	2.25

Figure 5: Testing the Model Under New Assumptions



Refining the model: Part 2 (Extension)
A careful examination of the graph suggests that the time values are becoming too small for larger values of the height. If this is indeed a pattern that holds for larger values of H obtained in supplementary experiments, the model will need to be reconsidered in order to take this information into account. Perhaps the object is experiencing some force that tends to slow its downward motion, such as atmospheric drag. If this is the case, then larger values of H must be considered.

6. Validate the Model by Testing in Real-World Situations

One of the reasons for developing models is to help us predict events in the real world.

In order to see if our model is adequate, students could use the model to predict the time of fall of objects when dropped from various heights (e.g., tennis ball, basketball, baseball). Having calculated a value, they could record data using the real items, graph the results, and comment on the validity of the model.

Iterative Nature of Model Construction

Model construction is an iterative (repeating) process. The flowchart below shows the steps to be taken.

Figure 6: Modelling Flowchart

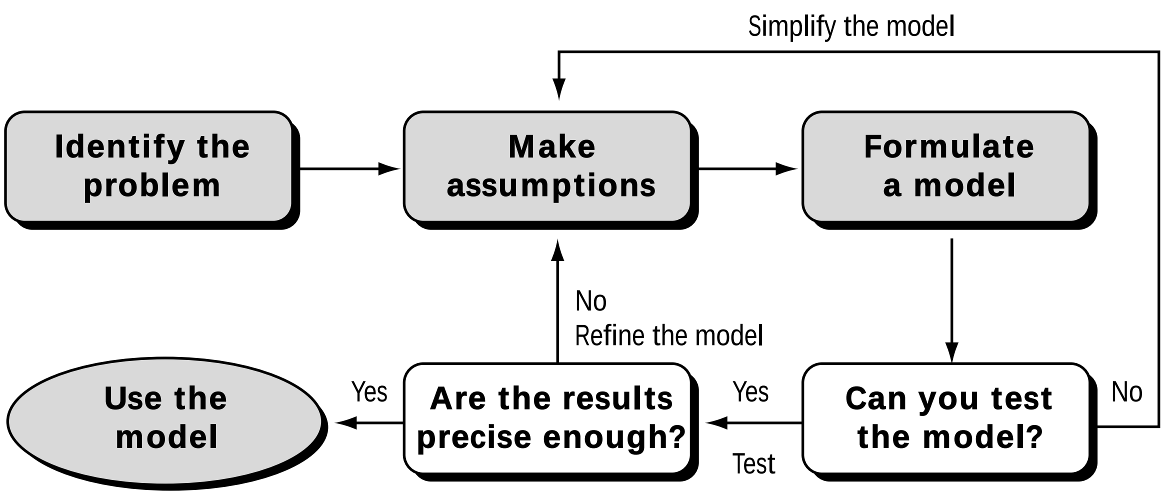


Table 5

Simplifying or Refining the Model as Required

Model Simplification	Model Refinement
• restrict problem identification	• expand the problem
• neglect variables	• consider additional variables
• combine the effects of several variables	• consider each variable in detail
• set some variables to be constant	• allow variation in the variables
• assume simple (linear) relationships	• consider nonlinear relationships
• incorporate more assumptions	• reduce the number of assumptions

Project A: Predicting Satellite Collisions

Project A: Predicting Satellite Collisions

Orientation

Time:

Phase I: Space Junk
Introduction and Review .... 1 class period
Research .... 2 class periods
Planning .... 1 class period
Phase II: Speakers
Gears .... 1 class period
Speakers .... 2 class periods
Beehives .... 2 class periods
Phase III: Iso- and Geosynchronous Orbits
Achilles and the Turtle .... 1 class period
Planets .... 2 class periods
Andromeda and the Liquid-Mirror Telescope .... 1 to 2 class periods
Big Bang .... 2 class periods
Autopsy of a Collision .... 1 + class periods

Instructional Strategies:

- Direct Instruction
- Case Study
- Group Work
- Issues Inquiry

Real-World Applications:

Some real-world applications of the skills learned in this project include:
- reading and understanding a scientific article
- projecting job trends, house prices, and business sales
- describing growth and decay of populations
- projecting the increased value of collectables like hockey cards, comic books, Beanie Babies, and Barbie dolls
- making good consumer decisions on the purchase of mountain bikes and stereo speakers
- using logarithmic scales like the Richter Scale
- using precision and accuracy
- optimizing the use of space

- packing camping gear more efficiently
- using exponents and appreciating their efficiency

Project Overview:

Figure 1 on the following page is a schematic to help you conceptualize Project A, Predicting Satellite Collisions.

Project Synopsis

In this project students learn more about the risk of collision between satellites, and between satellites and fragments of satellites or meteorites. The risk of collision has increased because of the dramatic increase of satellites placed into specific orbits. This issue is becoming an important problem that people should be prepared to deal with in the very near future.

Through reading a scientific article and doing research, students are introduced in a rigorous way to an real-world socio-technological issue. They are expected to read and gather information about the issue and to develop a critically-informed opinion about it. By doing this, they realize the importance of having a sufficiently developed mathematical and scientific background. In particular, they become aware that estimating the probability of a collision requires the extensive use of exponents and a working understanding of formulae involving rational numbers.

After reviewing the concepts related to the different representations of rational numbers, the concept development of the project begins with the exploration of the limitations of the set of rational numbers. By dealing with real-world situations, students are introduced to phenomena where continuity is involved. By modelling situations with formulae involving powers and by applying the laws of exponents, students expand their mathematical skills and abilities, improve their pattern representation skills, and deepen their understanding of an actual socio-technological issue.

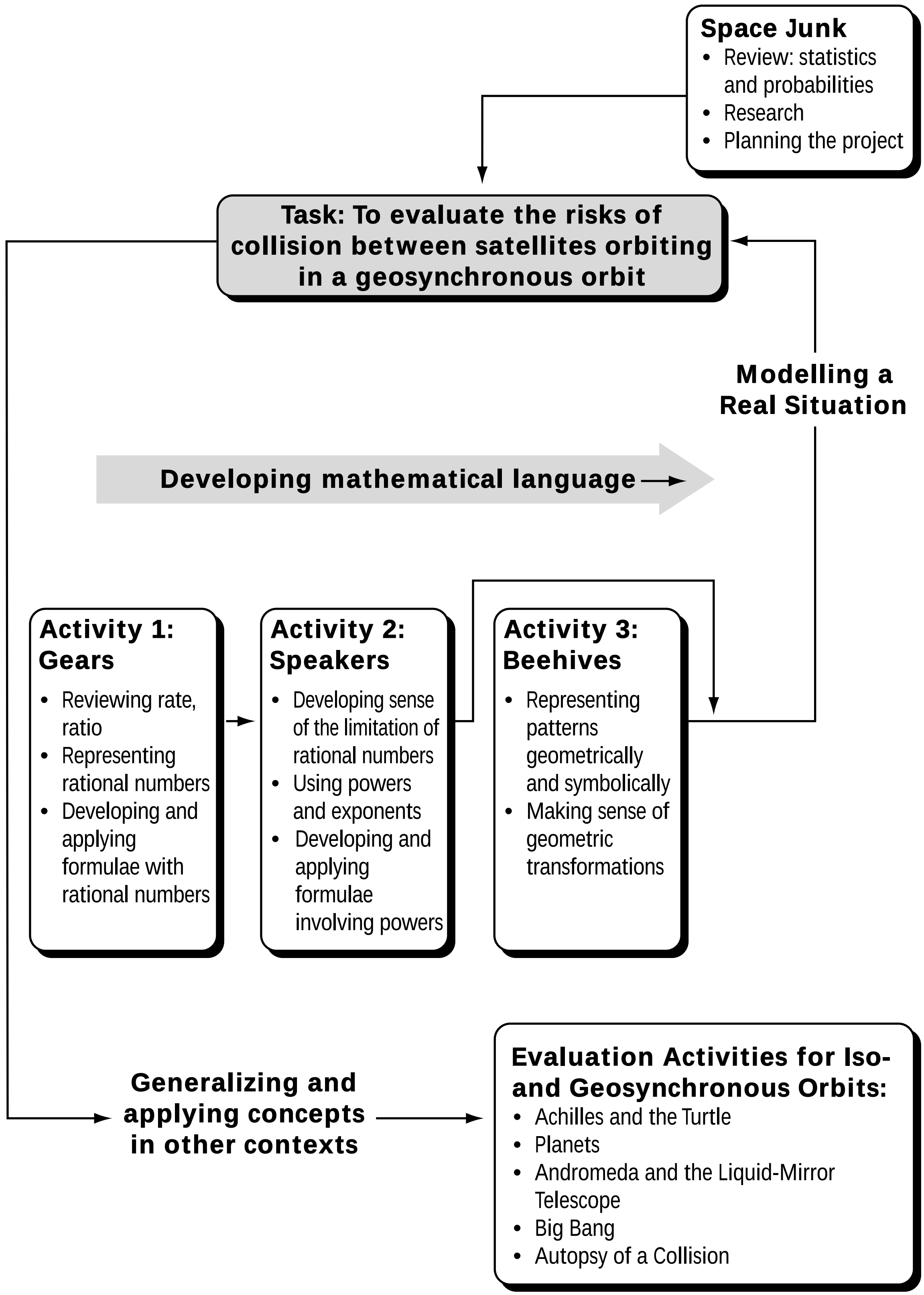
Phase Synopsis

The project is structured in the following way:

In Phase I: Space Junk, students read a scientific article in order to describe and analyse a situation. They then propose a plan of action based on their acquired knowledge.

In Phase II: Speakers, students model real situations with rational expressions and develop new skills and abilities needed to improve their model. By doing three activities they explore the limitations of rational numbers in order to deal with situations where continuity is involved. The activity sequence is designed so that students progressively realize the limitations of models using rational numbers.

Figure 1: Project A: Predicting Satellite Collisions



In Phase III: Iso- and Geosynchronous Orbits, students increase their awareness of the necessity of modelling situations with formulae involving powers to represent patterns. They refine their skills in manipulating algebraic expressions containing powers with integral components and rational bases as a simplification tool. In this phase, students focus on understanding the algebraic expressions rather than on developing manipulation skills. Students use calculators extensively to help them visualize, and are introduced to symbolic software in order to simplify algebraic expressions. In this phase students also propose a refined model by applying their new knowledge and identify further improvements that are needed to reflect more closely the actual data.

Phase I: Space Junk

Activity 1: Introduction and Review
Activity 2: Research
Activity 3: Planning

In this project students are asked to collect relevant data from as many sources as possible and to prepare a report on the cluttering of outer space. They are asked to present their findings in graphical form, and to discuss the possible consequences of this environmental problem.

There are two major objectives in Phase I. The first is to review Grade 8 concepts of number, pattern, statistics and probability. The second is to serve as a case study that helps most students develop new concepts and deepen their understanding of the problem. To assist them in their planning in Phase I, teachers may wish to consider the following options:

Option A:
Students are asked to prepare a report that includes the following:
- an introductory statement about the issue and an outline of the actual concerns;
- graphical presentations of the data that may include: population of satellites and debris versus size, population of satellites versus altitude and orbits, increase in the number of satellites over time;
- a discussion, illustrated by graphs, of the natural space cleaning processes; solar effects and return to earth; and
- a general conclusion.

Option B:
Students may also include:
- an outline of the mathematical and scientific concepts that would need to be developed in order to improve the model
- a graph illustrating a breakup and a collision
- a discussion of surveillance systems used today and their ability to predict collisions
- a flowchart or diagram showing different aspects of the problem

Space Junk is divided into three activities:

In Activity 1: Introduction and Review, the teacher introduces the project, indicates the importance of the issue, explains to students what is expected from them, and makes suggestions for the type of data that students need to collect.

The teacher suggests several sources of information for students.

This is also an opportunity for the teacher to informally assess whether students have the prerequisites to do this activity. Prerequisites include:
- knowledge and use of scientific notation
- knowledge of the concepts related to probability and statistical data
- skill at representing a set of data in graphical form in order to recognize patterns
- skill in using a graphing and/or a scientific calculator
- knowledge of force and energy (from the Science curriculum)

In Activity 2: Research, the teacher uses examples to demonstrate the advantages and disadvantages of using graphs. In particular, students are exposed to the limitations of extrapolating from a graph.

The teacher provides examples to emphasize the importance of using scientific notation and of modelling phenomena using formulae with rational numbers.

Students are expected to prepare their report from the information collected. The teacher helps students to:
- decide upon the kind of outcome they are seeking
- find the best way to plot their data on a graph in order to identify patterns; and
- determine the relative influence of various factors on the graphs

In Activity 3: Planning, students present their report to the class. The teacher establishes a sequential list of mathematical and scientific concepts that are helpful in understanding this particular issue as well as other similar issues.

At the conclusion of this first phase, the teacher presents the project plan and indicates the additional concepts that will be developed in Phase II: Speakers, and Phase III: Iso- and Geosynchronous Orbits.

Activity 1: Introduction and Review

Objective:

This activity has three main objectives. First, students will have a reasonable understanding of the key aspects of the issue. In particular, it is expected that they will be able to identify the factors involved in the increase in the amount of space debris and number of satellites. The second objective is the review of number concepts and operations acquired in Grade 8. The third objective is for students to gain confidence in their ability to read and comprehend scientific articles.

Materials:

- reprint of the article by Nicholas L. Johnson. “Monitoring and Controlling Debris in Space”, *Scientific American* (August 1998): 62-67.
- Student Activity: Space Junk Introduction and Review

Procedure:

Introduce the activity and ask students to read the Scientific American article. Assist them in gaining understanding by guiding discussion about key aspects.

Use the data presented in the article to:
- verify that students are able to use their calculators correctly, and
- review scientific notation, ratios and proportions, properties of the circle, projects of distance, mass, time, and velocity.

Have students answer the questions suggested in Student Activity: Space Junk Introduction and Review and complete the tables from the analysis of the 3-D graph in Figure 1.

Students may either produce a cardboard model of the 3-D graph or reproduce the graph using computer graphics.

Background Article:

> On June 3, 1996, the upper stage of a Pegasus rocket launched in 1994 broke up creating the largest debris cloud on record - more than 700 objects large enough to be tracked, in orbits from 250 to 2500 kilometers in altitude. This single event instantly doubled the official collision hazard to the Hubble Space Telescope that orbits just 25 km below. Further radar observations revealed an estimated 300 000 pieces of debris larger than four millimeters, big enough to damage most spacecraft. During the second Hubble servicing mission in February 1997, the space shuttle Discovery had to maneuver away from a piece of Pegasus debris that was projected to come within 1.5 kilometers. The astronauts also noticed pits on Hubble equipment and a hole in one of its antennas, that had apparently been caused by a collision with space debris before 1993. (Nicholas L. Johnson, “Monitoring and Controlling Debris in Space”, *Scientific American* [August 1998], 62-67).

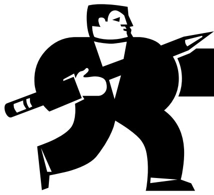
Additional Background Information:

Satellites revolve around the earth in specific orbits. The most overcrowded of these altitudes are 850 km, 1000 km, 1500 km, the semi-synchronous altitude of 20 000 km, and the geosynchronous altitude of 36 000 km in the plane of the equator. The lifetime of a satellite is quite short and no systematic and efficient way of disposing of the ‘dead’ satellites has yet been implemented.

Furthermore, satellites decay naturally, producing more and more debris. This debris collides and creates more debris. Even microscopic debris like paint flakes of 0.1 mm or less poses a risk of a dramatic collision. For example, the solar panels of the Hubble space telescope are punctured by paint flakes and solid-rocket motor slag. When this happens, the efficiency of the panels is greatly reduced. All of these factors contribute to increased risk of a serious collision between an active satellite and debris.

Currently, the probability that an average size satellite (10 m^2) will be destroyed is 1/50 000 per year and is increasing exponentially (in 1995, the probability was 1/100 000). This probability depends upon the density of satellites at a particular altitude. Actually, one satellite orbiting between 800 km and 1000 km is expected to be destroyed within the next 10 years.

The risk of collision is taken very seriously by those people responsible for the construction of the International Space Station. A piece of microscopic debris could be fatal to an astronaut working outside the station.

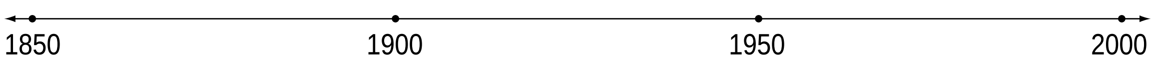


Student Activity: Space Junk Introduction and Review

Student Name: ____________________________ Date: ___________________

After reading the article by Nicholas L. Johnson. “Monitoring and Controlling Debris in Space”, *Scientific American* (August 1998): 62-67. Complete the following time line by indicating the number of identifiable objects in space at particular points in time.

Figure 2: Exploration of Space



Approximately 9000 objects (10 cm or more in length) are orbiting the earth. About 550 of these are active satellites. Fifty-four of these active satellites are equipped with electronuclear reactors. Electronuclear reactors contain a nuclear carburant, uranium 235 for Russian satellites, or plutonium 238 for North American satellites. This nuclear carburant has a mass of 1.45 tons.

Using the above information from the article, estimate the following:

- the ratio of active satellites to the total number of space objects
- the ratio of nuclear satellites to the total number of satellites
- the average mass of an orbiting object in kilograms
- the average mass of an active satellite in kilograms
- the average mass of nuclear carburant per nuclear satellite in kilograms
- the closest orbit to earth in kilometres
- the altitude of the geosynchronous orbit in kilometres

Student Activity: Space Junk Introduction and Review

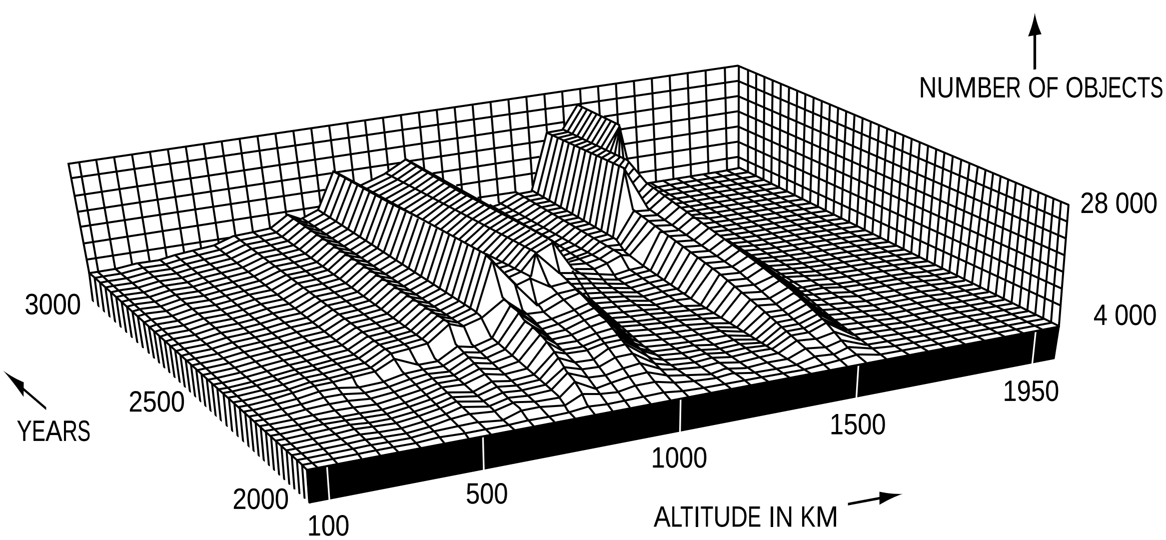
- the velocity of a satellite orbiting at 1000 km

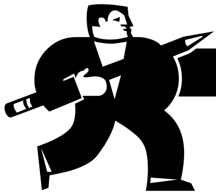
- the velocity of a satellite orbiting at 2000 km

- the velocity of a satellite orbiting at 36 000 km

Using information from Figure 3: Number of Objects in Space, complete Tables 1 to 4 and the comment areas for Tables 3 and 4.

Figure 3: Number of Objects in Space





Student Activity: Space Junk Introduction and Review

Table 1

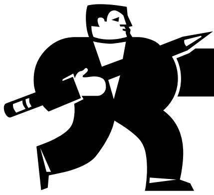
Space Junk in 1998

Altitude (km)	Number of Objects	Mass of Objects (tons)
100		
500		
750		
1000		
1250		
1500		
1750		
2000		
35 785		

Table 2

Space Junk Projections for 2050

Altitude (km)	Number of Objects	Mass of Objects (tons)
100		
500		
750		
1000		
1250		
1500		
1750		
2000		
35 785		



Student Activity: Space Junk Introduction and Review

Table 3

Increased Density of Space Junk 1997 to 2050

Altitude (km)	% Increase in Number of Objects	% Increase in the Number of Tons
100		
500		
750		
1000		
1250		
1500		
1750		
2000		
35 785		

Comments:

Table 4

Projections of Space Junk for the Year 2100

Altitude (km)	Number of Objects	Mass of Objects (tons)
100		
500		
750		
1000		
1250		
1500		
1750		
2000		
35 785		

Comments:

Activity 2: Research

Objective:

The objective of this activity is to help students be aware of the need to increase their knowledge and skills so that they can correctly interpret graphical representations of real-world data.

Materials:

- reprint of the article by Nicholas L. Johnson. "Monitoring and Controlling Debris in Space", *Scientific American* (August 1998): 62-67.
- access to Internet
- graphing calculator and computer with graphics software
- Student Activity: Planning the Research

Procedure: Part 1

Help students in their second reading of the *Scientific American* article and have them interpret the graphs.

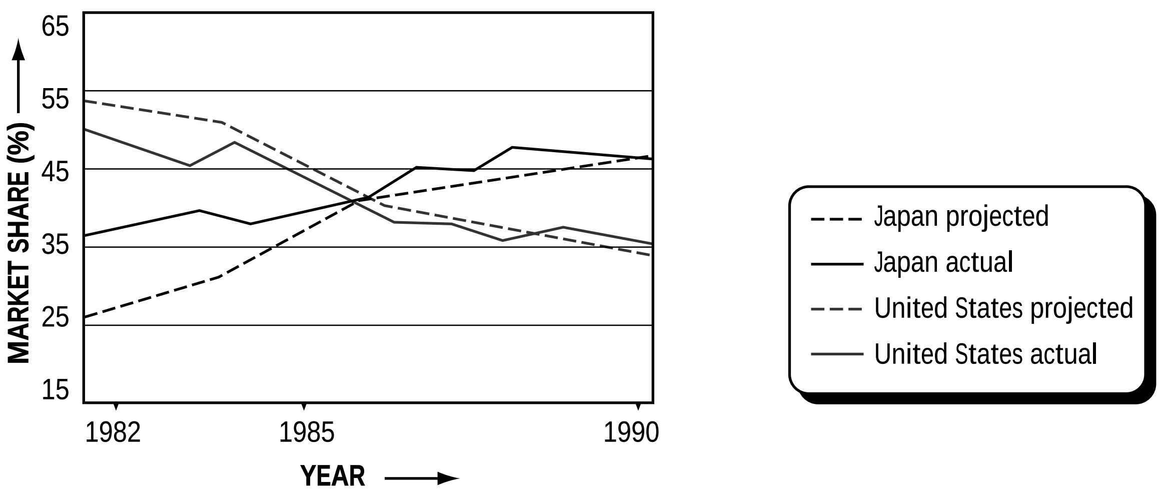
Tell students that extrapolating from graphs requires both a good knowledge of the various factors involved in a situation and on a good understanding of the relationships between and among the variables of a problem situation.

Introduce and comment upon the following situation:

> In 1989, a team of researchers at M.I.T. (Massachusetts Institute of Technology) prepared a report on Industrial Productivity. They examined the reasons American dominance of the high-technology industry was experiencing a serious competitive decline. Problems identified ranged from poor product quality to excessively long product cycles. The researchers presented the following graphical representation of the situation and projected the future of the industry to the year 2000.

Present an overhead of the graph, Figure 4: Market Share of Global Semiconductor Industry for Japan and the United States, 1982-1989. Have students use appropriate vocabulary to comment on the shape of the two lines: Japan Projected and United States Projected.

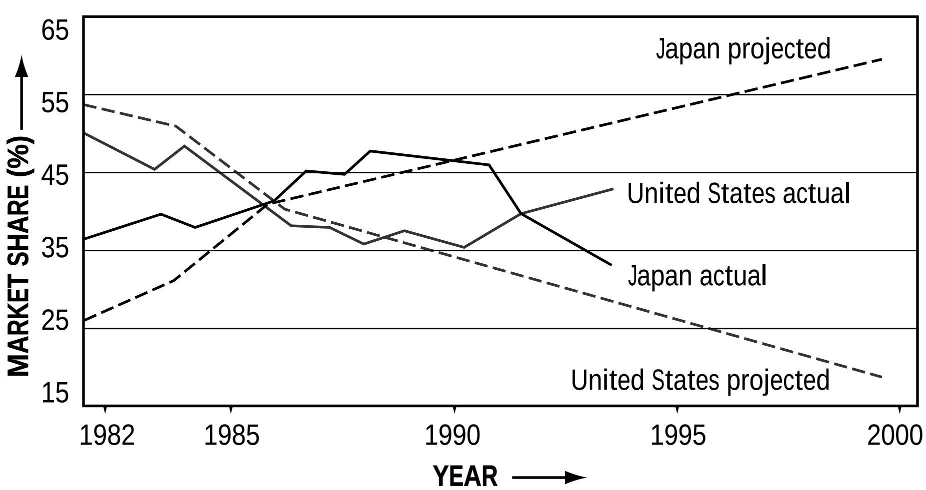
Figure 4: Market Share of Global Semiconductor Industry for Japan and the United States, 1982-1989



Adapted from: R.K. Lester, *The Productive Edge*: *How US Industries are Pointing the Way to a New Era of Economic Growth*, New York: W.W. Norton and Company, 1998.

Present an overhead of the graph, (Figure 5: Market Share of Global Semiconductor Industry for Japan and the United States) representing the actual situation in 1998 and superimpose this on an overhead of the previous graph (Figure 4: Market Share of Global Semiconductor Industry for Japan and the United States., 1982-1989).

Figure 5: Market Share of Global Semiconductor Industry for Japan and the United States



Adapted from: R.K. Lester, *The Productive Edge*: *How US Industries are Pointing the Way to a New Era of Economic Growth*, New York: W.W. Norton and Company, 1998.

Have students use general terms to comment upon the discrepancy between the projected situation in Figure 4 and the actual situation in Figure 5.

Ask students to identify some of the factors that were not considered in the projection but have since been revealed to be of crucial importance. The turning point for this is in 1991.

Procedure: Part 2

Use the previous example to stress that it is necessary to continue to identify all of the possible factors affecting the evolution of a situation and to determine how these factors are related. Use the flowchart of Modelling Situations as a guide to conducting this discussion.

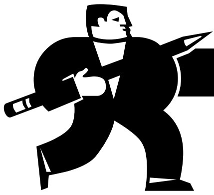
Have students identify the factors involved in the present situation and have them discuss the possibility of relationships among orbits, mass, energy and velocity. Demonstrate the revolution of an object about a fixed point in space, attached with different lengths of rope to relate the length to the velocity.

Ask students to recall and discuss the different types of satellites, their number, and their different orbits.

Inform students about the construction of the International Space Station and the risks of an accident. The Discovery Channel website provides additional information about the International Space Station.

Have students complete Student Activity: Planning the Research.

Ask students to discuss their projections from Figure 6: Estimate of Earth's Artificial Satellites in 1998 to Figure 7: Projection of Earth's Artificial Satellites and Debris for Year 2025. Then compare these with the results shown on the 3-D graph of Activity 1, Space Junk - Introduction and Review.



Student Activity: Planning the Research

Student Name: ______________________ Date: ______________

Complete the following:

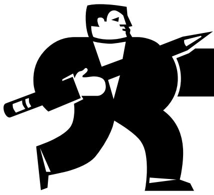
What are some reasons it is important to be able to monitor the amount of debris in Space?

What are some reasons it is important to be able to control the amount of debris in Space?

If you were in charge of a team having to propose solutions to the space junk problem in outer space, list and briefly describe some specific kinds of information you would need.

What are some of the ways that you could obtain this information?

What are some of the ways you would like to see this information presented? Why?



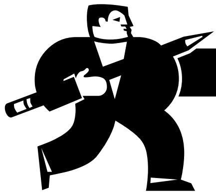
Student Activity: Planning the Research

Complete Table 5: Information.

Table 5

Information

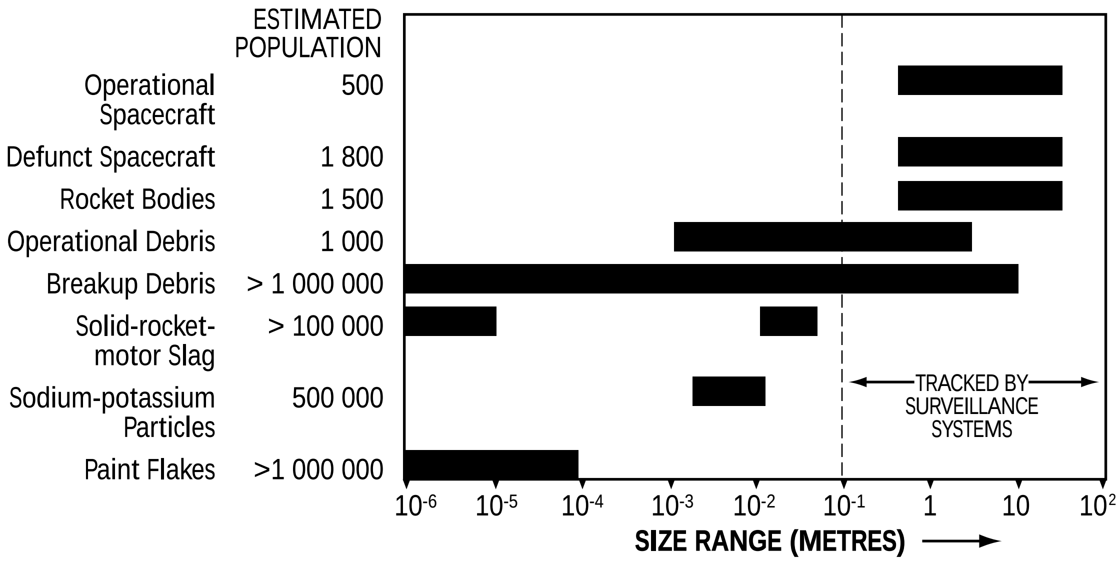
Kinds of Information to be Collected	Methods for Collecting the Information	How the Information will be Used

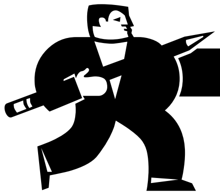


Student Activity: Planning the Research

The following bar graph (Figure 6: Estimate of Earth's Artificial Satellites in 1998) represents the 1998 estimated population of space craft and debris according to size.

Figure 6: Estimate of Earth's Artificial Satellites in 1998

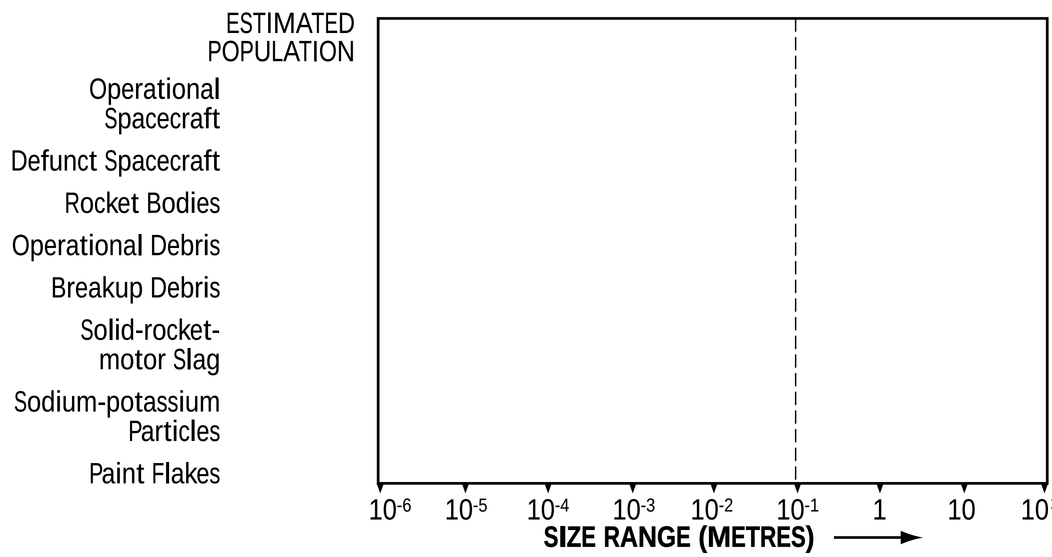




Student Activity: Planning the Research

Create a similar bar graph representing the situation in the year 2025. List and describe the factors you considered in creating your graph.

Figure 7: Projection of Earth's Artificial Satellites and Debris for Year 2025



Provide your reasons for each population estimate.

Operational Spacecraft

Defunct Spacecraft

Rocket Bodies

Operational Debris

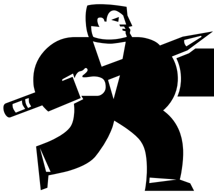
Breakup Debris

Solid-rocket-motor Slag

Sodium-potassium Particles

Paint Flakes

Comments:



Student Activity: Planning the Research

List and briefly describe all of the factors that you think are important to consider.

In Table 6: Risk Factors, regroup the factors according to whether they increase or decrease the risk of collisions.

Table 6

Risk Factors

Factors that Increase the Risks	Factors that Decrease the Risks

Activity 3: Planning

Objective:

Under the teacher's guidance, students prepare a flowchart for their Project Action Plan by completing the Self Evaluation Checklist and the Questions to Guide Your Project Planning.

Materials:

- Student Activity: Planning the Research
- graphing calculator
- computer with graphics software

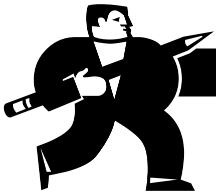
Procedure:

Ask students to describe the issue verbally. Provide guidance to them so that they come to realize that there are two aspects to the problem:
- an increase in the number of satellites and consequently an increase in the number of tons of debris, and
- a cleaning process whereby there is a natural return to earth of some debris due to the action of the sun.

Ask students to propose a method to determine the probability of a collision between debris and an operating satellite, spacecraft or Space Station. The method proposed should involve the concept of density of an orbit.

Ask students to develop a Project Action Plan Flowchart. In order to complete their Project Action Plan Flowchart, students are asked to identify the mathematical and scientific concepts they need to explore, and to describe their expectations regarding the application of these concepts in the present context. The creation of the flowchart should follow a brainstorming session about all of the factors involved and how these factors are related.

The teacher presents the plan and establishes its parallel with the flowcharts suggested by the students.



Student Activity: Planning the Research

Student Name: ___________________________ Date: __________________

1. Self Evaluation Checklist

Rate yourself on each of the following. Circle the number that corresponds with how well you are satisfied with your performance in each area. Put a check mark in front of areas in which you would like more help.

Key: 1-Not satisfied, 2-Need some help, ... 5-Very satisfied

Help?	Areas	Rating
❑	using negative exponents	1 2 3 4 5
❑	representing a number in scientific notation	1 2 3 4 5
❑	expressing a ratio in different ways	1 2 3 4 5
❑	representing and applying percents in fractional or decimal form	1 2 3 4 5
	performing operations with rational numbers in	
❑	fractional form	1 2 3 4 5
❑	decimal form	1 2 3 4 5
❑	using rates, ratios, proportions to solve problems	1 2 3 4 5
❑	using patterns to analyse and solve a problem	1 2 3 4 5
	representing patterns in the form of	
❑	graphs	1 2 3 4 5
❑	equations	1 2 3 4 5

2. Questions to Guide Your Project Planning

Clearly identify and describe the problem in writing. Be specific.

Phase II: Speakers

Phase Synopsis

The intent of this phase is to review and to reinforce the students' abilities regarding the two representations of the rational numbers - fractions and decimals - and to explore the limitations of the rational numbers. Students realize that, in some cases, reality is modelled with numbers that are not expressible in fractional form. This is the case with the spectrum of musical frequencies and with the limit polygons inscribed in a circle. In each case, students are exposed to a continuous situation that can be approached by successive approximations with a calculator or a computer. Students realize that numerical approximations can be made as close as they wish and that these are accurate enough for most applications.

Through a series of five activities, students practise their skills in order to model real situations in a variety of familiar contexts. To do this, they take advantage of their ability to recognize and represent relations between the different factors they have identified as playing a significant role in the situation.

In Activity 1: Gears, students are provided with an opportunity for review and introduction of modelling with formulae involving rational numbers. Students determine the gear ratios of a mountain bike and create a cardboard model of the gear mechanisms of a clock (Big Ben).

In Activity 2: Speakers, students learn about modelling with rational numbers and are introduced to the limitations of the set of rational numbers. Students compare different speakers by understanding frequency response curves provided by stereo speaker manufacturers. Using this information, students establish the mathematical models that will permit them to determine the frequency of a musical note from a discrete spectrum to a continuous spectrum. By re-scaling the intensity of a sound into decibels, they also explore the powers of 10.

Activity 3: Beehives allows students to evaluate the concepts covered in this phase. Students construct cardboard models of a beehive to help them evaluate their understanding of rational numbers.

Activity 1: Gears

Objective:

There are two major objectives in Activity 1. The first is a review of rate, ratio and proportion concepts as well as a review and reinforcement of pattern recognition skills. The second objective is an increased understanding of rational numbers, their representation in decimal and fraction forms, and their usefulness in real-world situations.

Materials:

- bicycle (10 , 15 or 18 speed)
- hooks
- rope
- chalk
- measuring tape
- cardboard
- scissors
- tape
- pins
- geometry kit
- Student Activity: Gears

Activity 1.1: Spur Gears

In this activity, students establish the relationship between the gear ratios of a bicycle and the distance corresponding to a complete revolution of the pedals. Determining this information helps them optimize their gear changing when operating their bicycles.

Procedure:

Have students measure the radius of the back wheel and the diameter (or radius) of each of the spur gears. A carpenter’s compass may be used for this.

If the weather permits and if space is available, have students measure the distance travelled on the ground during 10 complete revolutions of the pedals for each gear combination.

If the weather does not permit doing the activity outdoors, have students suspend the bicycle and measure the number of turns of the back wheel corresponding to a complete turn (360°) of the pedals for each of the 18 gear combinations. Make sure that the students hold the back wheel so that there is no free rotation during the measurements. Have students make a chalk mark on the tire so that they always start at the same place. Have students pedal backwards to position the pedal vertically before starting each new measurement.

To increase precision, students may measure the angles and distances of additional revolutions of the pedals and then divide by the number of revolutions. That is, three complete revolutions of the pedal gives five and one-third turns of the back wheel. This means that one revolution corresponds to 16/9 turns.

When they finish this task, have them complete the tables in Student Activity: Gears.

Ask students to develop a rule about the relationship between the chain ring diameters and the number of teeth (spurs) on each chain ring. To do this, they will need to do the following:

- examine all of the chain rings on the rear cog set;
- measure the diameter of each chain ring;
- measure the distance between the teeth on each chain ring;
- count the number of teeth on each chain ring; and
- determine the rule.

Establish a rule for the relationship between the diameter ratios and the ratios of the number of teeth.

Have students describe the relationship between the different ratios and the effort needed. Ask questions such as "How many more times do you need to pedal to go the same distance with different gear ratios?" In order to calculate the number of complete revolutions they need to pedal for a given distance, for example 150 meters, they may utilize Table 12: Gear Turns and Distances.

Have students develop a rule giving the number of complete pedal revolutions needed to go a given distance for a given gear ratio.

Activity 1.2: Angular Velocity

In this activity students determine the relationship between the gear radii and the angular velocity of a following gear in order to understand the mechanisms of a clock or a watch.

Have students construct cardboard models of spur gears like the ones illustrated in Figure 8 : Angular Velocity. Make sure that the dimensions (width and spacing) of the teeth are all the same. Each group of students constructs sets of gears of different radii.

Help students determine the number of complete turns that the gear rotates per second, for example, 1/3 turn per second. This is the angular velocity. Have them express angular velocity in terms of p per second, for example, 1p per sec, 1/4 p per sec, and revolutions per second or per minute, that is, r.p.s. or r.p.m.

Have students attach two gears using a piece of cardboard acting as an arm (see Figure 9: Rotating Gears).

Holding the larger gear in place, students rotate the arm with some angular velocity. If they rotate the arm too fast, the smaller gear will slip. Ask students to determine the angular velocity of the small gear. They estimate the answer, for example twice as fast, and determine the precise answer using a stopwatch.

In order to determine how fast the small gear is rotating (that is its angular velocity), ask students to complete Table 13: Velocities and then to develop a formula by trial and error. Encourage groups of students to collect results from other groups.

The answer is:
v_a is the angular velocity of the arm
v_s is the angular velocity of the small gear
R_L is the radius of the large gear
R_s is the radius of the small gear

the angular velocities are related by:

$v_S / v_a = (R_S + R_L)/ R_S$

Figure 8: Angular Velocity

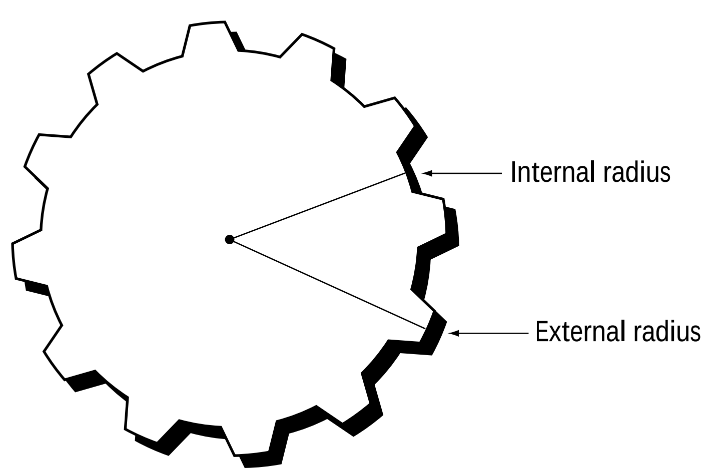
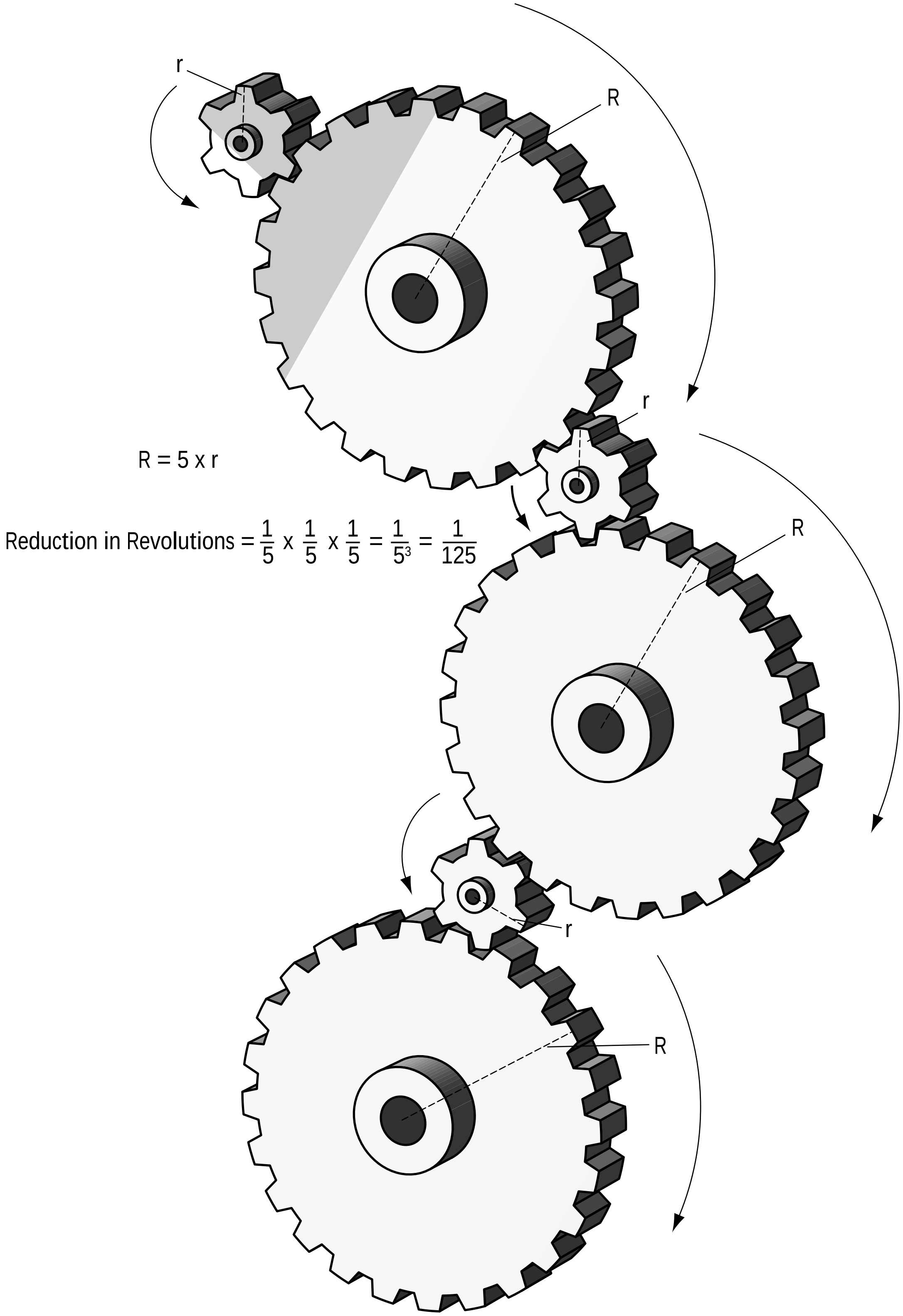


Figure 9: Rotating Gears



Challenge 1: Drive Shaft

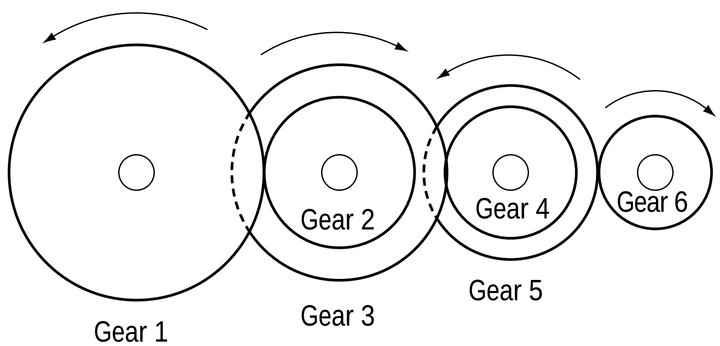
Two parallel shafts are centered 30 cm apart. If the drive shaft makes 80 r.p.m., find the diameters of wheels that work by rolling contact so that the driving shaft makes 240 r.p.m.

Apply this information about wheel diameters to determine the radii of a sequence of 5 gears needed to reduce the angular velocity by a factor of 100. For example, an electric motor has a r.p.m. of 1200 and one needs to rotate an arm with an angular velocity of 12 revolutions per minute or 1/5 of a turn per second.

Challenge 2: Big Ben

Old clocks and watches were built by assembling a series of spur gears to form a certain train. This activity demonstrates how this works. Students may choose to carry out research on Big Ben in order to appreciate the precision of the mechanism of this famous clock.

Figure 10: Spur Gears



Six spur gears are mounted upon parallel shafts. They engage as shown in Figure 10: Spur Gears. The number of teeth is as follows:

Gear #1: 84
Gear #2: 72
Gear #3: 48
Gear #4: 52
Gear #5: 36
Gear #6: 24

Find the angular velocity ratios v_2/v_1, v_3/v_1, v_4/v_1, v_5/v_1 and v_6/v_1

The last ratio, v_6/v_1 , is called the value of the train. It is equal to the product of the number of teeth in all the drivers, divided by the product of the number of teeth in all the followers.

What are the results if some of the intermediate shafts have one gear instead of two, and if one gear acts as both a driver and a follower?

Project Link:

Suppose that the orbit of the earth around the Sun is a circle whose radius is the average distance of the aphelion and the perihelion (see Tables 7, 8, 9, and 10: Physical Data for Planets).

Determine the average distance of the aphelion and the perihelion.

Determine the Earth's angular velocity and express it in different units.

Determine the velocity of the earth in orbit in km/sec.

In astronomy, physical data are often expressed in 'Earth Units', that is, by dividing the physical data of a planet by the corresponding data of the earth.

Express the following in Earth Units:

- the radii of the planets
- the periods of revolution in earth years
- the angular velocity in Earth Units.

Table 7

Physical Data for the Planets #1

Body	Radius (km)	Mass (kg)	Average Density (g/cc)	Semi-Major Axis of Revolution (km)	Aphelion (km)	Perihelion (km)
Sun	695950	1.991 E30	1.410			
Mercury	2433	3.181 E23	5.431	5.795 E7	6.986 E7	4.604 E7
Venus	6053	4.883 E24	5.226	1.081 E8	1.0885 E8	1.9737 E8
Earth	6371	5.979 E24	5.519	1.4957 E8	1.5207 E8	1.4707 E8
Moon	1738	7.354 E22	3.342	3.844 E5	4.055 E5	3.633 E5
Mars	3380	6.418 E23	3.907	2.2784 E8	2.4912 E8	2.066 E8
Jupiter	69758	1.901 E27	1.337	7.7814 E8	8.1580 E8	7.405 E8
Saturn	58219	5.684 E26	0.688	1.4270 E9	1.5045 E9	1.3495 E9
Uranus	23470	8.682 E25	1.603	2.8703 E9	3.002 E9	2.738 E9
Neptune	22716	1.027 E26	2.272	4.4999 E9	4.537 E9	4.463 E9
Pluto	5700	1.08 E24	1.65	5.909 E9	7.375 E9	4.443 E9
Icarus	0.53	5.0 E12	8.1	1.613 E8	2.949 E8	2.77 E7

Note: Icarus is asteroid # 1566

The exponents for large numbers are expressed as in the following example:
1.991 E30
is equivalent to
1.991×10^{30}

Table 8

Physical Data For the Planets #2

Body	Sidereal Period of Revolution (sec)	Equatorial Rotation Period (sec)	Average Solar Day (sec)	Rotational Velocity at Equator (km/sec)	Altitude of an Isosynchronous Satellite (km)
Sun		2.125 E6		0.6085	24054600
Mercury	7.602 E6	5.068 E6	1.5204 E7	0.003	237298
Venus	1.941 E7	R 2.107 E7	R 1.01 E7	0.0018	1534693
Earth	3.156 E7	8.616 E4	8.64 E4	0.4651	35767
Moon	2.361 E6	2.361 E6	2.551 E6	0.0046	86673
Mars	5.936 E7	8.864 E4	8.878 E4	0.2400	17049
Jupiter	3.743 E8	3.543 E4	3.543 E4	12.6568	87688
Saturn	9.296 E8	3.684 E4	3.641 E4	10.296	48797
Uranus	2.651 E9	R 3.881 E4	R 3.881 E4	3.8930	36370
Neptune	5.200 E9	5.665 E4	5.665 E4	2.5194	59506
Pluto	7.837 E9	5.523 E5	5.523 E5	0.0648	76510
Icarus	3.5338 E7	9.72 E4	9.72 E4	0.0000453	264

Note: The letter ‘R’ indicates a retrograde or opposite motion to that of the Sun/Earth system

Table 9

Physical Data for the Planets #3

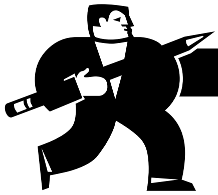
Body	Gravity (Mean) (cm/sec^2)	Gravity (Polar) (cm/sec^2)	Gravity (Equator) (cm/sec^2)	Escape Velocity (km/sec)	Orbital Velocity Perihelion (km/sec)	Orbital Velocity Aphelion (km/sec)
Sun	27372.0	27370.0	27369.0	617.23		
Mercury	357.8	357.8	357.8	4.1725	58.92	38.82
Venus	887.4	887.4	887.4	10.365	35.256	34.780
Earth	980.7	985.1	975.0	11.179	30.272	29.278
Moon	162.0	162.2	162.0	2.3735	1.075	0.964
Mars	374.0	376.5	355.7	5.0282	26.490	21.964
Jupiter	2601.0	2850.0	2260.0	60.238	13.700	12.435
Saturn	1117.0	1291.0	863.0	36.056	10.177	9.129
Uranus	1049.0	1156.0	937.0	22.194	7.116	6.490
Neptune	1325.0	1325.0	1297.0	24.536	5.472	5.383
Pluto	221.0	221.0	221.0	5.023	6.102	3.676
Icarus	0.120	0.357	0.069	0.00112	93.458	8.794

Table 10

Isosynchronous Satellite Data

Body	Altitude (km)	Orbital Velocity (km/sec)	Escape Velocity from (km/sec)	Solar Escape from (km/sec)
Sun	24055600	73.19	103.5	
Mercury	237301	0.297	0.420	67.64
Venus	1534720	0.459	0.650	49.53
Earth	35775	3.073	4.346	42.33
Moon*	86667	0.235	0.333	1.48
Mars	17032	1.447	2.046	34.18
Jupiter	89304	28.21	39.89	43.96
Saturn	50964	18.62	26.33	29.65
Uranus	36938	9.780	13.83	16.84
Neptune	59504	9.119	12.82	15.01
Pluto	76470	0.935	1.322	6.828
Icarus	264	?	?	?

Moon*: Escape from Earth



Student Activity: Gears

Student Name: ____________________________ Date: ___________________

Complete the following tables

Table 11

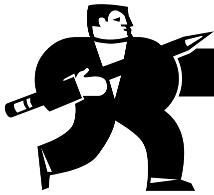
Dimensions

	Diameter (cm)	Number of Spurs
Front #1		
Front #2		
Front #3		
Back #1		
Back #2		
Back #3		
Back #4		
Back # 5		

Table 12

Gear Turns and Distances

Front	Back	Diameter Ratio	Spur Ratio	# Turns Back Wheel	Total Angle	Distance	Comments



Student Activity: Gears

Table 13

Velocities

Group ID	r_L	r_S	$(r_L + r_S)\ r_S$	v_a	estimated v_S	v_S

Formula :

Questions:

What is the angular velocity if the two gears have the same radius?

Are you surprised by this result? Draw a figure or roll a dime around a loonie to convince yourself.

What is the angular velocity if the radius of the small gear is half that of the larger gear?

What is the angular velocity if the radius of the small gear is five times smaller than that of the larger gear?

What is the angular velocity if the radius of the small gear is ten times smaller than that of the larger gear?

What is the rule?

What are some ways that you can modify the formula when you know the ratio of the two radii of the gears?

Activity 2: Speakers

Objective:

Students analyse the specifications of various speakers based upon the Frequency Response Curves information provided in manufacturers' brochures and booklets. Students perform a cost-benefit analysis based on their budget and their own musical requirements. At the end of this activity, students should be able to compare the features of different speakers.

Students apply concepts related to rational numbers and powers. They then apply formulae involving rational numbers and powers, and use mathematical models of physical situations.

Materials:

- manufacturers' information booklets on various types of stereo speakers
- metal strings of different lengths and compositions
- keyboard, piano, or other string instrument
- Student Activity: Speakers
- Student Activity: Range of Emitted and Perceived Sounds

Activity 2.1: Frequency Response Curve

When purchasing a set of stereo speakers, it is important to be able to identify the most important factors. In this activity students analyse the specifications and the frequency response curves relating the intensity of sound to the frequencies of speakers.

Procedure:

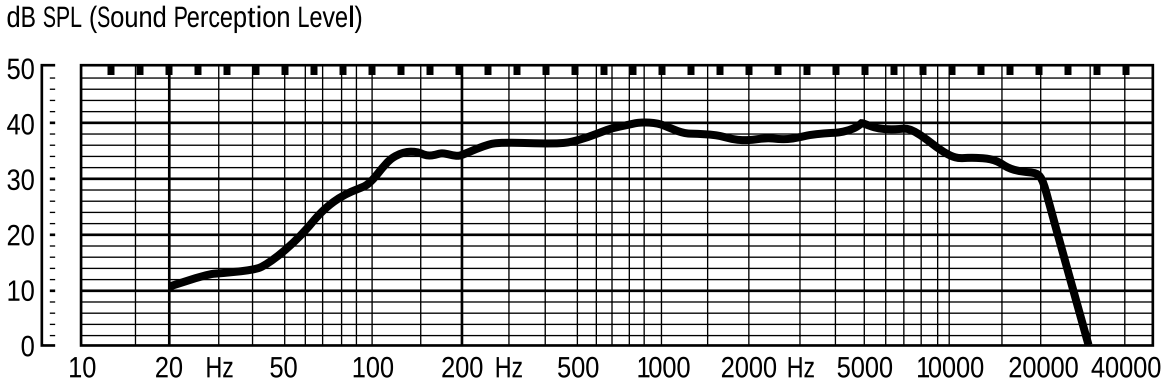
As preparation for this activity, ask students to go to music stores to obtain information about the various types of speakers on the market. The students should collect pamphlets that include information about the Frequency Response Curves (FRC) of the speakers, maximum power, power handling, frequency response range, and price.

You may invite a music store representative to make a brief presentation to the class about speakers and FRCs.

Select a typical FRC (Figure 11: Typical Frequency Response Curve) and project this onto an overhead screen. Ask students to comment on the features of the FRC graph by answering the following questions:

- What do the symbols dB and Hz mean?
- Why is the frequency scale not a linear one?
- Why is there a plateau between two frequencies?
- Why does the intensity drop rapidly after a certain frequency?
- Why is the intensity zero for some frequencies?
- Why do all the scales start at the same 20 Hz frequency?

Figure 11: Typical Frequency Response Curve



Have students complete a list of the features for all of the speakers they analysed (see Table 15: Speaker Specifications).

Tell students that they are going to learn more about these so that they can be better consumers when they choose to buy a set of speakers. To become better consumers, they need a better working knowledge of rational numbers and powers.

Present students with the two charts of Student Activity: Range of Sound and have them comment upon the charts.

Have students brainstorm different ways to produce musical sounds. Sounds are produced by vibrating strings, air forced through a pipe, vibrating membranes, and vocally. Give examples of musical instruments. Remind students that sound is characterized by a frequency. Help students define the frequency as the inverse of the period, that is, the time needed for a complete vibration. Invite students to estimate the period of a vibrating string and, then, to estimate the frequency of the vibrating string.

Present students with the frequency units (reciprocal sec, Hz, number of vibrations per second). Referring to the sound range charts in Student Activity: Range of Emitted and Perceived Sounds, ask questions such as:

- What is the range of periods of the sound emitted by humans?
- Are the sounds of cats of a lower or higher frequency than those made by dogs?
- What advantage would there be to a killer whale having a hearing range greater than its sound emission range? How many times greater is that range?

Activity 2.2: Vibrating Strings: The Violin

In this activity, students establish the relationship between the frequency of a vibrating string and the tuning of musical instruments. The frequency is derived from observations and experiments. Students also identify three characteristics of sound - intensity, pitch, and tone.

Procedure:

Before providing students with the formula for the frequency of a vibrating string (violin, piano,), ask them to identify the variables involved. They should mention the length, tension, diameter and the material of the string.

Tell students that rather than considering the nature and diameter of the string, it is preferable to characterize the string by its linear density. Linear density is the weight of a 1 metre length of string. For example, if 1 metre of string weighs 50 grams or 0.05 kilograms, the linear density would be 0.05 kilograms/metre.

Have students experiment with strings of different lengths and linear densities in order to determine how different variables affect the frequency. It is easier for students to estimate the period instead of the frequency and then to take the reciprocal.

Have students complete the qualitative table below (Table 14: The Effects of Different Variables on Frequency). Explain that the up and down arrows are qualitative measures and simply indicate increase or decrease in a variable, and a double arrow indicates a greater increase than a single arrow. For example, in line 1, we increase length and period, and we keep tension and linear density constant. This results in a frequency decrease.

Table 14

The Effects of Different Variables on Frequency

Length (*L*)	Tension (*T*)	Linear Density (*D*)	Period (*P*)	Frequency (*F*)
↑	constant	constant	↑	↓
constant	↑↑	constant	↓	↑
constant		constant		
constant	constant	↑		
↓	constant	constant		
↓↓	constant	constant		
constant	↓↓	constant		
constant	constant	↓↓		
constant	↓	↑↑		

Give students the formula for frequency (n)

$$n = k \ \frac{1}{2L}\sqrt{\frac{T}{D}}$$

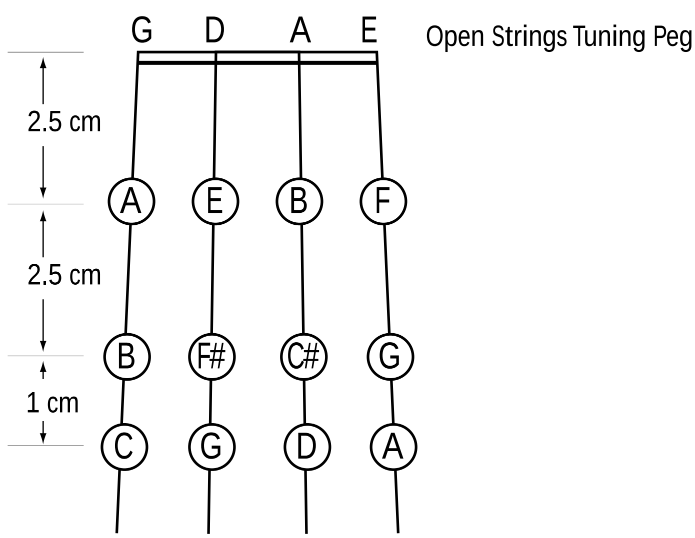
where T is the tension in Newton (kgm/s^2, D is the linear density in kg/m and L is the length in m, k is the number of Hz partially used (usually k = 1).

Have students verify that the units are correct by replacing each variable by its own unit. The frequency should then be expressed in Hz or reciprocal seconds (s^{-1}).

Have students apply this formula to a violin like the one illustrated in Figure 12: Open Tuning.

The length of the four strings between the tuning peg and the bridge is 26 cm.

Figure 12: Open Tuning



Ask questions such as:

The lowest note that can be obtained is the C_1 note (approximately 16 Hz) and the highest is the C_9 (approximately 16 700 Hz). The only instrument having this range is the organ.

- Can the human ear perceive those two notes?
- Can a dog perceive those two notes?
- Can a killer whale perceive those two notes??

How is the frequency changed when:

- pressing one string with the first, the second and the third finger?
- the tension of a string is decreased?
- the tension of a string is increased?
- the tension in a string decreases from 25 kg/mm^2 to 16 kg/mm^2? (answer: the second frequency will be 4/5 of the first)

Activity 2.3: Rational and Irrational Numbers

In this activity, students develop a formula to evaluate musical frequencies from a standard frequency used in orchestras to tune all the instruments. They are exposed to the limitations of the rational numbers.

Procedure:

For historical reasons, the musical sounds have been grouped into notes and scales. Each scale is divided into 12 notes (8 + 4):

C or B#
C# or D b
D
D# or E b
E or F b
F or E#
F# or G b
G
G# or A b
A
A# or B b
B or C b

Musicians use 10 successive scales (octaves) that is, a total of 121 notes covering the entire range of human audible sound from the extreme low to the extreme high. The lowest note C_1 has a frequency of approximately 16 Hz and the highest note C_9 has a frequency of approximately 16 700 Hz.

In two consecutive octaves, one note has a frequency that is exactly double the corresponding note of the previous octave. For example, in the second octave the frequency of the C_2 note would be approximately 16 x 2 = 32 Hz, in the third, C_3 has a frequency of 16 x 4 = 64 Hz, then 128, 256, 512, 1 024, 2 048, 4 096, 8 192, 16 384 Hz.

Ask students to use Student Activity: Tuning an Instrument in order to develop a formula that gives them a tool to determine the frequency of a given note from the frequency of the previous note of the same octave. (The answer is multiply by $\sqrt[12]{2}$). Guide students by giving them the following hint:

- suppose that the frequency of the next note is obtained by multiplying the frequency N by some number (call it x). Repeat the same process until you reach the same note in the next octave

$$N\,x\,x\,x\,x\,x\,x\,x\,x\,x\,x\,x\,x = Nx^{12}$$

because the frequency of the same note in the next octave is twice as large as the starting frequency.

$2N = Nx^{12}$
or $x^{12} = 2$
or $x = 2^{1/12} = \sqrt[12]{2}$

This number is not a rational number because any root of a number that is not a perfect square is not rational. Rational numbers may always be expressed as fractions.

Tell students that non-rational numbers can only be approximated. For example, students may use their calculators and perform the square root () function twenty times on the number 2 and then perform the SQUARE function twenty times but they will never get the number 2 exactly. In fact, after 11 operations, the rounding of the result has an effect on the answer.

Most musicians tune their instruments from a very specific note, the *A* 440. Ask students to devise a strategy in order to evaluate the frequency of the middle *C* on a piano. (Answer: 3 notes higher, that is, $2^{1/12}$ x $2^{1/12}$ x $2^{1/12}$ = $2^{1/4}$ = 1.1892071 x 440 = 523.25 Hz)

Students should use their calculators to estimate the value of $\sqrt[12]{2}$. Take the cubic root, then the square root. Take the square root again to obtain approximately 1.0594631

To get a better sense of the concept of irrational numbers, ask students to use different approximations of $\sqrt[12]{2}$, for example 1.1, 1.06, 1.059, 1.0595. Then they should use these approximations to evaluate the frequencies of a given note starting with the *A*440 note. For example, what is the frequency for the *A* in the 6th octave when you multiply 440 Hz twelve times by each of the approximations. Remember that the A_6 is exactly 880 Hz. Students will derive a sequence of closer approximations like:

440 x $(1.1)^{12}$ = 1380.91 Hz
440 x $(1.06)^{12}$ = 885.36 Hz

but never exactly 880 Hz.

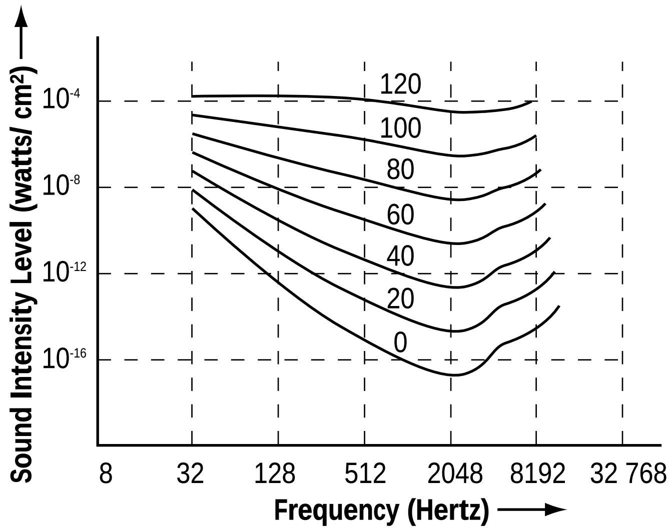
Activity 2.4: Decibels and Intensity of Sound

In this activity, students develop and use a formula to re-scale the units used in measuring sound intensity. This activity reinforces their ability to work with powers and with formulae involving powers.

Procedure:

On the overhead projector, superimpose a typical FRC (for example, Figure 11: Typical Frequency Response Curve) onto Figure 13: Levels of Sound Perception. Ensure that the frequencies of the scales in the two figures are enlarged to the same size. That is, the scales of the frequencies are the same in the two figures but the scales of the sound intensity are different, one is expressed in dB, the other in watts/cm^2)

Figure 13: Levels of Sound Perception



Tell students that when a sound travels through any medium, it carries energy. The amount of energy however is very small. For example, a full capacity stadium cheering loudly at a football game generates barely enough energy to warm one cup of tea.

The intensity of a sound is expressed in watts per square centimetre (or energy/sec/cm^2). That is, the intensity of sound is a power per unit area or an average rate of energy per unit area.

The ear is a very complex organ. It can perceive sounds once a certain intensity threshold is reached. Beyond a certain intensity, however, pain and damage to the ear drum result. Above and below this threshold, the ear cannot perceive any sound. The ear is also selective with respect to the frequency of the sound. The human ear can detect sounds that have a frequency between 20 Hz and 20 000 Hz. Outside this range, the sound cannot be perceived.

Referring to Figure 13: Levels of Sound Perception, discuss with students the features of these curves in relation to the FRC. Some aspects that can be discussed include:

- the threshold of hearing curve given for the intensity of each frequency
- the threshold of pain curve given for the intensity of each frequency
- the curves intersect and define a field where the sound is audible
- the scale of the frequencies is not linear (actually, it is a logarithmic scale) in order to cover a wide range of frequencies

Because of the human ear's extraordinary range of sensitivity (a range of over 1000 million million to 1), it is necessary to change the units of the intensity scale. The unit is called the decibel (after the Canadian inventor of the telephone, Alexander Graham Bell). The zero of the scale is 10^{-16} watts/cm^2.

This corresponds to the hearing threshold for the frequency 1 000 Hz and the intensity is then defined by the relation

$$I = 10^{-16} \times 10^{N/10} \text{ watts/cm}^2$$

N is called the number of decibels.

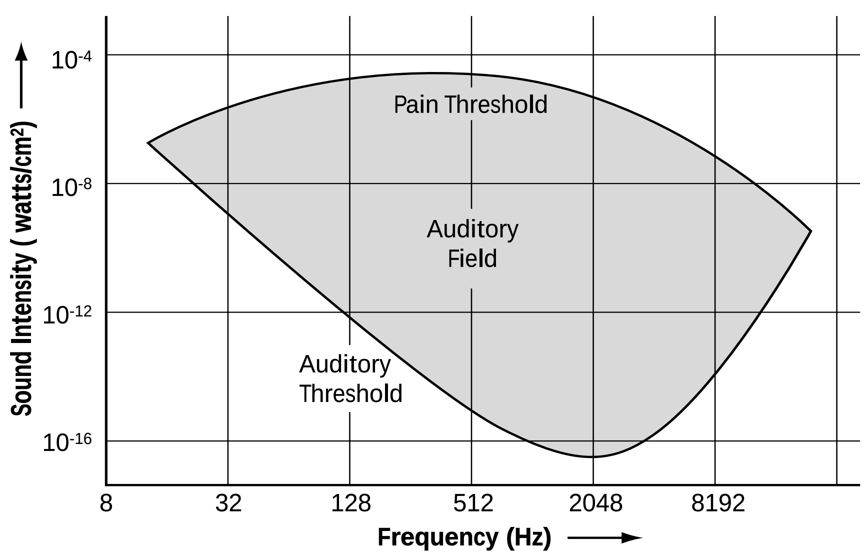
Have students use this formula to evaluate the intensities corresponding to 0 dB, 1 dB, 10 dB and 100 dB. Have students then relate these numbers to common sounds. Ask students to determine the number of dB corresponding to 10^{-13} (the sound of a whisper), 10^{-8} (a street with heavy traffic), and 10^{-1} watt/cm^2 (a jet plane with an afterburner).

Have students examine and complete Table 17: Sound Intensity Produced by Several Different Sources, in Student Activity: Speakers. Remind students to be careful about the units.

By examining the Figure 16: Levels of Sound Perception, students should note that the hearing threshold is not the same for all frequencies. As an extension, the students may examine and comment on the graph (Figure 14: Sensitivity of the Human Ear) of the intensity against frequency. Since the ear is not equally sensitive for all frequencies, the hearing threshold is not the same for all frequencies.

Have students compare this graph to the FRC so that they can determine the qualities of a stereo speaker and identify the stereo speaker performance that they cannot perceive.

Figure 14: Sensitivity of the Human Ear



Student Activity: Speakers

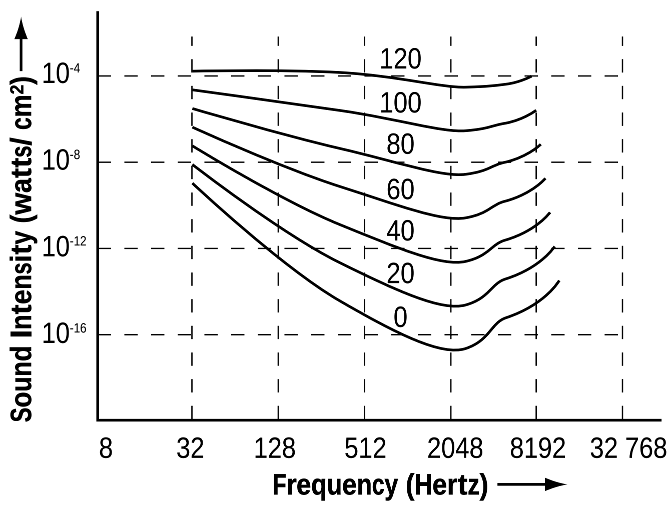
Student Name: ____________________________ Date: ____________________

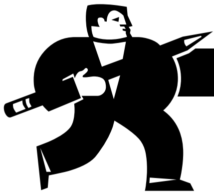
Table 15

Speaker Specifications

Speaker Model	Power Handling (W)	Maximum Power (W)	Frequency Range (Hz)	Plateau Height (dB)	Plateau Description	Price ($)

Figure 15: Levels of Sound Perception





Student Activity: Range of Emitted and Perceived Sounds

Student Name: ______________________________ Date: ____________________

Figure 16: Range of Emitted Sounds

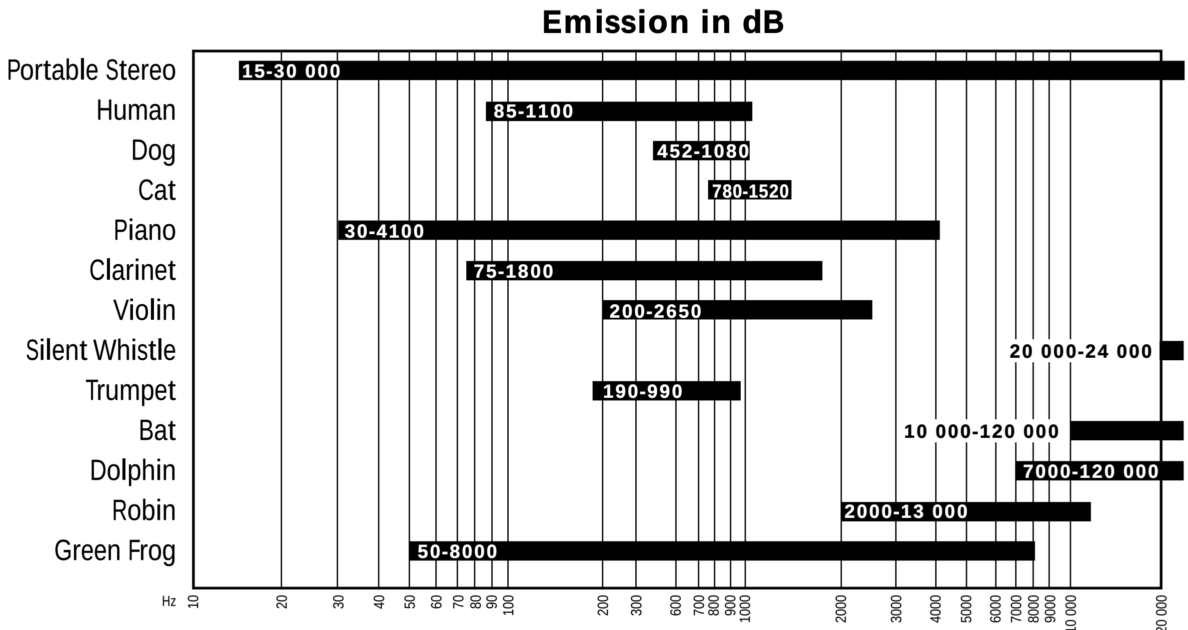
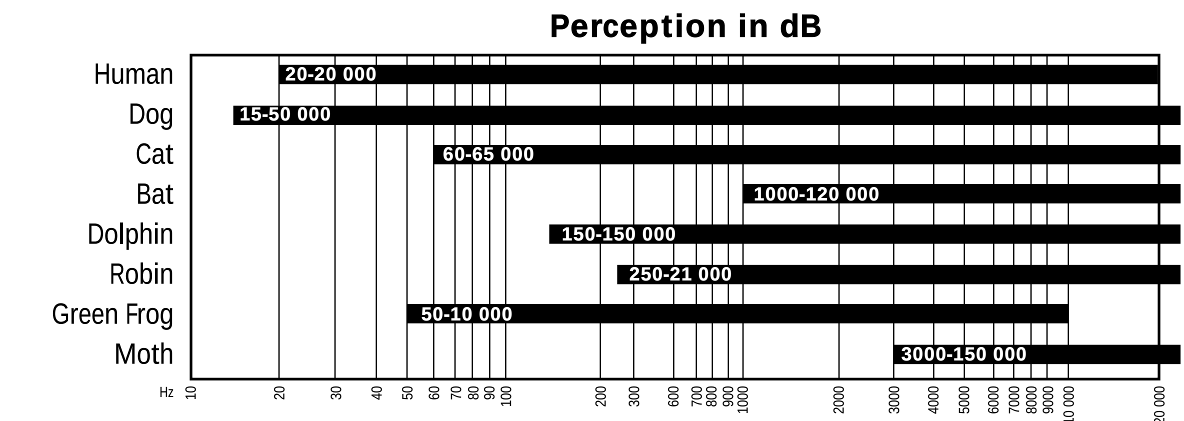


Figure 17: Range of Perceived Sounds

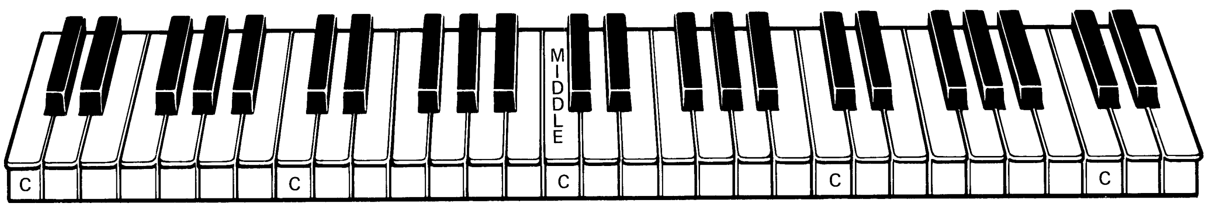


Student Activity: Tuning an Instrument

Student Name: ____________________________ Date: ____________________

On the following keyboard, identify the middle *C* note, that is, the *C* note in the middle of the keyboard.

Figure 18: Keyboard



Going to the left, identify the first *A* note - this is the *A*440 that is, the note that is supposed to be tuned exactly at a frequency of 440 Hz.

Identify all the notes for an entire octave (from *C* to *B* including the black notes).

Find all the *A* notes on the keyboard. What are the frequencies of these notes in Hz?

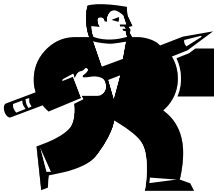
	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Hz:										

On a piano keyboard what is the frequency of the first possible audible *A* note from the left?

What is the frequency of the last possible audible *A* note to the right?

The first possible audible note produced by a church organ is a *C*, and the last possible note produced is also a *C*. How many notes are there on a keyboard? How many octaves? How many notes are there per octave?

From the *A*440, the next note to the right is an *A#*. You need to find the frequency of this note. Propose a strategy for doing this.



Student Activity: Tuning an Instrument

Use your result to estimate the frequency of middle *C*.

The frequency you have determined above is called an irrational number. A number that cannot be expressed as a fraction is an irrational number.

Use your calculator to determine the 4th root of 2

Use your calculator to determine the 8th root of 2

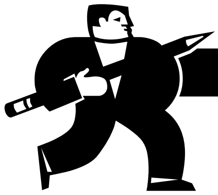
Use your calculator to determine the 6th root of 2

Use your calculator to determine the 12th root of 2

Use your calculator to determine the 24th root of 2

Numbers like $\sqrt{2}$ and π are irrational numbers and can only be approximated. Any square root of a number that is not a perfect square is an irrational number. You can never find an exact value for these numbers-only a close approximation. The approximation you determine depends on the calculator you use.

To convince yourself, do the following: use your calculator and take the square root of 2 twenty times. Then, use the square function and go back twenty times. Complete Table 16: Demonstrating the Limits of a Calculator, with your results. Compare your results with those of your classmates who have different models of calculators. Comment on the accuracy of the approximations.



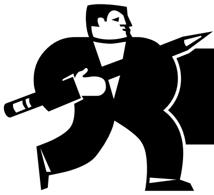
Student Activity: Tuning an Instrument

Table 16

Demonstrating the Limits of a Calculator

#	Number	$\sqrt{\ }$	Square
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

In what ways are the approximations you are able to obtain from your calculator good enough for most applications?



Student Activity: Sound Intensity Produced by Several Different Sources

Student Name: ____________________________ Date: ___________________

Complete Table 17: Sound Intensity Produced by Several Different Sources.

Table 17

Sound Intensity Produced by Several Different Sources

Source	dB	watt/m^2
Threshold of hearing	0	
Normal breathing	10	10^{-11}
Whispering (2 m)		10^{-10}
Quiet restaurant	50	
Normal conversation between two people		10^{-6}
Vacuum cleaner	80	
Loud music in an average room		10^{-3}
Rock concert	110	
Commercial airplane at takeoff	120	
Pain threshold	130	
Military airplane at takeoff		10^{2}
Rocket takeoff		10^{3}
Instant perforation of the ear drum	160	

Activity 3: Beehives

Objective :

Students realize that bees are able to use a given space in an optimal way. This is an optimization problem where a space is used to stock a maximum amount of a material, that is honey.

This activity is divided into five parts leading students progressively to understand a mathematical model that helps explain an optimization problem.

Activity 3.1: Concentric Squares
Students describe a pattern, represent it geometrically and then symbolically from a numerical representation.

Activity 3.2: Geometric Figures
Students review the properties of regular geometric figures.

Activity 3.3: Beehive
Students use their knowledge of the properties of geometric figures and their ability to represent a pattern in the construction of a cardboard beehive.

Activity 3.4: The Meeting
Students use their cardboard model to create a game that can be used to reinforce their knowledge of geometrical transformations in a plane.

Activity 3.5: Hexagonal Behive Cells
Students perform calculations on perimeter, area and volume to determine the optimal figure than can be used to construct a model.

Activity 3.1: Concentric Squares

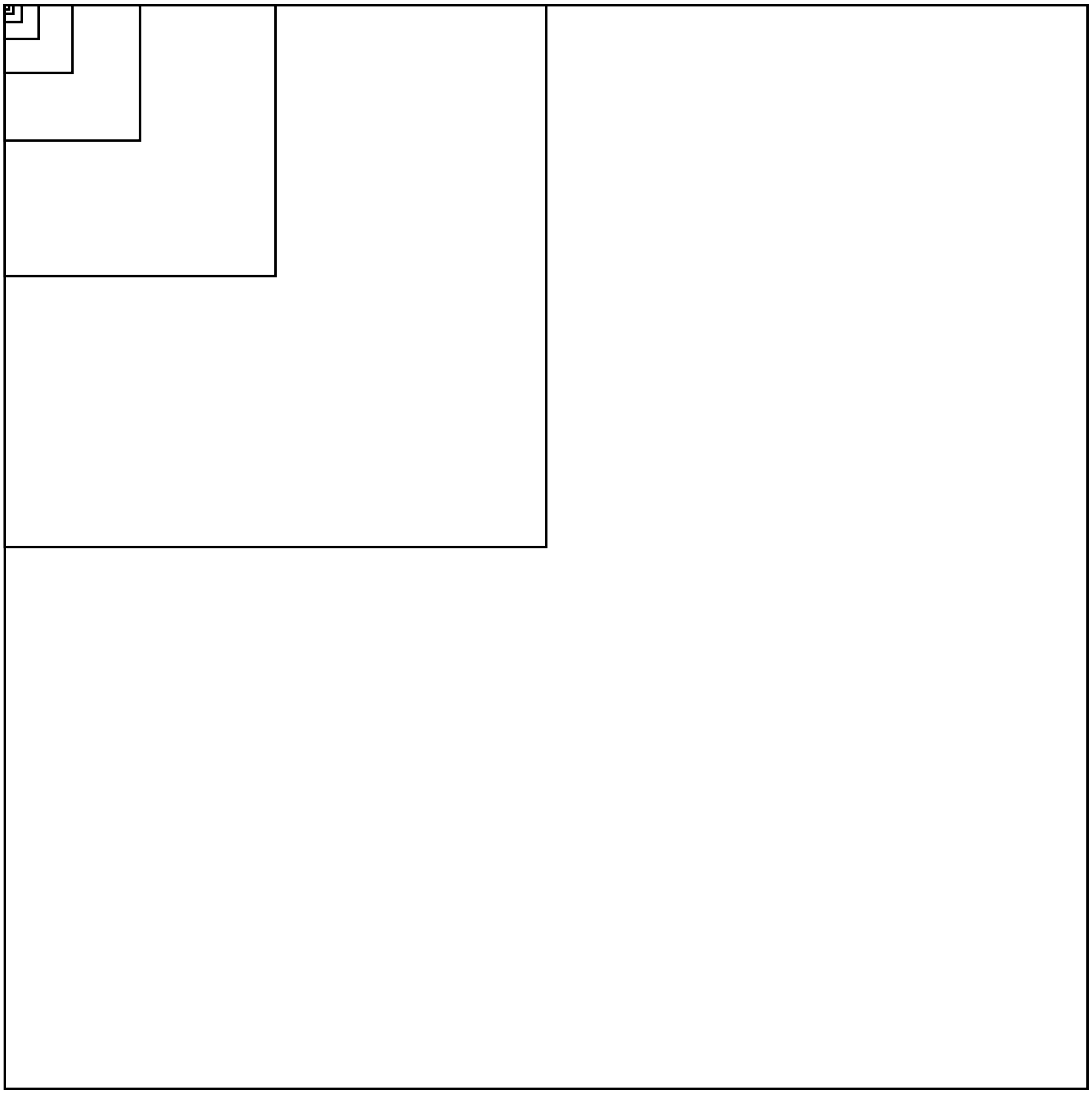
Objective:

The objective of this activity is to construct a series of concentric squares from a given square by halving the sides. The objective is to represent the recursive property geometrically and then algebraically.

Procedure:

Students construct a square of side 20 cm by using a square and a pair of compasses. See Figure 19: Concentric Squares below.

Figure 19: Concentric Squares



Students complete Table 18: Rules for Concentric Squares, and answer the questions in Student Activity: Concentric Squares.

Table 18

Rules for Concentric Squares

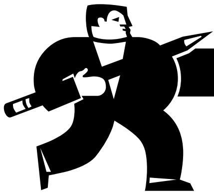
Steps	# Squares	Total # Squares	Rule
1	1	1	1 x 1 = 1
2	4	5	2 x 2 = 4 and 4 + 1 = 5
3	9	14	3 x 3 = 9 and 9 + 5 = 14
4	16		4 x 4 = 16 and 16 + 14 = 30 or 4 x 4 + 3 x 3 + 2 x 2 + 1 x 1 = 30
5	25	55	5 x 5 + 4 x 4 + 3 x 3 + 2 x 2+ 1 x 1 = 25 + 30 = 55
6			
7			
8			
9			
10			
11			
12			
20			20 x 20 + ... + 1 x 1 = 6 998 670
50			
100			
1000			

Have students compare the different ways their calculators operate when they evaluate the expression below.

$(n\ x^{2} +)$: $20x^2 + 19x^2 + ... = 6\ 998\ 670$

Have students verbally express the rule, graph the results and finally express the rule algebraically.

If n is the number of steps, the number of squares is $n^2 + (n-1)^2 + ...+1$



Student Activity: Concentric Squares

Student Name: ____________________________ Date: ___________________

Procedure:

- draw a square with 20 cm sides
- divide one side into 2
- construct a square inside the big square (this smaller square will have 1/4 the area of the original square with 20 cm sides)
- divide one side into 2
- construct a square inside the second square (this smaller square will have 1/16 the area of the original square with 20 cm sides)

Table 19

Rules for Concentric Squares

Steps	# Squares	Total # Squares	Rule
1	1	1	
2	4	5	
3	9	14	
4	16		
5			
6			
7			
8			
9			
10			
11			
12			
20			
50			
100			
1000			

Student Activity: Concentric Squares

Answer the following questions:

How do you obtain the numbers in column 2?

How do you obtain the numbers in column 3?

What is the number corresponding to 20 steps?

What is the number corresponding to 1000 steps?

Express the rule in your own words.

Express the rule in algebraic terms.

Activity 3.2: Geometric Figures

Objective:

The objective of this activity is to review the properties of basic geometric figures. Students use a pair of compasses to construct the basic regular polygons. In particular, they review concepts related to the angles and symmetries of basic regular polygons.

Procedure:

Make sure that students know how to use their compasses properly.

The first polygon is the hexagon that is obtained by taking the radius as the side. Ask students to compare this procedure to other ways of using a ruler and protractor. Refer to Project A for more details.

Ask students to draw the axes and to verify their accuracy by folding along the axes of symmetry.

For the regular dodecagon, use the compass to divide one side of the hexagon into two equal parts and join these together.

To obtain a square, join the vertices of the dodecagon three by three.

To obtain an equilateral triangle, join the vertices of the hexagon two by two, and so on.

Have students measure the sides of the polygon by using a pair of compasses. They should complete the Table 22: Geometric Figures in Student Activity: Geometric Figures. Ask students to discuss why this method is more accurate than a direct measurement with a ruler. See Table 20: Regular Polygons (Radius of Circle: 10 cm).

Have students extrapolate the perimeter of a circle by constructing a rule from the pattern observed in the Table 20: Regular Polygons (Radius of Circle: 10 cm). They should represent their rule in the form of a graph on their graphing calculator.

Students should realize that it now possible to construct a pentagon with this method. Ask students to design a method for constructing a pentagon. Have them draw and count the axes of symmetry and then enter this number in Table 21: Axes of Symmetry. Ask students to measure the angles of the pentagon and to record the values in Table 20: Regular Polygons. They may create a recursive rule for the angles in the same way. Have students compare the relationship between the number of sides and the angle values.

Table 20

Regular Polygons (Radius of Circle: 10 cm)

# Sides	Name	# Vertices	Length of Side (cm)	Perimeter (cm)
3	triangle	3	17.3	17.3 x 3 = 51.9
4	square	4	see calculation below	
6	hexagon	6	10	
8	octagon	8		
12	dodecagon	12	see calculation below	
21	?	24		

Note: Length of the sides of regular polygons

The angles in a regular polygon are all equal: multiply 180^o by the number of sides minus 2 and divide by the number of sides, for example:

(3 sides - 2) x 180^o/3 sides = (1) x 180^o/3= 60^o

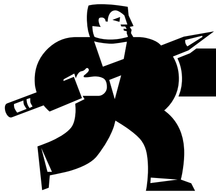
equilateral triangle: 1 x180^o /3 =60^o
square: 2 x 180^o/4 = 90^o
hexagon: 4 x 180^o/6 = 120^o
octagon: 6 x 180^o/8 = 135^o
dodecagon: 10 x 180^o/12 = 150^o

side of hexagon: length of side is given as 10 cm
half side of triangle: side of hexagon x 0.86 = 8.6 cm
side of dodecagon: half side of hexagon divided by 0.96 = 5/0.96 = 5.2 cm

Table 21

Axes of Symmetry

Name of Shape	Number of Vertices	Number of Axes
triangle	3	3
square		



Student Activity: Geometric Figures

Student Name: ________________________ Date: ________________

Draw a circle of radius x cm larger than 10 cm.
Construct a hexagon, a triangle, and so on by following your teacher's instructions.
At each step, name the polygon, count the number of sides and measure the side lengths and the angles. Enter your results below in Table 22: Geometric Figures:

Table 22

Geometric Figures

# Sides	Name	# Vertices	Side Length (cm)	Interior Angle (degrees)	Perimeter (cm)	Rule
3						
4						
6						
8						
12						

Propose a method to construct a pentagon.

Propose a method to construct a dodecagon.

What is the length of the pentagon side?

What is the length of the dodecagon side?

Student Activity: Geometric Figures

What is the perimeter of a regular 24-sided polygon?

What is the perimeter of a regular 36-sided polygon?

What is the perimeter of a regular 216-sided polygon?

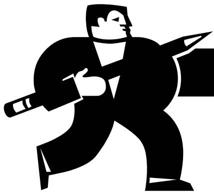
What is the perimeter of a regular 1296-sided polygon?

Draw all the axes of symmetry and complete Table 23: Number of Axes of Symmetry.

Table 23

Number of Axes of Symmetry

Name of Polygon	Number of Vertices	Number of Axes of Symmetry
triangle	3	3



Student Activity: Geometric Figures

For each polygon, count the number of angles, add all the angles, measure the angles and complete Table 24: Polygon Angles.

Table 24

Polygon Angles

# Sides	# Angles	Angle Measurement	Rule
3			
4			
5			
6			
8			
10			
12			

What is the interior angle of a 9-sided polygon?

What is the interior angle of a 27-sided polygon?

What is the interior angle of a 243-sided polygon?

What is the interior angle of a 729-sided polygon?

Explain and apply the rule you have derived.

Activity 3.3: Beehive

Objective:

In this activity, students conceptualize the properties of geometric figures and apply different recursive rules to fill a square of given dimensions.

Procedure:

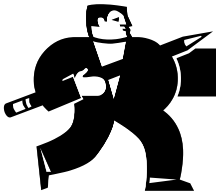
Have students draw and cut out a hexagon with a side of 5 cm using the method described earlier. They will use this model for drawing all the cells of the beehive.

Have students determine the center of a square piece of cardboard (50cm x 50cm) and then draw the two diagonals. Placing the model at the centre, they start tracing the beehive following the instructions, i.e., forming concentric rings of hexagons.

After two rings, students are asked to draw the axes of symmetry:
- one vertical through the centre
- one horizontal through the centre
- one axis at 30˚
- one axis at 60˚.

The axes will be used in the next activity, The Meeting. See Figure 23: Game Board.

Students complete the tables and the graphs in Student Activity: Construction of the Beehive, and answer the questions.



Student Activity: Construction of the Beehive

Student Name: ____________________________ Date: ___________________

Materials:

- sheet of solid cardboard
- scissors
- ruler
- pair of compasses

You are asked to:

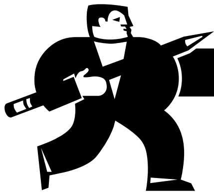
- Follow the instructions to construct the cardboard model of a beehive.
- Construct a table like Table 25: Total Cell Parameters to enter your data.
- After each ring use your own words to express the rules for calculating the number of cells, the total number of cells, and the perimeter.
- Represent on a graph how these quantities vary with the number of rings.
- Derive an equation of a relationship between the number of cells and the number of rings.
- Elaborate on the following questions:

What are some of the things that could happen if bees constructed a beehive with octagonal cells?

What are some things that could happen if bees constructed a beehive with pentagonal cells?

What are some things that could happen if bees constructed a beehive with square cells?

What are some reasons for the cells being hexagonal in form?



Student Activity: Construction of the Beehive

Table 25

Total Cell Parameters

Ring Number	Number of Cells Per Ring	Total Number of Cells	Number of Sides Per Ring	Total Number of Sides
1	1	1	6	6
2	6	7	24	30
3	12	19		
4	18			
5				
6				
12				
36				
100				
RULE				

Explain how you derived the expression for the rules.

How long does it take to fill the square if one ring is constructed every hour? Show your work.

How long does it take to finish the job if one cell is constructed every 10 minutes?

Estimate the actual number of cells in a beehive. Describe the process you used.

Student Activity: Construction of the Beehive

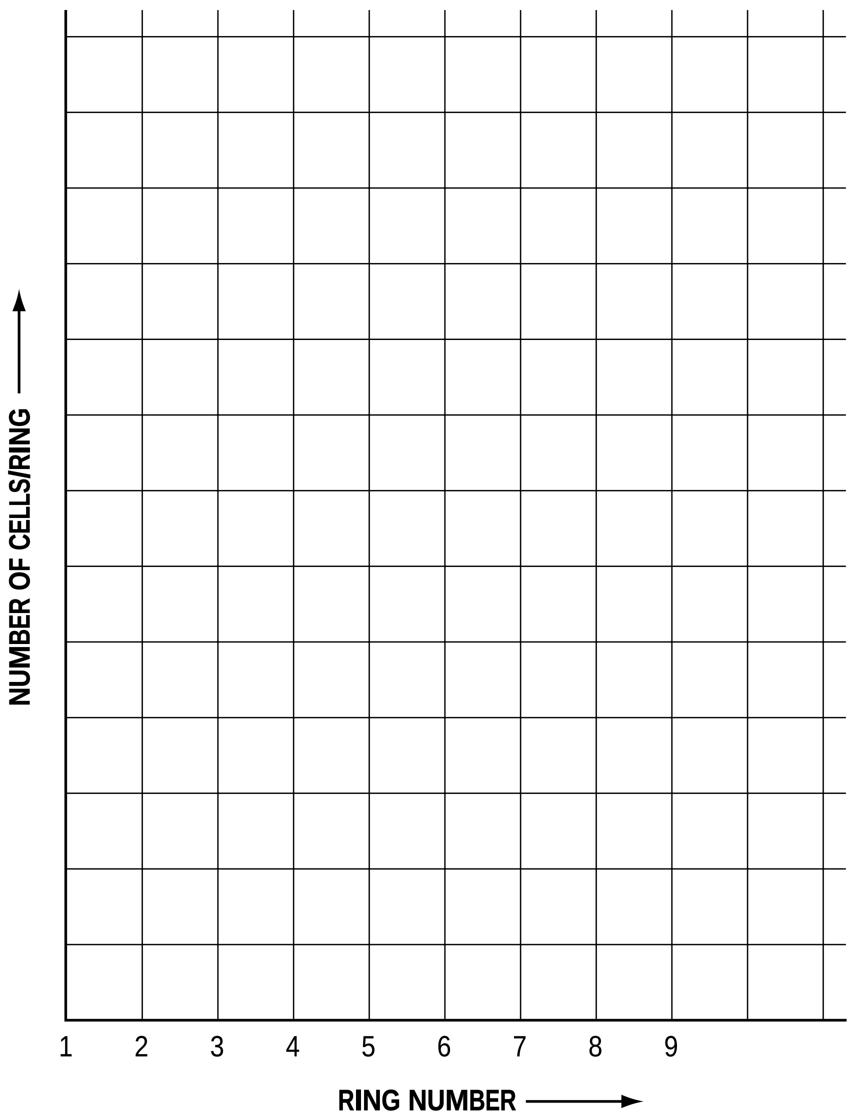
Estimate the dimensions of a real cell (side length, area, volume). Describe the process you used.

What is the total perimeter of the beehive?

Use the following blank graph to represent the number of cells, the total number of cells and the perimeter per ring. Use an *x* to plot the intersection of the number of cells and the ring number.

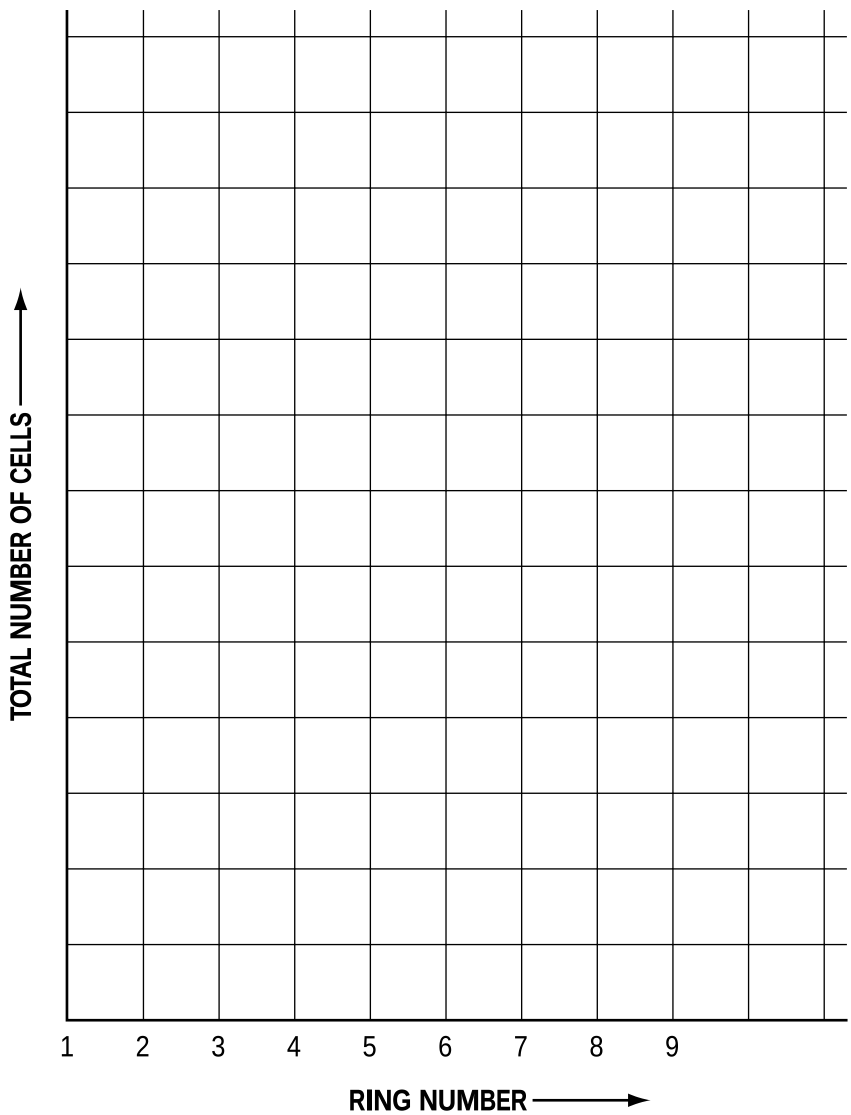
Student Activity: Construction of the Beehive

Figure 20: Number of Cells Per Ring



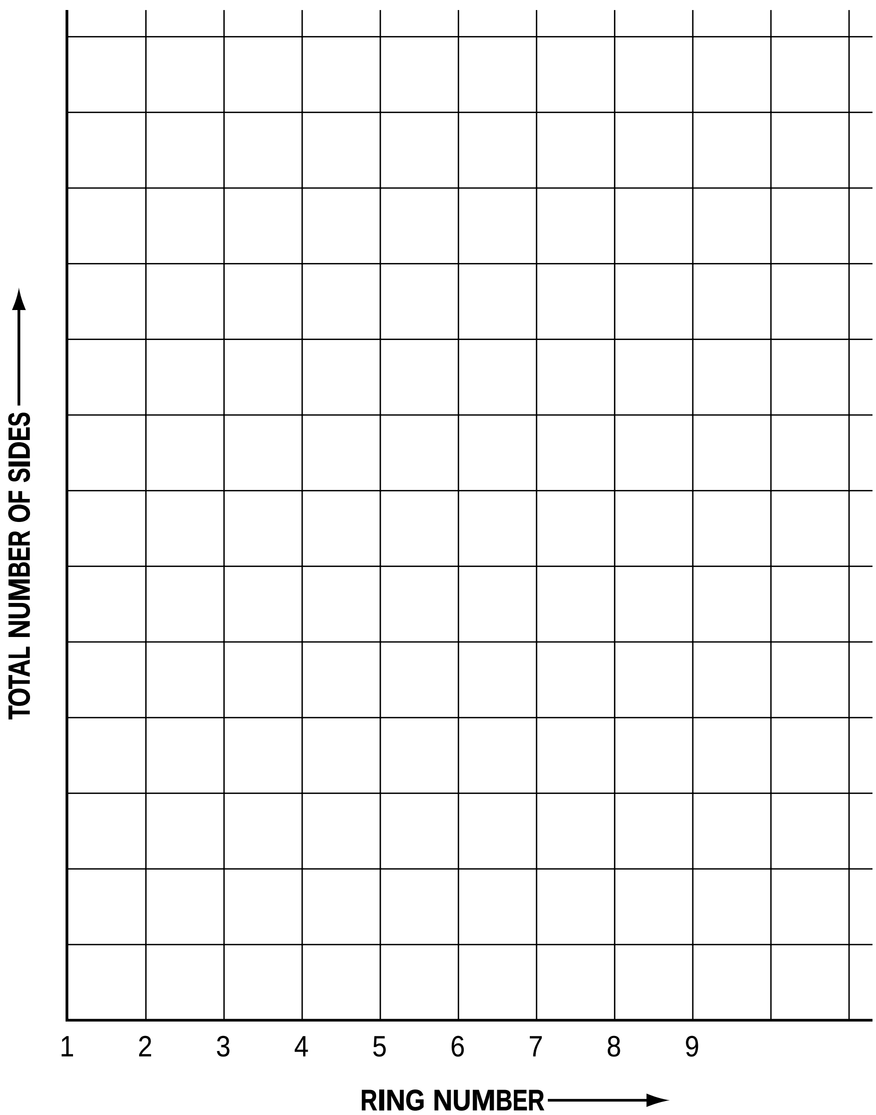
Student Activity: Construction of the Beehive

Figure 21: Total Number of Cells



Student Activity: Construction of the Beehive

Figure 22: Total Perimeter of Beehive



Activity 3.4: The Meeting

Objective:

The objective of this activity is a review of the concepts of displacement in the vertical, horizontal and diagonal planes.

Procedure:

The object of the game is to meet with a partner sitting in a given cell. Essentially students are playing the traditional naval combat game and the same rules apply as in the regular game.

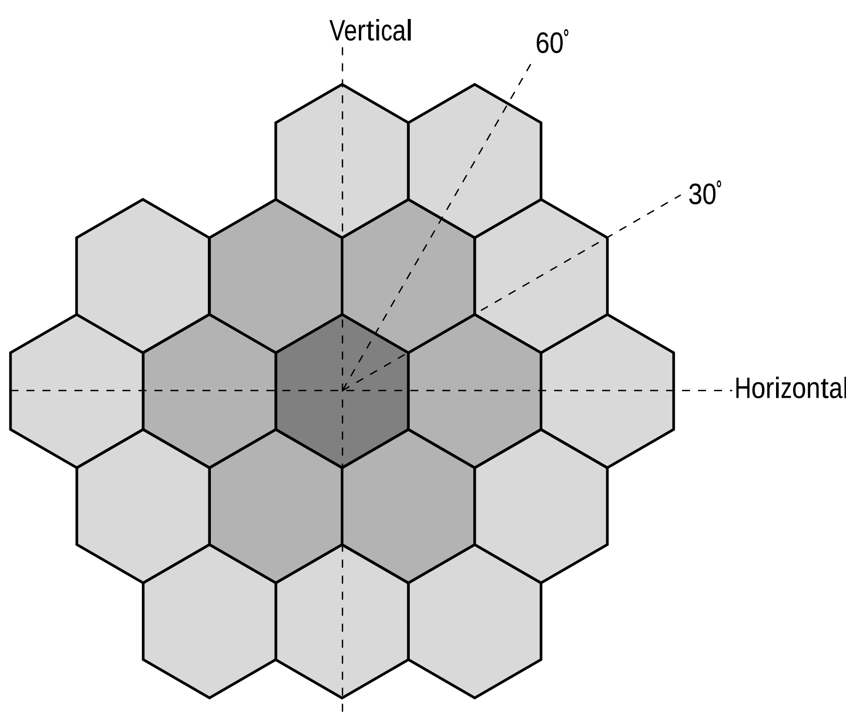
Constructing the Game Board

Have each student build their own game board. Instead of drawing a grid of squares on the game board, students fill the game board with hexagons. All students should have the same size hexagons on their game board and these should all be identified in the same way. The following directional labels should be added to the perimeter of each game board:

- horizontal
- vertical
- 30^o
- 60^o

The diagram below, Figure 23: Game Board, provides a guide of what the game board should look like.

Figure 23 : Game Board



Students can choose any game pieces they wish to mark their meeting place, for example chess pieces or checkers.

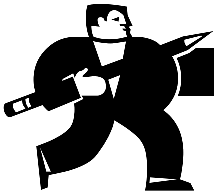
Finally, each game board must have a vertical backboard divider so that students cannot see the hexagon their opponent has identified as his meeting place.

Playing the Game

The objective of the game is for a player to reach his opponent's meeting place. The player who reaches the meeting place using the fewest number of moves is the winner.

Discuss and establish the game rules as a class

Students throw dice to determine the number of cells and the direction of movement. They describe their move to their opponent, for example: "I am moving left at a diagonal of 30˚, 6 cells from centre."



Student Activity: The Meeting

Student Name: ______________________________ Date: ____________________

Materials:

- game board with hexagonal spaces
- dice
- queen and a king from a chess game, or other game pieces

Procedure:

The objective of the game is for a player to reach her opponent's meeting place. The player who reaches the meeting place using the fewest number of moves is the winner.

Discuss and establish the game rules as a class.
Students throw dice to determine the number of cells and the direction of movement. They describe their move to their opponent, for example: "I am moving left at a diagonal of 30˚, 6 cells from centre."

Activity 3.5: Hexagonal Beehive Cells

Objective:

Why are beehive cells hexagonal? The objective of this activity is for students to measure length, perimeter and area to determine the optimal ratio, perimeter/area, for different tiling of a square. They derive a formula giving the ratio as a function of the number of sides for the regular polygons.

Procedure:

Have students draw and cut out models of different regular polygons (e.g., squares, pentagons, hexagons, octagons) all of them are inscribed in the same circle having a radius of 5 cm.

Ask them to derive the formula for the total area and the total perimeter in the same way as they did for the beehive problem. Prior to deriving the formula, ask them to graph the results.

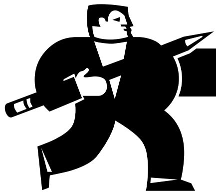
Ask students to comment on the way the variables are increasing.

Ask students to complete Table 25: Total Cell Parameters and Figure 24: Generic Graph.

Ask students to extrapolate for polygons with 10, 12, and more sides.

Ask them what would happen if they were trying to tile the area with circles.

Ask students to discuss why hexagons are used in nature in many different instances.



Student Activity: Why are Beehive Cells Hexagonal?

Student Name: ____________________________ Date: ___________________

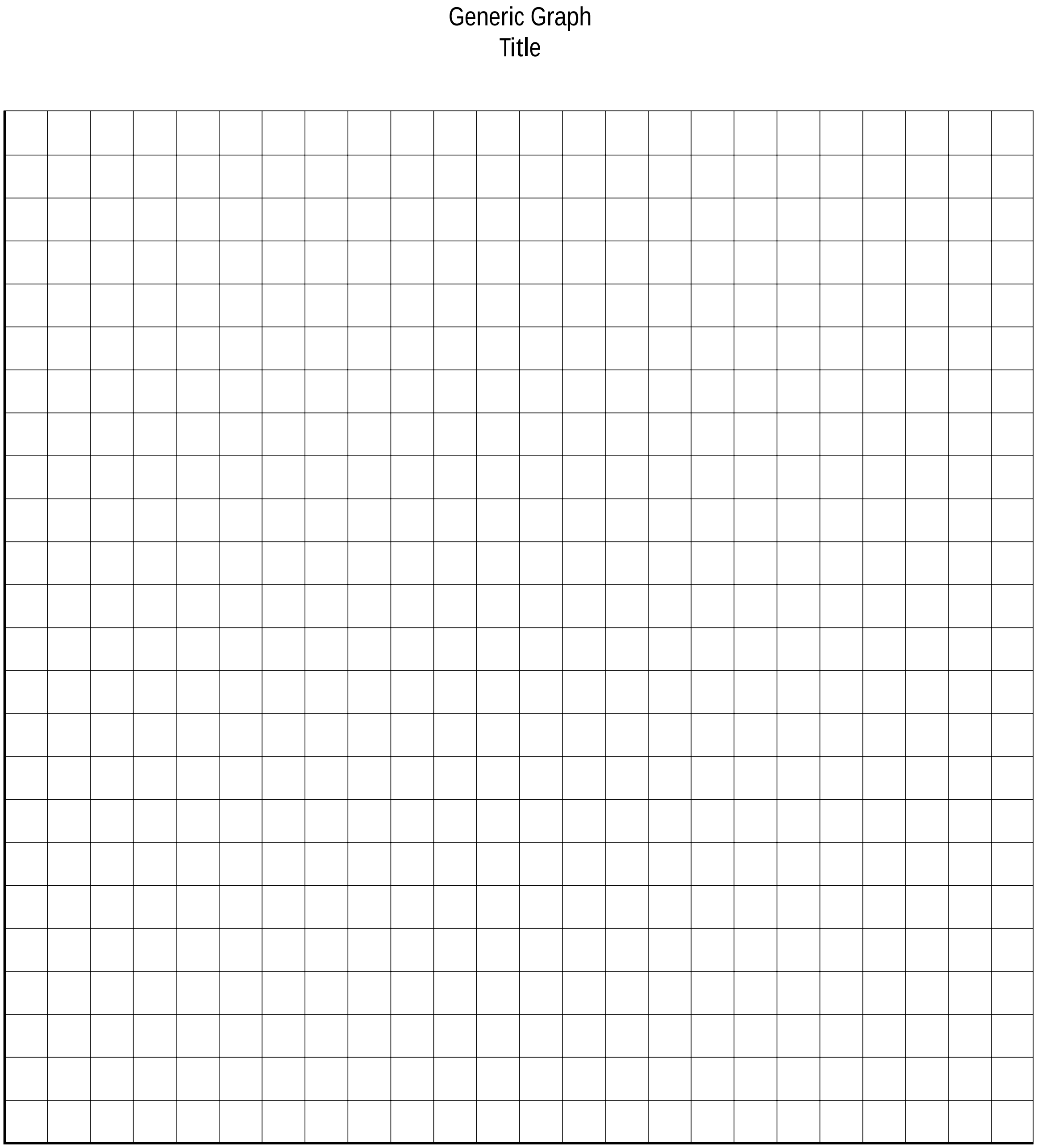
Materials:

- cardboard
- models of hexagons, squares, octagons and pentagons
- scissors

Procedure:

- Draw and cut out models of different regular polygons (e.g., squares, pentagons, hexagons, octagons) all of them inscribed in the same circle having a radius of 5 cm.
- Derive the formula for the total area and the total perimeter in the same way as you did for the beehive problem. Record your information using tables like the ones previously used to record your data.
- Use graphing software to draw the graphs in each situation and recopy on a blank graph like the one that follows.

Figure 24: Generic Graph



Phase III: Iso- and Geosynchronous Orbits

Activity 1: Achilles and the Turtle

Activity 2: Planets

Activity 3: Andromeda and the Liquid-Mirror Telescope

Activity 4: Big Bang

Activity 5: Autopsy of a Collision

Phase Synopsis:

In this evaluation phase students are provided with a number of opportunities to demonstrate the skills and knowledge they have acquired during the project. They are presented with real-world situations and must develop models with formulae containing powers and perform operations with a technological aid.

The objectives of this phase are:
- to increase student awareness of the necessity of modeling situations with formulae involving powers
- to reinforce student skills, and abilities to manipulate algebraic expressions containing powers with integral exponents and rational bases by using the Laws of Exponents
- to increase student skills in correctly using calculators with very large and very small numbers

As the teacher, you will need to determine the extent to which you will guide and monitor the activities. You may choose to use the flowchart of mathematical modelling as a guideline. At this point in the project, students have developed a sufficient number of concepts, and have the ability to select and conduct an activity of their choice. Five suggestions follow for such a problem-based learning activity.

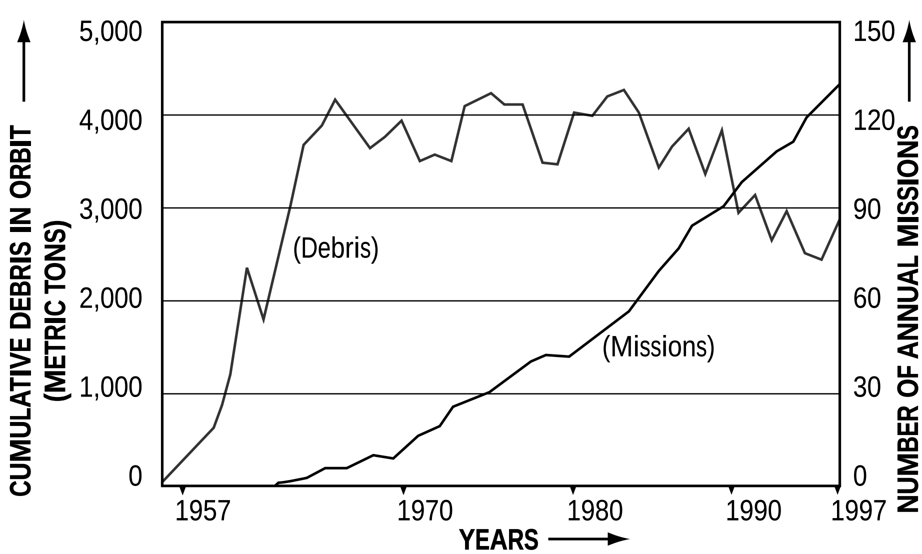
Students are asked to use the Internet to find necessary data and information, and to present their final report by making intensive use of new technology. The final report could be a class presentation or the preparation of an article for the school newspaper.

Provide students with the following four tables of data and three graphs. Offer challenges such as the following to assess their understanding of exponents, scientific notation and the Laws of Exponents:

- What is the ratio of the aphelions of Pluto and Earth?
- What is the volume of Venus?
- What is the mass of the Earth and the Moon combined?
- What is the difference between the period of revolution of the Earth and that of Neptune?
- Express the Earth's rotational velocity at the equator in km per year.
- Uranus rotates in the opposite direction to that of the Earth/Sun system. Explain.
- Estimate how long it takes for a fixed object on Uranus to be seen again from a fixed point on Earth.
- Estimate the ratios of the gravity at the equator for each planet with respect to Earth.
- Explain what escape velocity means. Calculate the ratio of the escape velocity on Icarus to that of Earth.
- Analyse and comment upon Figure 25: Cumulative Mass of Debris, and Figure 26: Earth Satellite Population.
- Define Solar Ratio Flux.

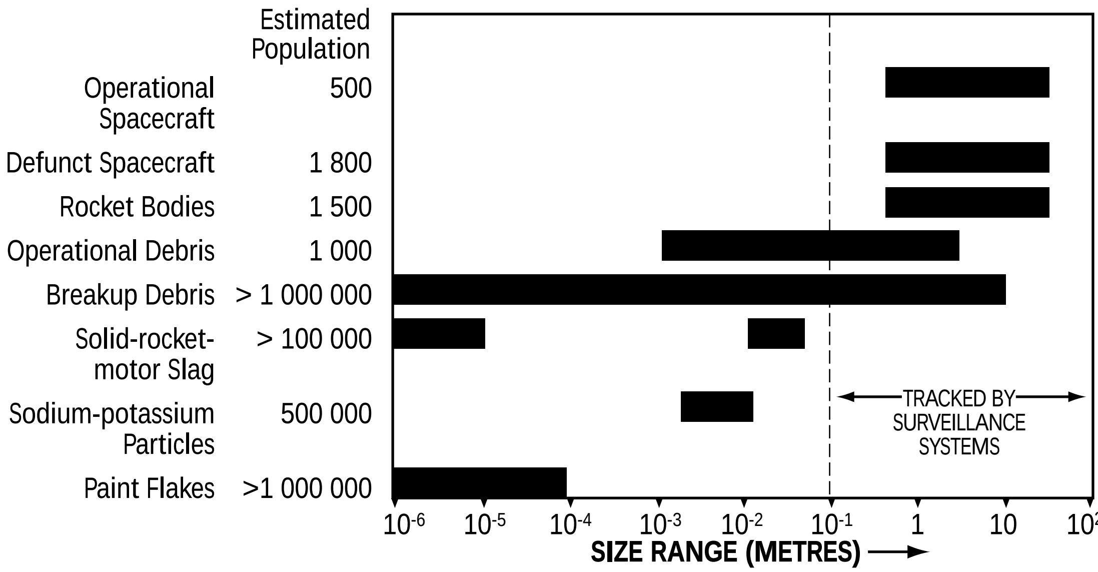
Using the information in Figure 27: Spatial Density, estimate the mass of satellites and debris at altitudes of 20 000 km and 40 000 km.

Figure 25: Cumulative Mass of Debris



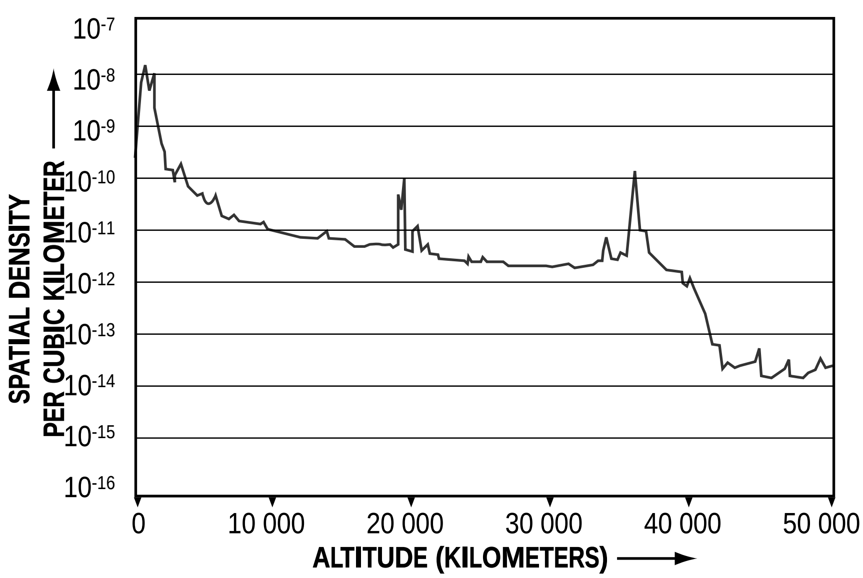
Adapted from: Nicholas L. Johnson. "Monitoring and Controlling Debris in Space", *Scientific American* (August 1998): 62-67.

Figure 26: Earth Satellite Population



Adapted from: Nicholas L. Johnson. "Monitoring and Controlling Debris in Space", *Scientific American* (August 1998): 62-67.

Figure 27: Spatial Density



Adapted from: Nicholas L. Johnson. "Monitoring and Controlling Debris in Space", *Scientific American* (August 1998): 62-67.

Activity 1: Achilles and the Turtle (1 hour)

Objective:

Using the concepts related to rational numbers and their limitations, students reproduce the experiments of Zeno in different contexts. In reproducing the experiments they discover a paradox of the Ancient Greeks. They realize the limits of the set of rational numbers.

Project product: Report on the concept of limit.

A. Situation

B. Variables

C. Assumptions

D. Model

E. Solving the Model

F. Testing the Model

G. Simplifying the Model

H. Refining the Model

I. Applying the Model

Activity 2: Planets (2 hours)

Objective:

Students develop and apply algebraic formulae in order to model the motion of planets and satellites (Kepler's Third Law).

Project product:

Mathematical models of the motion of planets and satellites.

Activity 3: Andromeda and the Liquid-Mirror Telescope

Objective:

From data obtained from the Dominion Astronomical Observatory and the Western Ontario Light Detection and Ranging Observatory, students perform measurements and calculations on galaxies.

Students propose ways to observe the sky and to calculate distances (or time) between stars.

Project product:

A report on different ways of observing the sky.

Activity 4: Big Bang

Objective:

Students evaluate their ability to use and apply expressions containing powers by exploring some aspects of the expansion of the Universe, the Theory of Relativity and Quantum Theory.

Project product:

Report on the various interpretations of the origin and the expansion of the Universe.

Activity 5: Autopsy of a Collision

Objective:

The objective of this activity is for students to evaluate the probability of a collision between two objects in a sequence of increasingly complex settings. They relate the probability of collision to the density of objects revolving around in the same orbit.

Project product:

- Report describing the various factors involved in predicting a space collision and proposing possible solutions to the problem, or
- Presentation: Monitoring and Controlling Debris in Space
- Class presentation or preparation of a documented article in the school newsletter, or a video presentation.

Project B: Determining the Relationship of Insulin to Diabetes

- **Orientation**
- **Project Synopsis**
- **Phase I: Diabetes**
- **Phase II: Tools for Monitoring Diabetes**
  Activity 1: Fuel
  Activity 2: Controlling Diabetes
  Activity 3: Modelling Two Competing Processes
- **Phase III: Solving a Regimen Problem**

Project B: Determining the Relationship of Insulin to Diabetes

Orientation

Time:

Phase I: Diabetes
Activity 1: Diabetes Research .... 1 to 2 class periods
Phase II: Tools for Monitoring Diabetes
Activity 1: Fuel .... 1 to 2 class periods
Activity 2: Controlling Diabetes .... 2 class periods
Activity 3: Modelling Two Competing Processes .... 1 to 2 class periods
Phase III: Solving a Regimen Problem .... 1 class period

Instructional Strategies:

- Direct Instruction
- Case Study
- Group Work
- Issues Inquiry

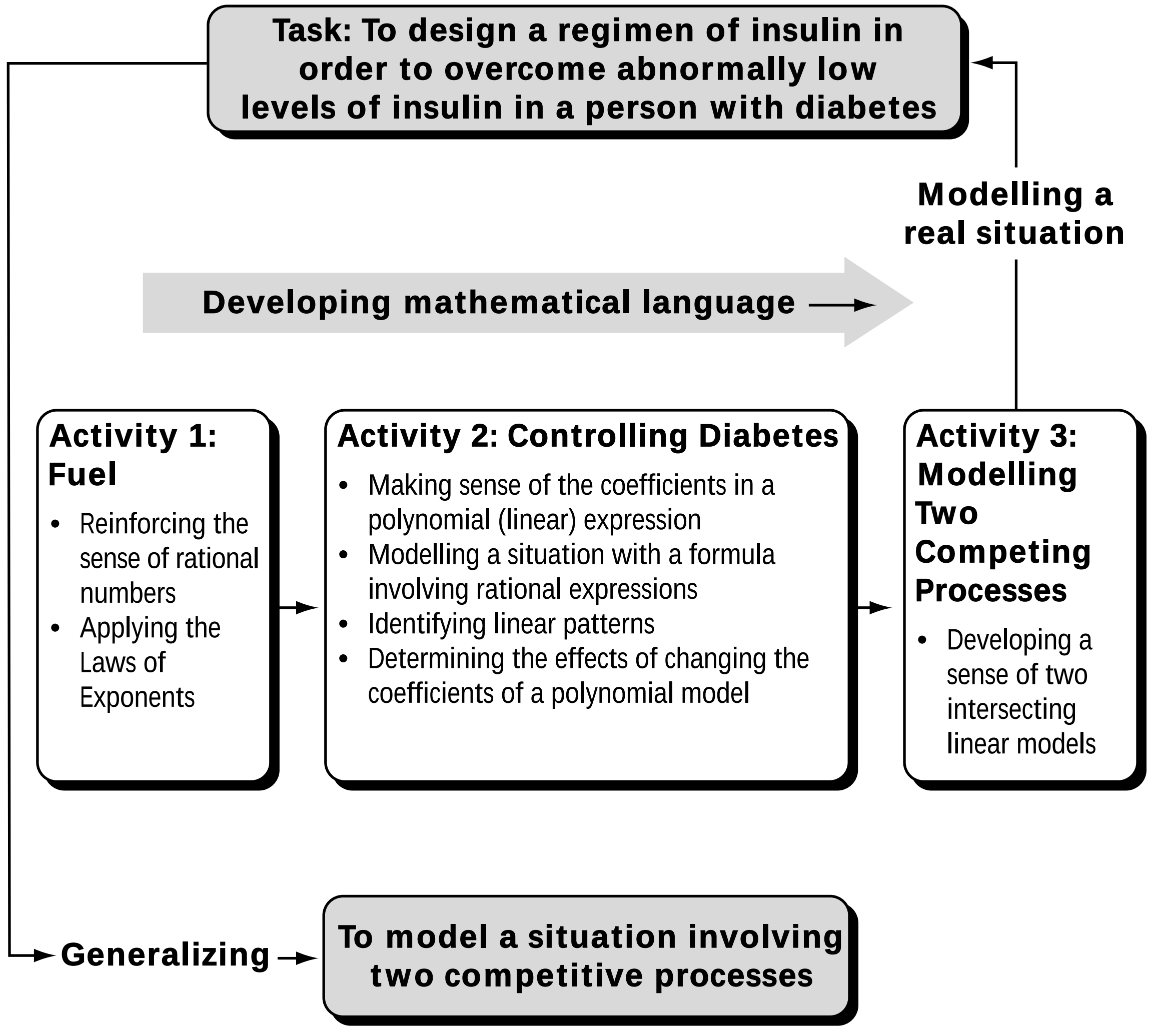
Real-World Applications:

Some real-world applications of the skills learned in this project include:
- representing and interpreting health data in the form of graphs
- making better decisions about health based on the interpretation of health data
- determining the optimal price for concert ticket sold by scalpers

Project Overview:

Figure 1 on the following page is a schematic to help you conceptualize Project B: Determining the Relationship of Insulin to Diabetes.

Figure 1: Project B: Determining the Relationship of Insulin to Diabetes



Project Synopsis

The objective of this project is to reinforce student skills in pattern recognition and in performing calculations with rational numbers and powers while developing their consciousness about diabetes.

Students identify and graphically represent patterns. These patterns are then applied to make predictions and draw conclusions.

Students analyse two competitive processes (input and output) with linear models in order to develop their understanding of polynomial coefficients and their significance.

Students are evaluated on their skills and abilities in:
- statistics, that is, establishing the correlation between two variables (Activity 1: Diabetes Research)
- number concepts and number operations, that is using rational numbers and powers in a real context by applying the laws of exponents (Activity 1: Fuel)
- patterns, variables and equations that is, model a real situation with polynomials and solving first-degree equations (Activity 2: Controlling Diabetes, and Activity 3: Modelling Two Competing Processes).

This project has three phases.

In Phase I: Diabetes, students are introduced to the topic of diabetes and assigned the task of collecting more information about the disease.

Phase II: Tools for Monitoring Diabetes has three major activities, Activity I: Fuel; Activity 2: Controlling Diabetes; and Activity 3: Modelling Two Competing Processes. In Activity 1: Fuel, students identify the variables that are involved in diabetes development. Students relate the causes, the diagnosis and the treatment of diabetes. In Activity 2: Controlling Diabetes, students recognize the patterns representing the change in the variables (sugar and insulin) over time. Students also have an opportunity to design regimens. Finally, in Activity 3: Modelling Two Competing Processes, students learn about the biochemical properties of insulin, and its role in the metabolism of sugar and in regulating the blood glucose level by injection. Students are assigned the tasks of designing other situations where two competing processes are involved and of modelling the result.

Phase III: Solving a Regimen Problem is the evaluation component of this project. In this phase, the mathematical and scientific concepts are evaluated from reports prepared by groups of students. These reports include: a description of the preparation of a regimen of insulin injections from a table of blood sugar readings; answers to questions about the impact of changes in the diet; and the social and economic implications of diabetes.

Phase I: Diabetes

Activity 1: Diabetes Research

Objective:

The objective of this activity is for students to become familiar with the nature of diabetes, the vocabulary used, and the physiological effects of the illness. Students collect and represent data related to the occurrence of diabetes and analyse the results in terms of two variables. You may also choose to do Phase I as part of the science course or in collaboration with the science teacher.

Note: Before you begin this project, check with your school administrator to determine whether there is any school, district or Ministry of Education policy governing the use of body fluids for testing purposes. Some activities require authentic data from humans. For sources of human physiological data, consult local health officials and/or diabetes organizations.

Materials:

- information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centres
- access to Internet
- prepared microscope slide of a pancreas cell(s)
- Student Activity: Diabetes Research
- access to a nurse, physician, dietitian or student with diabetes

Procedure:

Diabetes is a condition in which the body cannot properly use the energy from food eaten. When food is ingested, some of it is broken down by the digestive system into forms of sugar, one of which is called glucose. Glucose is the main fuel of the body. It enters the bloodstream and goes to the cells where it is used as energy or where it is stored in the liver for future use.

Insulin is a chemical made and stored in specific cells of an organ called the pancreas. Refer to Figure 2: Where Insulin is Made. Insulin is needed to help sugar leave the bloodstream and enter cells of the body: liver cells following digestion, and all body cells when human activity levels rise. Refer to Figure 3: Microscopic Sections of Pancreas and to Figure 4: Intake of Glucose Into an Islet Cell.

Diabetes develops when the body cannot:
- make any insulin;
- make enough insulin; or
- properly use the insulin it makes.

Refer to Figure 5: What Happens if Insulin is Not Produced or if Insulin Does Not Work to see the impact on a cell of the above conditions.When diabetes develops, glucose stays in the blood, is not being stored in the liver and is excreted in the urine. Body cells do not get the glucose they need to produce smaller energy packets. Instead the body starts burning stored fat to make energy. When fat is broken down by the digestive system, poisons form called ketones. The body tries to rid itself of the ketones by sending them out in the urine. A buildup of ketones in the body can be dangerous and lead to a problem called ketoacidosis.

Invite a student with diabetes, a nurse, or dietitian to make a brief presentation that includes a videotaped demonstration of how a blood glucose reading is taken using a meter or a strip.

Provide a microscope preparation of the pancreas showing the scattered Islets of Langerhans. Have students draw the Islets and their cells, and estimate the dimensions of the pancreas and the Islets.

Have students do Student Activity: Diabetes Research to guide their investigation. Then ask them to present their findings in a short report.

Divide students into small groups and have each group select two variables. Students can represent the relation between these two variables as a graph or a diagram. Have them describe the type of relationship between the two variables by determining if there is a strong correlation and if there is a direct or inverse relationship.

Figure 2: Where Insulin is Made

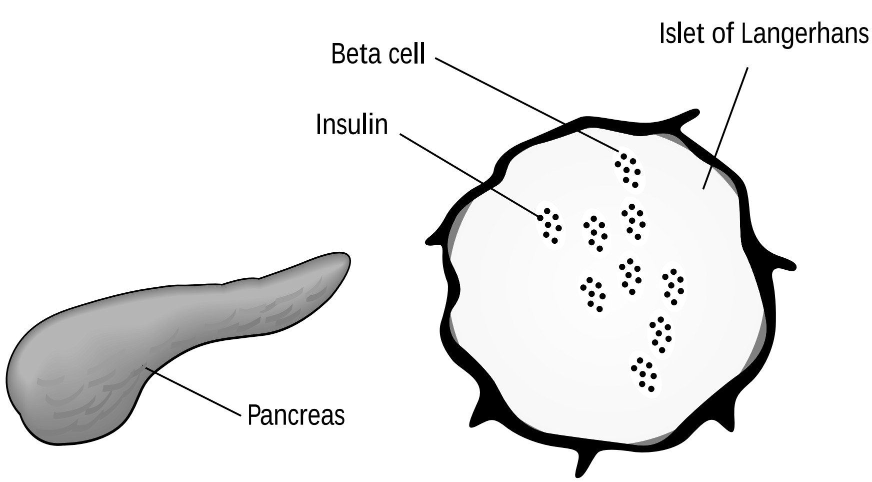
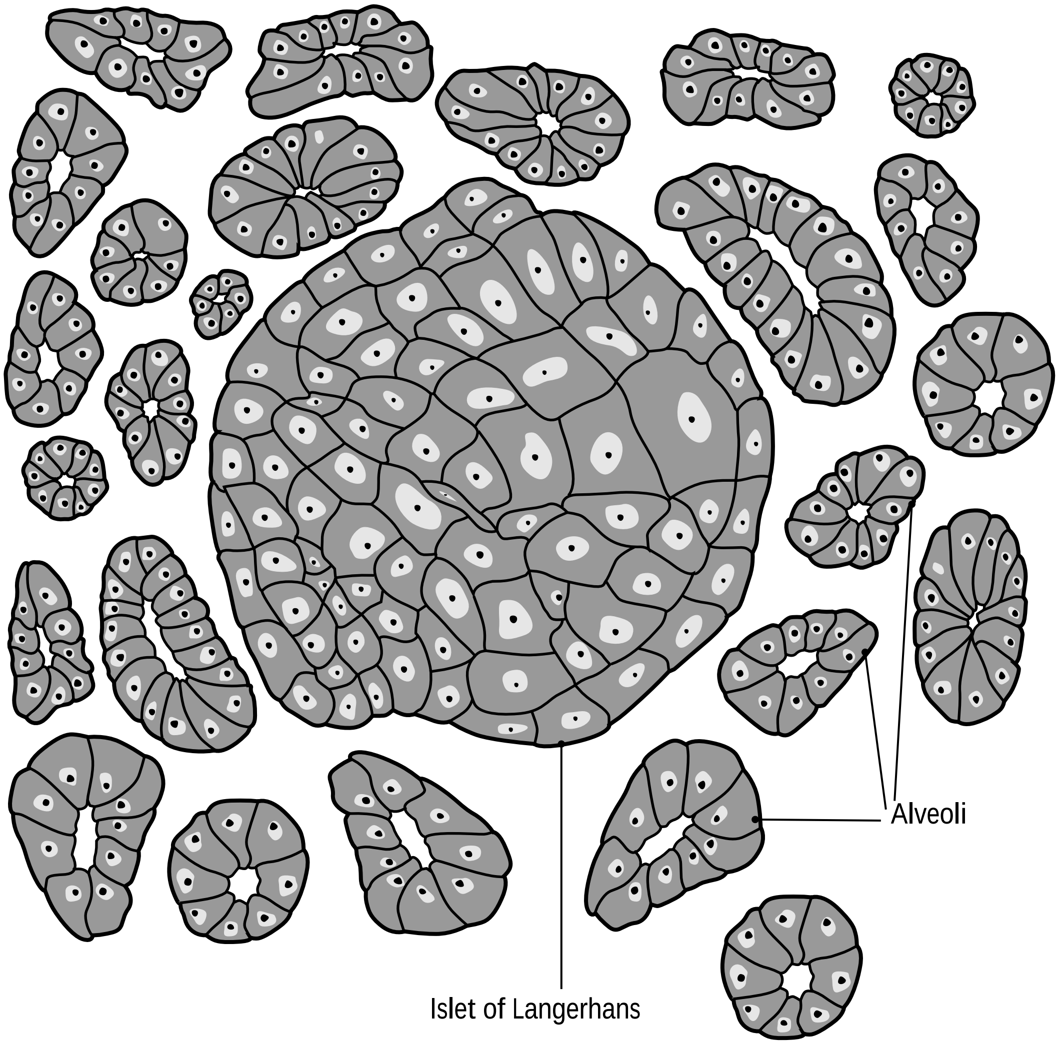
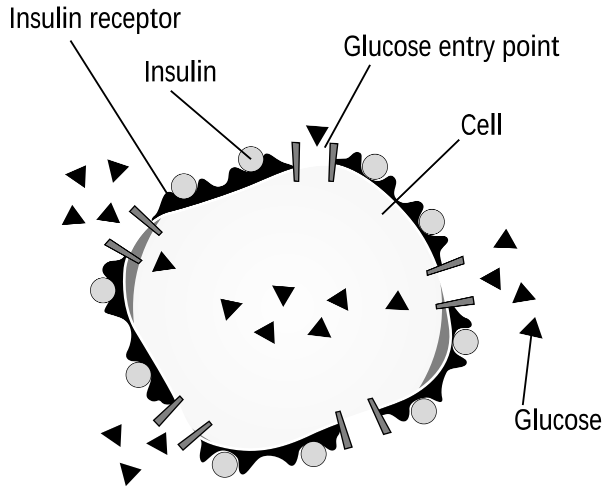


Figure 3: Microscopic Sections of Pancreas



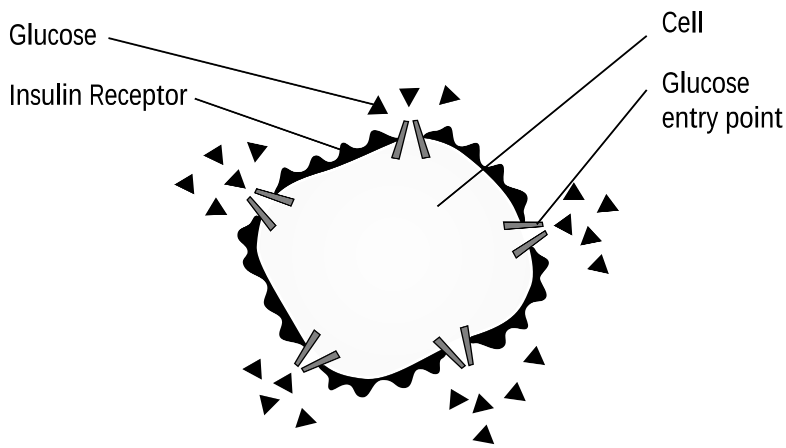
Note: the boundary of this ‘thin’ section region defines the Islet.

Figure 4: Intake of Glucose Into an Islet Cell

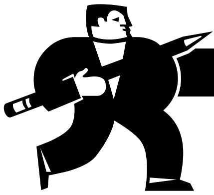


Note: this diagram shows insulin promoting the intake of glucose into an Islet cell.

Figure 5: What Happens if Insulin is Not Produced or if Insulin Does Not Work



Glucose entry denied when:
- receptor does not accept insulin
- insulin is not present
- receptor sites are damaged



Student Activity: Diabetes Research

Student Name: ______________________________ Date: ____________________

1. Research: What is diabetes?

- What is diabetes?
- What are the causes of diabetes?
- What are some of the characteristics of people who have diabetes, for factors such as age, gender and geographic location? Are there some groups of people who have a higher than average incidence of the disease? If so, why might this be?
- What is the number of Canadians with diabetes?
- What is the percentage of Canadians with diabetes?
- How many people have Type I diabetes and how many have Type II? Complete Table 1: Comparing Type I and Type II Incidence by Age Group. Complete Figure 6: Type I and Type II Incidence by Age Group.

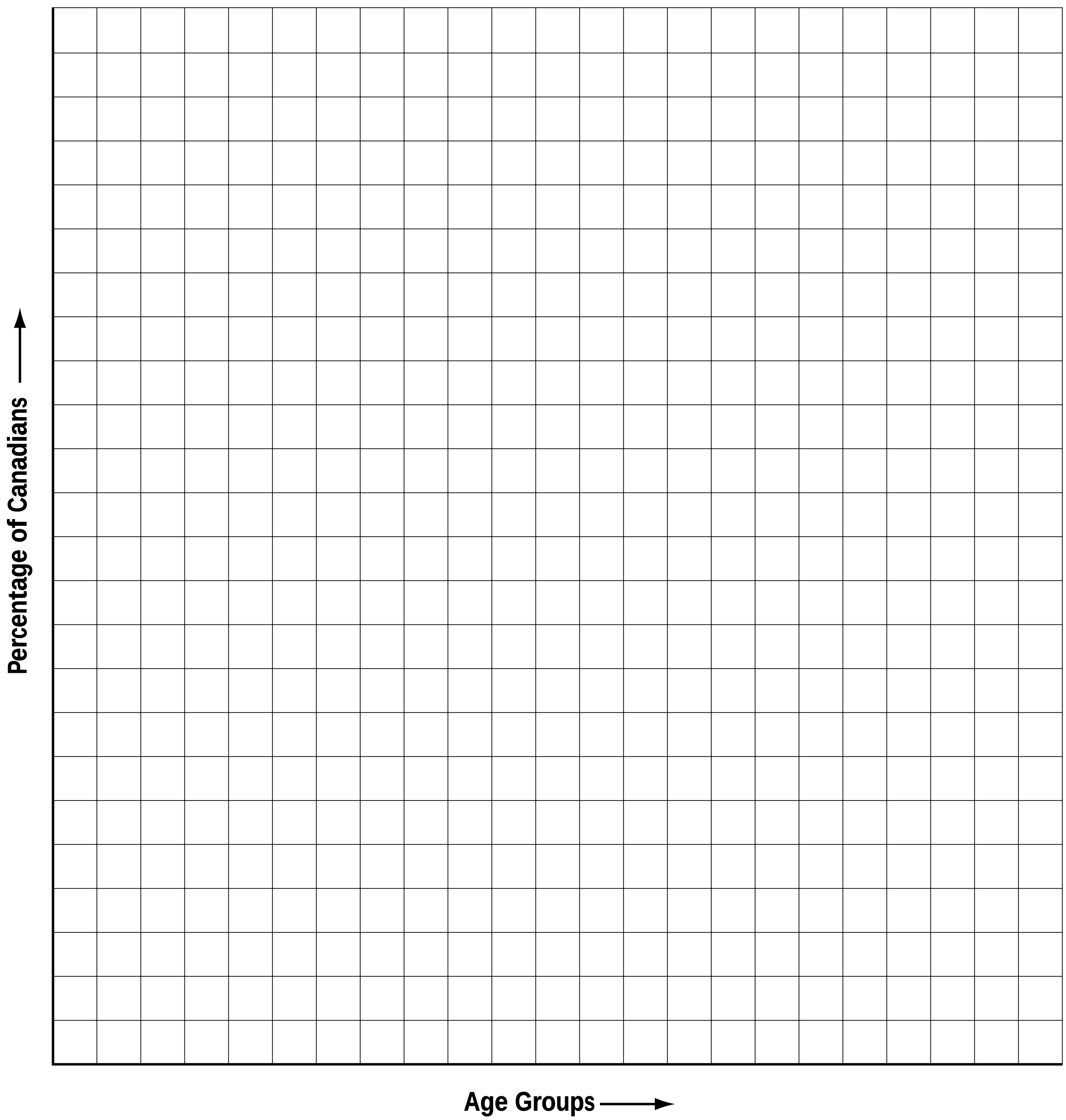
Table 1

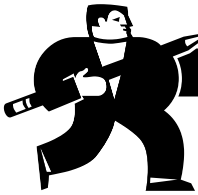
Comparing Type I and Type II Incidence by Age Group

Age Group	Type I Diabetes	Type II Diabetes

Student Activity: Diabetes Research

Figure 6: Type I and Type II Incidence by Age Group





Student Activity: Diabetes Research

- What are the symptoms of diabetes?

- What are the consequences to the individual of having diabetes?

- What are the financial costs to the individual, the province and the nation?

- What are the costs involved in the treatment of diabetes? How do these costs compare to the treatment costs 50 years ago?

- Is there a cure for diabetes? What are some recent developments from medical research?

- What are some of the ways that people cope with diabetes?

- What is the significance of 1-800-BANTING, used as the toll-free number for the Canadian Diabetes Association.

Student Activity: Diabetes Research

Comment on the following terms related to diabetes

- Pancreas:

  Description:

  Dimensions:

  Diagram:

- Islet of Langerhans:

  Description:

  Dimensions:

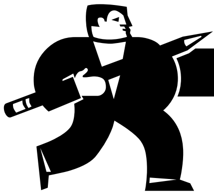
  Diagram:

- Beta cell:

  Description:

  Dimensions:

  Diagram:



Student Activity: Diabetes Research

- Describe the role that Banting and Best played in the history of diabetes research.

- Insulin:
  Definition:

  Chemical formula:

- Glucose:
  Definition:

  Chemical formula:

- Sugars:
  Definition:

  Chemical formula generalized:

- Ketones:
  Definition:

  Chemical formula generalized:

Student Activity: Diabetes Research

- Carbohydrates:
  Definition:

Chemical formula generalized:

- Type I Diabetes:
  Definition:

- Type II Diabetes:
  Definition:

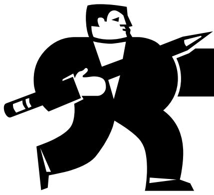
2. What are the complications of diabetes?

- Low blood sugar (Hypoglycemia)
  Causes:

  Signs of mild hypoglycemia:

  Ways to treat low blood sugar:

  Ways to avoid low blood sugar:



Student Activity: Diabetes Research

- High blood sugar (Hyperglycemia)
  Causes:

  Signs of hyperglycemia:

  Ways to treat high blood sugar:

  Ways to avoid high blood sugar:

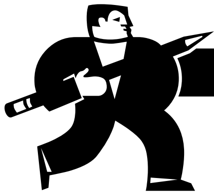
- Ketoacidosis
  Causes:

  Signs of mild ketoacidosis:

  Ways to avoid ketoacidosis:

  Ways to treat ketoacidosis:

  Description of long term complications:



Student Activity: Diabetes Research

3. Statistics

Name and describe any two variables related to diabetes.

Describe the kind of relationship you predict between the two variables for a given set of conditions. Explain.

Represent their relationship to each other in a diagram.

Complete Table 2: Values of the Two Variables, by recording the values of the two variables for example, over time. Properly identify the units of each of the variables.

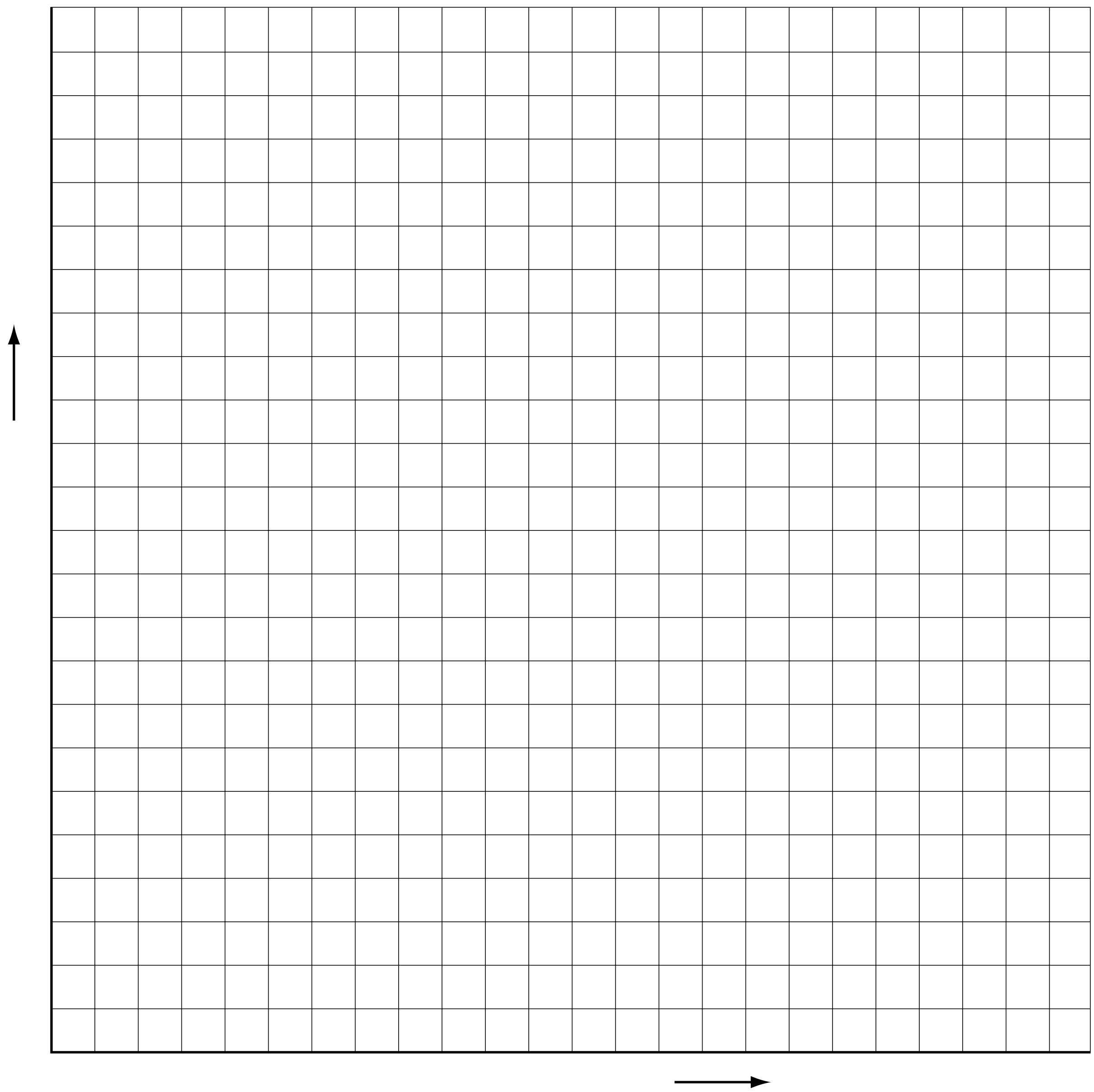
Table 2

Values of the Two Variables

Variable #1	Variable #2

Student Activity: Diabetes Research

Create a scatter graph in the space below for the two variables and analyse the relationship. Were your predictions correct?



Phase II: Tools for Monitoring Diabetes

Activity 1: Fuel

Objective:

Students perform operations with rational numbers and powers. They develop simple linear models representing two competing processes in order to develop their understanding of the coefficients of a polynomial model representing a real situation.

Materials:

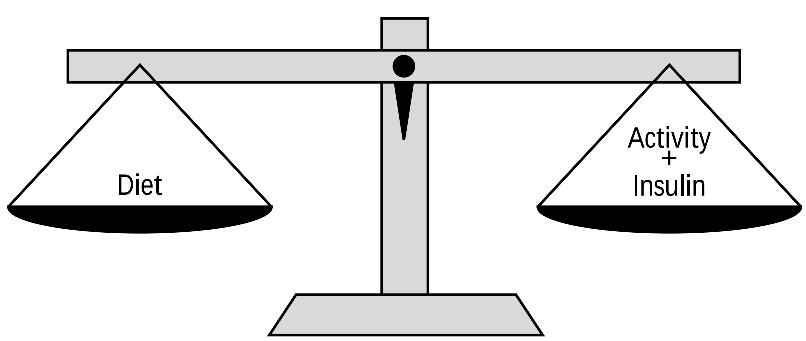
- information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centres
- Student Activity: Fuel
- graphing calculator or mathematics software for example Mathcad and Mathematica
- access to a nurse, physician, dietitian, or to a student with diabetes
- copies of diabetes diaries
- Good Health Eating Guide Poster or equivalent

Procedure:

Students learn about controlling the glucose level in the bloodstream by identifying three factors involved in a relationship when fuel is burned:
1. diet
2. activity
3. insulin.

See Figure 7: Balancing Act below.

Figure 7: Balancing Act



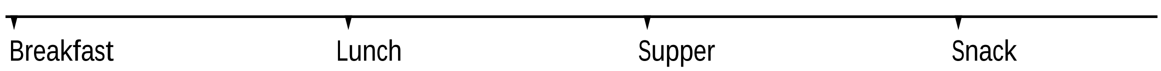
These factors contribute to the fluctuations in the blood glucose level. People with Type I diabetes need to adjust the glucose level by manipulating these three factors. To do this they must understand the effect and contribution of each of the factors.

Have students use the 24-hour timeline (see Figure 12: 24-Hour Timeline in Student Activity: Fuel) to show what they expect the glucose level to be in the blood of a normal person under normal conditions with the proper amount of insulin. This is called a qualitative estimation, since students can only estimate whether the level is higher or lower.

Ask students to include meals, sleep time, exercise, and stress on a timeline and to estimate the effect these factors have on the glucose level.

For example, in Figure 8: Typical Eating Pattern for Individuals in the Human Population below, it is apparent that there is a regular interval of food intake. In Figure 9, Blood Glucose (mmol/L) Normal Pattern, the curve represents a typical pattern for blood glucose. Comment upon the fluctuations during the day.

Figure 8: Typical Eating Pattern for Individuals in the Human Population



Tell students that the blood glucose level is expressed in mmol/L. This unit is used by doctors and nurses when referring to diabetes and concentrations of some blood constituents. (One mmol, millimole, is 0.001 of one mole.)

Tell students that one mole of glucose weighs 180 g (if applicable, refer to the Activity 2: Controlling Diabetes for the chemical properties of glucose and for the concept of mole). Tell students that chemists express concentrations using a unit called the mole (mol) instead of grams.

Indicate that the mole of a substance is the number of elementary particles (6.023×10^{23}) of that particular substance. Have students perform some calculations using the concept of moles. For instance, what is the number of moles of one marble when one marble weighs 10 g? What is the number of moles in a rice grain when one grain of rice weighs 1 g? What is the number of moles in a grain of sand when one grain of sand weighs 0.1 g?

Before eating, the normal level of blood sugar (glucose) is 4-6 mmol/L. Eating raises the level (1 or 2 hours after a meal, the blood sugar can go up to 8mmol/L); exercising lowers the level because it utilizes glucose (energy). After a meal, the blood glucose level increases. When this happens, insulin goes from the pancreas to the blood to help the extra glucose enter the liver's cells. Figure 10: Insulin Rate (in U/h) Normal Response, shows how the body sends extra insulin into the blood to look after the sugar that has been absorbed into the bloodstream after a meal.

Figure 9: Blood Glucose (mmol/L) Normal Pattern

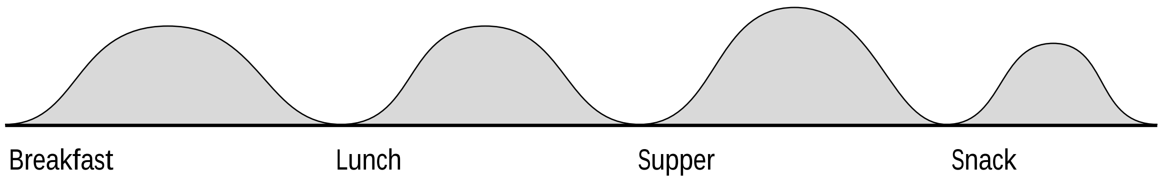
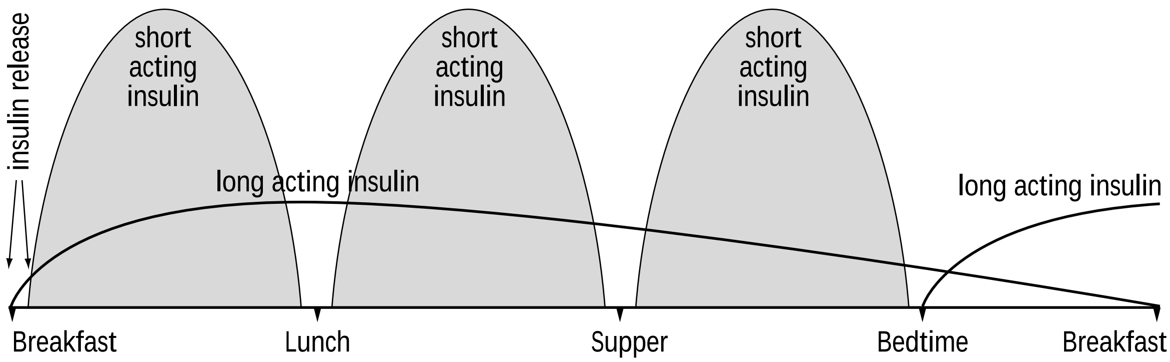


Figure 10: Insulin Rate (in U/h) Normal Response



Ask students to determine the amount of glucose (in g) normally present in one litre and in 10 litres of blood. (Ten litres is the average volume of blood in adults.)

180 g is the weight of 1 mole of glucose

0.18 g is the weight of 1 mmol of glucose

0.18 x 4-6 mmol/L = 0.72 g to 1.08 g of glucose in 1 litre of blood

or 7.2 g to 10.8 g of glucose in the entire circulatory system.

Provide students with the following problem about burning calories.

When one mole of glucose (that is, 180 g) is burned by oxygen in cells, the process releases 2820 kJ (kilojoules) of heat energy in the form of chemical bonds of ATP.

If your glucose reading rises to 8 mmol/L within the next hour after breakfast and comes down to 4 mmol/L after 3 hours, how many calories did you store in your liver as glucose?

Students should represent the situation on a graph to make sure they understand the situation and then make the unit conversions.

From the above example, 0.18 g of glucose is the weight of 1 mmol so,

0.18 g/mmol x 8 mmol/L = 1.44 g/L.

In 10 L, one has 1.44 g/L x 10 L = 14.4 g of glucose present in the entire system after 1 hour.

After 3 hours, half of the glucose remains, that is, 7.2 g and, therefore, 7.2 g of glucose has been stored. How much energy does this represent?

Note that: 1 Joule = 0.23901 calories
1 kJ = 239.01 calories
1 calorie = 1/0.23901 J
= 4.184 J

As indicated earlier 180 g releases 2820 kJ or
2820 x 239.01 cal = 674 008.2 calories or
674 kcal, therefore,
7.2 g releases 674 kcal x 7.2 g/ 180 g = 27 kcal of energy when burned.

Discuss the results with your students in relation to the daily amount of energy required by an average adult (1500 kcal at rest in a warm room to 3000 kcal for a person doing average work) .

Ask students:
- "Why are the results so low?"
- "Does the result make sense?"

You can provide them with some clues to the answer:
- the process is continuous
- the glucose is only one source of energy; other carbohydrates, fats and proteins also produce energy while they are metabolized
- glucose is at the end of the chain of decomposition of carbohydrates; intermediate steps already produced energy.

These values are approximately the same as the amount of energy consumed by a 100-watt light bulb operating for a 24-hr period.

Table 3, below, shows the fuel value of some common foods.

Table 3

Fuel Value of Some Common Foods

Food	Protein (grams)	Fat (%)	Carbohydrate (%)	Fuel Value (kcal/g)
Apples	0.4	0.5	13	0.64
Beer	1.0	0	4.0	4.8
Bread (white, enriched)	9.0	3.0	32.0	2.8
Cheese (cheddar)	28.0	37.0	4.0	4.7
Eggs	13.0	10.0	0.7	1.6
Fudge	2.0	11.0	81.0	4.1
Green beans	1.0	0	5.3	2.2
Hamburger	22.0	30.0	0	3.6
Milk	3.3	4.0	5.0	0.74
Peanuts	26.0	39.0	22.0	5.6
Average Fuel Energy (kcal/g)	**4.1**	**9.3**	**3.8**	

The fuel energy value of carbohydrates is approximately 4 kcal/g of carbohydrate (3.8 in Table 3 above) and the fuel energy value of fats is approximately 10 kcal/g (9.3 in Table 3 above).

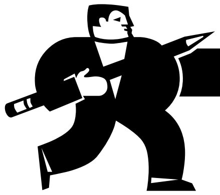
To maintain body weight, a normal active, healthy adult must take in about 130 kJ of food energy per kilogram of body weight per day. Have students prepare a daily diet model (Student Activity: Modelling the Blood Sugar Level) and compare the data in Table 3: Fuel Value of Some Common Foods, above with the required number of calories to maintain their body weight. Make sure that students first convert the energy from kJ to kcal.

Tell students that although exercise is great for improving tone and firming up sagging tissues, it is not an effective way to lose weight. A one hour brisk walk over average terrain uses only about 700 kJ of stored energy. This is about one quarter of the energy contained in a quarter-pound hamburger in a bun.

Note: the popular term calorie is actually a kilocalorie and is sometimes written calorie (1000 cal = 1 Cal).

Have students complete the Figure 11: Blood Sugar Level Over Time, in Student Activity: Modelling the Blood Sugar Level in a more rigorous fashion from the data students have provided through their own research. Insist that only carbohydrates are considered in glucose production and that the lower part of the graph showing the input of insulin (time and quantity) be included. Tell students that insulin is measured in Units of insulin per hour (U/h). As a rule, a person needs about 0.3 U of insulin per kilogram of weight per day. For example, a 50 kg person would need 15 Units a day.

Note: if students are unable to locate blood glucose data, then make it available to them. They should be encouraged, however, to research plausible sources.

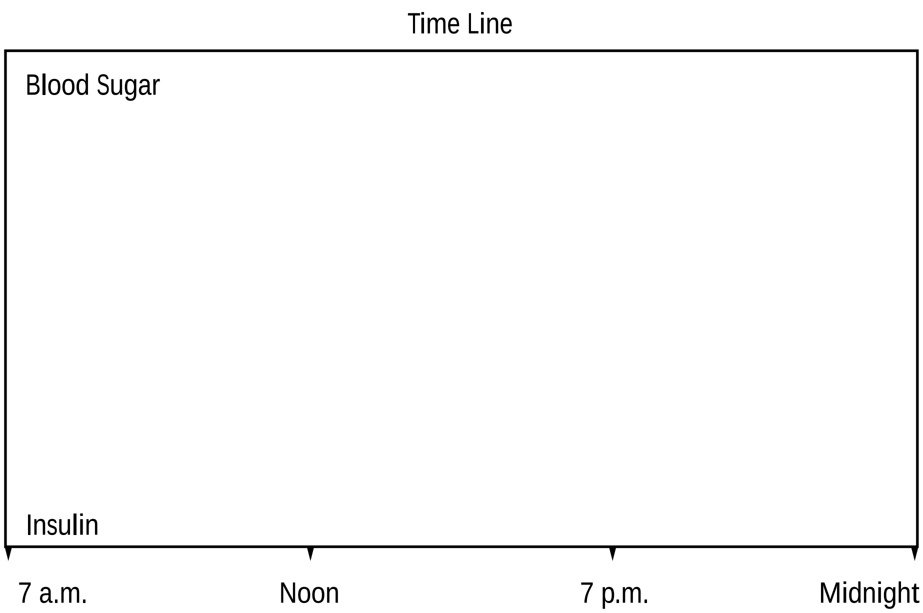


Student Activity: Fuel

Student Name: ____________________________ Date: ___________________

In Figure 11: Blood Sugar Level Over Time, below, use a pencil to sketch your data regarding changes in blood glucose level in relation to meals and activities. Represent the situation as accurately as possible. At the base of the timeline show the changes in the insulin input that are needed to take care of the sugar. The scales do not need to be precise.

Figure 11: Blood Sugar Level Over Time



How many grams of glucose are there in the blood (10 litres in all the body) when the reading of the blood sugar indicates 5.5 mmol/L?

1 mol of glucose weighs _________ g
1 mmol of glucose weighs _________ g
In one litre, 5.5 mmol of glucose weighs _________ g
In 10 litres, there are _________ mmol glucose and _________ g glucose

How many grams of glucose have been burned and transformed into energy between each of the two readings in Table 4: Amount of Glucose Burned below? Again assume a 10 litre volume of blood in the body.

Student Activity: Fuel

Table 4

Amount of Glucose Burned

#	Reading 1	Reading 2	Grams of Glucose Burned
1	8.4 mmol/L	4.8 mmol/L	
2		4.8 mmol/L	6.0 g
3	10.4 mmol/L		8.4 g

For each of the three situations above, estimate the amount of energy produced and express this in kJ and kcal in Table 5: Amount of Energy Produced.

Table 5

Amount of Energy Produced

#	Grams of Glucose Burned	Energy (kJ)	Energy (kcal)
1			
2			
3			

Use the Good Health Eating Guide Poster or equivalent to plan a daily diet. Use the above data to prepare a 24-hour timeline indicating only the blood sugar level contribution of each of the sources of carbohydrates expressed in mmol/L. You will use this graph later to prepare an insulin regimen.

First, estimate the weight of blood sugar produced by 10 g of carbohydrate, and then, convert this into the number of mmol/L. When you have completed the conversions, complete Table 6: Amounts of Blood Sugar Produced.

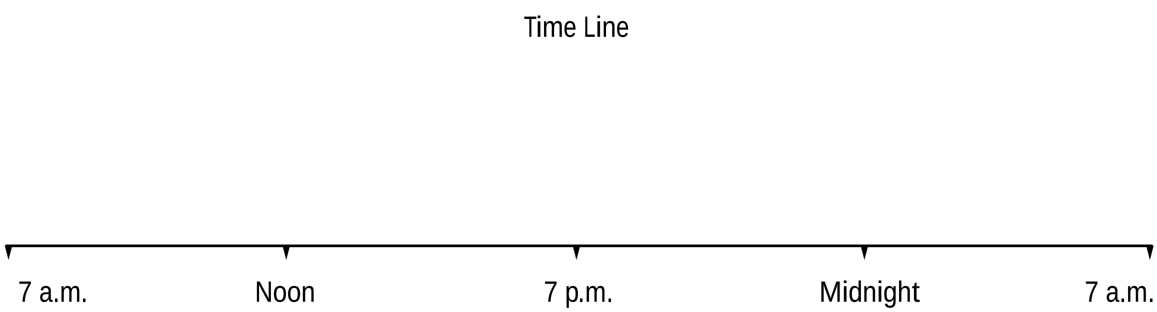
Student Activity: Fuel

Table 6

Amounts of Blood Sugar Produced

Time						
Time						
Starch						
Fruits						
Milk						
Sugars						
Total mmol/L						

Figure 12: 24-Hour Timeline



Activity 2: Controlling Diabetes

Objective:

The students develop simple linear models representing two competing processes in order to develop their understanding of the coefficients of a polynomial model representing a real situation.

The students solve problems related to insulin deficiency by using their knowledge of patterns and relations. They propose insulin regimens and solve problems related to insulin adjustment in cases of low and high readings, illness, exercise and special circumstances.

Materials:

- information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centres
- Student Activity: Modelling the Blood Sugar Level
- Student Activity: Planning the Day
- graphing calculator or mathematics software for example Mathcad and Mathematica
- access to a nurse, physician, dietitian, or to a student with diabetes
- copies of diabetes diaries
- copy of Good Health Eating Guide poster or equivalent

Procedure:

Ask students why people with diabetes regularly monitor their blood glucose (before meals and at bedtime, 1 to 4 times a day).

Diabetics generally use a diary to record their blood glucose levels along with insulin doses and comments. Comments might include information about delayed meals, extra snacks or having the flu.

Provide students with anonymous diary information from a diabetic source where blood glucose levels have been monitored during a normal day. They must record the measurements on the timeline in Student Activity: Modelling the Blood Sugar Level. They also need to identify and record times of meals, snacks, and exercise on the timeline.

Modelling Blood Sugar Level

Provide students with examples in Student Activity: Modelling the Blood Sugar Level and have them interpret each of the curves. These examples represent different patterns of variation in blood sugar at different times throughout the day. Students are asked to find the equation for each curve.

For example, in Example 1 the blood glucose level at 7 a.m. is 5 mmol/L. Blood glucose is identified below by the letter g. The function represented by an equation would read:

$g = 5$ (for $t = 7$)

In Example 2, the blood sugar level increases from 5 mmol/L at 7 a.m. to 7.5 mmol/L at 12 p.m., a period of 5 hours. Have students determine that the equation is:

$g = 0.5\ t + 5$ (for $t > 7$and $t \leq 12$) or ($7 < t \leq 12$)

In Example 3, the level decreases from 7.5 mmol/L to 5.5 mmol/L over a period of 4 hours. The equation should read:

$g = -0.5t + 7.5$ (for $t > 12$ and $t \leq 4$) or ($12 < t \leq 4$)

In Example 4, the level increases from 5.5 mmol/L to 8.5 mmol/L within an hour. The equation should read:

$g = 3t + 5.5$ (for $t > 4$ and $t \leq 5$) or ($4 > t \leq 5$)

Ensure that students understand the significance of the coefficients of the independent variable; that is, negative means decrease, positive means an increase in glucose level; the larger the coefficient is, the bigger the increase (input).

Have students compare available readings to these observations and then comment on the differences in terms of meals, snacks and exercise.

Have students solve the problems in the Student Activity: Modelling the Blood Sugar Level. In these problems they must add or subtract two competing processes, that is input and output.

Modelling Insulin Release

Have students set a target range for blood glucose control from their 24-hour timeline glucose level established in the previous activity. They should assume that the body is not releasing any insulin into the bloodstream. The recommended target is 4 to 7 mmol/L of glucose.

Distribute some insulin manufacturer's pamphlets describing how insulin works. There are several types of insulin having different types of action. See Figure 13: Action Times of Insulin.

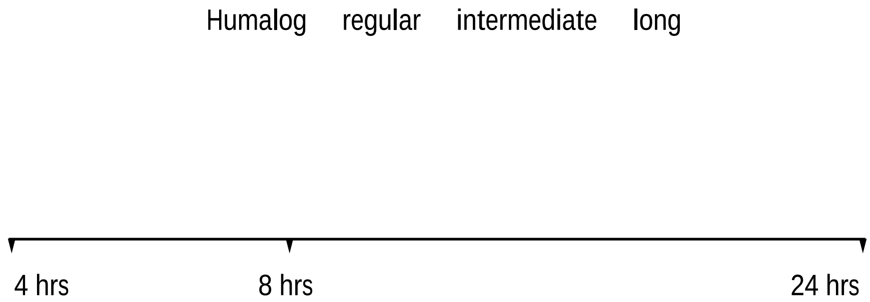
Rapid acting insulin (Humalog or Lispro) begins to work 5 to 15 minutes after injection, peaks in one hour and has a duration of 3 to 4 hours.

Short acting insulin (Regular or R or Toronto insulin) begins to work one-half hour after injection, peaks in 2 to 4 hours and has a duration of 6 to 8 hours.

Intermediate acting insulin (also N or NPH or L-insulin) begins to work 1 to 3 hours after injection, peaks in 6 to 12 hours and has a duration of 18 to 24 hours.

Long acting insulin (or U-insulin) begins to work 4 to 6 hours after injection, and has a duration of 24 to 28 hours.

Figure 13: Action Times of Insulin



Have students determine equations representing each of the four types of functions for insulin. To do this they need to simplify the model to a linear one by doing the following:

- time 0 to the time insulin begins to work: constant level zone
- time insulin begins to work to peak: increasing zone (linear)
- time insulin stops increasing to time insulin stops working: decreasing zone (linear)

For example, where the release of insulin is denoted by I, the rapid acting insulin could be modelled by the equations shown below.

$I = 0$ (for $t \geq 0$ and $t \leq 0.4$) or ($0 \leq t \leq 0.4$)

$I = 40/3\ t$ (for $t \geq 0.4$ and $t \leq 1$) or ($0.4 \leq t \leq 1$)

$I = -10/3\ t$ ($t \geq 1$ and $t \leq 4$) or ($1 \leq t \leq 4$)

40/3 represents a rate of increase in the insulin level.
-10/3 represents a rate of decrease in the insulin level.

Provide students with some typical insulin regimens like those illustrated in Figures 14, 15, and 16 below and have them comment on the patterns in relation to the increase in blood sugar (input).

Figure 14: One Injection Regimen

One Injection: regular and intermediate acting insulin

Breakfast blood glucose test reflects the tail-end action of the intermediate acting insulin given the day before

Lunch blood glucose test reflects the action of the regular insulin

Supper blood glucose test reflects the action of the intermediate acting insulin

Bedtime blood glucose test reflects the action of the intermediate acting insulin

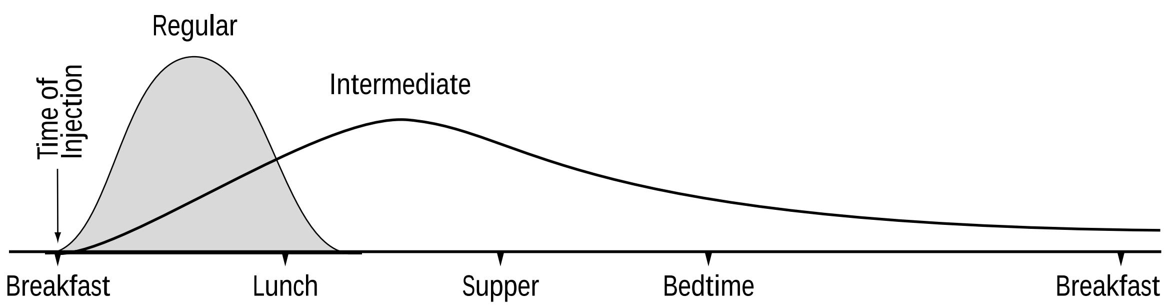


Figure 15: Two Injection Regimen

Two Injections: regular and intermediate acting insulins

Breakfast blood glucose test reflects the action of the intermediate acting insulin given the evening before

Lunch blood glucose test reflects the action of the morning regular insulin

Supper blood glucose test reflects the action of the morning intermediate acting insulin

Bedtime blood glucose test reflects the action of the supper time regular insulin

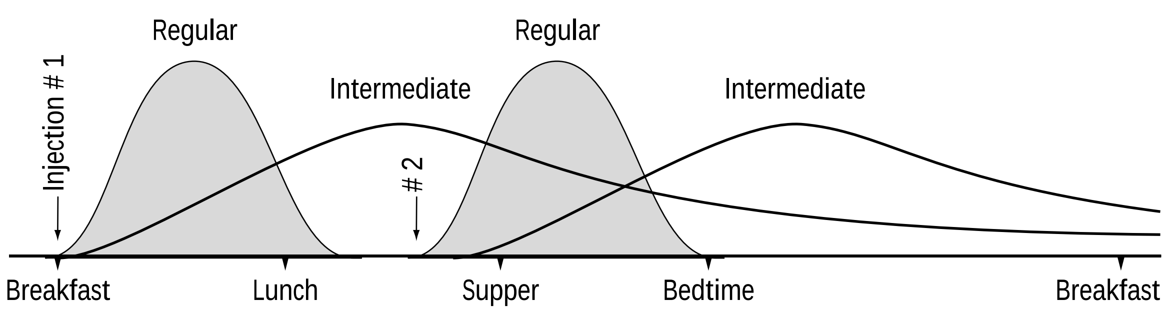
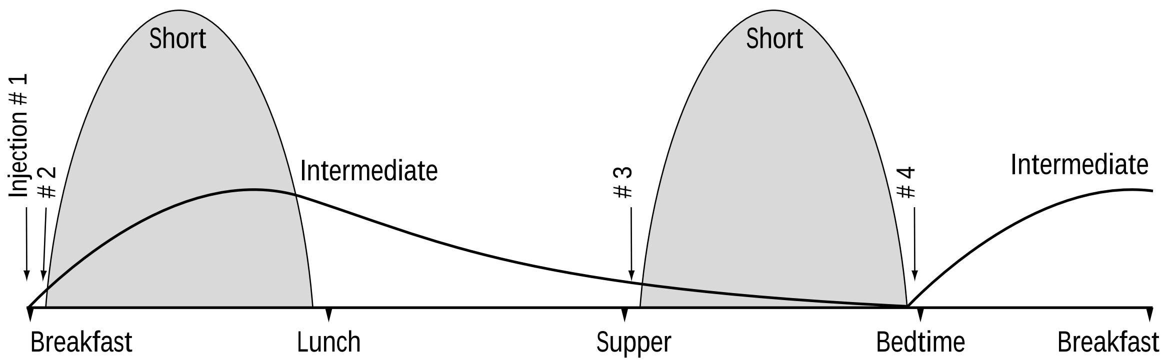
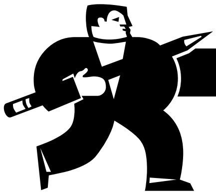


Figure 16: Four Injection Regimen



Have students prepare (Student Activity: Planning the Day) an insulin regimen from the blood sugar pattern they have prepared in the former activity in order to maintain the glucose level in the 4-7 mmol/L range all day (24 hours). Provide several copies of the Student Activity: Planning the Day to each student so that they can propose different options for insulin regimens.



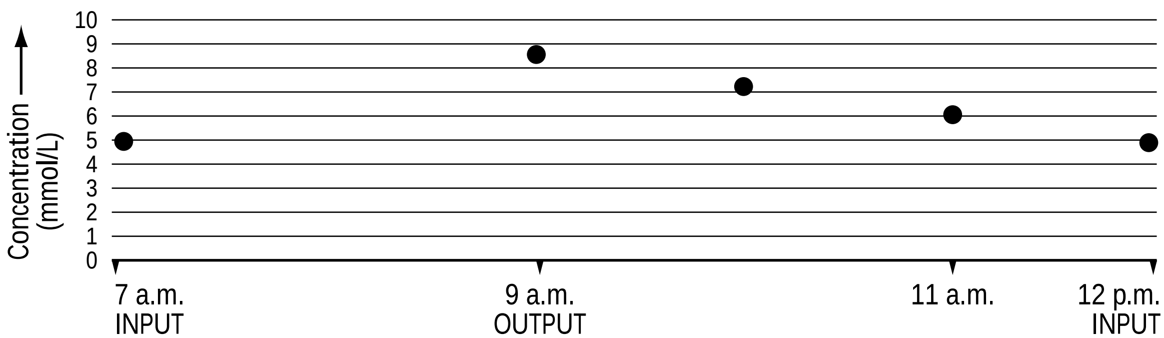
Student Activity: Modelling the Blood Sugar Level

Student Name: ______________________ Date: ________________

Determine the mathematical model that represents the evolution of blood levels sugar in the eight activity pages that follow. Use the example below to help determine each model you create.

Example of Modelling the Evolution of Blood Sugar

Data:
Before 7a.m., the reading is 5.0 mmol/L
(INPUT) Breakfast is at 7a.m.
The reading at 9 a.m. is 8.5 mmol/L
(OUTPUT) At 10 a.m., the reading is down to 7.25 mmol/L
At 11 a.m., the reading is 6.0 mmol/L
(INPUT) At lunch time (12 p.m.), the reading is 5.0 mmol/L
These data points are plotted below:



Equations:
Before 7a.m. : $g = 5$ mmol/L

Between 7a.m. and 9 a.m., the level increases by 3.5 mmol/L over a period of 2 hours that is, 3.5/2 per hour:
$g = 5 + 3.5/2\ t = 5 + 1.75\ t$

Between 9a.m. and 11 a.m. :
$g = 8.5 - 2.5/2\ t = 8.5 - 1.25\ t$

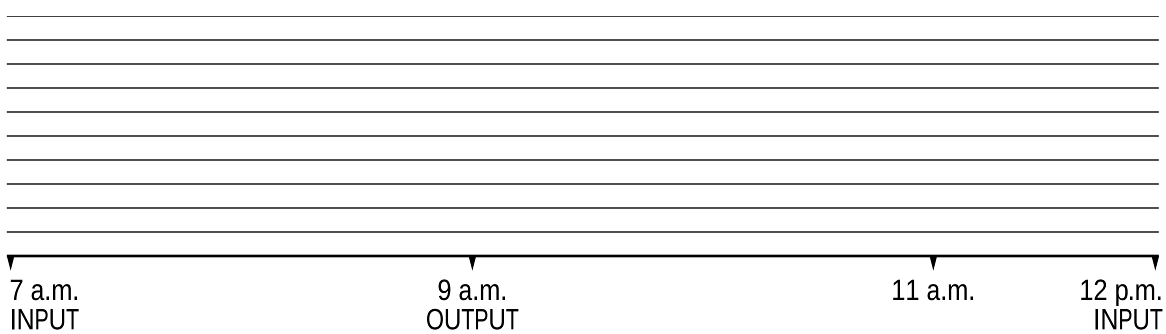
Between 11a.m. and 12 a.m. :
$g = 6 - 1\ t = 6 - t$

Checking:
Use the third equation to calculate the level at 10 a.m.:

$g = 8.5 - 1.25 \times 1 = 8.5 - 1.25 = 7.25$ mmol/L

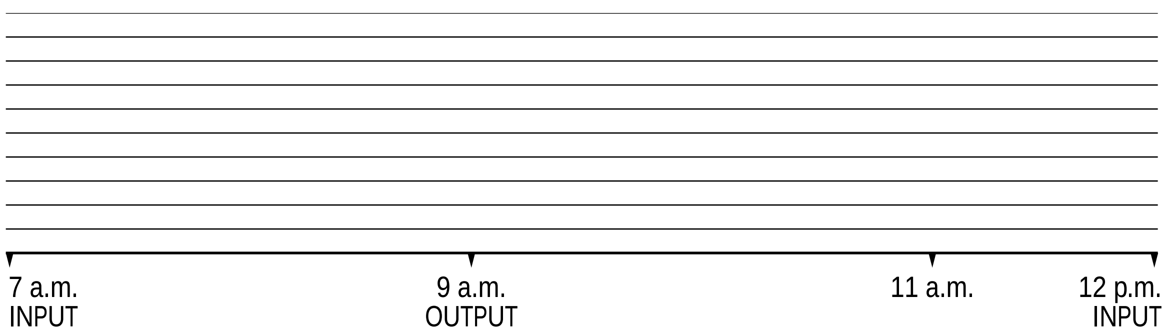
Student Activity: Modelling the Blood Sugar Level

Example Model #1



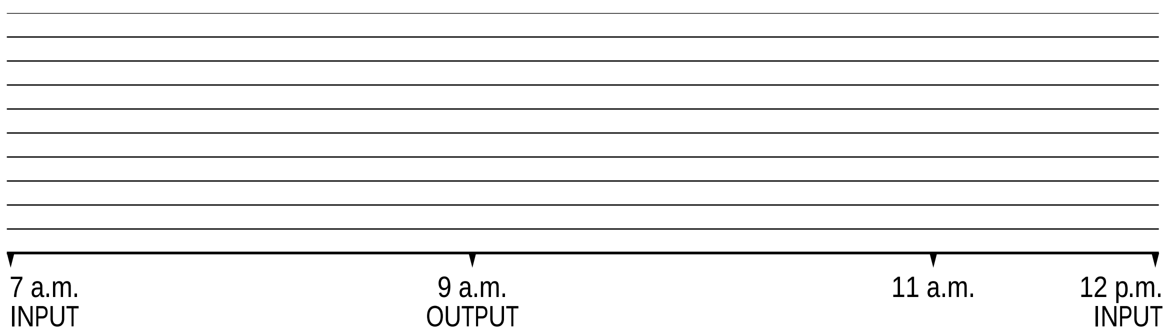
Student Activity: Modelling the Blood Sugar Level

Example Model #2



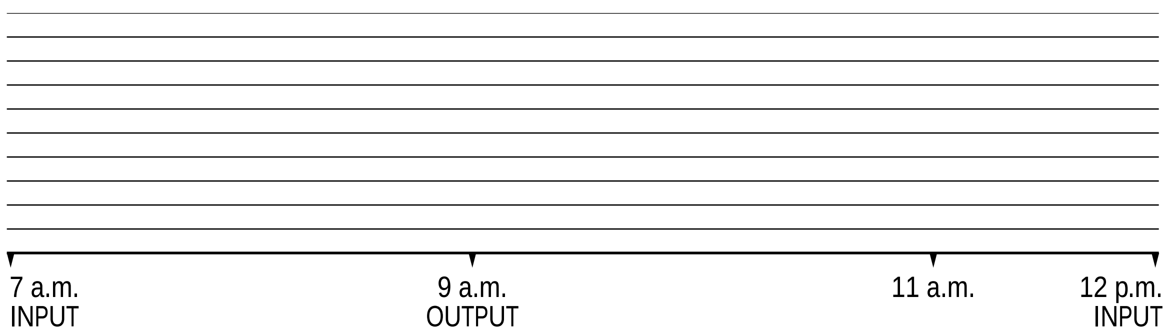
Student Activity: Modelling the Blood Sugar Level

Example Model #3



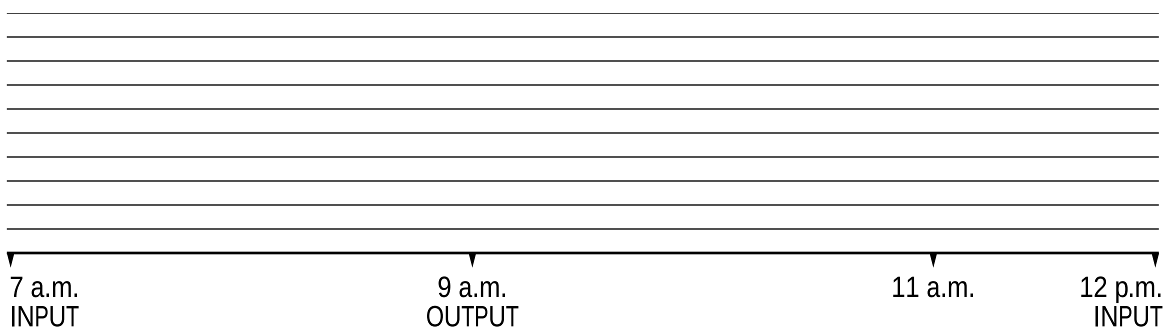
Student Activity: Modelling the Blood Sugar Level

Example Model #4



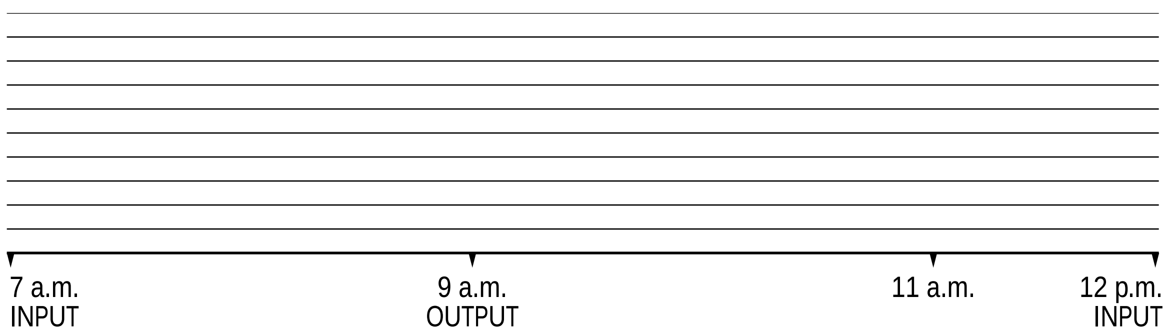
Student Activity: Modelling the Blood Sugar Level

Example Model #5



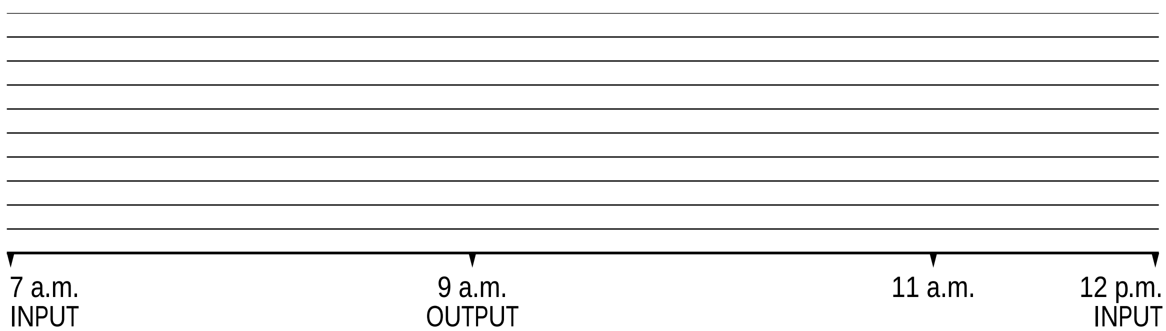
Student Activity: Modelling the Blood Sugar Level

Example Model #6



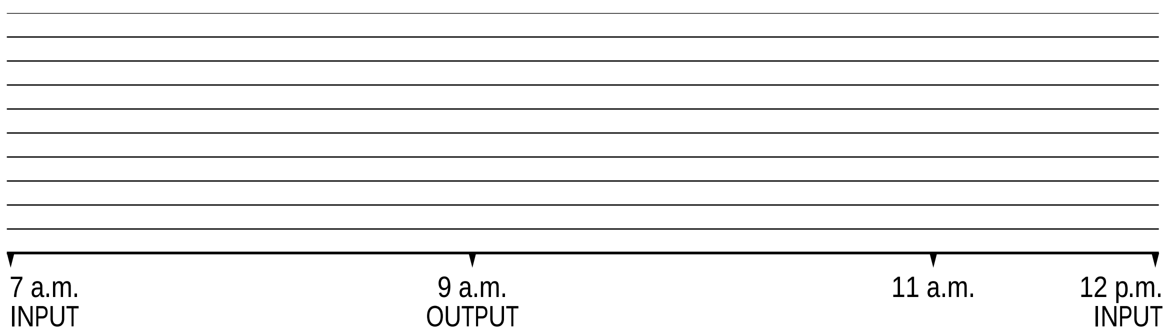
Student Activity: Modelling the Blood Sugar Level

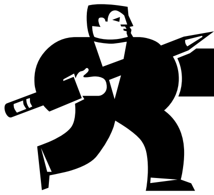
Example Model #7



Student Activity: Modelling the Blood Sugar Level

Example Model #8





Student Activity: Planning the Day

Student Name: ______________________ Date: ________________

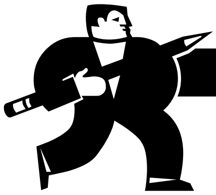
Use the pattern of Student Activity: Fuel showing the changes of blood sugar over a 24-hour period and propose a regimen by combining the different types of insulin. Your target is 4 to 7 mmol/L for the entire period. Suggest different options. Submit your proposals to a nurse or a physician for comments.

Obtain a diabetes diary. These are available from most major drug store chains. Compare this diary to the one provided by your teacher.

Insulin Regimen: Planning a Day

7 a.m.	9 a.m.	11 a.m.	12 p.m.
INPUT	OUTPUT		INPUT

Comments:



Student Activity: Planning the Day

Follow the guidelines below to adjust the insulin:

If the insulin dose is
- less than 8 units, adjust it by 1 unit at a time
- between 8-20 units, adjust it by 2 units at a time
- greater than 20 units, adjust it no more than 10%. For example, if the dose is 30 units, adjust by 3 units.

Adjusting for Low Readings

- A reading of 3.5 mmol/L glucose or less is a low reading.
- Use the chart *What is your Blood Sugar Level?* A low glucose reading is called an Insulin Reaction. An insulin reaction can be treated by having 10 g of carbohydrate (2 teaspoons of sugar or a half cup of juice).
- A low reading can result from a delayed meal, missed portions of food and/or extra activity.
- When unexpected insulin reactions occur, the insulin dose must be decreased.

Adjust the following situation:
- Mary is taking 5 units of regular and 15 units of intermediate insulin in the morning. The glucose readings have been between 4 and 6 mmol/L.
- One day, an unexplained insulin reaction occurred before lunch. What is the solution?

Adjusting for High Readings

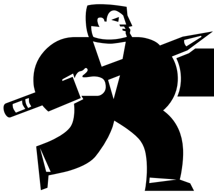
Suggest a solution for each of the following three situations:

Situation #1: Dose:
- 7 units of regular acting insulin and 18 units of intermediate acting insulin in the morning
- 4 units of regular acting insulin and 8 units of intermediate acting insulin before supper

Glucose readings in mmol/L are:

Day	7 a.m.	Noon	6 p.m.	10 p.m.
Monday	8.0	4.8	14.8	9.8
Tuesday	6.9	6.2	11.3	9.4
Wednesday	7.1	7.6	15.9	11.2

Solution:



Student Activity: Planning the Day

Situation #2: Dose:
- 30 units of intermediate acting insulin in the morning
- 18 units of intermediate acting insulin before supper

Glucose readings in mmol/L are:

Day	7 a.m.	Noon	6 p.m.	10 p.m.
Monday	10.9	8.9	16.4	12.3
Tuesday	17.2	12.6	11.1	10.9
Wednesday	15.8	13.2	12.7	14.5

Solution:

Situation #3: Dose:
- 10 units of rapid acting insulin (Humalog) and 32 of intermediate acting insulin in the morning
- 8 units of rapid acting insulin (Humalog) before supper
- 12 units of intermediate acting insulin at bedtime

Glucose readings in mmol/L are:

Day	7 a.m.	Noon	6 p.m.	10 p.m.
Monday	4.8	4.8	12.9	4.2
Tuesday	5.2	8.9	14.5	6.9
Wednesday	8.6	9.2	16.0	5.0

Solution:

Activity 3: Modelling Two Competing Processes

Objective:

The objective of this activity is for students to develop simple linear models representing two competing processes. This helps develop their understanding of the coefficients of a polynomial model representing a real situation. The students solve problems related to insulin deficiency by using their knowledge of patterns and relations. They propose insulin regimens and solve problems related to insulin adjustment in situations of low and high readings, illness, exercise and special circumstance. They extend their knowledge by designing a situation where two competing processes are involved.

Materials:

- information pamphlets obtained from the Canadian Diabetes Association and local Diabetes Education Centers

Procedure:

1. Introduction

Assume that y represents a population and t represents time. An example of an increasing process is one in which the size of population grows, while a decreasing process is one in which the population decays.

When these two processes (both linear) are combined, the overall result is the sum of the two linear models.

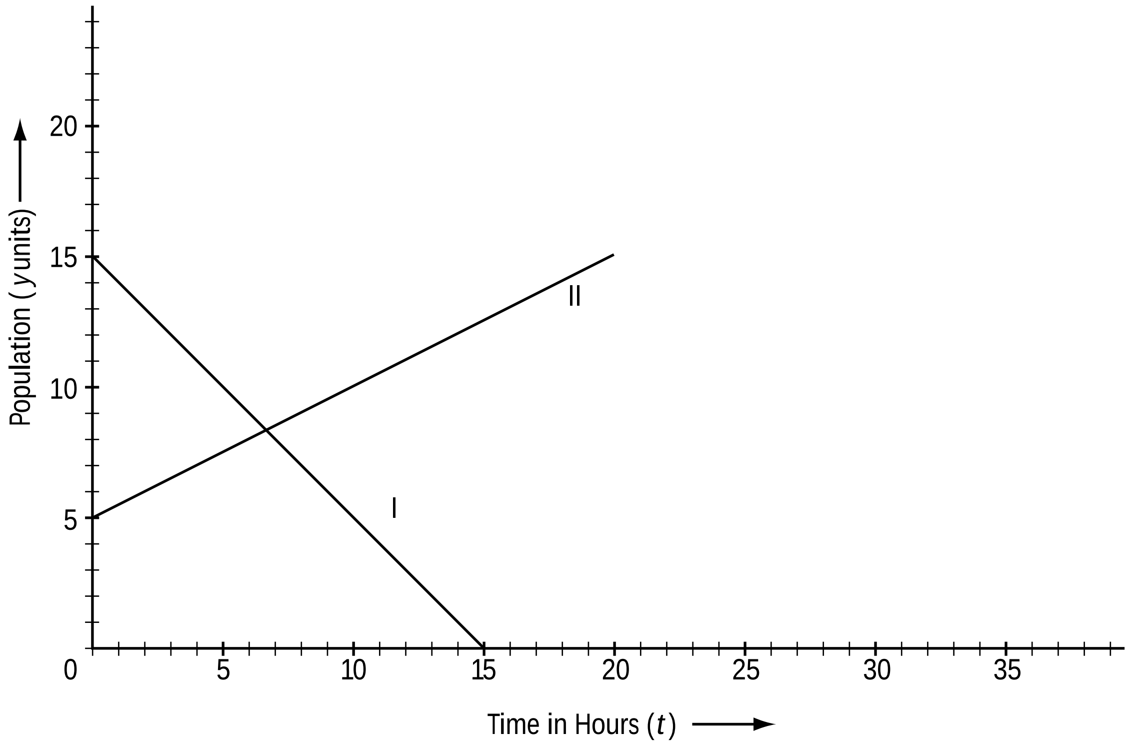
Figure 17: Increasing and Decreasing Processes illustrates two processes:

I: $y = -t + 15$ (decreasing at a rate of -1 U/h from 15)

II: $y = 0.5t + 5$ (increasing at a rate of 0.5 U/h from 5)

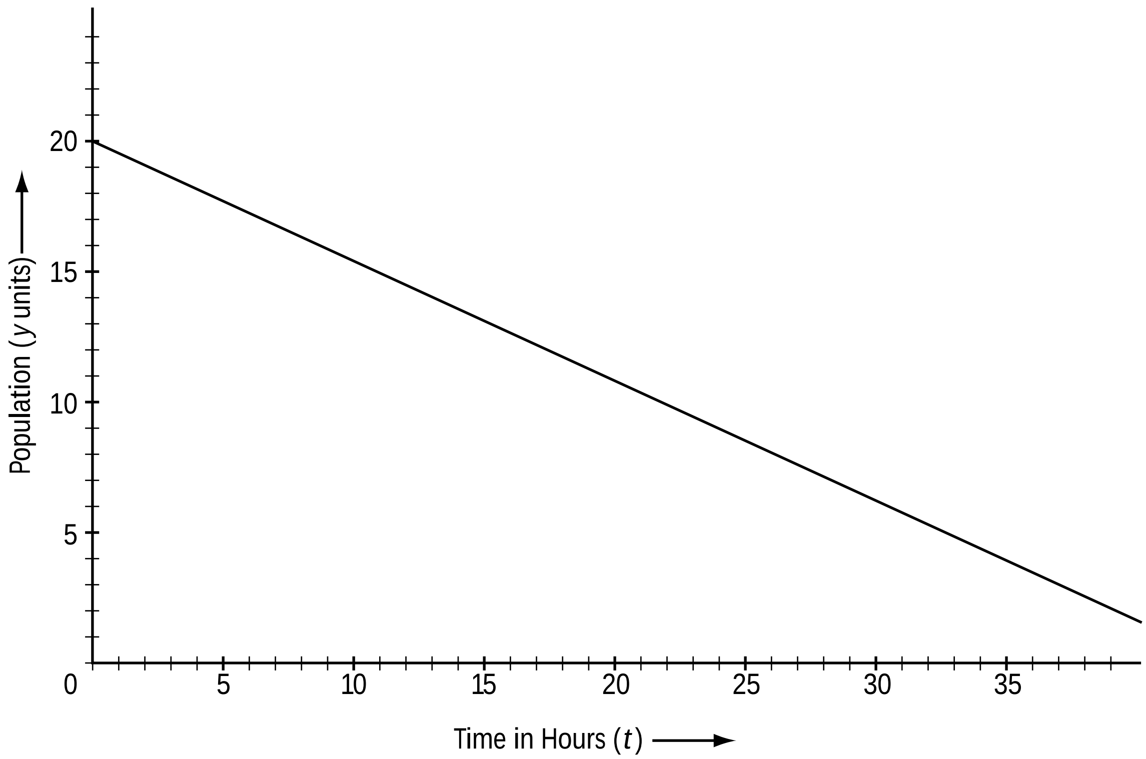
Overall process (Figure 18: Overall Process)
$Y = -t + 15 + 0.5t + 5 = -0.5t + 20$ (decreasing at a rate of -0.5 U/h from 20)

Figure 17: Increasing and Decreasing Processes



At the beginning, the total population is the sum 5 + 15 = 20 units. The overall population varies from the contribution of the two processes, one increasing, the other decreasing. For example, at point $t = 4$, process I provides 11 units and process II provides 7 units for a total of 18 units.

Figure 18: Overall Process



Typical questions:

- When will the population be zero? (Answer: $t = 40$)
- What would be the model of a decreasing process such that the population remains constant all the time? (Answer: the coefficient of t in the decreasing process must be equal and of opposite sign)
- What would be the model for the decreasing process such that the overall population is always increasing? (Answer: anything larger than -0.5)
- What does it mean when a process model has a zero coefficient for t?

2. Deriving the Equation of a Linear Model

A linear model may illustrate the ratio (proportionality) between two quantities, where one quantity varies at a constant rate with respect to the other. A linear model is the simplest way to model a relation between two quantities and is represented graphically by a straight line. Symbolically, the model is represented by the equation $y = mx + b$ where m is the rate of change of y with respect to x, and b is the starting point where the situation is analysed.

For example, the length of a spring at rest is 10 cm. Under the action of a force of 5 units, the length is 15 cm. Assuming that a linear model is appropriate for this situation, what is the equation?

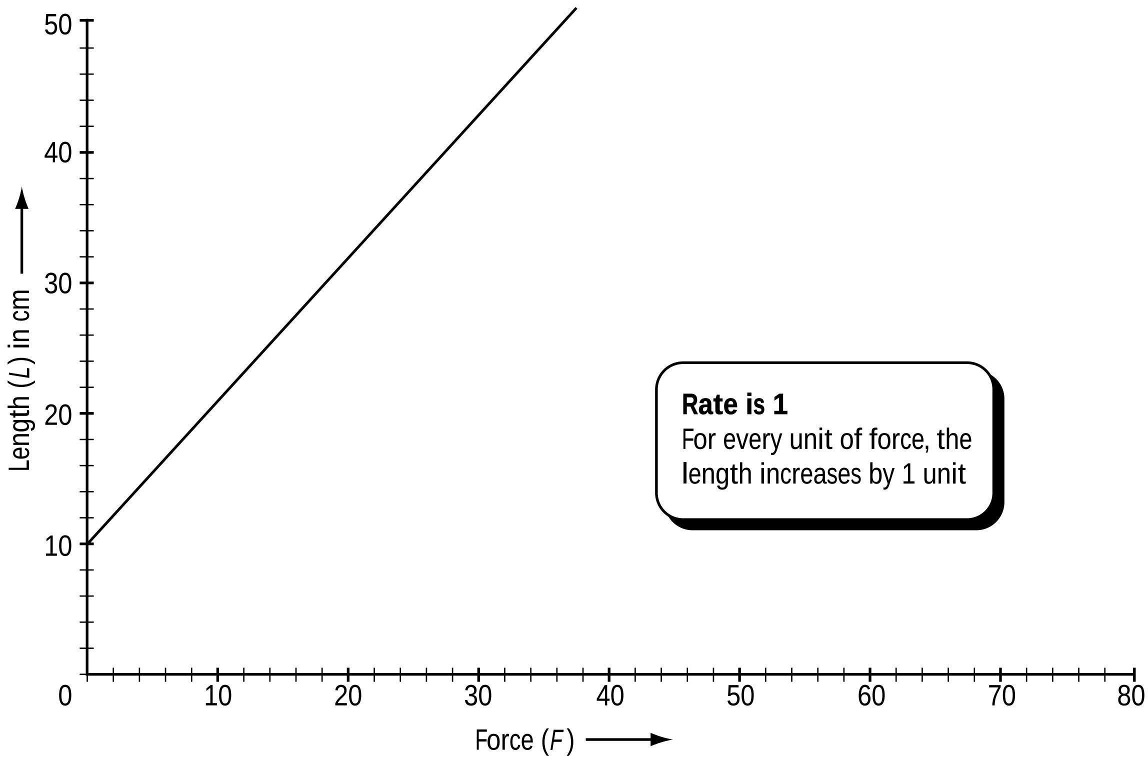
Call L the length of the spring, F the force and R the length of the spring at rest. L is proportional to F. $L = mF$ where m is a constant term. If $F = 0$ (rest), $L = R$. Together, the linear model looks like:

$$L = mF + R$$

Determining m: for $F = 5$, $L = 15$. Substitute to obtain $m = 1$

Now $L = F + 10$. For example, when the force is 20 units, the length is 30 cm.

Figure 19: Graphing the Relationship Between Length and Force



3. Modelling the Action of Insulin Over Time

W is the time at which the insulin begins to work
T is the time it takes for the insulin to reach a peak (from the time of injection)
N is the number of units of insulin at the peak
C is the duration the insulin remains constant at peak
i is the level of insulin in units of insulin
t is the time in hours from the injection of the insulin

The process can be broken down into four phases (see Figure 20: Modelling the Action of Insulin Over Time, Example 1).

Phase I: time W needed for the insulin to be working

Equation: $i = 0, 0 \leq t < W$

Phase II: reaching the peak of N units at a rate m over a period T

Equation: $\mathrm{i} = mt + b$, $W \geq t < T$

At $t = W, i = 0$

At $t = T, i = N$

$$m = \frac{N}{T\text{-}W}$$

$$b = -\frac{NW}{T\text{-}W}$$

$$i = \frac{N}{T\text{-}W}t - \frac{NW}{T\text{-}W}$$

Phase III: remains constant at N during a time C

Equation: $i = N$

Phase IV: decreasing at a rate M

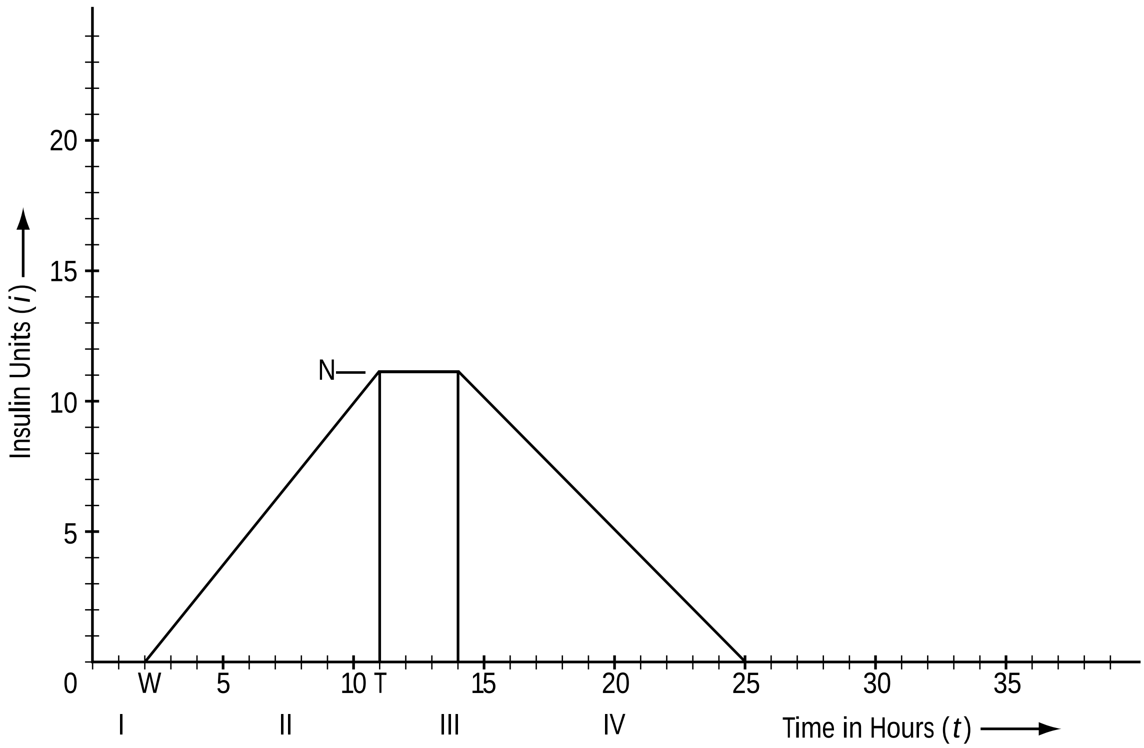
Equation: $i = Mt + d$

At $t = T + C, i = N$

At $t = 2T + C, i = 0$

$$i = -\frac{N}{T}t + \frac{N(2T+C)}{T}$$

Figure 20: Modelling the Action of Insulin Over Time: Example 1

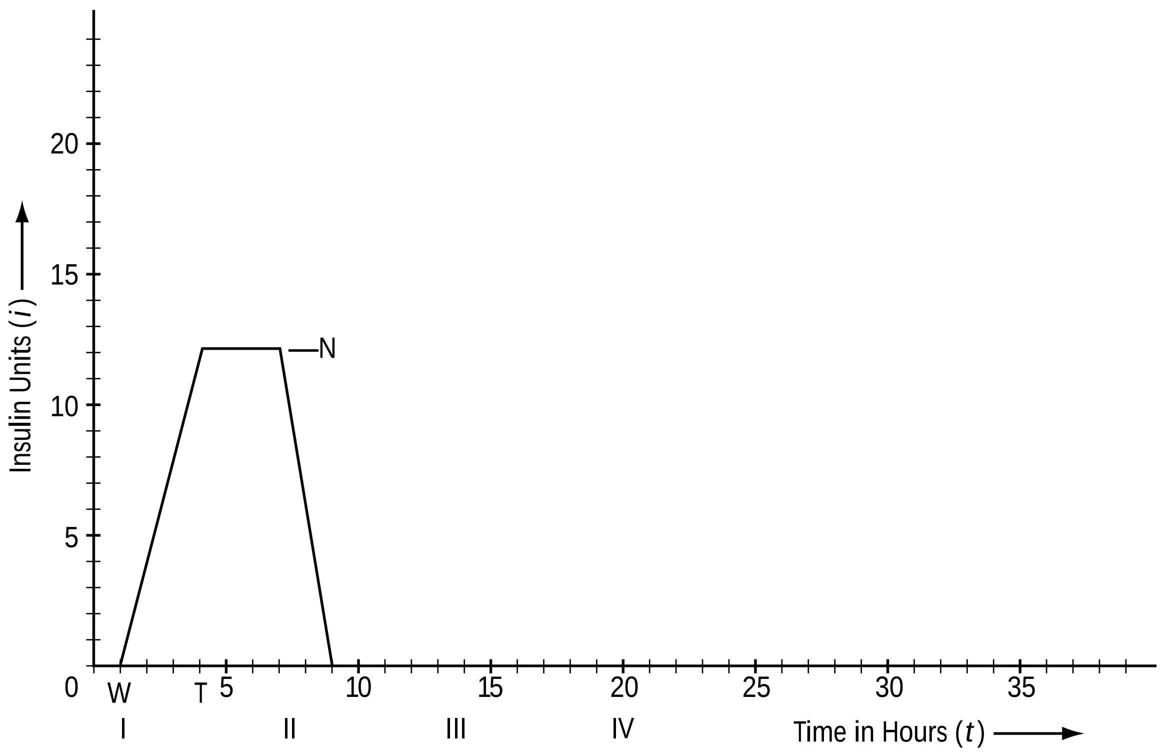


See Figure: 21: Modelling the Action of Insulin Over Time: Example 2.
Example 2:

N = 12 units
W = 1 hour
T = 4 hours
C = 2 hours

Phase I: $i = 0$
Phase II: $i = 4t - 4$
Phase III: $i = 12$
Phase IV: $i = -3t + 30$

Figure 21: Modelling the Action of Insulin Over Time: Example 2

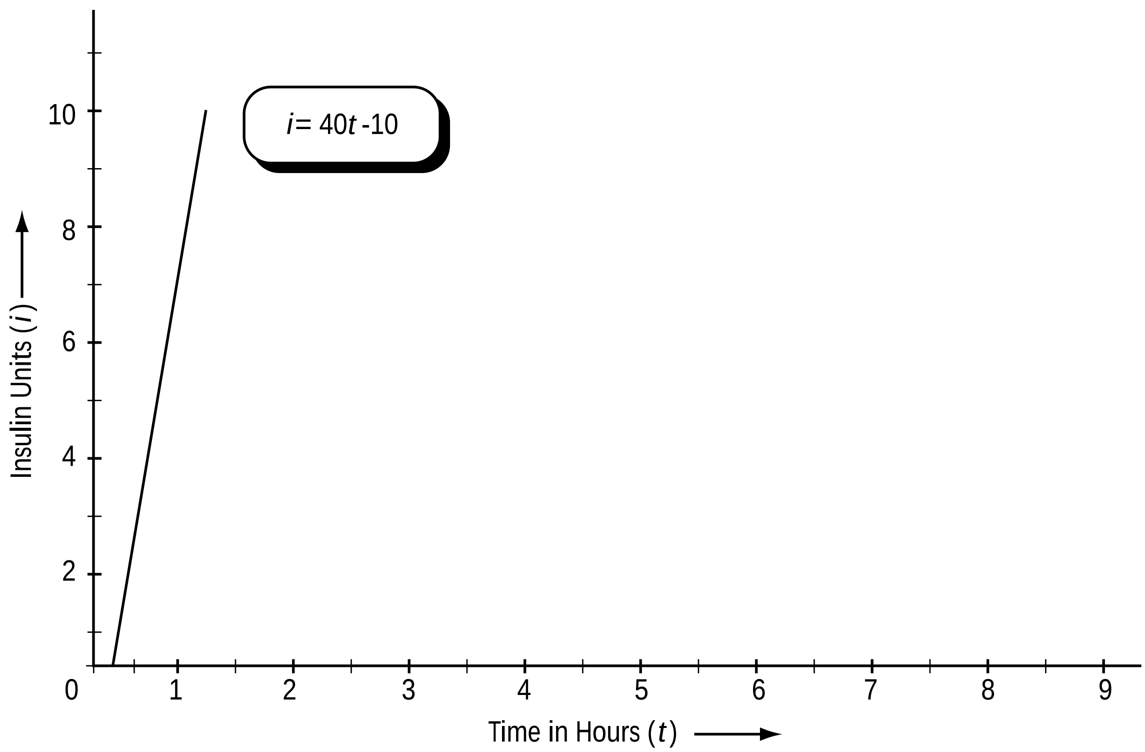


4. Types of Insulin

Rapid Acting Insulin (Humalog)

Peak is at 10 units
Time to begin to work is 1/4 hour
Time to reach the peak is 1 hour

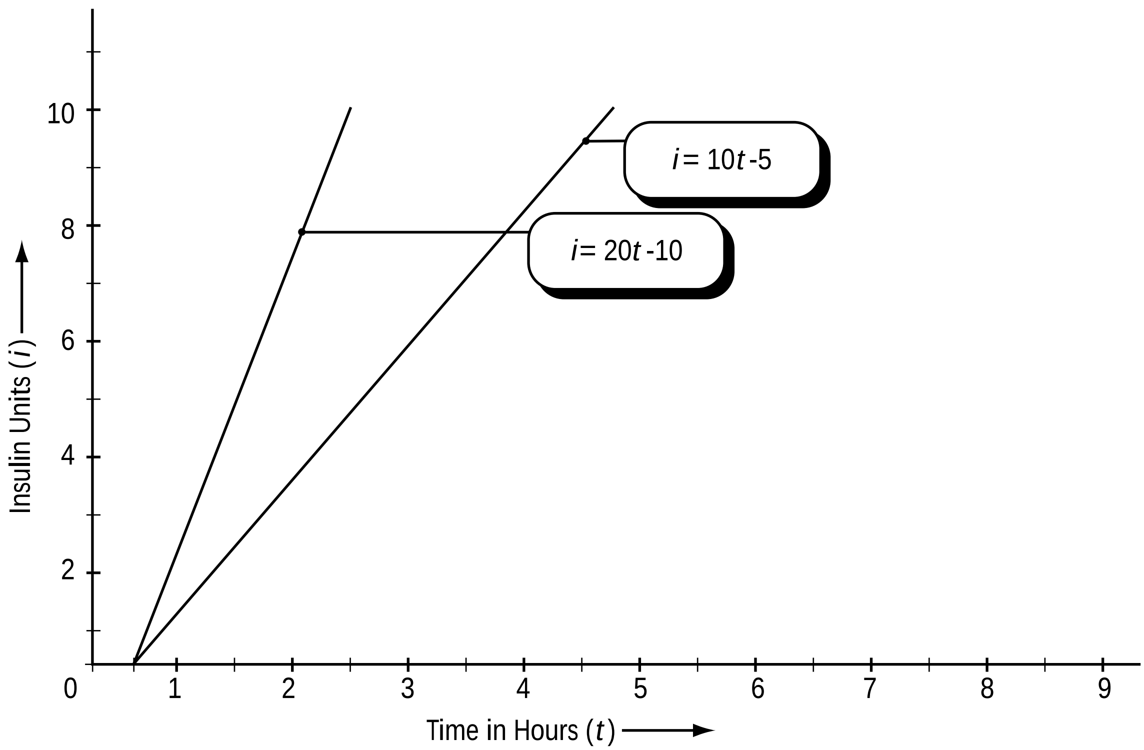
Figure 22: Rapid Acting Insulin



Short Acting Insulin (Toronto)

Peak is at 10 units
Time to begin to work is 1/2 hour
Time to reach the peak is 2 to 4 hours

Figure 23: Short Acting Insulin

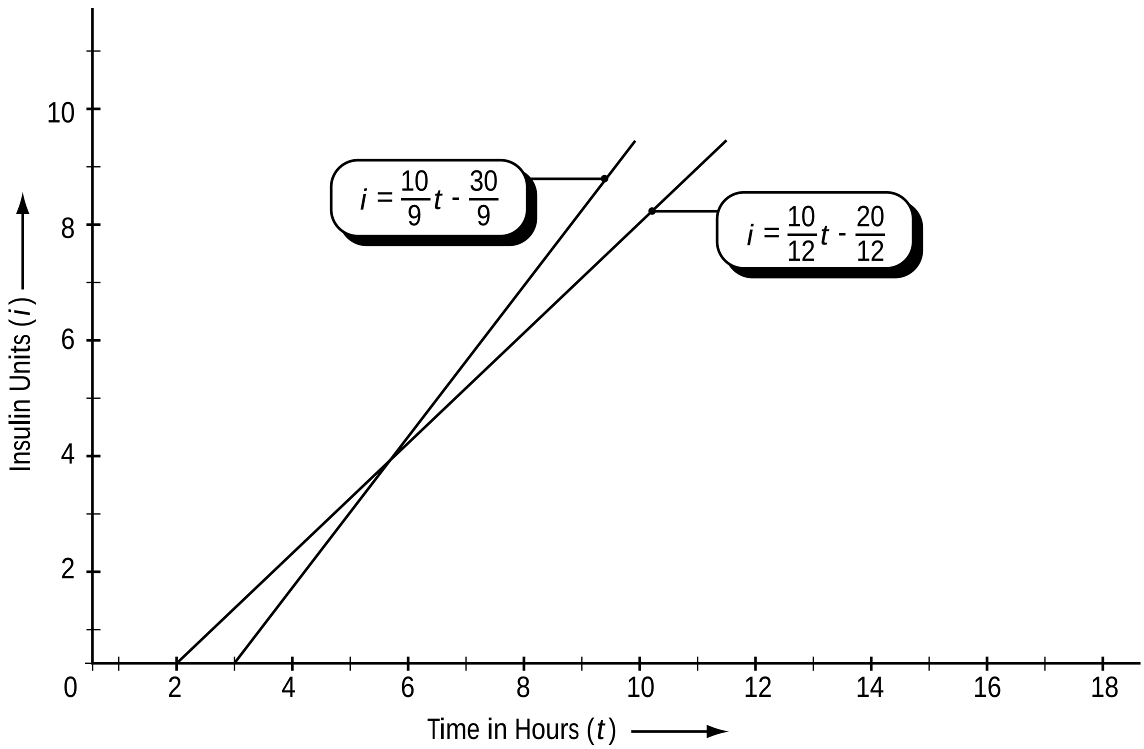


Intermediate Acting insulin (NPH)

Peak is at 10 units
Time to begin to work is 1to 3 hours
Time to reach the peak is 6 to 12 hours

In example 1, N = 10 units, W = 3 hours and T = 9 hours
In example 2, N = 10 units, W = 2 hours and T = 12 hours

Figure 24: Intermediate Acting Insulin



Question: At what time are the insulin levels the same?

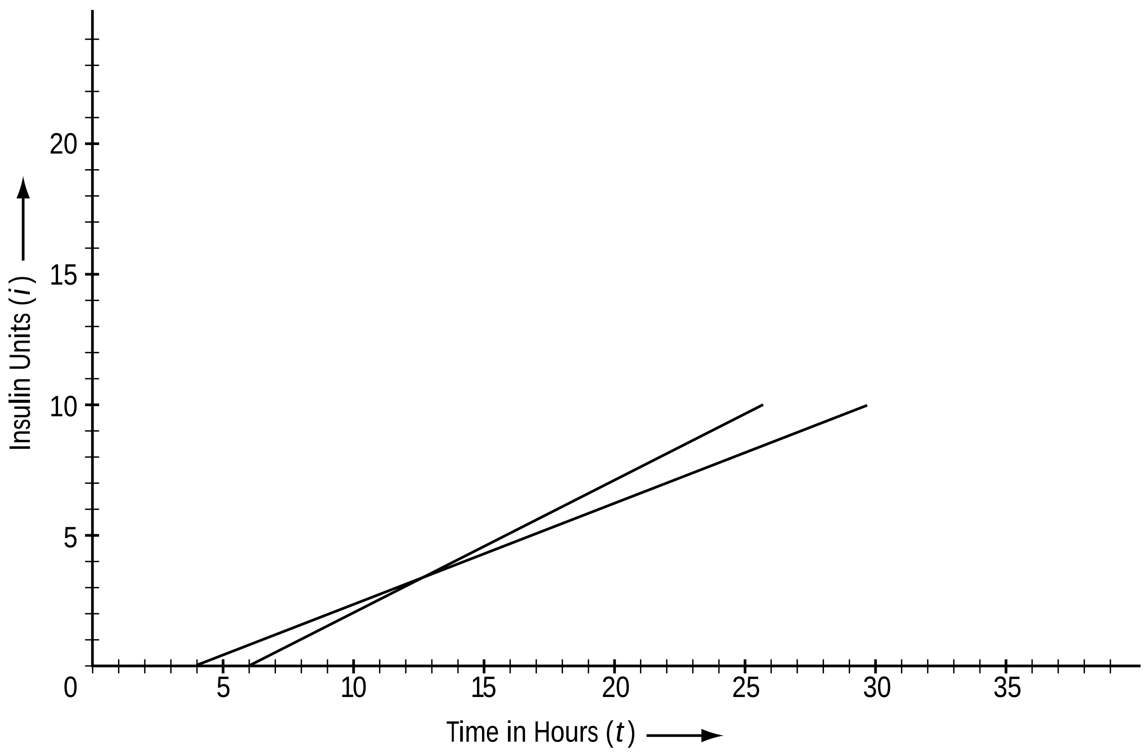
Long Acting Insulin (U-Insulin)

Peak is at 10 units
Time to begin to work is 4-6 hours
Time to reach the peak is 24-28 hours

Example 1 : N = 10 units, W = 4 hours, T = 28

Example 2: N = 10 units, W = 6 hours, T = 24 hours

Figure 25: Long Acting Insulin

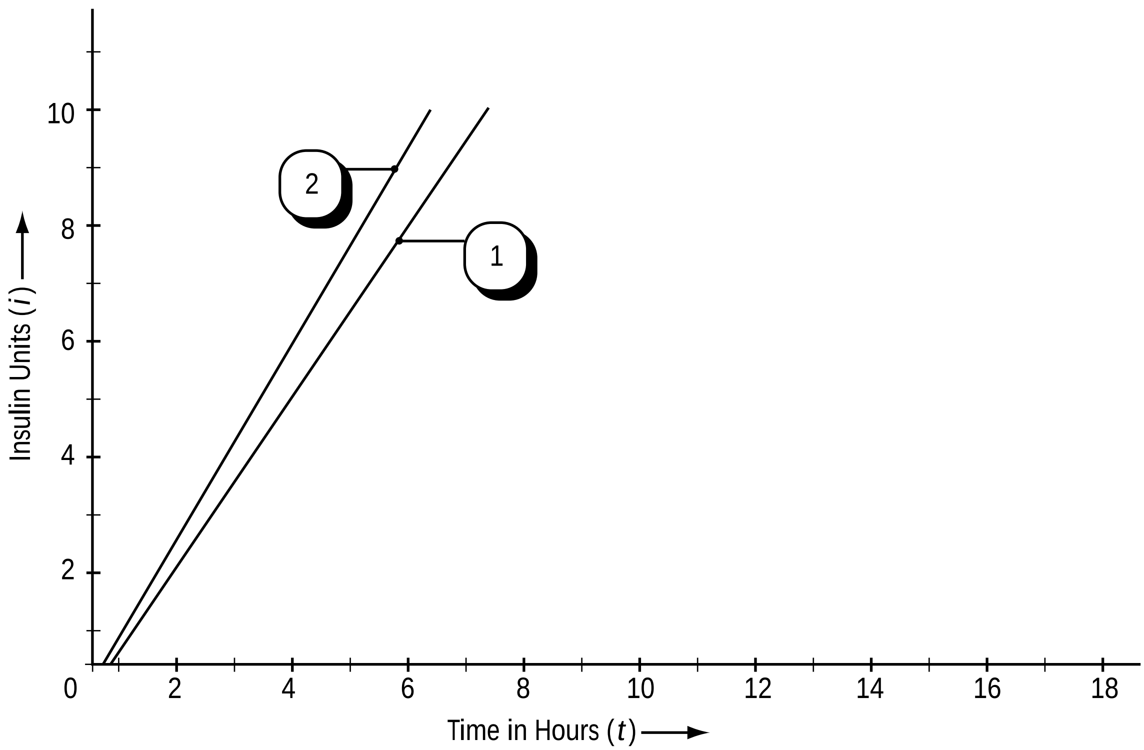


Question: At what time are the insulin levels the same?

5. Typical Insulin Problems to Solve

On the following graph determine *W* and *T*, given *N* =10 units.

Figure 26: Typical Insulin Problem to Solve



Answer the following:

Which insulin begins to work more quickly?

Which insulin reaches a peak more quickly?

How long does it take for insulin I to reach a peak?

6. Combined Action of Different Insulins

Suppose a patient is injecting a short acting insulin (10 UI) at the same time as 10 UI of long acting insulin.

The short acting insulin begins to work about 0.5 hour and peaks after 3 hours. The level remains the same for 3 hours.

The long acting begins to work after 5 hours and peaks after 12 hours. The level remains the same over a period of 8 hours.

Graph the level of insulin over a 24-hour time period.

Data

Short acting:
N =
W =
T =
C =

Long acting:
N =
W =
C =

Equations
Short acting Insulin:
Phase I: $i = 0$
Phase II: $i = 4t$
Phase III: $i =$
Phase IV: $i =$

Long acting Insulin:
Phase I: $i = 0$
Phase II: $i = 10t / 7$
Phase III: $i =$
Phase IV: $i =$

During the period between 6 a.m. and 10 a.m., the two insulins are combined. The short acting insulin is in phase IV and long acting insulin is in phase II. Add the two equations to determine the change in insulin levels between 6 a.m. and 10 a.m.

Figure 27: Combined Action of Different Short Acting Insulins

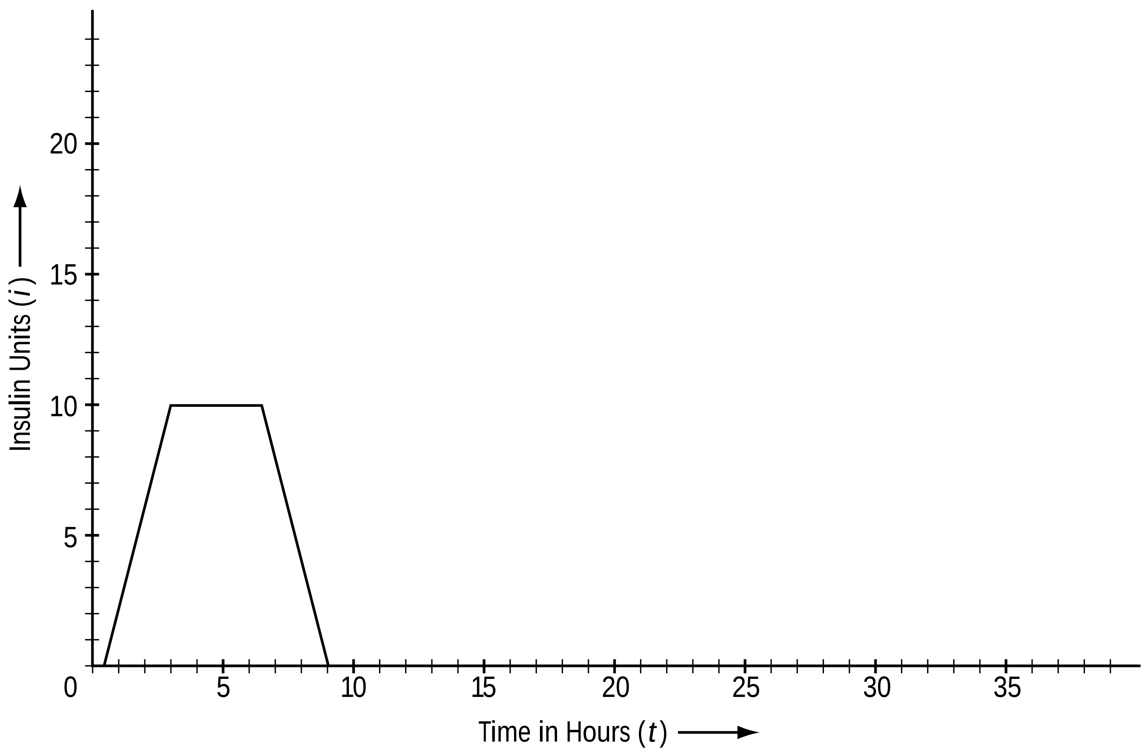
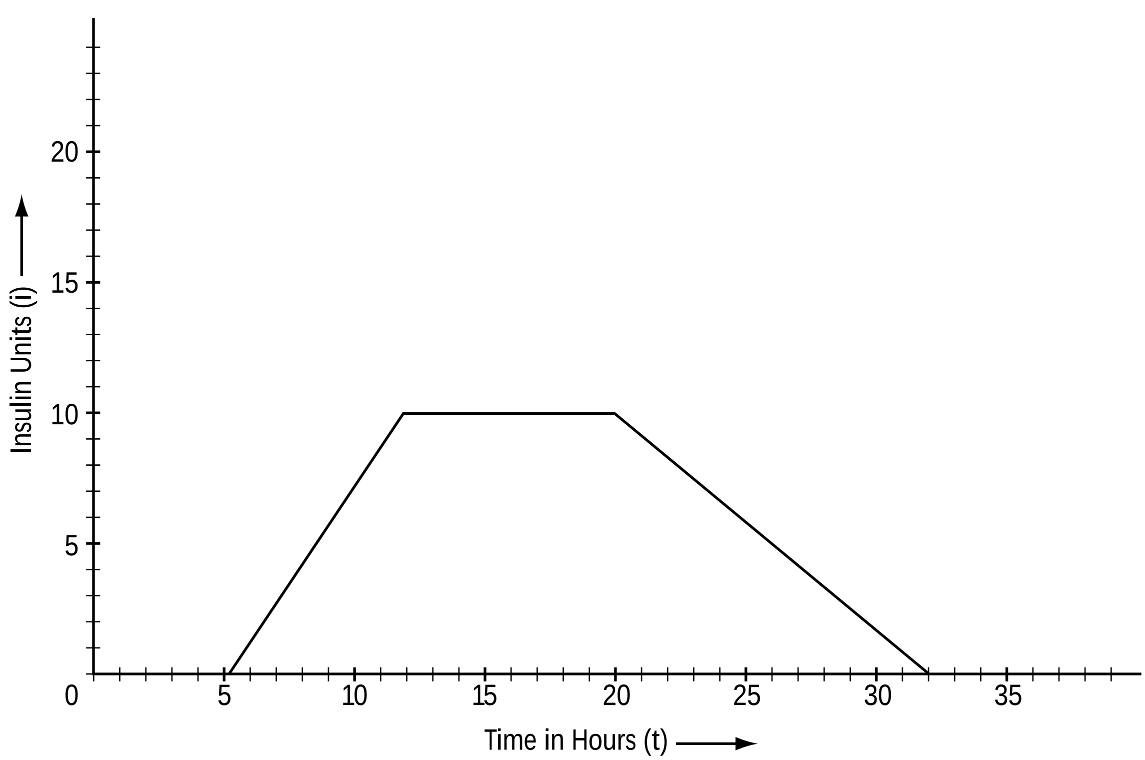


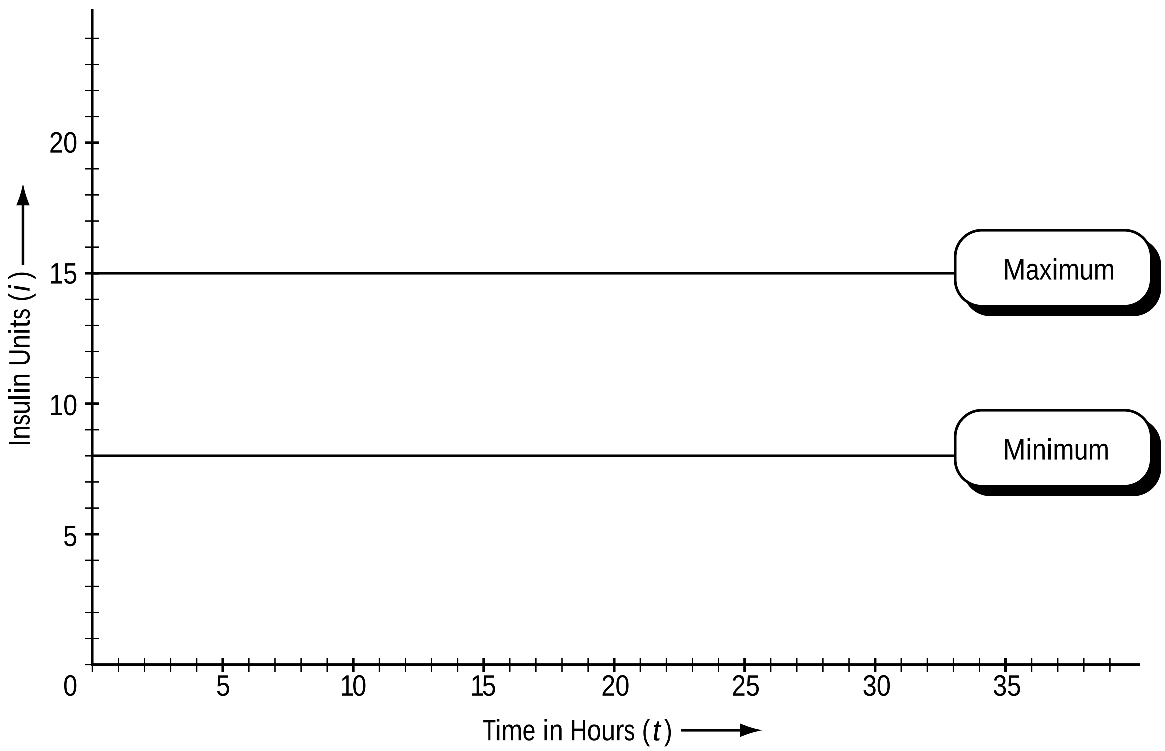
Figure 28: Combined Action of Different Long Acting Insulins



Evaluation Problem

By combining the different types of insulin, design an insulin regimen so that 8 units are released in the blood within 1/2 hour and remain within the limits of 8 to 15 units over a period of 24 hours.

Figure 29: Designing an Insulin Regimen



Phase III: Solving a Regimen Problem

Procedure:

In this phase, mathematical and science concepts are evaluated from reports prepared by individual students or groups of students. These reports may include a description of diabetes, preparation of a regimen of insulin injections from a table of blood sugar readings, answers to questions and changes in the diet, and the social and economic implications of diabetes.

Note:
This project has been prepared with the help of Dr. Susanne Voetman, M.D., The Medicine Cabinet, Nanaimo, B.C., and Norma E. Gomerich and Elly Hay, Diabetes Education Centre, Nanaimo Regional General Hospital, Nanaimo, B.C.

Project C: Planning for a Teen Recreation Centre

- **Orientation**
- **Project Synopsis**
- **Phase I: Introduction**
- **Phase II: The Survey**
  Activity 1: Graphing
  Activity 2: Relating Statistical Variables
  Activity 3: Linear Relationships
  Activity 4: Evaluation
- **Phase III: Application**
  Activity 1: Application of Concepts to Community Needs
  Activity 2: The Plan

Project C: Planning for a Teen Recreation Centre

Orientation

Time:

Phase I: Introduction .... 1 class period
Phase II: The Survey
Activity 1: Graphing .... 1 class period
Activity 2: Relating Statistical Variables .... 1 to 2 class periods
Activity 3: Linear Relationships .... 1 class period
Activity 4: Evaluation .... 1 class period
Phase III: Application
Activity 1: Application of Concepts to Community Needs .... 1 class period
Activity 2: The Plan .... 1 class period

Instructional Strategies:

- Case Study
- Field Study
- Group Work
- Issues Inquiry

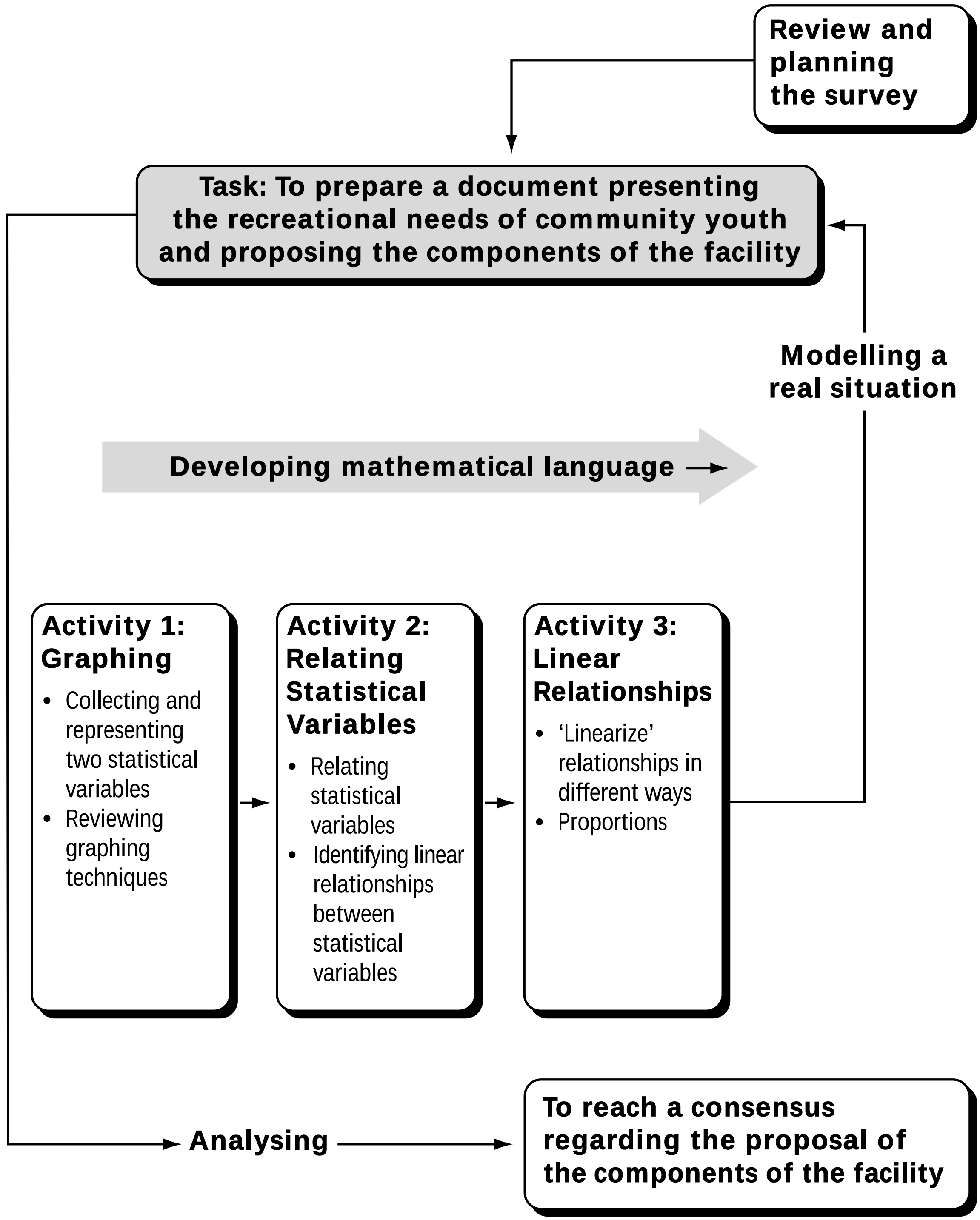
Real-World Applications:

Some real-world applications of the skills and concepts learned in this project include:
- interpreting and analysing wildlife management and fisheries survey data
- judging the quality of survey questions
- interpreting and judging the results of a survey
- designing surveys and undertaking research

Project Overview:

Figure 1 on the following page is a schematic to help you conceptualize Project C, Planning for a Teen Recreation Centre.

Figure 1: Project C: Planning for a Teen Recreation Centre



Project Synopsis

In this project, students prepare a document presenting the recreational, cultural, and physical needs of youth that can be met through a teen community recreation centre. The document is to include a description of the components of the facility and their relative location.

In this project groups of students investigate the relationship, if any, between two variables related to the use of a recreational facility; create and interpret scatterplots; and draw appropriate conclusions relating to the planning of a multi-use facility.

This project can be done as a self-contained project or it can be used as the starting point for a much larger project that would incorporate Project D: Drawing a Scale Map for a Teen Recreation Centre and Project E: Building a Scale Model for a Teen Recreation Centre.

In either case, students are expected to design a survey, collect and present survey data, analyse the results, and make the appropriate recommendations regarding the facility.

This project begins with a set of concept development activities and then moves on to the application of these concepts for determining the factors that can affect the location and the components of the facility.

During the concept development activities, students analyse sets of experimental data in terms of two variables using a graphing calculator (if available) and determine if there is a correlation between the two variables.

Students are then expected to collect their own data from the community and apply their knowledge to determine and represent eventual relationships from scatterplots in order to prepare a rigorous analysis.

Phase I of this project gives the teacher an opportunity to present the project, stress its relevance in other fields of human activity, and let students know what is expected of them. The students and teacher prepare a plan of action where the steps are identified and reasons for them are given. The class is divided into groups and the teacher assigns tasks to each group.

The teacher should determine whether a review is necessary. The areas to be reviewed may include: methods of collecting information, data presentation using a graph, and analysis of statistical data. Some students may also need to review how to use their calculator to perform statistical calculations and draw graphs.

In Phase II students use the survey as a data collection method. They determine relationships between two variables from the examination of scatterplots, and explore linear, non-linear, and random relationships.

In Phase III, students develop a plan based on survey results and their analyses. Students are asked to reach a consensus on the location of the facility and on the different components including swimming pool, library and theatre. They identify and explore the relevant mathematical concepts they would need in order to prepare and submit a professional-looking proposal to the proper authorities.

Mathematical concepts developed in projects D and E are given below.

- In Project D, Drawing a Scale Map for a Teen Recreation Centre, students are introduced to triangulating techniques in surveying. They prepare a scale map from actual measurements.
- In Project E, Building a Scale Model for a Teen Recreation Centre, students develop and apply their spatial abilities to solve construction problems. They construct three-dimensional models from plans.

Phase 1: Introduction

Objective:

In this phase, students formulate a picture of the recreational needs of youth in their community. To make this picture as accurate as possible, students need to determine if there are relationships between youth activities and other variables such as their age, gender, the distance to existing facilities from their home, and the time required to travel.

Students also need to determine the extent to which their habits would change if a new facility was to meet their needs through better accessibility, and particular components of a teen recreation centre.

In order to investigate these relationships accurately, students explore scatterplots and lines of best fit by inspection and, if available, by using a graphing calculator.

Note: Before you begin this project, check with your school administrator to determine whether there is any school or district policy of which you need to be aware regarding students conducting or participating in surveys.

Materials:

- graphing calculator
- computer with graphing software

Procedure:

Introduce the project and stress its relevance and importance in other fields of human activity. Inviting a city councillor, a survey analyst, an insurance consultant, or a police officer to speak to your class can provide students with information about real-world applications for the mathematics they are studying.

Informally evaluate the current mathematical knowledge of your students and prepare a review if necessary. This review may include the following:
- the concept of population and sample (what is the population to be surveyed and what would be an appropriate sampling method);
- how to formulate statistical questions clearly;
- how to use appropriate methods of collecting and displaying data; and
- how to identify bias in data collection and presentation.

Use examples to introduce the concept of a statistical relationship between two variables. Mention to students that they are going to explore this particular aspect of statistics. Select one example where two variables are strongly related and another example where they are not related.

Prepare an action plan with students, and indicate what is expected from them at the end of the current project. The action plan is to include:

- the steps involved including contacting schools, obtaining a list of students, deciding how to sample; and
- the type of interview to be used, for example personal or telephone.

In discussion with the students, decide:

- the survey outcomes
- those specific questions that are appropriate
- those community members who are appropriate to interview

Provide examples of questionnaires and surveys.

Divide the class into groups and assign specific tasks, that is one or two specific components of a teen recreation centre.

- Encourage students to prepare supplementary questions related to their topic. For example, if their topic is to investigate the need for a library, ask students to provide questions such as: “Are you currently using a library? How many times a week do you use the library?”
- Ask students to represent their results in the form of graphs before they proceed to the planning session.
- During the process of collecting and representing the results, students should be reminded to stay focussed on the purpose of this activity, that is, to form a picture of student needs related to a teen recreation facility.

Phase II: The Survey

Objective:

The objective of this phase is for students to explore the relationship between two statistical variables, and to learn ways of representing and interpreting data.

Procedure:

Distribute Student Activity: Graphing. Students use this activity to explore the relationship between two statistical variables by representing data obtained from various sources on a graph. Using Figure 2: Number of Accidents Versus Age Group, they analyse the relationship between the variables.

As an extension, students explore ways of creating a linear relation.

How do you decide if two events or situations are related? If so, how do you describe the way(s) in which they are related?
For example:
- What is the relation (if any) between age and car accidents, or age and number of speeding tickets?
- Is there a relation between the distance you live from a swimming pool and the number of times you use the pool, or that distance and the time required to travel to the pool?
- Is there a relation between the hour of the day and the water consumed in your house?
- Is there a relation between the number of hours you spend on the telephone and the size of your shoes?

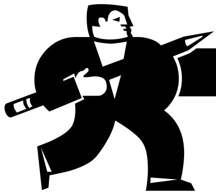
Ask students to formulate questions for which they would like to find answers. Examples of these might include:
- Does the occurrence of accidents increase with age?
- Is there a particular age group in which there is a greater probability of having more than one accident in a year?

Then ask students to identify all of those variables they would need to consider in order to answer their question.

Students then plot their data as a graph by choosing the two variables. Example of pairs of variables are:
- zero accidents and age group
- number of accidents in one year and the 40-49 age group
- total number of accidents and a particular age group

The students should informally describe the relationship between the two variables. For example, is one variable always increasing, not related, increasing regularly, increasing up to a certain point and then decreasing with respect to the second variable?

Ask students to use the same graph region to represent three different relations, each with two or more pairs of variables. Ask them to describe the relationships between the pairs of variables when three relations seem to follow the same pattern: increasing, reaching a peak, decreasing and increasing again



Student Activity: Graphing

Student Name: ______________________ Date: ________________

Activity 1: Graphing

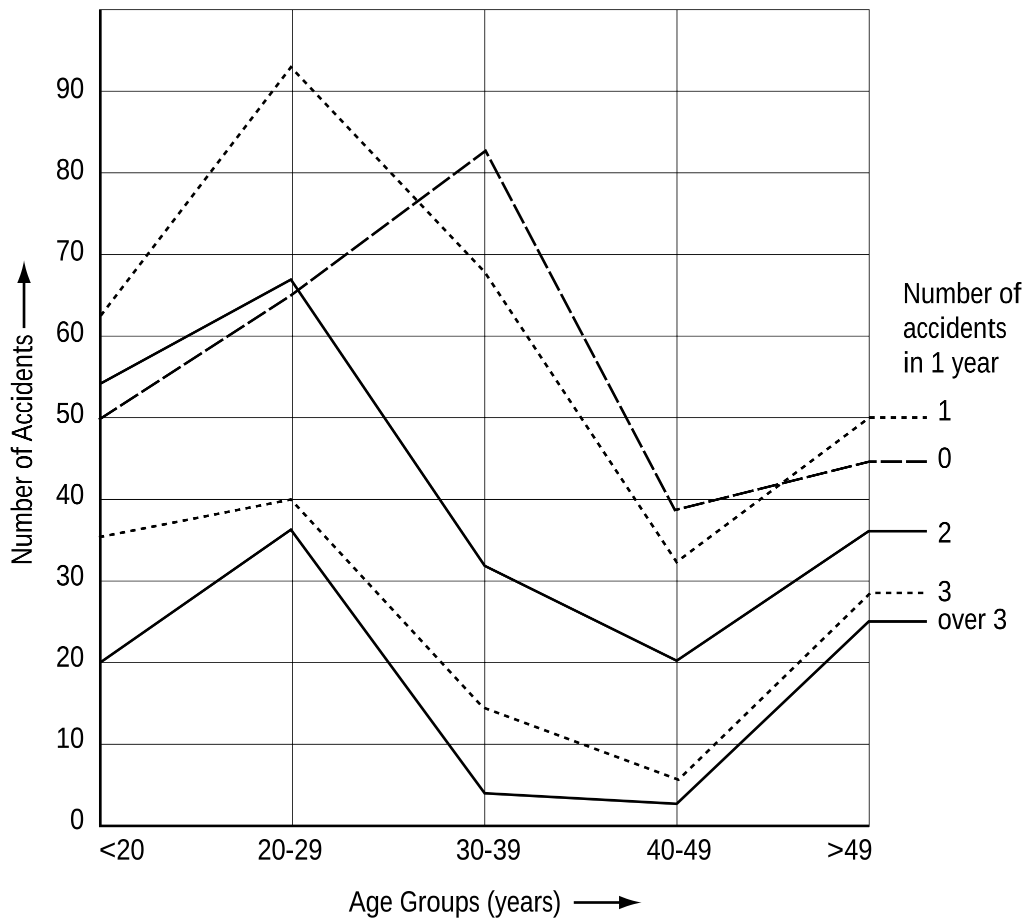
The following data are from an insurance company. One thousand drivers were selected at random in a particular city to determine if there was a relationship between age and number of accidents in a year.

Table 1

Relation Between Age Group and Accidents

	Accidents in 1 year				
Age	**0**	**1**	**2**	**3**	**Over 3**
Under 20	50	62	53	35	20
20-29	64	93	67	40	36
30-39	82	68	32	14	4
40-49	38	32	20	7	3
Over 49	43	50	35	28	24

Figure 2: Number of Accidents Versus Age Group



Ask students to answer the following:

- What is the probability for a driver chosen at random, of being under 20 years old <u>and</u> having 3 accidents in 1 year?

answer: $\left(\dfrac{35}{1000} = 0.035\right)$

- of being 30-39 years old <u>and</u> having 1 or more accidents in 1 year?

answer: $\left(\dfrac{68+32+14+4}{1000} = 0.118\right)$

- of having no accidents in 1 year?

answer: $\left(\dfrac{50+64+82+38+43}{1000} = 0.277\right)$

- of being under 20 years old <u>or</u> having 3 accidents in 1 year?

answer: $\left(\dfrac{53+62+50+35+20+40+14+7+28}{1000} = 0.309\right)$

In preparation for the next class, ask students to prepare a set of questions regarding car accidents for which they would like to obtain an answer. Students may be able to obtain statistics from ICBC, insurance companies, the RCMP, or their municipal police.

Some suggested questions are:
- Is it true that drivers are more likely to be involved in a car accident in the first six months after they obtain their driver's license than at any time later?
- What is the relationship between gender and the number of car accidents in a lifetime?
- Is it true that fewer red cars are involved in accidents than grey cars?
- What is the relationship between the number of accidents and the age of the car?
- What is the relationship between the number of speeding tickets and the age of the car?
- What is the relationship between the number of speeding tickets and the make of the car?
- What is the relationship between the number of speeding tickets and the age of the driver?
- What is the relationship between the number of speeding tickets and the gender of the driver?

Determine the extent to which students can:
- formulate problems clearly and concisely;
- select appropriate variables; and
- select proper sources of information.

Activity 2: Relating Statistical Variables

Objective:

The objective of this activity is to provide an opportunity for students to relate statistical variables and show these relationships graphically.

Materials:

- graphing calculator

Procedure:

Suggest to students that they design supplementary questions concerning two apparently unrelated variables (such as the number of speeding tickets versus car colour, or the number of accidents versus the date of the month).

Verify with your students that their questions have been formulated properly. If necessary, help students to reformulate them. The questions should address a specific situation involving two variables.

Give some examples of poorly formulated questions, for example, questions that:

- are too general
- have situations involving more than two variables
- have situations that are too specific with respect to the available information

Verify that students have collected the necessary information. If not, suggest that they make some reasonable guesses about what is missing.

Help students construct their graphs by suggesting what variable should be identified on the horizontal axis and what variable should be on the vertical axis.

Stress the fact that although the layout is sometimes not really important, almost always for presentation purposes it is better to position the graph in a particular way. As an example, for Figure 2: Number of Accidents Versus Age Group, explain that the researcher was seeking the number of accidents as a function of (dependent upon) the age groups rather than the reverse. The dependent variable is usually plotted on the vertical or y-axis.

Insist on the appropriate choice for the units represented on the axes. That is, the units selected should result in the data points occupying most of the space on the graph. Have students make a rough sketch of their graphs with pencil and paper in order to avoid surprises.

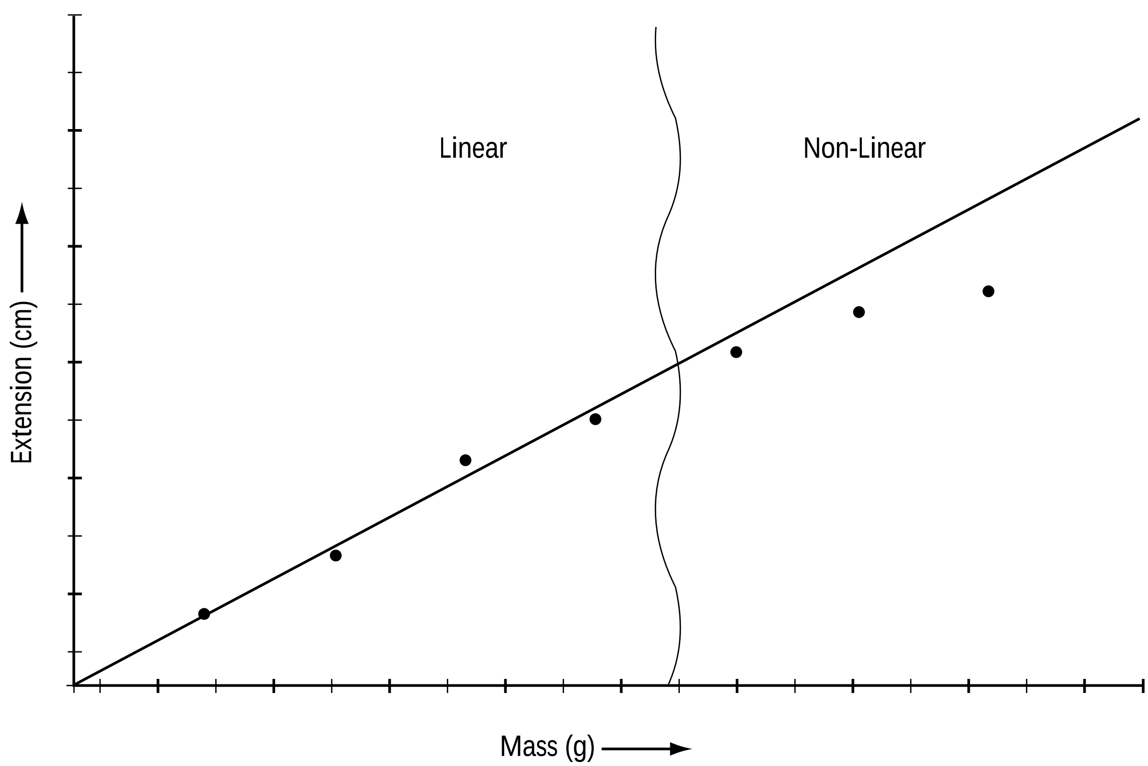
When their rough sketches are complete, ask them to informally describe the relationships between the variables (e.g., linear, no correlation, not constant, geometric) and then have them use a ruler, flexible ruler or snake to sketch the lines more accurately. Finally, have them draw the graphs with the help of a calculator or computer.

Have students determine the number for domains for the graph where the relation seems to be linear. Have students clearly identify the domains.

The teacher may consider the following activity to explore the linearity of a relation between two continuous variables:

- Have students measure the extension of a spring attached to various weights. This can be done by using an exerciser in the gym, a kit obtained from the physics lab, or a set of elastics attached to a support.
- Students measure the extension of the spring and complete Table 6: Springs. This information is then used to draw a graph.
- The vertical extension of the spring is plotted against the mass of the weights attached to the spring as in the graph shown in Figure 3: Elasticity of a Spring.
- Ask students to use a ruler to draw a line joining the points of the graph. They should then determine the domain where the relation seems to remain linear and the domain where the relation ceases being linear. A discontinuity may occur when the spring or the elastic is stretched too far.

Figure 3: Elasticity of a Spring



Activity 3: Linear Relationships

Objective:

The objective of this activity is to learn more about linear relationships between variables.

Materials:

- graphing calculator

Procedure:

In some cases, it is possible to demonstrate a linear relationship simply by changing the scale or by modifying one of the variables. An example of this is squaring one variable if the graph looks like a parabola such as that for a falling object.

Have students play with some graphs. By using their calculators and manipulating or changing scales such as squaring or taking the square root of one variable, they may be able to produce a linear graph or approximate one.

In other cases, students may use some techniques such as the median-median line or linear regression to demonstrate a linear relation.

The following example can help you introduce the median-median technique informally to your students:

- The Federal Department of Fisheries and Oceans has recorded the length and the mass of a sample of prawns fished in June 1998. The results are presented in Table 2: Length and Mass of Prawns.
- Distribute a copy of Table 2: Length and Mass of Prawns to the students and ask them to prepare a scatterplot as Figure 6: Prawns.
- Ask students to comment on any obvious relationships between the two variables. Then have them regroup the prawns into three categories based on mass: small, regular, and jumbo. Students need to decide upon the borders separating the three domains.
- Ask them to determine the average length and the average mass within each domain and to plot those results.
- Have students compare their regroupings with other students. As a class, have them decide the best regroupings in terms of the linearity of the line joining the three points of the graph.

Table 2

Length and Mass of Prawns

Length (cm)	Mass (g)	Length (cm)	Mass (g)
7.5	25.7	9.1	27.3
7.6	25.9	9.2	28.0
7.6	26.7	9.5	27.8
7.6	26.8	9.5	27.8
8	26.0	9.5	28.0
8	26.5	9.5	28.3
8	26.8	9.9	27.3
8	27.1	9.9	27.8
8.5	26.3	9.9	27.9
8.5	26.5	10	28.3
8.5	27.5	10.8	28.5
8.5	27.8	11	28.6
9	26.6	11.5	28.1
9	26.7	12	28.5
9	27.0	12	28.7
9	26.8	12.5	28.5
9.1	26.4	13	28.9

Introduce linear regression with the following example of data obtained from a bank. Table 3: Annual Savings and Annual Income compares annual savings with annual family income.

Table 3

Annual Savings and Annual Income

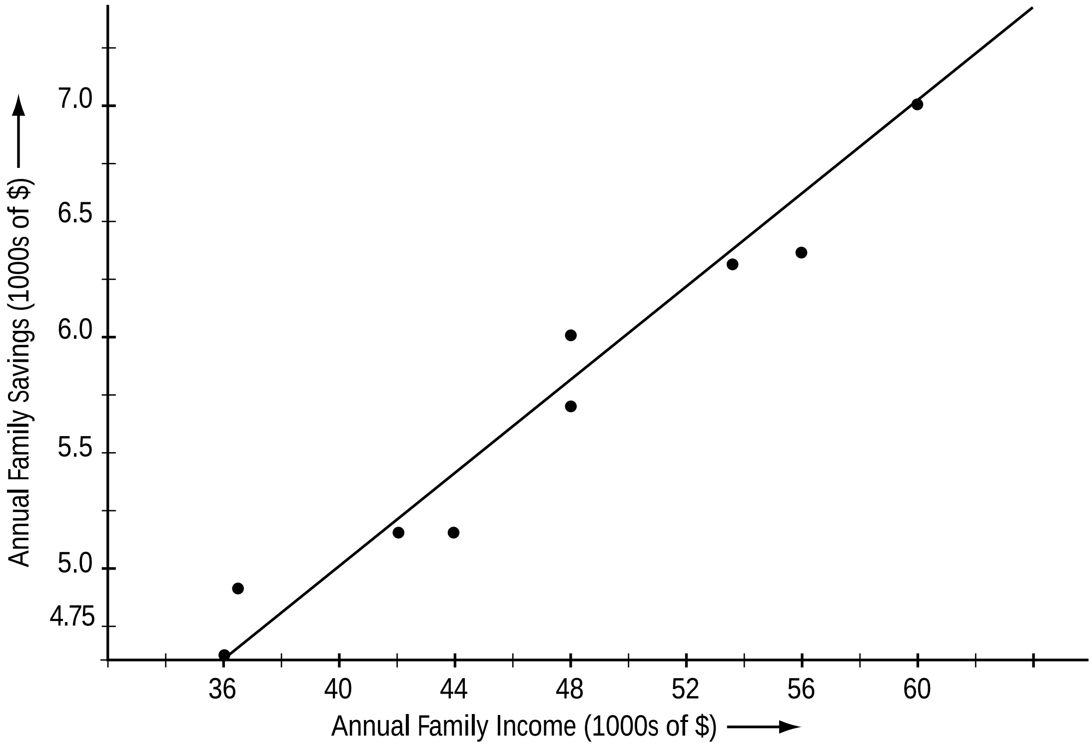
Annual Income (1000s of $)	Annual Savings (1000s of $)
42	5.2
48	6.0
36	4.6
56	6.3
60	7.1
54	6.3
44	5.2
48	5.7

Ask students to plot the data and guess the type of curve that would best fit the scatterplots. This curve should come as close as possible to all the points of the diagram. Suggest that they use a ruler and that they make sure that there as many points to the right as there are to the left of the ruler's edge. Suggest that they try to 'weight' the distances between the points and the ruler.

The result has been calculated below by using a linear regression in Figure 4: Line of Best Fit Using Linear Regression.

Do not have students use their scientific calculators at this point because they are not able to understand the meaning of the equation of the regression line. It is, however, worthwhile to mention that there are systematic ways of evaluating the line of best fit.

Figure 4: Line of Best Fit Using Linear Regression



Students may collect data such as arm span versus shoe size or height versus arm span, using themselves and other students.

Have students perform the measurements and then have them prepare a graphical representation (plot the data and produce scatterplots).

Ask students to use Student Activity Figure 7: Generic Graph to record their measurements and prepare a graph.

Have students discuss the graph to determine the nature of the relationship — whether or not it is linear.

Have students regroup the scatterplot data in different ways and have them determine the median of each subgroup.

Have them plot the medians. This time the relationship may be more linear than it was originally.

Students should try different regroupings until they obtain a model that can be considered linear.

Encourage students to use their calculators to perform these operations (most calculators can generate regression lines).

Activity 4: Evaluation

Objective:

The objective of this activity is to determine the extent to which students are able to:

- properly identify a given problem and the variables;
- accurately formulate the questions that are being addressed;
- collect appropriate data;
- prepare a table representing the data;
- construct a graph representing the data (graphs should include axes, units, points on the graph, and a title);
- correctly use a calculator or computer;
- identify the existence of any relationship between two variables;
- describe the relationship correctly and concisely;
- determine a linear domain, if applicable;
- propose ways of making a linear relationship (no calculations); and
- relate the interpretation of the graph with the questions that were designed originally, or modify the original problem by taking into account the results of the graph analysis.

Materials:

- graph paper
- graphing calculator
- computer with graphing software

Procedure:

Present the following list of pairs of variables fromTable 4: Suggested Pairs of Variables (General) to students and have them pick two pairs. You may choose to add more pairs to the list.

Table 4

Suggested Pairs of Variables (General)

Variable 1	Variable 2
Heart rate	Oxygen consumption
Heart rate	Age
Day of the week	Number of speeding tickets
Number of houses sold	Month of the year
Life expectancy	Each year after 1900
Distance between two major cities	Cost of airline ticket
Value of the Canadian dollar/US dollar	Each month after 1996
Height of Grade 9 students	Mass of Grade 9 students
High school drop-out rate	Each year since 1990
Month of the year	Level of water consumption
Colour of car	Number of accidents
Self-employed workers	Each year since 1990
Number of single family houses/condominiums sold	Price of single family house/condominium
Distance from school	Time required to travel to school
Percentage of voters for a party in an election	Number of seats in the legislature
Percentage federal taxes	Taxable income
World record for the 100 m dash	Age of the record holder
Level of blood glucose	Hour of the day

Table 5

Suggested Pairs of Variables (Project-Related)

Variable 1	Variable 2
Number of hours spent in a library	Grade level
Number of hours/week spent on a leisure activity	Distance from the house to the location where the activity takes place
Number of hours/week spent on a leisure activity	Number of hours spent on the phone with friends
Distance from the house to the location where the activity takes place	Time needed to travel from the house to the location where the activity takes place
Number of hours spent on the phone with friends	Day of the week
Number of hours/week spent on a leisure activity	Time needed to travel from the house to the location where the activity takes place
Number of hours/day spent on a leisure activity	Day of the week
Number of hours/week spent on a leisure activity	Age

Ask students to formulate one or two questions about the data set they are going to prepare.

Have them collect data from the appropriate information sources or to design and conduct an experiment that allows them to collect the necessary data. They should mention the source of their information or the details of their experiments.

Students then represent their data in the form of a table and that of a graph. Students may use Student Activity, Figure 7: Generic Graph for this purpose. Encourage students to use available technology for their graphs.

If applicable, ask students to draw the line of best fit using the available technology.

Ask students to comment on the relationship between the two variables and to present a rigorous written analysis. This analysis should include a description of the two variables, a description of the relationship in terms of linearity or non-linearity, the eventual domains of linearity, and the way the relationship could be made linear if it does not appear to be.

Ask students to prepare a short report on their findings. Remind students that the criteria described at the beginning of this activity are used to evaluate the activity.

Phase III: Application

Activity 1: Application of Concepts to Community Needs

Objective:

At this point, students have the necessary tools to conduct their survey and to begin an analysis of any correlation among the pairs of project variables.

At the end of this activity, the students present a description of the recreational, cultural, and physical needs of youth in the community from the survey they conducted among 13-16 year olds. This includes collecting data, representing and analysing the results, and making recommendations.

Materials:

- blank table and graph forms
- graphing calculator
- computer with graphing software

Procedure:

Ask students to create as many pairs of two variables as the ones presented in the previous activity and record these pairs using Table 7: Suggested Pairs of Variables (General).

Ask them to design as many questions as necessary to be able to produce a rigorous analysis of the recreational needs of community youth.

Students are expected to present their results in the form of tables and graphs, to analyse the results in terms of two variables, and to make recommendations to the class. Students may wish to design their own data tables and use Figure 7: Generic Graph to record their results.

The results should be presented by gender, age, area of residence in the community, distance from existing facilities, time required to travel from home to the existing facilities, and so on.

The results obtained may be used in other activities as examples of determining relationships between two variables.

Insist on clarity and conciseness in the questions they use to design the survey:
- one single topic at a time
- precision of the questions
- simplicity
- two variables involved at a time

Student Activity: Springs

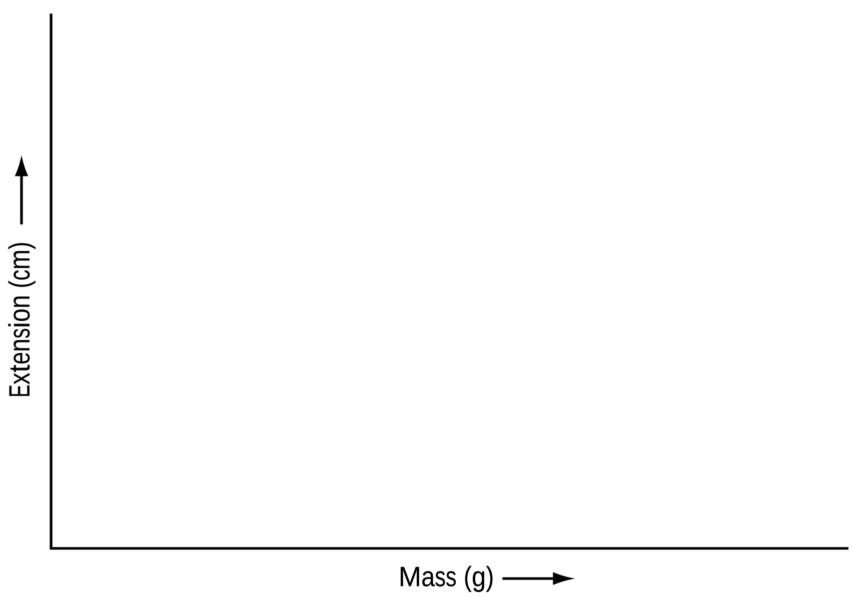
Student Name: ____________________ Date: ______________

Table 6

Springs

Mass (g)	Extension (cm)

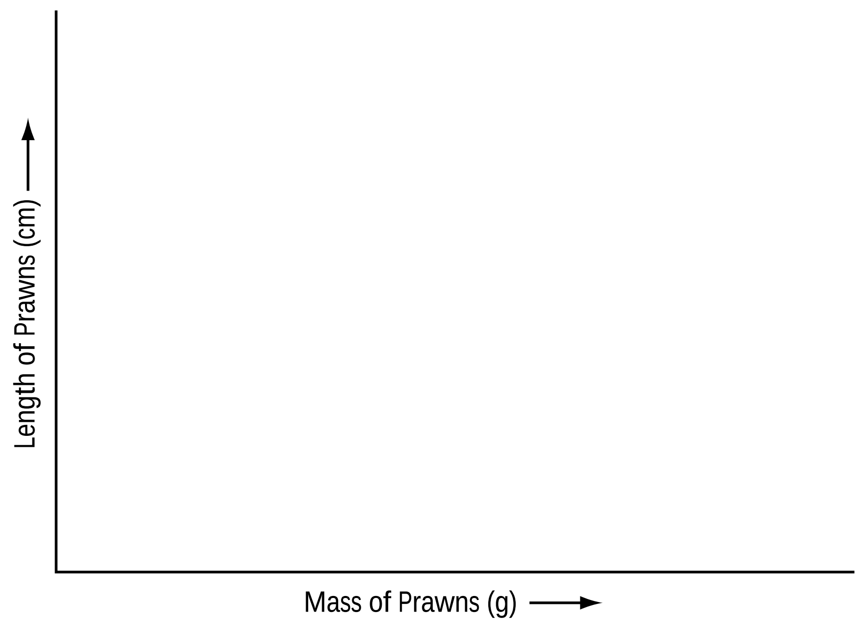
Figure 5: Springs

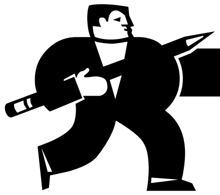


Student Activity: Prawns

Student Name: ____________________________ Date: ___________________

Figure 6: Prawns



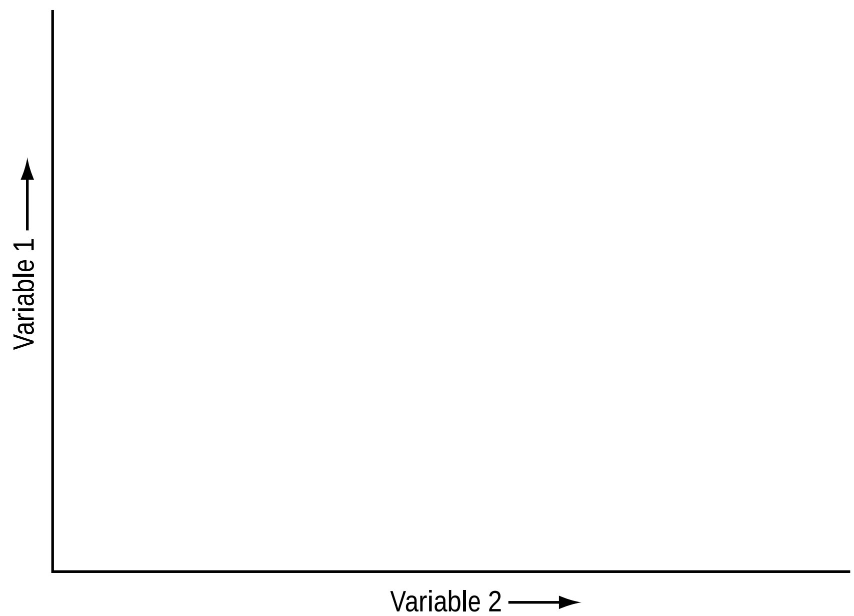


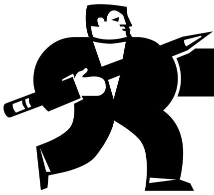
Student Activity: Generic Graph

Student Name: ____________________________ Date: ___________________

You should be provided with several copies of Figure 7 below.

Figure 7: Generic Graph





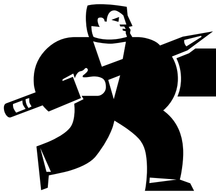
Student Activity: Generic Formats

Student Name: ______________________ Date: ______________

Table 7

Suggested Pairs of Variables (General)

Variable 1	Variable 2



Student Activity: Example of a Survey

Student Name: ____________________________ Date: ___________________

1. Preparing the survey

General information
- School:
- Grade:
- Contact Name:
- Number of students:
- Other:

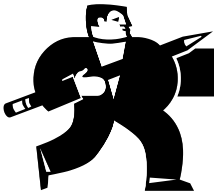
Main topic
- What is the specific information your group has been asked to obtain?

Supplementary information
- What additional information do you think is important to obtain and why?

2. Collecting Data

Determine the type of survey to be distributed for the collection of the data, for example, telephone interview, face-to-face interview, or questionnaire.

Population and Sample size
- Describe the population your group has been asked to survey:
  - age
  - gender
  - other characteristics
- Describe the number of students to be surveyed (sample size):
  - age
  - gender
  - other characteristics
- Describe the percentage of the population:
  - age
  - gender
  - other characteristics



Student Activity: Example of a Survey

Brainstorm the specific information related to your task that you would like to obtain during the interview, for example, age, swimming pool use, public library use.

Decide upon a format for each question. For example,

- Do you think that a 50m swimming pool is important for our community?

Not Very Important 1 2 3 4 5 Very Important

Organize and regroup your questions into sections and identify the purpose of each section. Organizing and regrouping your questions before you do the survey helps you compile the results later.

For example,

Section 1: General information
- What is your age?
- Gender: M F
- Where do you live?

Section 2: .... (currently, I ....)
- Are you enrolled in ... ?
- How many hours do you spend on .... ?

Section 3: ..... (if ..., I would ....)
- use ... 3, 4, ... times a week,

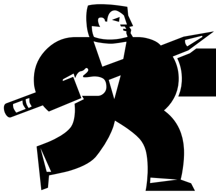
Section 4: ..... (if I have to choose between ... and ..., I would prefer ...)

Tips

Before writing the final questionnaire check with your teacher to see if there is a school or school district policy regarding the kinds of questions you can ask or regarding student participation. Once you have this information, test out (pilot) your questionnaire with some friends. Use their feedback and the results to adjust and fine tune the questions. Submit the questionnaire to your teacher for final comments and suggestions before you actually conduct your research.

Make the required number of copies of the questionnaire.

Use a checklist to make sure that you are ready to conduct the survey.



Student Activity: Example of a Survey

If you decide to conduct interviews, do not influence the answers. For example, allow the same amount of time for each interview to take place, ask each person the questions in the same order using the same tone of voice.

3. Presenting and analysing results

Select the most appropriate format for presenting your survey results. Explain your selection.

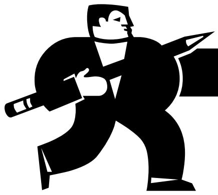
Compile the results by section and construct tables (see samples) before drawing the graphs. Construct a table for each section of questions and write a title for the table. See the example on the following pages.

Make all the calculations including averages and draw conclusions.

What are some conclusions you can make from your graphs?

What are the recommendations of your group as a result of your survey?

Sample
Suppose your group was given the task of determining the need for a swimming pool. You may consider presenting your results in the form of a table like the one below.



Student Activity: Example of a Survey

Table 8

Swimming Pool Survey Results

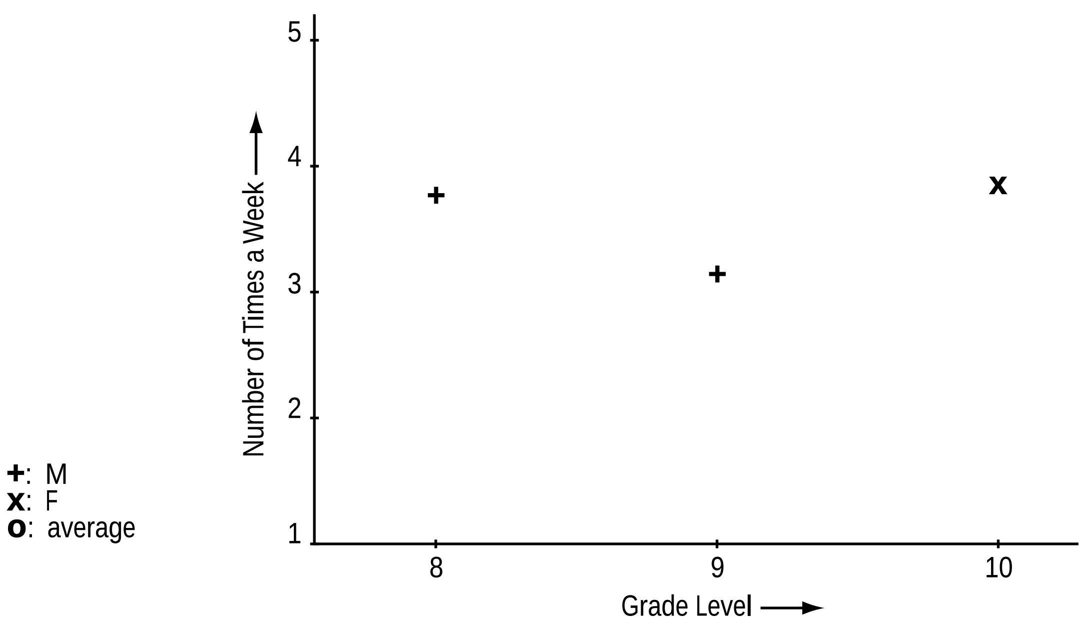
Grade	M	F	Rating Scale 1	2	3	4	5	Average Rating Males	Average Rating Females	Overall Average Rating	Remarks
8	6	5	0	1	2	2	1	3.5	3.8	3.75	
9	4	7	0	1	2	2	1	3.0	3.2	3.1	
10	6	7	0	1	2	2	1	3.7	4.0	3.85	
Total/Ave.	16	19	0	3	6	6	3			3.57	

Rating Scale Key:

1: not important at all
2: not very important
3: somewhat important
4: important
5: very important

Present your data on a graph and clearly label the axes.

Figure 8: Frequency of Student Use of Swimming Pool



Draw your conclusions and make recommendations.

Activity 2: The Plan

Objective:

Students are asked to reach a consensus regarding the choice of features for the facility and their relative location in order to meet the needs expressed in the survey.

Students also decide how to distribute the tasks amongst themselves (forming specialized groups for each of the activities).

Materials:

- survey results from each group

Procedure:

Guide students in their decision-making process. Ensure that their interpretation is consistent with the results of the survey.

Impose a time limit on the discussions. Students should reach a decision within the class period and a first draft of the proposal should be completed as an assignment for the next class.

Prior to the second period allowed for the planning, prepare a format where the different components are identified and the tasks are clearly defined, Ensure that students know who is doing what, when, and how. Also, the format should explain the purpose of the activities, the usefulness of the corresponding mathematical concepts, and how are they are related to the project.

The students form groups. Each group's tasks are described specifically with respect to the project. Each group should complete each of the activities while keeping its own group's specific task in mind.

Project D: Drawing a Scale Map for a Teen Recreation Centre

- **Orientation**
- **Project Synopsis**
- **Phase I: Triangulation**
  Activity 1: Triangulation
  Activity 2: Pythagorean Relation
- **Phase II: Solving the Problem**
- **Part 1: Solving the Problem Using Trigonometry**
  Activity 1: Measuring a Tree
  Activity 2: Revisiting the Accident
  Activity 3: Ramping Up
  Activity 4: Brain Surgery
  Activity 5: Ancient Greece
  Activity 6: Mapping
- **Part 2: Solving the Problem Using Geometry**
  Activity 7: Revisiting the Tree
  Activity 8: Measuring Grade
  Activity 9: Mapping Using Similarity
  Activity 10: Mapping Using Congruence
- **Phase III: Preparing the Scale Map**
  Activity 1: Sketch Mapping
  Activity 2: The Scale Map

Project D: Drawing a Scale Map for a Teen Recreation Centre

Orientation

Time:

12 to 13 hours

Instructional Strategies:

- Direct Instruction
- Field Study
- Group Work

Real-World Applications:

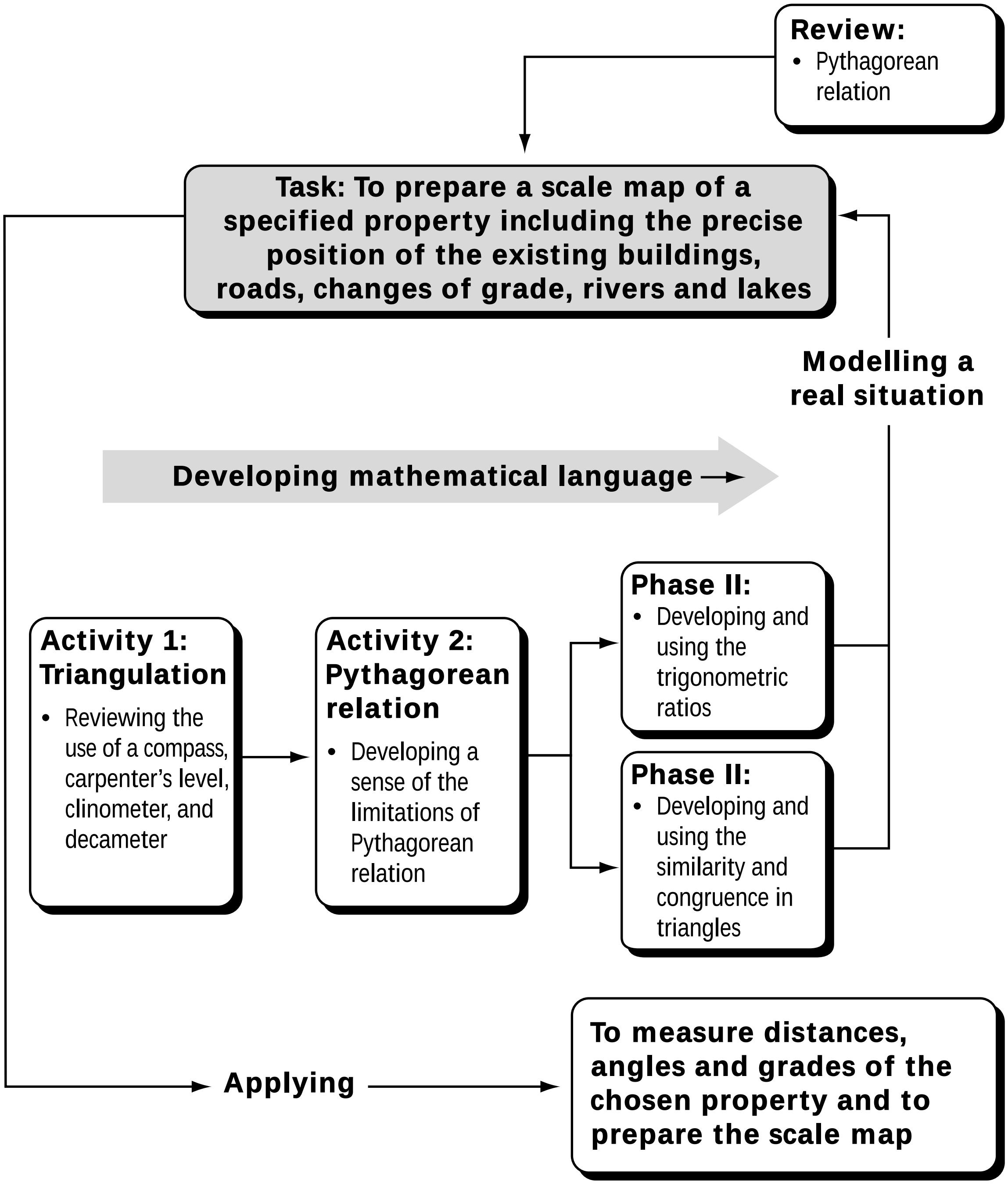
Some real-world applications of the skills and concepts learned in this project include:

- describing activities that are done by surveyors, engineers, architects, timber cruisers, dress designers and interior designers
- using a scale map for outdoor recreation activities and travel
- adapting patterns for sewing and quilting
- designing the layout for chairs and tables in the school gym for a dance
- designing a bird house or stereo cabinet

Project Overview:

Figure 1 on the following page is a schematic to help you conceptualize Project D: Drawing a Scale Map for a Teen Recreation Centre.

Figure 1: Unit D: Drawing a Scale Map for a Teen Recreation Centre



Project Synopsis

The class is expected to prepare a scale map of the specified property. The scale map must include major features, for example, existing buildings, roads, changes of grade, rivers, lakes.

The objective of the project is to provide students with the trigonometry and geometry tools necessary to draw a scale map.

In Phase I students learn more about the problems associated with drawing a scale map through activities that apply triangulation and the Pythagorean relation.

In Phase II, students learn about trigonometric and geometric solutions throughout the course of the activities.

Finally, in Phase III students draw both a sketch map and their final scale map.

Phase I: Triangulation

Objective:

The objective of this phase is for students to learn more about the challenges associated with drawing maps. In particular, they learn about triangulation and the Pythagorean relation.

Materials:

- pair of compasses
- 2 m graduated pole
- decameter and/or rope
- stand for level
- two carpenter's levels
- protractor
- clinometer (for an example, refer to Figure 4: Clinometer)
- drawing instruments (that is, pens, pencils, ruler)

Procedure:

Ideally, the property to be surveyed for the project should have an irregular shape and surface. Obstacles like bushes, a river, a lake, or existing buildings add interesting complexity to the activity ensuring that students develop indirect measurement strategies. Students should include measurements of distances, angles, and levels.

Ask students how they might measure the distance and the direction of a line between two points when one point is not visible from the other, or where one point is not accessible.

Relate their suggestions to the project by giving examples of the need to find indirect measurement methods in situations where one point is not accessible for direct measurement.

Suggestions for introducing the concepts
- Borrow a video on surveying or invite a surveyor to do a class presentation.
- Briefly describe the historical development of indirect methods such as trigonometry. The term 'trigonometry' is derived from the ancient Greek terms for 'trigone' for triangle and 'metric' for measure. The measuring techniques involving triangles were developed by the ancient Greeks, mainly to calculate distances and angles in astronomy. Find illustrations of this to show your students.

Give examples where triangulation is necessary and stress the similarity of the problems (indirect measurement) among the examples.

- Loggers often need to estimate the height of a tree before cutting it down to determine where the tip of the tree will fall, or whether the tree will hit power lines or other trees. Loggers use an instrument called a 'clinometer' to estimate the height of trees. Show the students a clinometer or explain how to build one.
- In surveying, one has to measure the distance and the direction between two points that are either not accessible, or not visible at the same time. Surveyors use trigonometric techniques all the time. (If available, show a theodolite and explain its use by surveyors.) This was the case during the construction of the Railways in the 19th century. Recently, during the construction of the Chunnel, (the tunnel under the English Channel that connects England and France), surveyors and engineers needed to use indirect measurement techniques.
- Traffic accident investigators need to apply indirect measurement techniques to determine the speed of vehicles involved in accidents or to prepare scale diagrams as evidence in court. Show illustrations or pictures of an accident and the scale diagram that has been presented in court.
- Astronomers cannot walk the distance between two planets so they rely on indirect measurement methods. Show the illustration of how Eratosthenes measured the radius of the Earth in 195 BC.
- Brain surgeons cannot directly measure the distances and the angles in the brain of the patient before operating with a laser. Show the illustration of the measurements done on a brain before surgery.
- Dentists need to rely on indirect measurements when analysing the X-rays of a patient's jaw in order to design braces. Show an X-ray obtained from a dentist and the construction of triangles that helps the dentist decide upon the shape of the braces.
- In navigation, determining the exact location of a boat requires a very good understanding of trigonometry. Show an example. Demonstrate the use of a sextant as well if there is one available to you.

Ask students to find other examples where triangulation techniques are necessary (see Student Activity: Research).

Ask students to answer the following questions.
- What is triangulation?
- Why is it easier and more precise to measure angles rather than distances?
- How are the trigonometric ratios used in surveying?
- Why is similarity and congruence of triangles so important in surveying?

Activity 1: Triangulation

Objective:

The objective of this activity is for students to use triangulation as a means of measuring something they cannot see.

Materials:

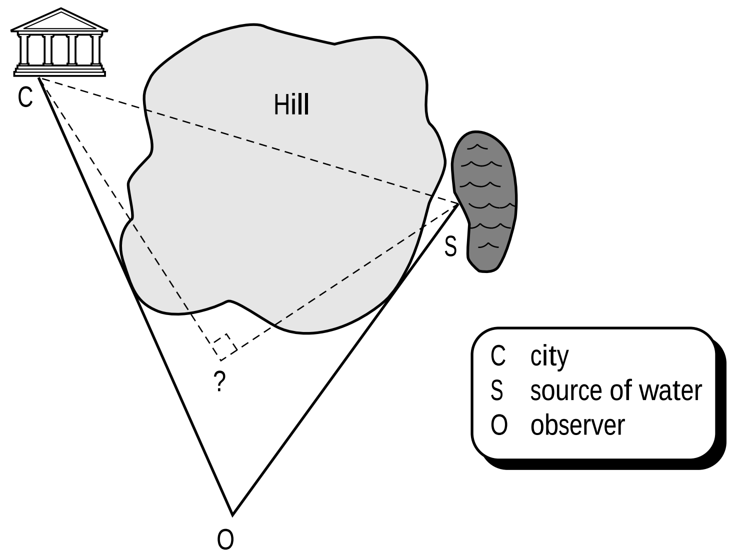
- pair of compasses
- 2 m graduated pole
- decameter and/or a rope
- stand for level
- two carpenter's levels
- protractor
- clinometer (for an example, refer to Figure 4: Clinometer)
- drawing instruments (that is, pens, pencils, ruler)

Procedure:

Because of an increasing population, a certain city of ancient Greece found its water supply insufficient. Residents decided that water had to be channelled from a source in the nearby mountains. Unfortunately, a large hill intervened, and there was no alternative to tunnelling. Working both sides of the hill, the tunnellers met in the middle as planned (see Figure 2: Water Supply in Ancient Greece). How did they do this?

How did planners determine the correct direction to ensure that the two crews would meet? How would you have planned the job? Remember that the Greeks could not use radio signals or telescopes, for they had neither. Nevertheless, they devised a method and actually succeeded in making their tunnels from both sides meet somewhere inside the hill.

Figure 2: Water Supply in Ancient Greece



Essentially, the problem is this: How do the tunnellers determine the direction and the length of the would-be line of sight between the two points *C* and *S* when a hill intervenes?

Ask students to list and describe concepts and techniques they already have in order to solve such a problem. Suggest that they use the Pythagorean relationship for it permits the calculation of the third side of a right triangle. This implies that a right triangle whose hypotenuse is the line of sight *CS* is to be constructed. How do they draw such a right triangle? What do they know? How many different right triangles can they construct? How would they actually do this in a field?

This takes care of the length but the directions are not yet determined. What is missing? Students realize that they need to measure an angle. Present different right triangles having the same hypotenuse demonstrating to students that the angles differ. Introduce the necessity of determining the relationships between sizes of angles and lengths of the sides of triangles.

Now present a different situation (see Figure 2: Water Supply in Ancient Greece) where it is not possible to construct a right triangle such that the observer, *O*, can see both points *C* and *S* (if it were possible the line *OC* or *OS* would pass through the hill!). The triangle *OCS* is not a right triangle.

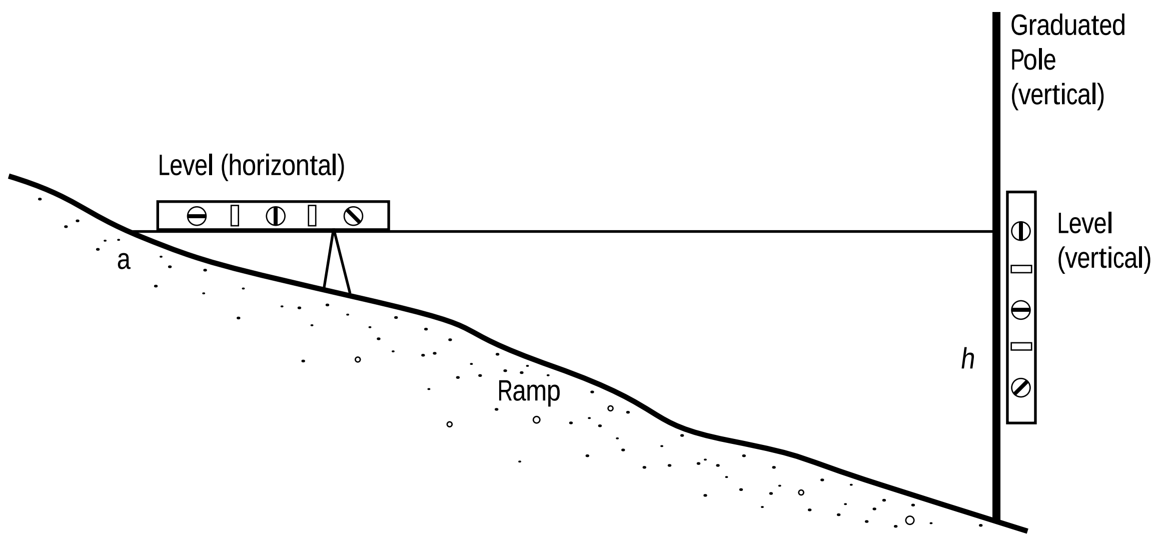
Ask students how they would approach this new situation. Introduce the need for the concept of similarity of triangles and show students it is possible to calculate the ratio of the corresponding sides. Through discussion help students understand that the corresponding angles in similar triangles are congruent.

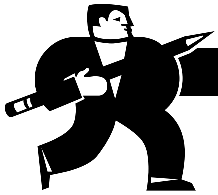
Outdoor Demonstrations:

Demonstrate, in the school yard, how to use a compass, a carpenter's level, a clinometer, and a decameter. For example, have the class identify two points, *C* and *S*, such that *C* cannot be seen from *S* (using a building or a hill). Walk away from the obstacle with a compass until you can see *C* and *S* with a right angle. Ask students to measure *OC* and *OS*. From *O*, walk along a circular arc (traced with the rope) making sure that the angle remains 90° and ask students to measure the distances.

Demonstrate, in the school yard, how to measure a slope with the two levels (see Figure 3: Levels) and how to use the clinometer to measure an elevation.

Figure 3: Levels





Student Activity: Triangulation

Student Name: ____________________________ Date: ___________________

What is triangulation?

In this activity, you will learn how to measure distances and angles for points that do not allow direct measurements. This is the case when the Pythagorean relation cannot be used.

You will derive useful formulae to determine the length of a side of a right triangle. These formulae are the trigonometric ratios of a right triangle.

The methods you are going to learn are necessary for measuring distances and angles in open fields. These methods are used regularly by surveyors, traffic accident analysts, navigators, and even brain surgeons and dentists.

Why do triangulation?

In the logging industry, the height of a tree is often estimated before it is cut down. Questions such as, “Where will the tip of the tree fall?” or “ Is the tree going to hit power lines or damage another tree?” need to be answered.
In surveying, the angle and the direction between two points that are either not accessible or not visible at the same time need to be measured.

- Traffic accident investigators need to apply indirect measurement techniques to determine the speed of vehicles involved in accidents or to prepare scale diagrams as evidence in court.
- Astronomers cannot walk the distance between two planets so they depend on indirect measurement methods.
- Brain surgeons cannot directly measure the distances and the angles in a patient’s brain before operating with a laser.
- Dentists need to rely on indirect measurements to analyse the X-rays of a patient’s jaw in order to design braces.

Materials:

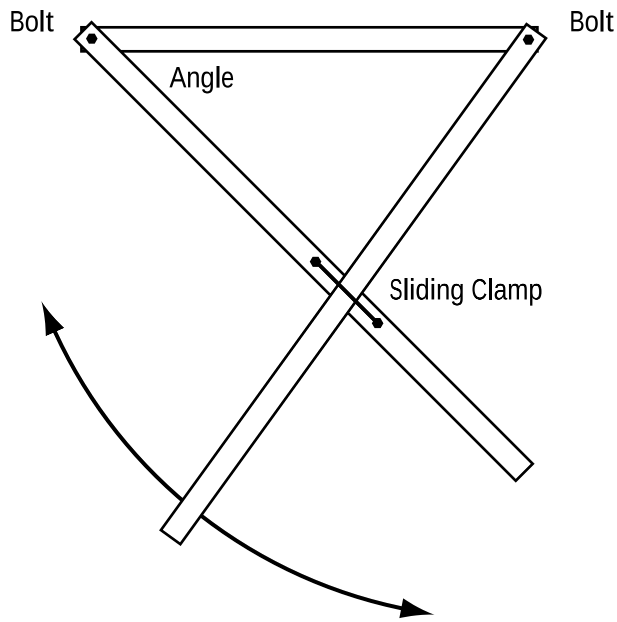
You will need the following materials and equipment. Make sure that they are available and that you know how to use them properly. Ask your teacher if necessary.

- pair of compasses
- 2 m graduated pole
- decameter and/or a rope
- stand for level
- two carpenter’s levels
- protractor
- clinometer (see Figure 4: Clinometer)

Student Activity: Triangulation

- drawing instruments (that is, pens, pencils, ruler)
- graph paper

Figure 4: Clinometer

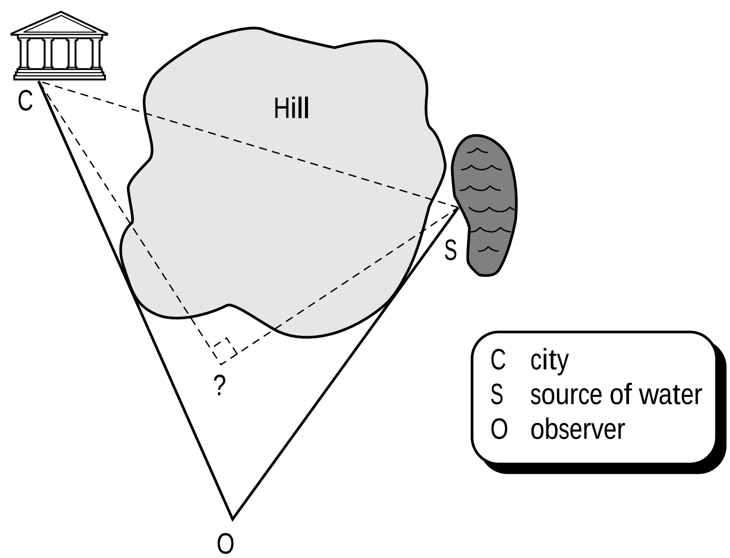


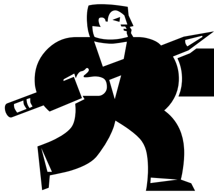
Propose a Strategy to Solve the Following Problem:

Due to an increasing population, a certain city of ancient Greece found its water supply insufficient. It was decided that water had to be channelled in from a source in the nearby mountains. Unfortunately, a large hill intervened and there was no alternative except to tunnel through the hill. Working from both sides of the hill, the tunnellers met in the middle as planned. (See Figure 5: Water Supply in Ancient Greece, below.) How did they do this?

Student Activity: Triangulation

Figure 5: Water Supply in Ancient Greece





Student Activity: Research

Student Name: ______________________ Date: _______________

Using the Internet, a magazine or a newspaper, find one picture of a situation where distance or length is important but cannot be measured directly. Your picture can be a drawing or a photograph. Paste the picture into the space below, describe the measurement, and draw a sketch of how the measurement can be made using triangles.

Picture:

This is a picture of:

The source of this picture is:

Description of the measurement:

Sketch of how the measurement can be made by using triangles:

Activity 2: Pythagorean Relation

Objective:

The objective of this activity is to practise using the Pythagorean relation.

Materials:

- student activity sheets

Procedure:

Review the Pythagorean relation if necessary. Confirm that students:
- understand the Pythagorean relation [Shape and Space - Measurement I (Mathematics 8)]; and
- can use their calculators to compute square roots with decimals.

Suggest the following situation:

If the lengths of two sides of a right triangle are known, the third side may be calculated using the Pythagorean relation. The Pythagorean relation follows:

In a right triangle, the square of (on) the side opposite the right angle is equal to the sum of the squares of (on) the other two sides

$$c^2 = a^2 + b^2$$

or $c = \sqrt{a^2 + b^2}$

In Figure 6: Accident Investigation #1, the height of the cliff a has been measured as 18.75 m. The car landed 9.14 m from the bottom of the cliff, a distance b. The length of the hypotenuse c would be:

$$c = \sqrt{351.56 + 83.54}$$

$$c = \sqrt{351.56 + 83.54}$$

$$c = \sqrt{435.10}$$

$$c = 20.86 \text{ m}$$

Verify that students can identify the hypotenuse and the right angle sides (legs) of the triangle. Also ensure that they can enter the proper data and use the calculator's square root function.

Remind students that the length of the hypotenuse is longer than either of the other sides. Ask them why this is so. Then have them estimate the result from the figure.

Make sure that they use their calculators properly and that they are able to round the result (in this case, two decimals). Can they calculate the result with a minimum of operations?

Reinforce with the following problem:

When the length of one side of a right triangle is known and the length of the hypotenuse is also known, the length of the remaining side can be calculated. A traffic accident investigator has measured, by using a tape, the embankment (hypotenuse c of the right triangle) and the height of the embankment (side a of the triangle).

c = 20.86 m
a = 18.75 m

Using the Pythagorean relation, determine that b = 9.14 m

Make sure that the students are able to manipulate the formula properly. Ask them to estimate the result (from Figure 6: Accident Investigation #1) and from the fact that the answer needs to be smaller than the hypotenuse and the other side of the right angle). You could mention that if they do the wrong subtraction, they will end up with a negative number and the calculator will indicate that with an 'ERROR' message.

When students suggest situations where the Pythagorean relation can be used, ensurethat they realize these situations have one aspect in common—two sides of a right triangle are known.

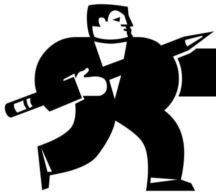
You may use an evaluation form to assess the prerequisite skills and knowledge of your students. An example, Table 1: Assessing the Prerequisites, is on the following page.

Table 1

Assessing the Prerequisites

Code	Criteria	Rating	Comments

The Code column refers to the requisite learning outcomes in the matrix of the prerequisites in Appendix A. Additional criteria may be added as required.



Student Activity: Pythagorean Relation

Student Name: ______________________ Date: ______________

If the lengths of two sides of a right triangle are known, the third side can be calculated by using the Pythagorean relation.

In a right triangle, the square of the side opposite the right angle is equal to the sum of the squares of the other two sides

$c^2 = a^2 + b^2$

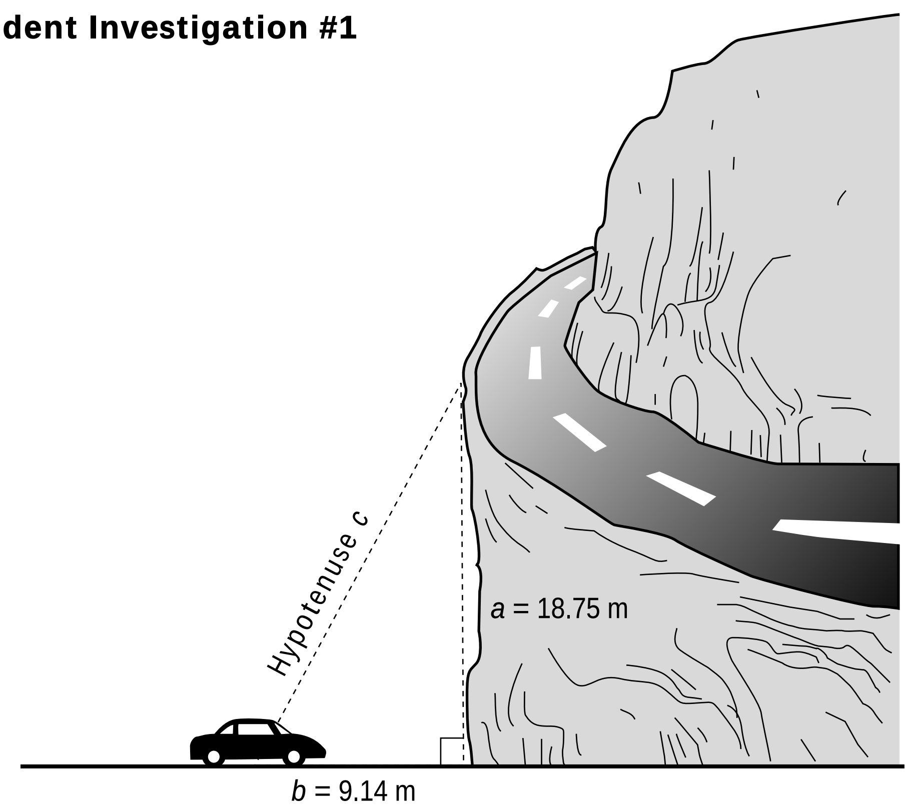
or $c = \sqrt{a^2 + b^2}$

Accident Investigation Example #1

In Figure 6: Accident Investigation #1, below, the height of the cliff is 18.75 m and is represented by a.
The car landed 9.14 m from the bottom of the cliff. This distance is b.
The length of the hypotenuse c would be:

$c =$ ______________________

Figure 6: Accident Investigation #1



Student Activity: Pythagorean Relation

Accident Investigation Example #2

When the length of one side of a right triangle is known and the length of the hypotenuse is also known, the length of the remaining side may be calculated using the Pythagorean relation.

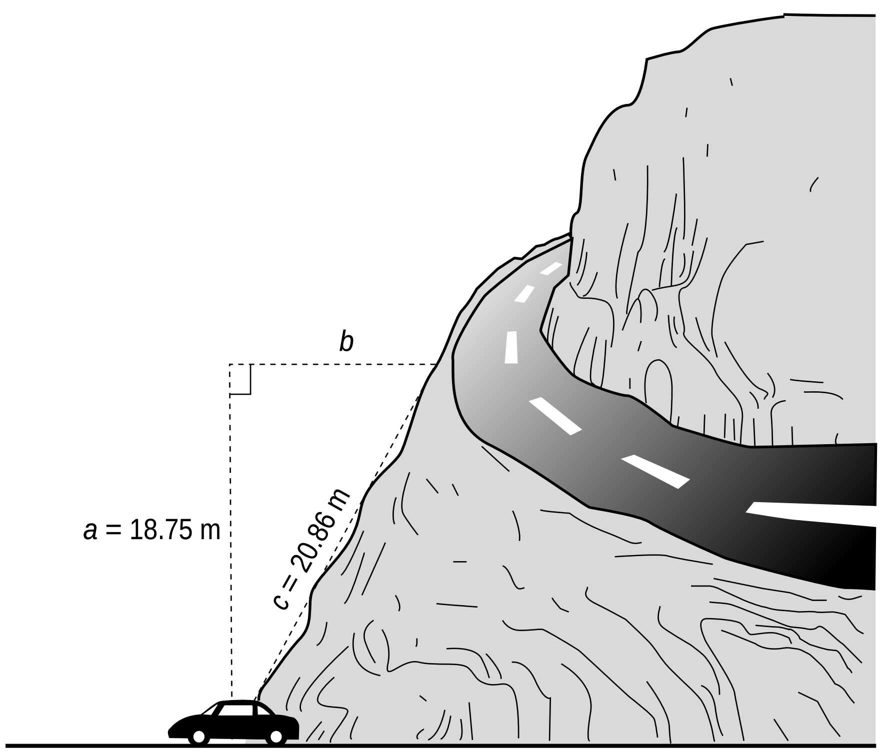
As shown below in Figure 7: Accident Investigation #2, a traffic accident investigator used a tape measure to determine the length of the embankment (hypotenuse c of the right triangle). The height of the embankment (side a of the triangle) was obtained from records kept during the time of road construction.

c = 20.86 m
a = 18.75 m

Use the Pythagorean relation to determine *b*.

b = ____________________

Figure 7: Accident Investigation #2



Student Activity: Pythagorean Relation

Describe another situation where the Pythagorean relation can be used. Draw a diagram.

Describe a situation where the Pythagorean relation cannot be used.

What are some of the ways in which these two situations are different?

Phase II
Part 1: Solving the Problem Using Trigonometry

Objective:

The objectives of Phase II are twofold. The first objective is to guide students in the development of the tangent ratio formula from a real-world situation. Students record measurements, graph the relations, and then develop the formula. There is a motivation to develop the formula because students discover that it is impossible to solve some problems using the Pythagorean relation.

The second objective of this activity is to have students realize the limitations of the tangent ratio in solving concrete problems and to introduce other trigonometric ratios. Once an angle and any two sides of a right triangle are known, the tangent ratio method is not the best way to solve the right triangle. It is possible to solve the problem indirectly by using the Pythagorean relation.

This activity can be used to reinforce or evaluate the learning outcomes relating to Patterns and Relations. See Appendix A: Correlation Between Prescribed Learning Outcomes and Projects.

Materials:

- clinometer
- decameter
- protractor
- pair of compasses
- 2 carpenter's levels
- stand
- 2 m pole
- scientific calculator

Procedure for Tangent Ratio:

Students have identified situations where the Pythagorean relation cannot be used. Such a situation might be to measure the height of a pole or a tree. Encourage students to find reasons that make such a calculation necessary. An example might be that a diseased tree must be cut down and the faller wants to make sure that, in falling, it will not crash into another tree, or fall on another object or person. Encourage students to estimate the height of the tree. One of the ways they might do this is by comparing the height of the tree to the known height of a nearby structure.

Verify that students can use their clinometer properly. Verify that they have used Table 2: Distances and Angles (Student Activity: Measuring a Tree) to record the distances measured with the decameter and the angles measured with the protractor and clinometer.

Can students deduce that the angle at a long distance will eventually be 0° and that the angle at the tree base is 90°?

Note the details students use to express the relation between the distance and the angle. Do they realize that the relation is not linear, that is, if the distance doubles, the angle does not double? Moreover, do they realize that the way the angle changes depends on the distance they measure? Far away from the tree, the angle varies a small amount with distance but when they are close to the base of the tree it varies a large amount.

Encourage students to use the information above to help them while they plot angles versus distances. They should visualize that the distance is growing longer and longer as they approach 0°. Ask them, "What happens at 0°?"

This representation of a relation is more abstract than the numerical representation and some students may have difficulties. Help them compare this relation with a linear relation on the same graph and show them how they differ.

For some students, you will have to use numbers to represent the relation in words.

Demonstrate how to use the $\boxed{\text{TAN}}$ mode on the calculator. Note that calculators differ. On some calculators, the angles can be expressed in degrees, grad, or rad. Make sure that students use the $\boxed{\text{DEG}}$ mode. They should round the result to 3 decimals places.

While completing Table 3: Tan, encourage students to first complete the first and third columns (angles and distances, respectively) and then use a minimum of operations on the calculator:

Angle $\boxed{\text{TAN}}$ x Distance $\boxed{=}$

What is the meaningful number of decimals (or number of significant figures) in the product? The tangent has 3 decimals and the distance had 2 decimals. In this case the answer should be rounded to 2 decimal places.

The results in the last column should be constant. This is the height of the tree. Observe how the students deduce this result. The angle and the distance vary. The only fixed measure is the height of the tree.

Students should be able to express the relationship in their own words. For example:

"If I multiply the distance from the tree by the tangent of the angle, I obtain the height of the tree no matter what the distance is."

Students should also be able to express the relation in the form of a formula. If d is the distance, A is the angle, and H is the height, then

$$d \times tan(A) = H$$

Give students several angles and ask them to calculate the tangent ratio on their calculators. Ask them to explain why tan 90° gives an error on the calculator (refer to the graph).

Encourage students to manipulate the tangent ratio formula in order to obtain an expression for *d* and an expression for tan(A):

$$d = \frac{H}{tan\ (A)} \quad \text{and} \quad tan(A) = \frac{H}{d}$$

Some students may prefer to write the formula in terms of adjacent and opposite sides of the right angle. Ask them to record this on their Student Activity sheet:

$$tan(A) = \frac{\text{opposite side}}{\text{adjacent side}}$$

Ask students to solve the three problems posed in Diagrams 1, 2, and 3 on the Student Activity: Measuring a Tree, and to imagine real situations that can be represented by the diagrams. They should solve the problem from Activity 2 of the Student Activity: Measuring a Tree for homework.

Assessment:

While students are engaged in the activity, informally evaluate how well they are able to:
- assess the relevance of this activity to the overall project;
- identify situations where the Pythagorean relation cannot be used to solve a particular concrete problem;
- identify situations where the tangent ratio can be used to solve a concrete problem;
- express the tangent ratio as the ratio of the opposite side to the adjacent side of right triangle;
- use their calculators to correctly find the tangent of an angle;
- round the result to three decimal places; and
- manipulate the tangent ratio formula to find one quantity in terms of the other two.

Student Activity: Measuring a Tree

Student Name: ____________________________ Date: ___________________

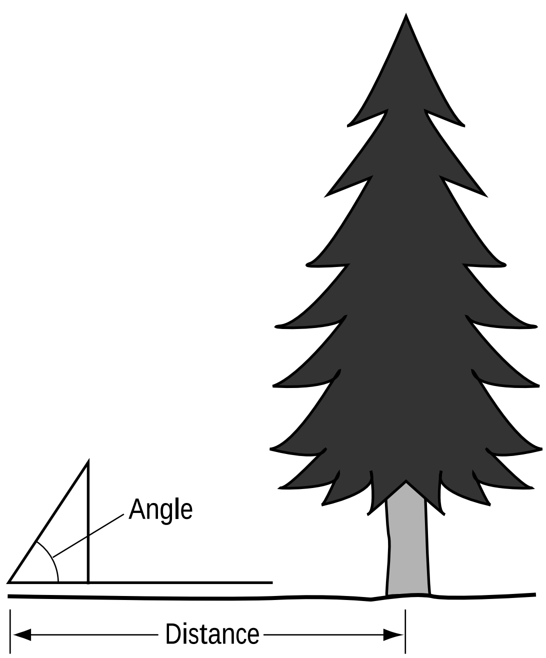
Activity 1: Measuring a Tree

Measuring the height of a tree or a pole is a situation where the Pythagorean relation cannot be used.

What are some reasons you might need to know these heights?

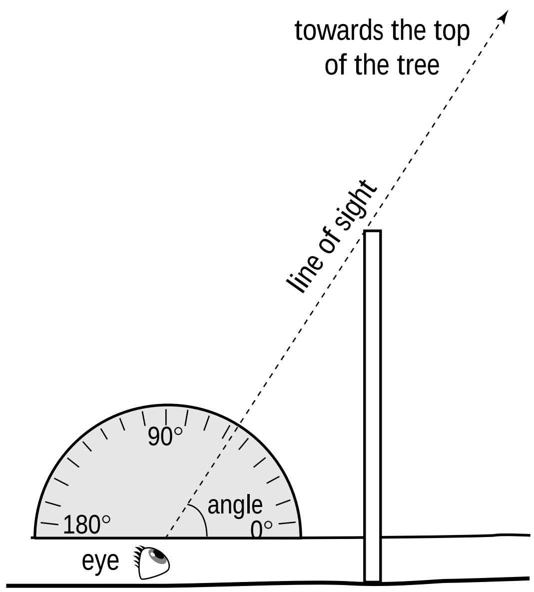
With your partner, choose a tree, pole, or building on the school property. Estimate the height of the object you chose:________________metres

Figure 8: Viewing the Tree



Student Activity: Measuring a Tree

Figure 9: Using a Clinometer



Walking away from the tree, notice how the angle at which you see the top of the tree changes (Figure 8: Viewing the Tree). Use your clinometer and the protractor (as indicated in Figure 9: Using a Clinometer) to determine the angle at the base of the object, at 10m, and at 25m.

at the base of the object ______________________

10 m away ______________________

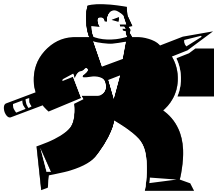
25 m away ______________________

Walk away from the tree so you see the top of the object under an angle of 60°, 45°, 30° and measure the distance you walked.

60° _____________ m

45° _____________ m

30° _____________ m



Student Activity: Measuring a Tree

Table 2

Distances and Angles

Distance (metres)	Angle (degrees)
10 meters	
	45˚

What are some things you notice about the distance/angle relationship in Table 2: Distances and Angles?

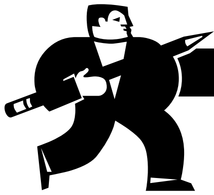
Questions:

What is the angle at a horizontal distance of 500 m?

What is the horizontal distance if the angle is 90°?

What is the horizontal distance if the angle is 0°?

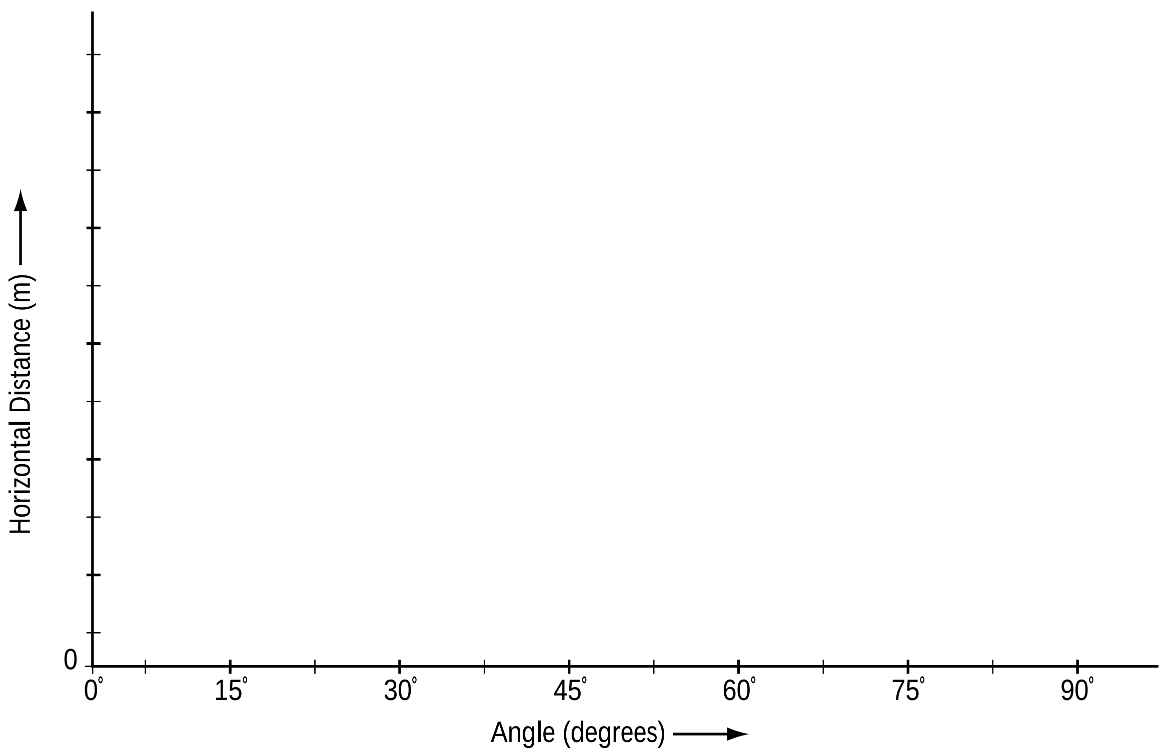
Use your own words to describe the change of the angle with respect to the change of the distance? (e.g., if the distance doubles, what happens to the angle, and is this a linear relation?)



Student Activity: Measuring a Tree

On the grid below, plot the angles against the distances as recorded in Table 2: Distances and Angles.

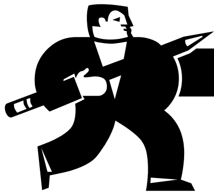
Figure 10: Relation Between Distances and Angles



What are some things you notice about the relationship between the horizontal distances and the angles?

Describe how the angle varies with the distance.

Use the TAN mode on your calculator for the angles in Table 2: Distances and Angles and record your results in Table 3: Tan, below.



Student Activity: Measuring a Tree

Table 3

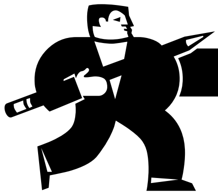
Tan

Angle (degrees)	TAN of Angle (3 decimal places)	Distance (metres)	Product (m) (TAN of Angle x Distance)

What is the meaning of the product calculated in the last column?

Use your own words to describe how angle *A*, the horizontal distance from the bottom of the tree, *d*, and the height of the tree, *H*, are related.

Write an equation to describe the relationship.

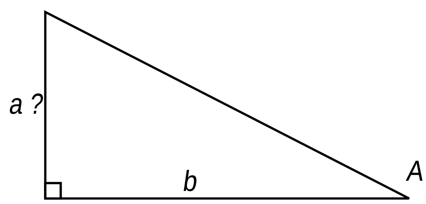


Student Activity: Measuring a Tree

Develop formulae to calculate the distances and angle A in the diagrams below.

In Diagram 1, you know the adjacent side, b, and the angle A, and you want to determine the opposite side, a. Use the space provided to show the steps you used to determine the formula expressed in terms of a.

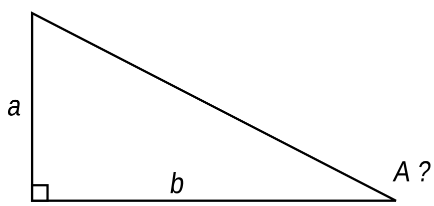
Diagram 1



Formula: a = ___________

In Diagram 2, you know the adjacent side, b, the opposite side, a, and you want to determine the angle A. Use the space provided to show steps.

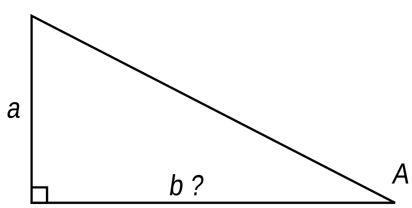
Diagram 2



Formula: $\angle A$ = ___________

In Diagram 3, you know the opposite side, a, and the angle A, and you want to determine the adjacent side b. Use the space provided to show steps.

Diagram 3



Formula: b = ___________

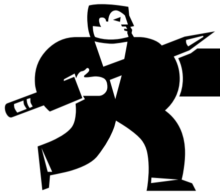
Tangent ratio in a right triangle:

Describe a real-world situation to illustrate Diagrams 1, 2, and 3 above.

Diagram 1:

Diagram 2:

Diagram 3:



Student Activity: Revisiting the Accident

Student Name: ___________________________ Date: __________________

Activity 2: Revisiting the Accident

(using an angle and the Tangent Ratio to measure a distance)

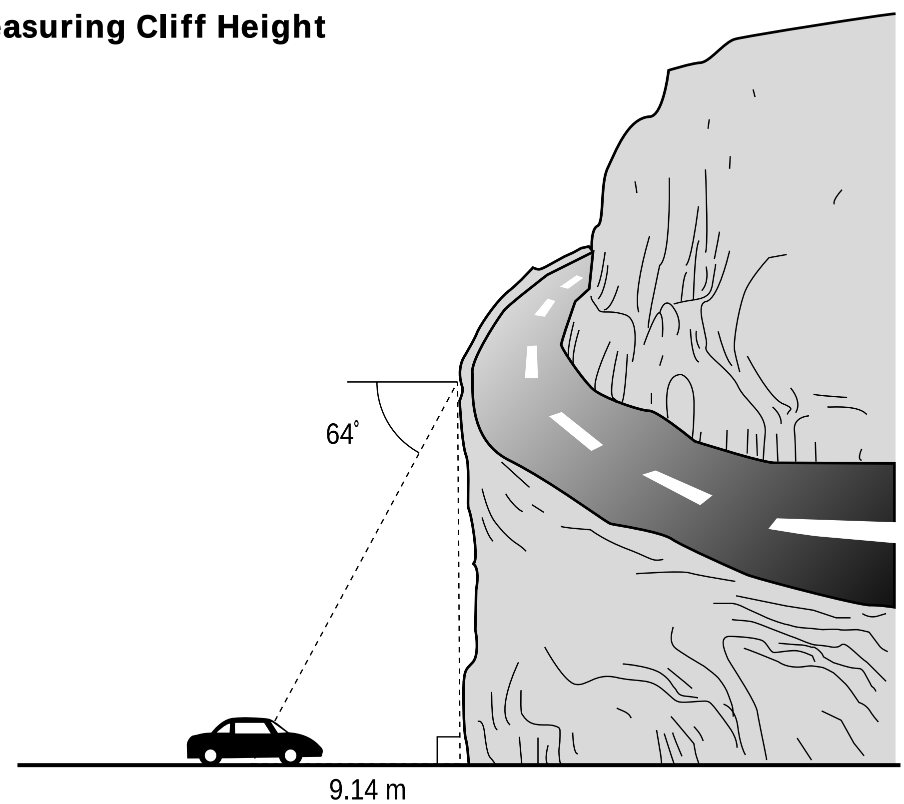
The following measurements were taken after an accident (see Figure 11: Measuring Cliff Height, below).

The vehicle left the highway and landed 9.14 m from the base of the cliff.

The angle from the horizontal to the point of landing, *A*, was 64°. It was measured with a clinometer.

In order to draw a scale diagram, the investigator needs to determine the height of the cliff. What is the vertical height of the cliff? What is the formula you used?

Figure 11: Measuring Cliff Height



Procedure for Sine and Cosine:

A major concern at this point, is to ensure that students are able to decide which ratio of sides is the most appropriate to use in a given situation. It is also important to mention that any ratio can be used but extra work may be required.

Introduce students to inverse trigonometric functions and have them use their calculators to determine an angle given the sine or the cosine ratio when they have to solve a concrete problem such as determining an angle of elevation or the angle of depression of an access ramp. (See Figure 12: Determining Angle of Depression).

Ask students to brainstorm some reasons for needing to know the angle of a ramp or a road, for example, a provincial construction regulation. Have students find other situations that can be modelled by a right triangle where the hypotenuse and one side are known.

Suggest the following situation: "What direct information can be obtained using radar (not height-finding) and how is the missing information determined?" The radar screen indicates the direction and the distance of an airplane but not the elevation (Figure 12: Determining Angle of Depression).

Have students determine the connection between these two situations and explain why none of the previous methods is directly applicable. Ask students to solve the problem indirectly by combining the tangent ratio and the Pythagorean relation. In Figure 12: Determining Angle of Depression,
h is the opposite side of angle A (measured quantity);
a is the adjacent side of angle A (not a measurable quantity); and
d is the hypotenuse (measured quantity).
From the tangent ratio, $h = a \times \tan(A)$ but a is unknown.

From the Pythagorean relation, $d^2 = a^2 + h^2$, so that $a = \sqrt{d^2 - h^2}$.
Now you are back to the tangent ratio method.

NOTE: there are two values for a (positive and negative). Students could test this with their calculator, by taking a number, taking the square root, changing the sign, and squaring the result.

If there is time, students can also derive the sine or cosine ratio in the same way they derived the tangent ratio.

Ask students to find a ramp or road and to determine the angle of elevation. A ladder could be used instead of a ramp or road. What are the direct measurements they can take? How is this related to the project?

Students should model the situation with a right triangle and correctly name the different sides of the triangle: a and h are the sides of the right angle, d is the hypotenuse.

Have students calculate the ratio, $\frac{\text{Height}}{\text{Distance}}$, at different distances and complete Table 4: Ramping Up. This is the ratio of the opposite side h (numerator) and the hypoteneuse, d, and is called the 'SINE' ratio for angle A. It is abbreviated 'sin A'. Introduce the cosine ratio in the same way.

Show students how to use the 2nd mode [OR: depending on the calculator, SHIFT or INV] followed by the SIN function on their calculator. Explain that they are doing the reverse, that is, if they know the value of the ratio, they can determine the corresponding angle.

Ask students to use both their own words and then a formula, to describe the relation between the height measured on the pole (opposite side to angle *A*), the distance on the ramp (hypotenuse), and the angle of the ramp (*A*).

Insist that students determine the appropriate ratio to be used in given situations. Have students work on the four diagrams found in the Student Activity: Ramping Up, and ask them to find alternate ways by combining two methods.

By using their calculators, have students realize that if the sine ratio of an angle = cosine ratio of an angle, then the angles are complementary. Have them practise with different values.

Enrichment: Accuracy and Rounding

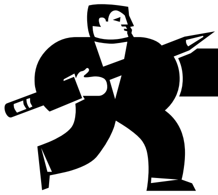
Have students use the calculator to calculate the sine and cosine ratios of different angles (accurate to one decimal place) near 0°, 45°, and 90°. Then have students discuss how the two ratios vary. They should notice that near 90° the sine varies slowly while the cosine varies rapidly and that the opposite occurs near 0°. What can they deduce about the ratios that would be useful in different situations? What can they say about the rounding of the ratios?

Assessment:

Have students propose a strategy for solving cliff heights (Student Activity: Pythagorean Relation) before taking measurements in the school yard and preparing the scale diagram. This activity could be used as an assessment for both the tangent ratio and the other trigonometric ratios, and for reviewing the concept of scaling. Refer to the Grade 8 Mathematics curriculum - Shape and Space; Measurement and Transformations. You may need to have students recall the concepts of enlargement and reduction, and how to use scaling factors.

Evaluate how well students:
- correctly identify the measurable quantities and the unknown ones;
- select another appropriate method to solve the problem;
- model the problem with the correct formula;
- use the correct functions on their calculators;
- make the appropriate rounding and approximations;
- transfer and apply their knowledge to other situations;
- relate their knowledge to the overall project;
- choose an acceptable scaling factor;
- accurately use the drawing instruments; and
- accurately transpose the scale map back to reality.

Note: For Student Activity: Mapping, Activity 6: Mapping, provide students with copies of Blackline Master 5 as needed.



Student Activity: Ramping Up

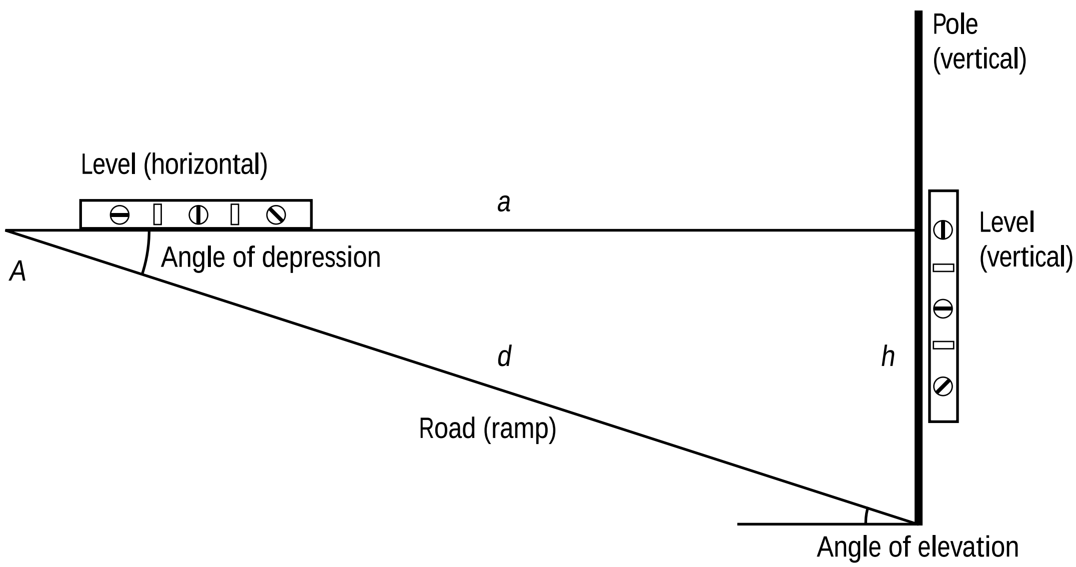
Student Name: ____________________________ Date: ___________________

In some situations, neither the Pythagorean relation nor the tangent ratio can be used because some distances or angles cannot be measured.

Activity 3: Ramping Up

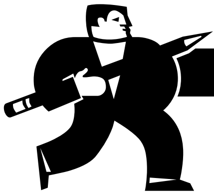
The following problem is an example of such a situation (see Figure 12: Determining Angle of Depression).
Determine the angle of depression of an access ramp by using a pole and a carpenter's level.

Figure 12: Determining Angle of Depression



What are some situations where you may need to determine the angle of depression of a ramp or a road?

Why can't you solve this problem by using the Pythagorean relation or the tangent ratio?



Student Activity: Ramping Up

With your partner, find a ramp or a road. You will need the 2 m graduated pole, the decameter, and the carpenter's level. Describe how you could measure the angle of depression.

What distances can you measure directly? Record these. Remember to use the same units (centimetres or metres) when you record the distances.

Model the situation with a right triangle and name the different sides of the triangle (where a and h are the sides of the right angle, d is the hypotenuse).

Repeat the same operation two more times at different places along the slope. Record your measurements below in Table 4: Ramping Up.

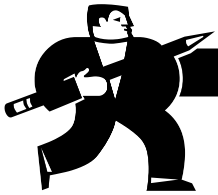
Calculate the ratio, $\frac{\text{Height}}{\text{Distance}}$. This is the ratio of the opposite side h to the hypoteneuse. This ratio is called the 'SINE' ratio corresponding to the angle A. It is abbreviated $\sin A$.

Press $\boxed{\text{2nd}}$ [OR: depending on the calculator, $\boxed{\text{SHIFT}}$ or $\boxed{\text{INV}}$] followed by $\boxed{\text{SIN}}$ on your calculator and enter the results below in Table 4: Ramping Up.

Table 4

Ramping Up

Ramp Distance (*d*)	Pole Height (*h*)	Ratio *h*/*d*	



Student Activity: Ramping Up

Describe the relationship among the height measured on the pole (opposite side), the distance on the ramp (or road) (hypotenuse), and the angle of depression of the ramp (or road) (A).

Write the formula describing the relation between h, d, and angle A.

The sine ratio can be used to solve problems where two quantities can be measured. List all the combinations of two quantities that can be measured directly so that you can then determine any of the other quantities that cannot be measured:

Measured quantities   Calculated quantity

and

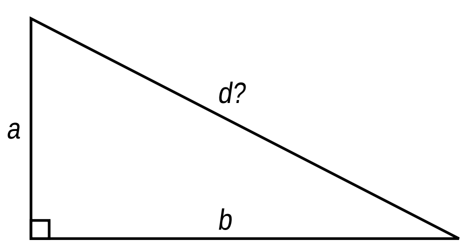
and

and

In each of the following diagrams:

- describe a situation in the real world that is like the one shown in the diagram;
- determine the quantities that are measurable;
- decide what method (Pythagorean relation, tangent ratio, sine ratio) to use to calculate the 'non-measurable' quantity; and
- calculate the 'non-measurable' quantity.

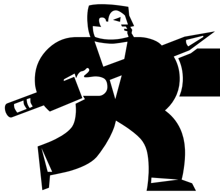
Diagram 1



Real Situation:

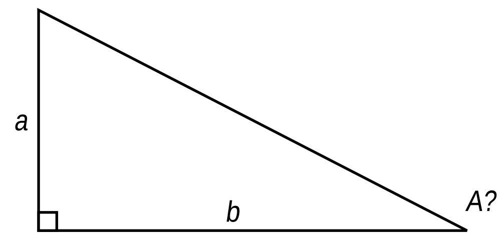
Method:

Solution:



Student Activity: Ramping Up

Diagram 2

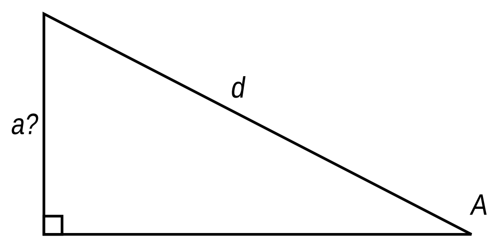


Real Situation:

Method:

Solution:

Diagram 3

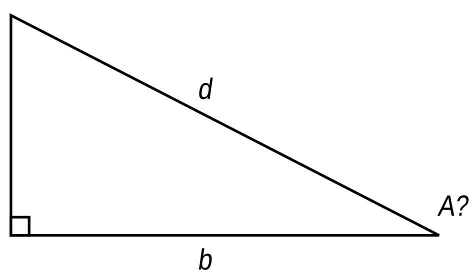


Real Situation:

Method:

Solution:

Diagram 4



Real Situation:

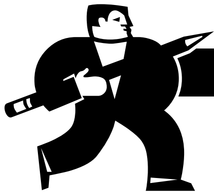
Method:

Solution:

In Diagram 4 above, none of the previous methods can be used directly. You may have found a combination of two methods. If not, try to combine any two of the other methods. This is approach is both complicated and time consuming.

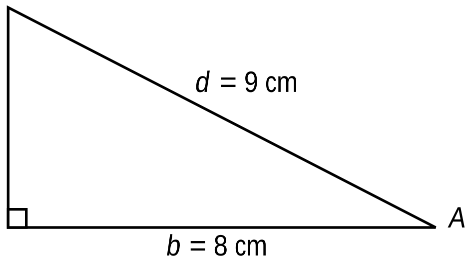
Describe some of the ways that this situation differs from the previous one?

In problems where the hypotenuse *d* and the adjacent side *b* are known the ratio $\frac{b}{d}$ is called the cosine ratio, abbreviated *cos*. (Figure 13: Cosine Ratio)



Student Activity: Ramping Up

Figure 13: Cosine Ratio



What are some real-life situations where it is advantageous to use the cosine ratio?

For Figure 13: Cosine Ratio, use your calculator to determine the angle whose cosine ratio is $\frac{b}{d}$. This time, press $\boxed{\text{SHIFT}}$ [OR $\boxed{\text{2nd}}$, or $\boxed{\text{INV}}$] followed by $\boxed{\text{COS}}$.

(Answer: 27.3°)

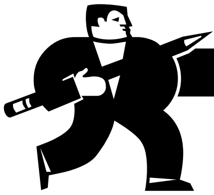
Use your calculator (or a trigonometric table) to determine the following ratios.

sine ratio = 0.500: angle = ___________ °
cosine ratio = 0.500: angle = ___________ °

sine ratio = 0.501: angle = ___________ °
sine ratio = 0.505: angle = ___________ °
sine ratio = 0.510: angle = ___________ °

cosine ratio = 0.101: angle = ___________ °
cosine ratio = 0.105: angle = ___________ °
cosine ratio = 0.110: angle = ___________ °

How do you describe the accuracy of the measurements?



Student Activity: Ramping Up

How does an error in the calculation of the ratio affect the size of the angle?

Use your calculator to evaluate the sine and the cosine ratios of the following angles. Round your answers to three decimals and record in Table 5: Sine and Cosine Ratios.

Table 5

Sine and Cosine Ratios

Angle	Sine	Cosine
0		
1		
5		
44		
45		
50		
85		
86		
90		

Is the accuracy of the measurements more or less important near 90° or near 0°? Explain.

Student Activity: Brain Surgery

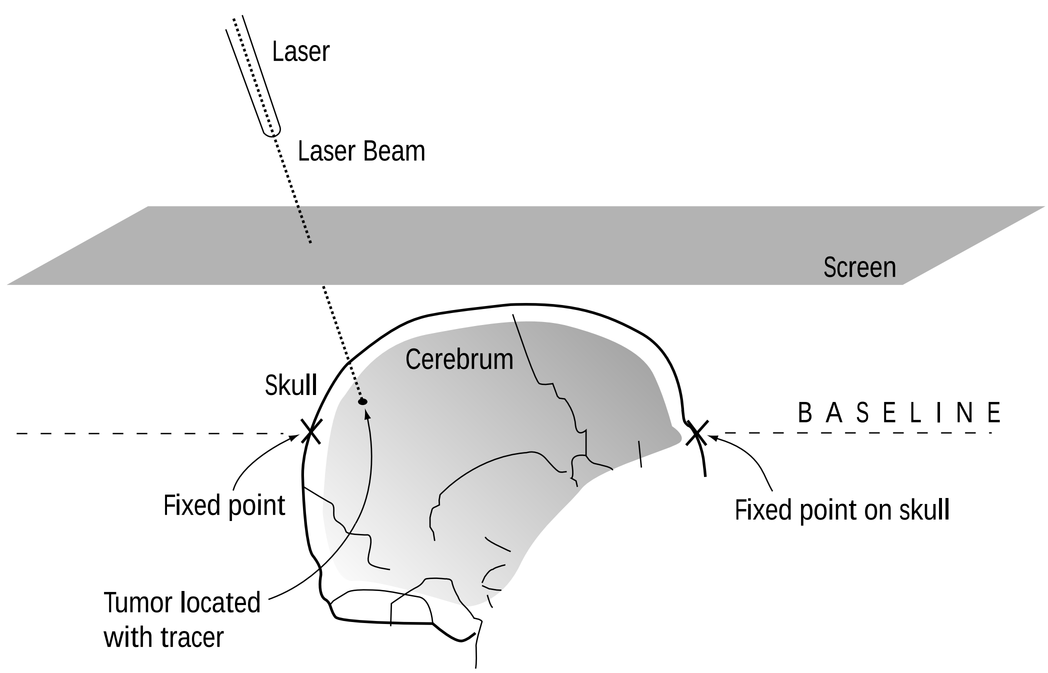
Student Name: ______________________________ Date: ____________________

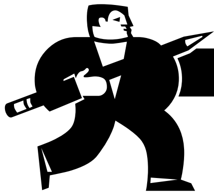
Activity 4: Brain Surgery

Brain surgeons need to direct a laser beam with very high precision in order to remove a tumour without damaging the healthy parts of the brain. The tumour is located with a 'tracer' and the distance from the tracer to a screen is measured. Refer below to Figure 14: Laser Brain Surgery. A baseline is drawn by using two fixed points on the skull.

Model the situation with a right triangle and identify, on the diagram, the direct measurements you can take and the measurements you must deduce in order to direct the laser beam.

Figure 14: Laser Brain Surgery





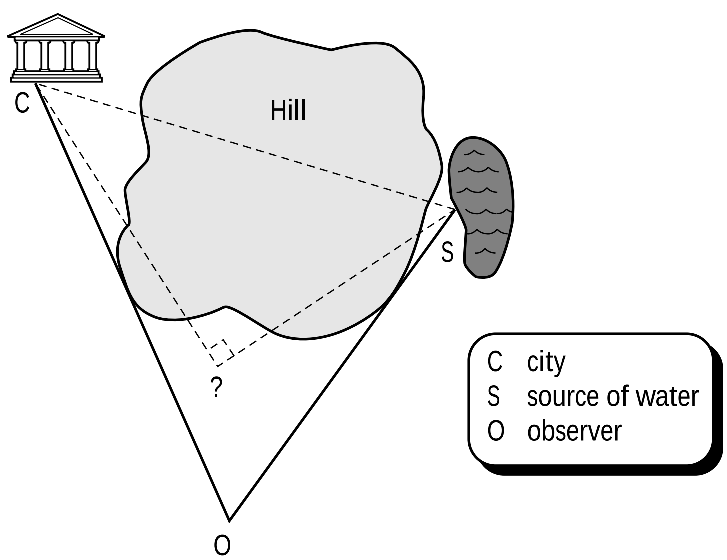
Student Activity: Ancient Greece

Student Name: ____________________________ Date: ___________________

Activity 5: Ancient Greece

Due to an increasing population, a certain city of ancient Greece found its water supply insufficient. Water had to be channelled from a source in the nearby mountains. Unfortunately, a large hill intervened and there was no alternative to tunnelling. Working both sides of the hill, the tunnellers met in the middle as planned. (See Figure 15: Water Supply in Ancient Greece.)

Figure 15: Water Supply in Ancient Greece



How did the planners determine the correct direction to ensure that the two crews would meet? How would you have planned the job? Remember that the ancient Greeks could not use radio signals or telescopes. Nevertheless, they devised a method and actually succeeded in making their tunnels from both sides meet somewhere inside the hill.

Use your knowledge of trigonometric ratios to solve this problem.

Student Activity: Mapping

Student Name: ______________________ Date: ______________

Activity 6: Mapping

Identify two points, *C* and *S*, where *C* cannot be seen from *S* such as points along the sides of a building or a hill.

Walk away from the obstacle with a compass until you can see both *C* and *S* with a right angle. (See your teacher for the proper use of a compass.)

Measure *OC* and *OS* with the decameter.

From *O*, walk along a circular arc making sure that the angle remains 90° and measure the distances.

Sketch the locations and record below all the information for your scale diagram. You need to decide on references (a point and a direction) so you can place them on the scale diagram.

Scale Diagram
Use the scale map pages provided by your teacher.

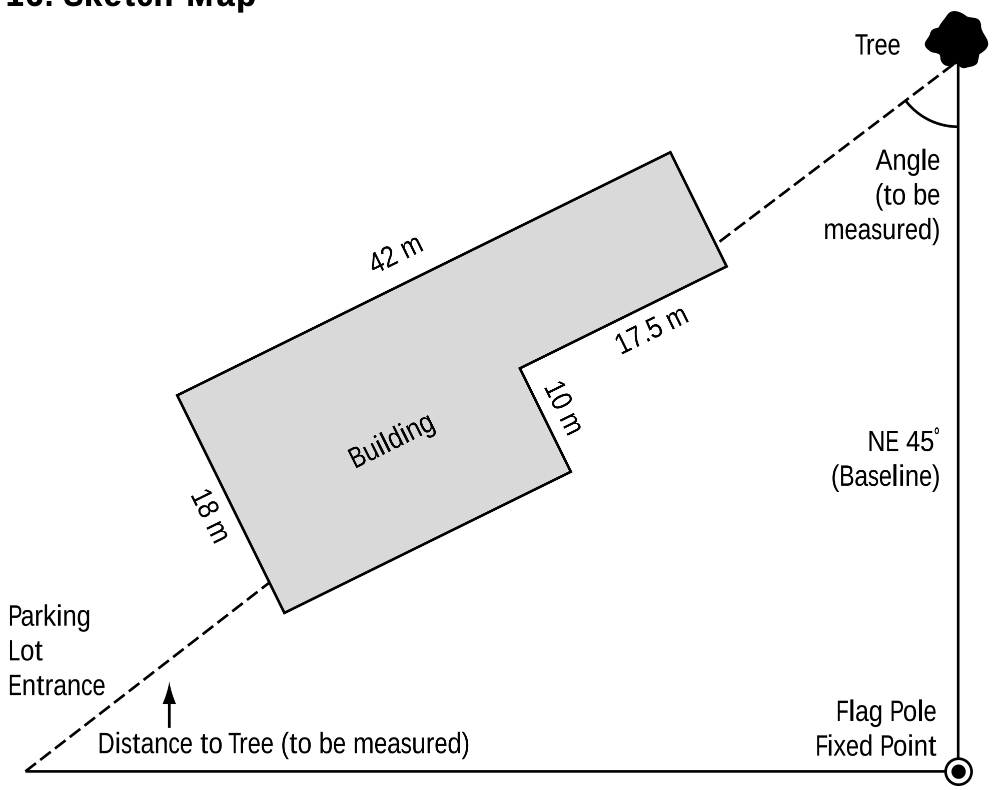
Select the scale you want to use. Your selection depends on how large the distances are on the field. For example, if the distances are quite large, let 1 mm on the diagram = 1 m in actual distance; for smaller distances, let 1 cm on the diagram = 1 m in actual distance.

On your paper, indicate the 'fixed point' and the 'baseline' first. From your sketch, you are able to decide where to place the fixed point and the baseline in order that the entire situation can be drawn on the paper. (See Figure 16: Sketch Map.)

Indicate the direction of the baseline (in degrees) from your measurements and then indicate all the measurements by using the scale.

Student Activity: Mapping

Figure 16: Sketch Map



Phase II
Part 2: Solving the Problem Using Geometry

Objective:

The objective of Phase II: Part 2, is to develop solutions to the problem by using geometry, particularly similarity and congruence.

Materials:

- rope
- decameter
- levels
- stand
- pole
- posts
- flags (or red cones)

Procedure: Similarity of Triangles

The intent of this activity is to introduce students to the concept of similarity of triangles. Students are asked either to determine the unknown height of a tree (by constructing similar triangles) or to determine the grade of a road and then, to solve the problem of the ancient Greek City in a different way. They discover the conditions under which two triangles are similar through direct measurement.

In the first situation, the triangles are right triangles. They then extend the similarity conditions to any triangles in the last situation.

These concepts become helpful in their project because they provide alternate methods for determining indirect measures in a field when right triangles cannot be constructed.

In the Student Activity: Measuring Grade, students experimentally determine the ratios of corresponding sides of two similar triangles. Students note the congruence of the corresponding angles and deduce that the ratios of corresponding sides are equal numerically. Students then express the equality of the ratios formally and use this property to solve the problem.

If there is not enough time to do this activity outside, students can use the results of the measurements from Phase II Activity 1: Measuring a Tree.

Measuring the Grade of a Roadway

Students revisit the concept of similarity in measuring the grade of a ramp (see Phase II Activity 3: Ramping Up). They can use the results found in that activity, if time is a problem.

Similar triangles can be used to measure the grade or elevation of a roadway using a carpenter's level (see Figure 3: Levels).

The percentage grade is the vertical rise or the fall of the road over a horizontal distance of 100 m.

In Figure 21: Measuring Grade, the triangles *BTR* and *BDF* are similar. The board *BT* is 3 m (or 300 cm) long, the fall is 30 cm. Over a distance of 100 m, the fall will be 10 m. This is called the 'percentage grade' of the road. The grade is 10%.

By using the tangent ratio, $\tan(B) = 0.1$. Using the inverse tangent function on the calculator gives 5.71°.

Measurement in the School Yard (Part 2) (Partial evaluation)

This activity illustrates the inverse process. That is, it goes from a representation to reality. It also provides students with a new tool for making measurements in their project.

Referencing Figure 23: Mapping Using Similar Triangles #2 the two points, *C* and *S*, are such that *C* cannot be seen from *S* and, also, a right triangle cannot be constructed. Here, it is not possible to construct a right triangle such that the observer, *O*, can see both points *C* and *S*.

In this new situation, the triangle *SOC* is not a right triangle. Can students apply the method of similarity? How are students approaching this new situation?

Have students draw a scale diagram as precisely as possible using the field measurements from their portfolio. To help them understand how crucial accuracy is in a scale diagram, have students estimate:

- how an error of 2 mm on the diagram would be transposed in the field if their scale is 1 cm : 1 m (answer: 20 cm)
- how an error of 1° (with the protractor) would be reflected in the field over a distance of 35 m.

Students construct similar triangles and use the property of similarity to determine where *E* would be on line joining *O* to *C* such that the line *ED* is parallel to the line *CS*. They should notice the congruence of the angles and apply the equality of the ratios of sides to determine the precise direction and length of line *CS*.

Review the conditions of similarity of triangles by recapping the congruence of the angles and the proportionality of the corresponding sides. Indicate to students that there will be three cases.

Mention to students that in their project they will likely have to use this method because it is not always possible to construct right triangles in the real world.

Procedure: Congruence of Triangles: How to Measure an Object's Distance from the Shoreline

If necessary, review transformations, that is, translation, rotation, and reflection. During transformations, the angles and distances are conserved (by translating a triangle, one obtains a 'congruent triangle').

Congruent triangles are a special case of similar triangles. Congruent triangles have ratios 1 : 1, that is, all corresponding sides are equal and the scaling factor is 1. Congruent triangles are triangles that are equal in all their corresponding sides and angles.

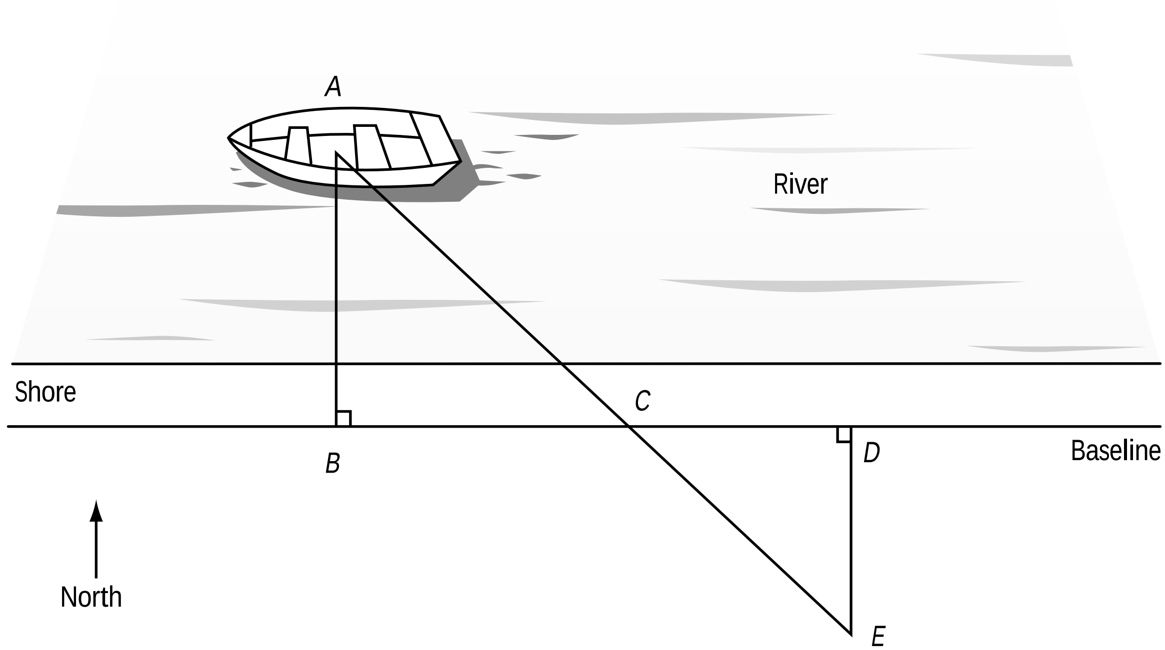
The task is to estimate the position of a boat in a lake, a river, or other area that is difficult to reach. A real-world example is that of a coast guard need to estimate the distance from shore to a boat in trouble.

The distance may be measured indirectly by using a baseline along the shoreline, and sighting and preparing two congruent triangles. You may use Figure 17: Measuring Distance From the Shore, below as a rough guide to the following construction:

- Lay a baseline *BD* of any convenient length. At point *B*, the baseline is at a right angle (90°) with the line of sight to the boat at *A*.
- Find the baseline's midpoint and mark it *C*. The lines from *B* to *C* and *C* to *D* are equal in length.
- From *D*, proceed at an angle of 90° (south in this example) to a point where *C* may be sighted in direct line with point *A*. Mark this point *E*.
- Measure the distance from point *D* to point *E*. This represents the distance the boat is out in the river (*BA*) because in congruent triangles, all corresponding sides and angles are equal.

As a reinforcement activity, ask students to determine a way to solve the problem if there are some restrictions on the length of the baseline. For example, some obstacles restrict the construction of a baseline that is long enough to construct congruent triangles. Again reference Figure 17: Measuring Distance From the Shore, below.

Figure 17: Measuring Distance From the Shore



This is a good opportunity to place the students in a problem situation where they have to apply their knowledge of similar triangles.

Students should recognize that the two triangles are similar and explain that corresponding angles are congruent.

Students find the corresponding ratios and evaluate the unknown distance from the proportionality:

$\frac{ED}{AB} = \frac{DC}{BC}$ implies that ED = AB $\left(\frac{DC}{BC}\right)$.

For example, if DC = 3 times BC, then DE = 3 times AB.
Make sure the students have A, E, and C on the same line.

Evaluation Criteria

To what extent can students in a given situation:
- demonstrate that there is often more than one method that may be used
- choose the most appropriate and precise methods from a range of methods
- apply their knowledge of indirect measurement methods
- use measurement instruments with accuracy and drawing equipment with precision
- manipulate similarity ratios to determine an unknown measure
- perform calculations with precision and round results properly
- determine the reasonableness of results
- explain the impact of imprecision of field measurements on scale diagrams

Student Activity: Revisiting the Tree

Student Name: ______________________ Date: ______________

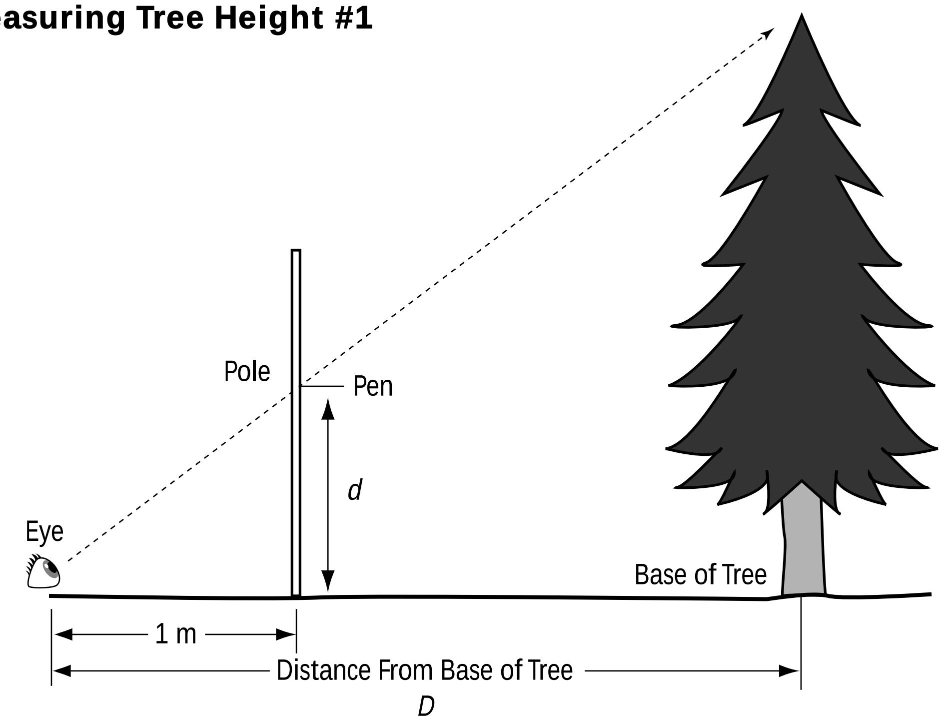
Activity 7: Revisiting the Tree

Sometimes it is more convenient to use techniques that do not involve trigonometric ratios.

Here is a situation where you can determine the height of a tree without using the tangent ratio as you did in the first activity.

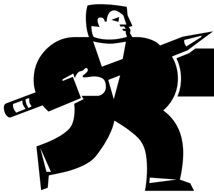
At some distance away from the tree, lie down on the ground and ask your partner to hold the 2 m pole vertically at a distance of 1 m from your eyes (see Figure 18: Measuring Tree Height #1).

Figure 18: Measuring Tree Height #1



Look up at the top of the tree and ask your partner to move a pen along the pole until you see the pen and the top of the tree on the same line. Record the distance of the pen on the pole and measure how far you are from the base of the tree.

Repeat this at different distances from the tree, always keeping the pole 1 metre away from you, and record your results in Table 6: Measuring Tree Height.



Student Activity: Revisiting the Tree

Table 6

Measuring Tree Height

Distance of the Pen on the Pole (d)	Distance From the Tree (D)	Product (D x d)

What are your conclusions?

Repeat the same procedure placing the pole at 2 m and 3 m from your eyes.

Calculate, in each case, the product d x D when the pole was positioned:
at 1 m, d x D =

at 2 m, d x D =

at 3 m, d x D =

What is the product of d x D when the distance of the pole from eyes is:

0.2 m? d x D =

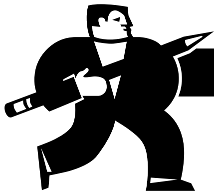
0.5 m? d x D =

5 m? d x D =

Let x represent the distance between your eyes and the pole (Figure 19: Measuring Tree Height #2 below).

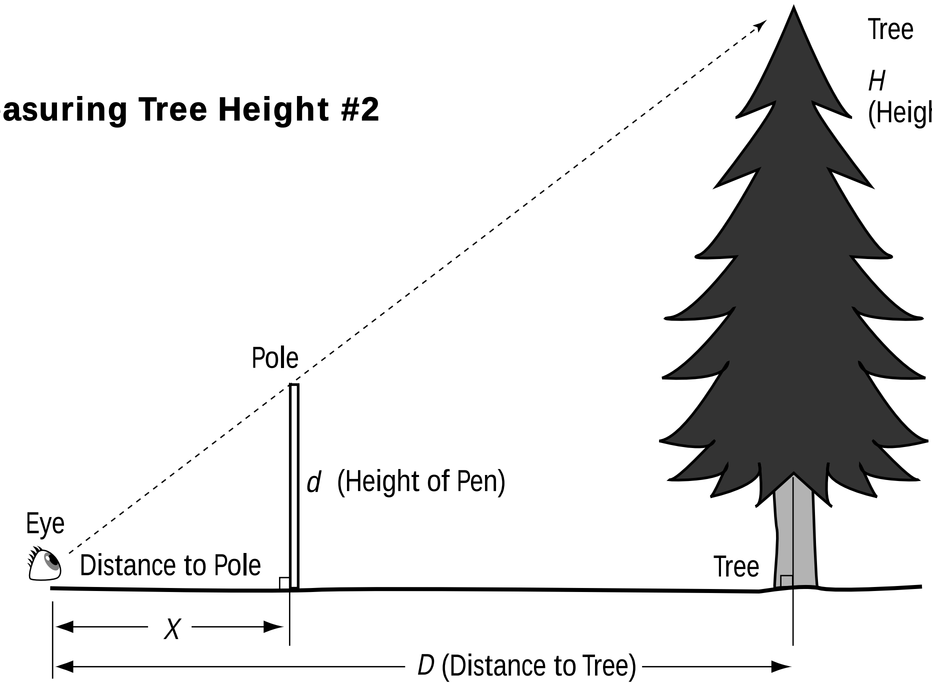
In your own words, relate x, d, D, and the height of the tree, H.

Express this relation in the form of an algebraic function (formula)?



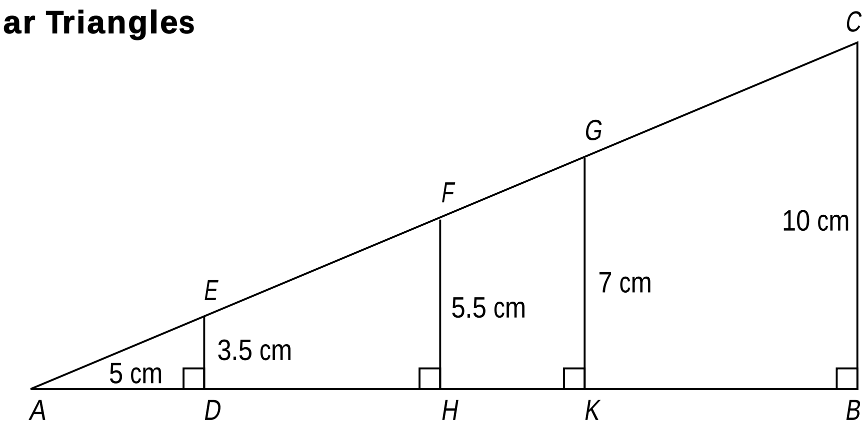
Student Activity: Revisiting the Tree

Figure 19: Measuring Tree Height #2



For the Figure 22: Similar Triangles, what can you say about angles E, F, G, and C?

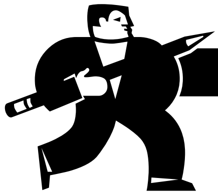
Figure 20: Similar Triangles



Triangles ADE, AHF, AKG, and ABC each have equal corresponding angles. Triangles with each pair of corresponding angles equal are called 'similar triangles'.

The corresponding sides of similar triangles are in the same ratio. To convince you, measure the lengths with a ruler and evaluate the ratios. For example, measure AD and AH, ED and FH, and AE and AF. Calculate the ratios $\frac{AD}{AH}$, $\frac{ED}{FH}$, and $\frac{EE}{AF}$.

$\frac{AD}{AH} =$ $\qquad\qquad$ $\frac{ED}{FH} =$ $\qquad\qquad$ $\frac{AE}{AF} =$



Student Activity: Measuring Grade

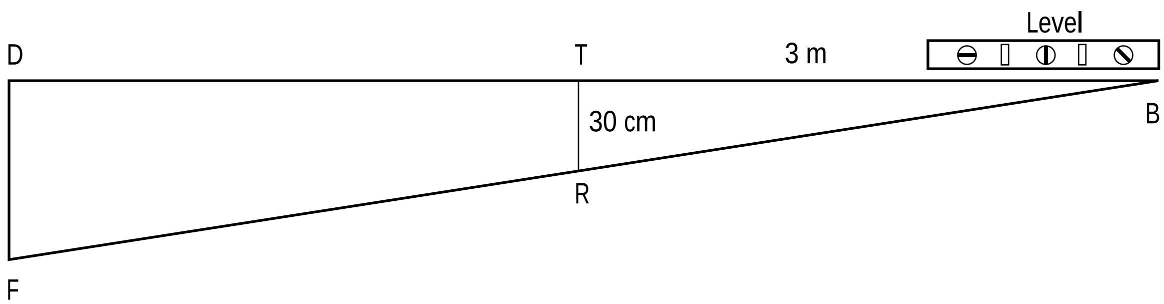
Student Name: ____________________________ Date: ___________________

Activity 8: Measuring

Similar triangles can be used to measure the grade or elevation of a roadway with a carpenter's level (see Figure 21: Measuring Grade).

The percentage grade is the vertical rise or the fall of the road over a horizontal distance of 100 m.

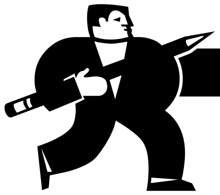
Figure 21: Measuring Grade



In the figure, triangles *BTR* and *BDF* are similar triangles having the same shape but not the same size . The board (*BT*) is 3 m (or 300 cm) long, and the vertical fall (*TR*) is 30 cm. What is the vertical fall (*DF*) for a distance of 100 m?

This ratio expressed as a percentage (%) is called the 'percentage grade' of the road.

Determine the depression angle of this road. What is the trigonometric ratio you used to determine this?



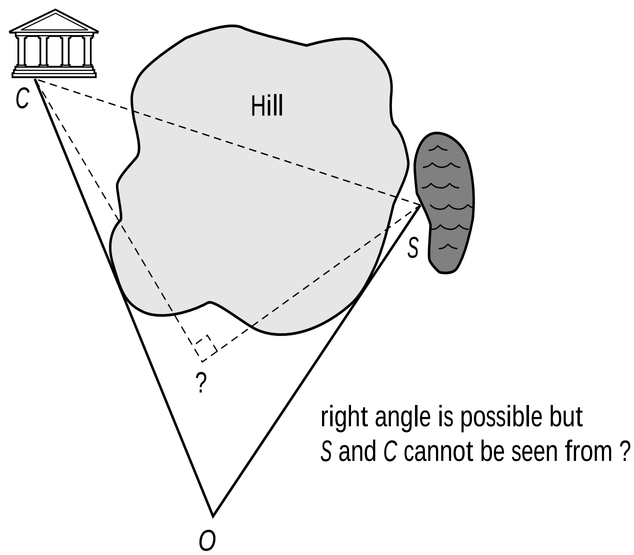
Student Activity: Mapping Using Similarity

Student Name: ______________________________ Date: ____________________

Activity 9: Mapping Using Similarity

Assume that the two points *C* and *S* are such that *C* cannot be seen from *S* and where you cannot construct a right triangle *SOC* when observing both *C* and *S* from *O* (Figure 22: Mapping Using Similar Triangles #1).

Figure 22: Mapping Using Similar Triangles #1



Student Activity: Mapping Using Similarity

It is not possible in this situation to construct a right triangle such that the observer, *O*, can see both points *C* and *S* (such a triangle would have the line *OC* or *OS* passing through the hill).

In this situation, the triangle *SOC* is not a right triangle. Can the method of similarity be applied?

How can you approach this new situation?

In the school yard, walk away from a building until you are able to see both points *C* and *S* (see Figure 23: Mapping Using Similar Triangles #2 below).

Measure *OC* and *OS* with the decameter.

From *O*, walk a certain distance (somewhere between the building and *O*) in the exact direction of *S*, call this point *D*. Measure and record this distance *OD*.

Draw a scale diagram of the situation.

On the scale diagram, use the property of similarity to determine where point *E* would be on the line joining *O* to *C* so that line *ED* is parallel to line *CS*.

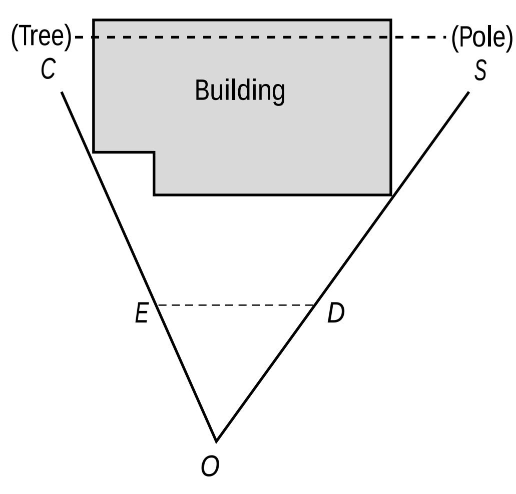
Do you see a way of using the idea that *ED* and *CS* are parallel? Discuss this with students in your group.

Similarity is not a property for right triangles only. The same rules apply for all triangles that do not have right angles, that is, for scalene and isosceles triangles.

Similarity gives you a way to calculate an unknown side of a scalene triangle when another side is known and two corresponding sides of a similar triangle are known.

You will have an opportunity to apply this feature in another activity.

Figure 23 : Mapping Using Similar Triangles #2



Student Activity: Mapping Using Congruence

Student Name: ____________________________ Date: ___________________

Activity 10: Mapping Using Congruence

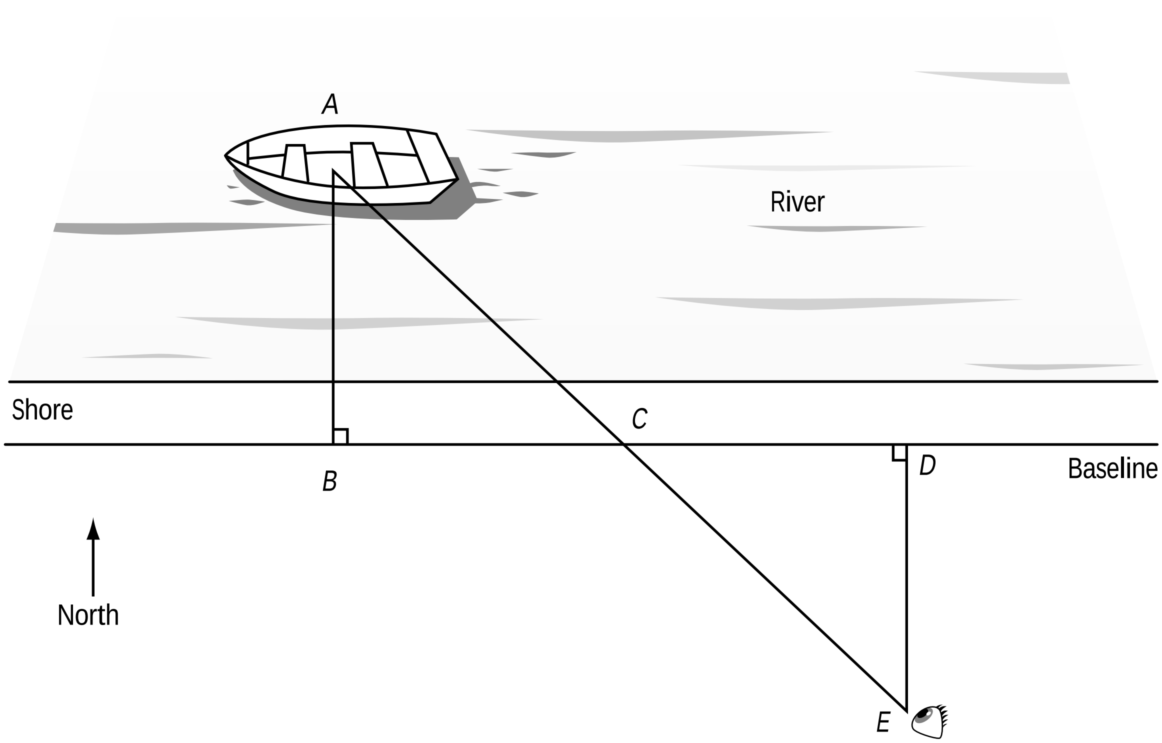
Congruent triangles are those equal in all their corresponding sides and angles.

The position of a boat in a lake, river, or other area that is difficult to reach may be measured indirectly by using a baseline along the shoreline, sighting, and drawing two congruent triangles.

Example (Figure 24: Mapping Using Congruent Triangles, below):

- Lay a baseline, *BD*, of any convenient length. The end of the baseline, *B*, must be such that *BD* is at right angles (90°) with the line of sight from *BD* to the boat at *A*. (Assume A lies north of *BD*.)
- Find the midpoint of the baseline and mark it *C*. *BC* and *CD* are equal in length.
- From *D*, proceed at an angle of 90° (south, in this example) to a point where *C* is sighted in direct line with point *A*. Mark this point *E*.
- Measure the distance from point *D* to point *E*. This represents the distance from shore the vehicle is out in the river. This is true because for congruent triangles, all corresponding sides and angles are equal.

Figure 24: Mapping Using Congruent Triangles



Phase III: Preparing the Scale Map

Objective:

The objective of this phase of the project is the preparation of the scale map. This phase provides students with the opportunity to apply the trigonometric and geometric concepts they have developed over the course of this project. These include how they use their knowledge both to approach new situations in the field and to apply it in different contexts.

Materials:

- graph paper
- ruler
- pair of compasses
- sharp-pointed pencil

Procedure:

Keep in mind the learning outcomes covered during this phase of the project, in particular those that address: Problem Solving, Mathematical Reasoning, Group Skills, Communication Skills and Attitude. The use of the checklist of Appendix A may be helpful.

Groups of students walk the property taking note of points of interest, structures that can serve as reference points, and compass directions.

Remind students to use a compass to record North and the baseline direction.

Students mark the property limits by placing posts at every corner and numbering them. The first step in the sketch will be to draw the property boundaries.

Students sketch the relative positions of objects such as buildings, fences, roads, and bushes and anticipate the points they will have to locate. For each point, they should note what is visible and what is not visible. For example, in Figure 25: Diagram of a Sketch Map, post Z is visible from posts U and R but not from post V.

Back in the classroom, students can now divide the tasks. For instance, one group could be responsible for the boundaries of the parking lot, another responsible for locating a building, and another for the levels, and so on.

All students should participate in determining the location of the boundaries of the property.

Each group develops a strategy. For an example of a strategy, refer to Table 7: Measurements.

Observe the students and advise them about the best strategy. Note how they approach the problem and how they apply the concepts of trigonometry to find and justify ways to perform indirect measurements. (See Figure 25: Diagram of a Sketch Map.)

Tell students that more than one strategy is possible. Encourage them to find the most appropriate and most reliable strategies.

Evaluate their strategies and make comments and corrections before sending them out in the field.

In the field, check if students are using their equipment properly and following the strategies they developed.

Encourage student groups to exchange information about previously located points.

Ensure that students are aware that measuring a distance on an uneven surface leads to inaccuracy (because the surface is not flat).

Help students construct their scale diagrams using construction of similar triangles.

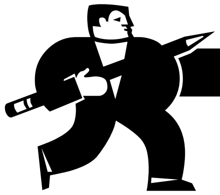
Back in the classroom, students use graph paper, a ruler, a pair of compasses, and a sharp-pointed pencil to draw the scale map. Using a pencil with a broad or dull point may cause inaccuracy.

Remind students to indicate the scale and the legend on their scale maps. Encourage them to use a straight edge and a pair of compasses (instead of using a ruler) to record distances on the map.

Students should proceed first with the most obvious points and continue by using the results obtained in their Table 7: Measurements. Regularly check the sketches produced by other groups to avoid mistakes.

Some points have been used in the measurements more often than necessary. In some cases, you will observe some discrepancies. Rely on the most precise measurements.

For indirect measurements, have students prepare separate maps including all the constructions and calculations.



Student Activity: Sketch Mapping

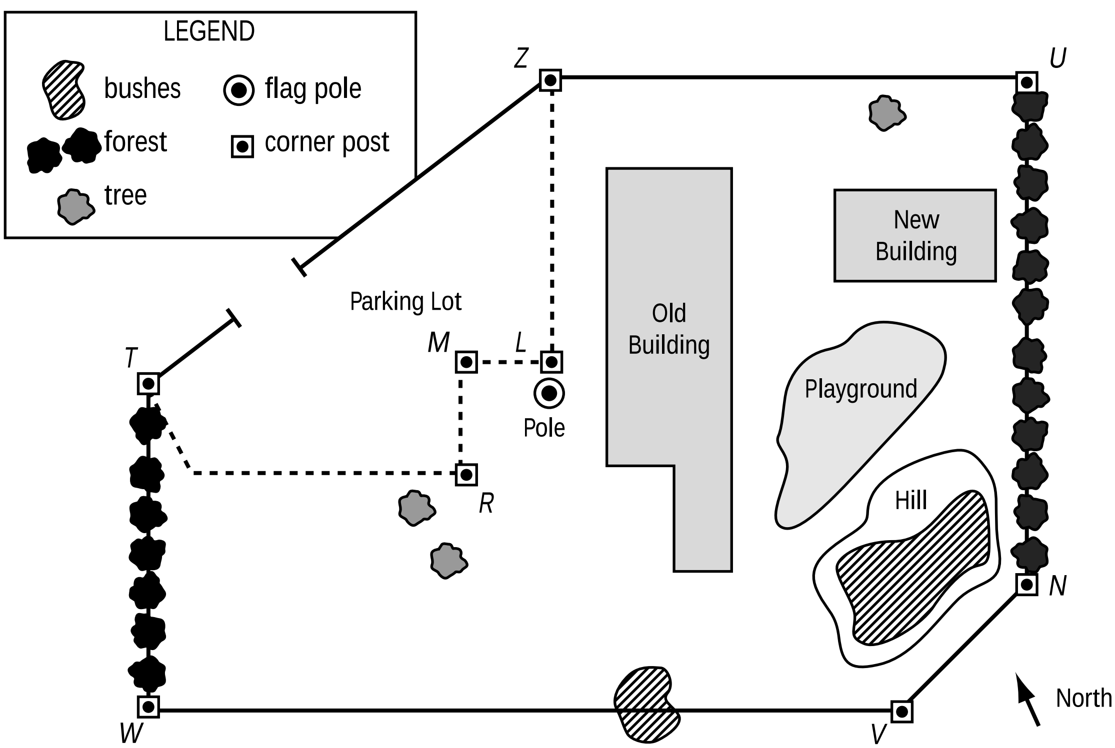
Student Name: ____________________________ Date: ___________________

Activity 1: Sketch Mapping

Go to the site and walk the property, taking note of every interesting point such as a building or hill.

On a piece of paper, sketch the boundaries of the property. The boundaries might include roads, fences, trees and rivers. Roughly indicate the interesting points you have noticed. This sketch does not need to be precise. Use symbols to identify the particular points. Make sure you identify North precisely. See Figure 25: Diagram of Sketch Map below.

Figure 25: Diagram of Sketch Map

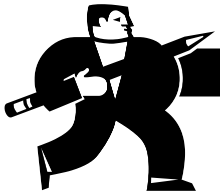


Procedure:

The tasks are divided among the groups (one group may locate the old building, another group is responsible for locating the new building, and so on).

All groups are responsible for the boundaries of the property.

Take a few minutes to develop and elaborate upon a strategy within your group.



Student Activity: Sketch Mapping

A good strategy would include the following operations:

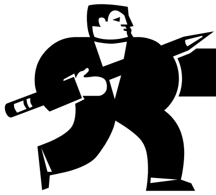
Sample:
- Determine all the points, for example buildings and hills that you need to locate on the map. Mention if they can be seen from each of the other points. Points could include: property corner posts, old buildings, new buildings, playground, shrubs, and hills.
- Find a fixed point, for example a flag pole.
- Find a fixed direction (this will be your baseline). Use the fixed point and another reference point, note the direction with a compass as precisely as possible. For example, the direction from the flagpole to the corner pole, L, is 40° East (N 40° E).
- Then you need to locate the directions and the distances of the property boundaries. This must be done accurately because you may use some of these points later as secondary reference points (points for verification).
- You should use a table to record your measurements (See Table 7: Measurements).

From the flag pole, you can see the corner poles at *W*, *T*, *R*, *M*, *L* and *Z*. Determine direct measurements with the decameter and the compass. For other corners, you need to use indirect measurements.

Table 7

Measurements

Point	Distance to Fixed Point (*m*)	Direction from North	Method
Corner *L*	5	40 E	
Corner *M*	9	330 E	
Corner *R*	4	260E	
Corner *Z*	35	40 E	
Corner *U*			
Corner *N*			



Student Activity: Sketch Mapping

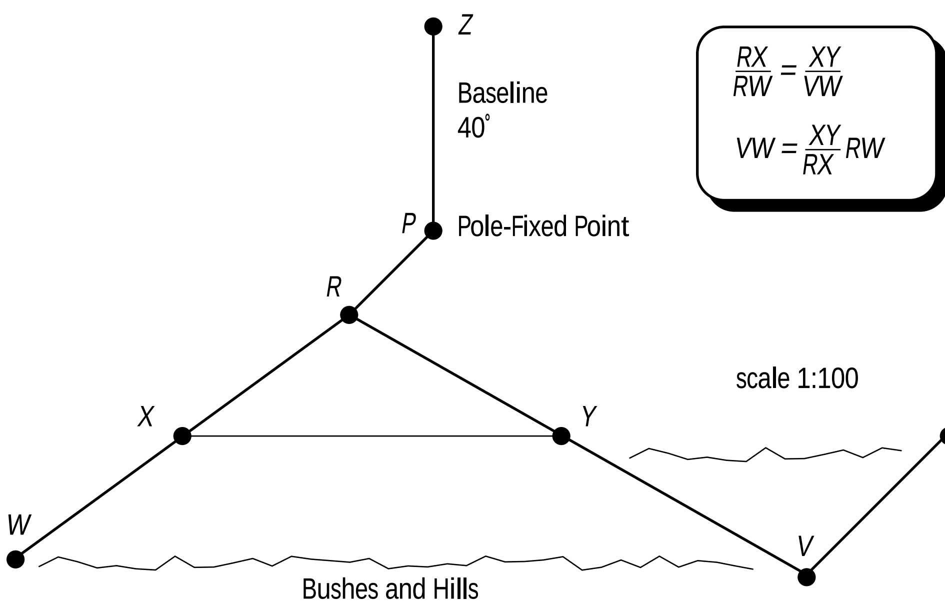
Detailed procedure to locate corner *U*, given location of pole, *P*, and corner *Z*:

- right triangle is visualized with pole, *Z* and corner *U*
- distance pole to corner *Z* is known: 35 m with decameter
- distance corner *Z* to corner *U* is measured: 42 m with decameter
- distance pole to corner *U*: calculated using Pythagorean relation 54.67 m
- direction: angle *ZPU*: tangent ratio *ZU/ZP* - calculator - 50.2°

Procedure to locate corner *N*: (A suggested strategy follows; others are possible. See Figure 26: Locating Corner 3 Construction)

- corner *N* cannot be seen from any other located points but can be seen from corner *V*, but corner *V* cannot be seen from corner *W* for bushes and hills
- distance *VW* cannot be measured directly
- corner *R* can be seen from corner *V* and distance *VW* can be measured
- *WR* can be measured and *W* can be seen from *R*
- construct triangle *RWV* and use similarity of triangles technique to locate corner *V*
- construct triangle *VWN* and use similarity of triangles to locate corner *N*

Figure 26: Locating Corner 3 Construction



Student Activity: Sketch Mapping

Field Measures

For each point you need to locate, devise a similar action plan with your partners.

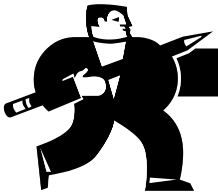
During this operation, take turns within your group doing the different tasks. These tasks include: measuring distances, angles and levels; noting the results; and double-checking the measures. Make sure that your group follows the strategy.

Record the measurements in the field book and include as much detailed information as you can. This will be helpful during the drawing of the scale map.

Exchange information with the other groups and double-check your results against theirs. You may save time by using points located by another group.

Measure the distances accurately. The decameter must be held as accurately as possible in a vertical position. You may tie the rope first between the two points and follow the rope with the decameter. Measurements that use only the rope can be inaccurate because the rope can stretch.

When measuring distances, make sure that you measure only flat surfaces. Measuring distances between two points on an uneven surface leads to major error.



Student Activity: The Scale Map

Student Name: ____________________________ Date: ___________________

Activity 2: The Scale Map

Materials:

- Scale Map blank sheet
- graph paper
- ruler
- pair of compasses
- sharp-pointed pencil

Procedure:

From the sketch, place the fixed point and the baseline in such a way that the whole map can be drawn on the paper. Indicate North and the direction of the baseline. Prepare a legend that everybody is able to understand.

Show the scale at the bottom of the plan. It is a good idea to draw the baseline long enough for accuracy and properly divided. Then you can use your pair of compasses to measure distances from the graduated line.

Now add the most obvious points and continue by using the results obtained in your Table 7: Measurements. Compare your sketch with those of the other groups on a regular basis to avoid mistakes.

Some points have been measured more than necessary. You can use this extra information to double-check your measurements. In some cases, there will be some discrepancies. You should rely on the measurements you think are most precise.

For indirect measurements, prepare separate maps that include all the constructions and calculations.

Project E: Building a Scale Model for a Teen Recreation Centre

Project E: Building a Scale Model for a Teen Recreation Centre

Orientation

Time:

6 to 7 hours

Instructional Strategies:

- Learning Centres
- Math Lab
- Direct Instruction
- Group Work
- Independent Study

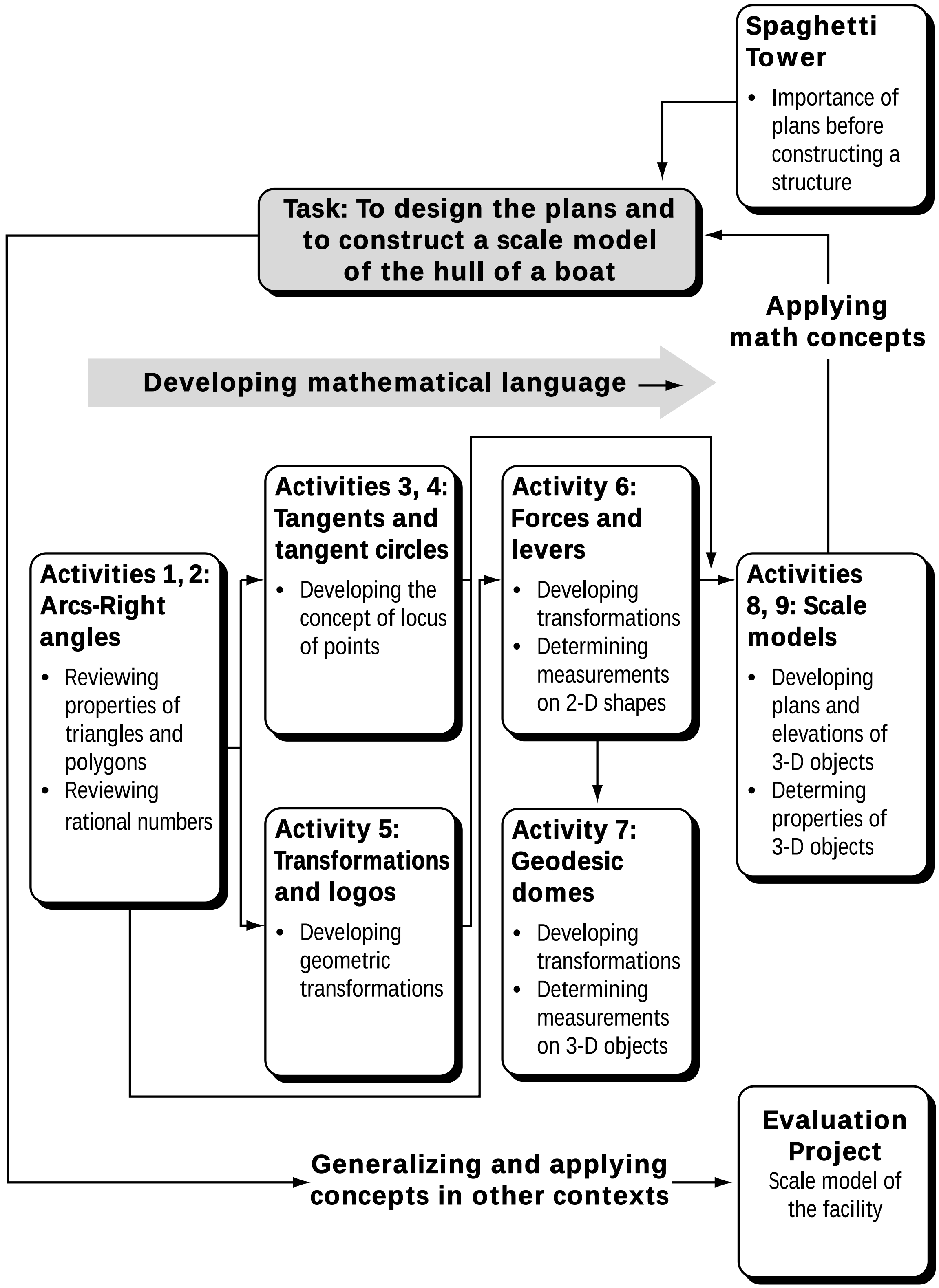
Real-World Applications:

Some real world applications of the skills and concepts learned in this project include:
- maintaining accuracy and precision in construction
- employing accuracy and precision in scale drawings and maps
- constructing a model of the stage sets required for a school play
- building a model railway or doll house to scale

Project Overview:

Figure 1 on the following page is a schematic to help you conceptualize Project E: Building a Scale Model for a Teen Recreation Centre.

Figure 1: Project E: Building a Scale Model for a Teen Recreation Centre



Project Synopsis

This project aims to familiarize the students with the design of precise and accurate scale plans of three dimensional objects by using their abilities with spatial relations.

An additional objective of the project is to provide the students with some mathematical tools related to geometric properties of solids and geometric transformations. These mathematical tools help students simplify the task of constructing 3-D objects from a set of plans and vice versa.

In Phase I, students construct a tower. They draw a scale plan, construct the tower, then compare their plan to reality. This process increases student awareness about some of the elements required in construction.

Phase II is the concept development phase of this project. In this phase, students move from reality to representations of reality while they:
- reinforce their understanding of the properties of plane figures;
- acquire a working understanding of the concept of locus of a point in preparing designs and constructing various curves;
- integrate the concepts related to forces, and relate them to the geometry of the triangle by constructing different models of trusses; and
- use co-ordinate geometry and geometric transformations to simplify construction.

In Phase III, the application part, students move from representations of reality back to reality while they:
- prepare a set of scale plans; and
- use scale plans to construct scale models.

Students apply concepts to prepare a scale plan of a prescribed device. That is, they go from reality to a representation of reality. Then the students construct a device from plans, that is, they go from a representation to reality.

Understanding and designing the plans of a 3-D object, or building a project from a set of plans are skills students will use throughout their lives. In this project, students prepare a scale plan of a support structure (e.g., sawhorse, trestle, or truss) or a lifting structure (e.g., crane or cantilever bridge).

Have students research and bring images of devices similar to those identified above. Using a set of plans created by another group of students, students construct a scale model of a full scale structure able to lift a given weight to a given height or to support a given weight. The plans should be complete and precise enough to permit the construction without any misunderstanding between designers and builders.

Divide the class into an even number of student groups. Initially, groups prepare the scale plans of the device. Then, after having exchanged the plans, students either build scale models of the device or a full scale structure from the plans submitted by another group.

The weight that the device is supposed to lift or support, the height to which the device is supposed to lift the weight, and the particular characteristics of the structures have been determined by the planners and are clearly indicated on the blueprints.

This project can be viewed either as a self-contained project or as a step in the production of a scale model of a Teen Recreation Centre by combining this project with Projects C and D. In the first case, students are asked to produce a report at the end of the project on the construction of the lifting or supporting structure. Later, students may specialize in the production of the scale plans of a building or a structure of their choice. They complete a cardboard scale model as part of the class project of developing a proposal for a recreation facility.

Phase I: Introduction

Objective:

The objective of this phase is for students to appreciate and understand the importance of having a design and plan in hand before building a structure. Before constructing a bird feeder, stereo cabinet, dog house, or even renovating a room, it is wise to design a scale plan of the project.

A scale plan needs to be as precise as possible and flexible enough to allow room for changes during the construction phase. In any event, it is important to have a plan and the materials and equipment you need.

Sometimes it is worth constructing a scale model from the plans in order to have an idea of the final product. Students may experience some surprises if they do not make a scale plan and/or scale model. It could happen that if they decided to rearrange their room and they made only a rough sketch of what they wanted, they might end up with something completely different and unexpected (a desk may not fit, for example).

Activity 1: Constructing a Tower

Objective:

The objective of this activity is for students to develop a scale plan prior to starting a construction project. By constructing a tower of spaghetti and marshmallows, students learn first-hand the importance of scale plans.

Materials:

- one bag of marshmallows
- one package of spaghetti
- measuring tape

Procedure:

This activity is best done by groups of students working together.
Have students construct a tower using spaghetti and marshmallows. The structure should be able to support a certain weight (for example, the minimum requirements might be that it is to support the weight of a tennis ball and be higher than 15 cm). See Figure 2: Spaghetti and Marshmallow Tower below.

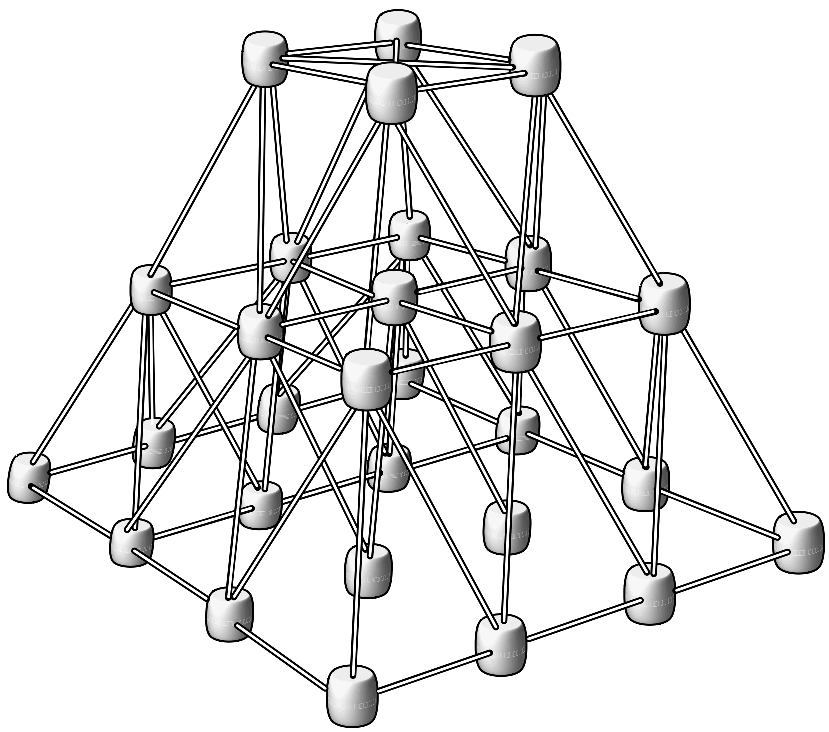
This activity can take the form of a class contest whose rules have been established previously by the students. For instance, one point is awarded for each extra centimetre over 15 cm, or 5 points if the structure can support a basketball.

This kind of activity can be used to stress the importance of having a proper plan before starting any construction project.

Have students draw a rough sketch of what they want to construct and ask them to follow their plan precisely.

After they have completed the activity, discuss how the construction has taken place. Students use the Student Activity: Spaghetti and Marshmallow Tower to record their findings.

Figure 2: Spaghetti and Marshmallow Tower



After the contest, have students relate the tower projects to projects they completed in the past. These may include: room renovation, shelf, bird feeder or dog house construction. Ask them to comment upon some of the difficulties they experienced while constructing their projects.

Tell students what they need to produce at the end of this project and the parameters they need to consider.

Help students establish evaluation criteria for the final product. These criteria should consider the level of their understanding and their ability to apply relevant mathematical concepts.

If necessary, briefly review measurement techniques for triangles and polygons. Students should have a working knowledge of how to measure the perimeter, area, and volume of regular polygons and simple 3-D objects. They should also be able to use and apply reduction and enlargement.

Outline the different concepts students are going to explore and relate them to the project. Establish an action plan with the students by identifying the steps necessary both to produce an accurate scale plan of a complex 3-D object and to construct a 3-D object from the plans provided by another group.

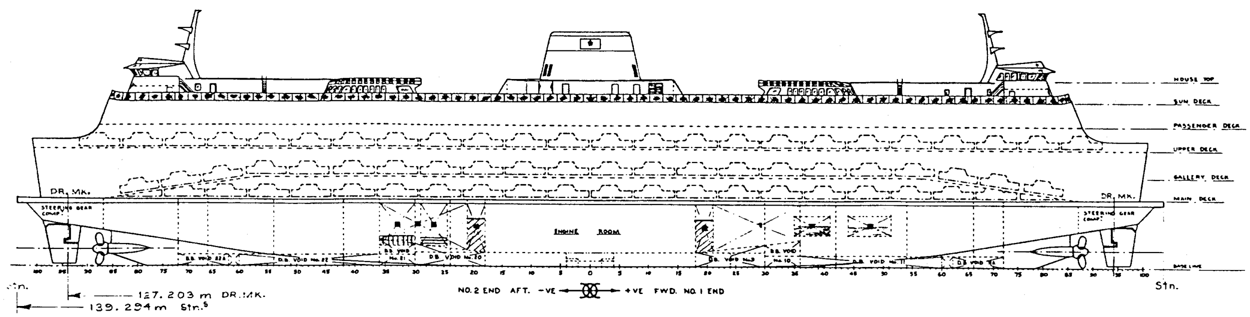
With the entire class, decide upon the characteristics of the devices. Have students then establish the requirements of their own device. For example, what has to be lifted, to what height, or what object the structure needs to be able to support.

Divide the class into an even number of groups of two or three students and assign the tasks.

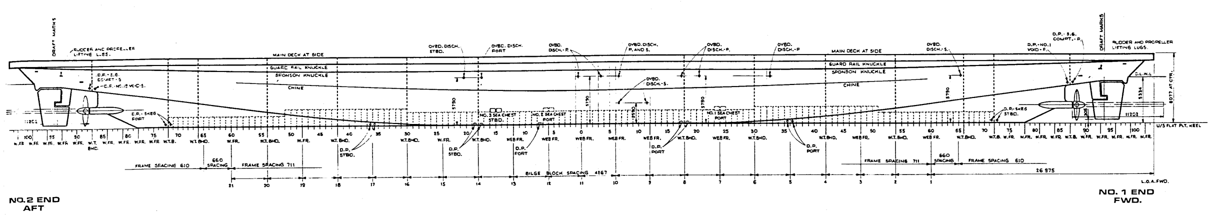
To introduce the concepts for this project you may:
- present a sample of a scale plan. This may be obtained from an architect or a boat builder. With the students, discuss the features of the plans: why it is important for the person in charge to have a plan, and the elevations of the structure.
- ask an architect or a contractor to make a brief presentation to the class about how they make and use plans, and have the students prepare a series of questions.
- obtain website pictures of lifting devices used in construction of historical structures like the pyramids in Egypt, temples in Ancient Greece, and cathedrals of the Middle Ages.

Figure 3: Plans for a Ferry

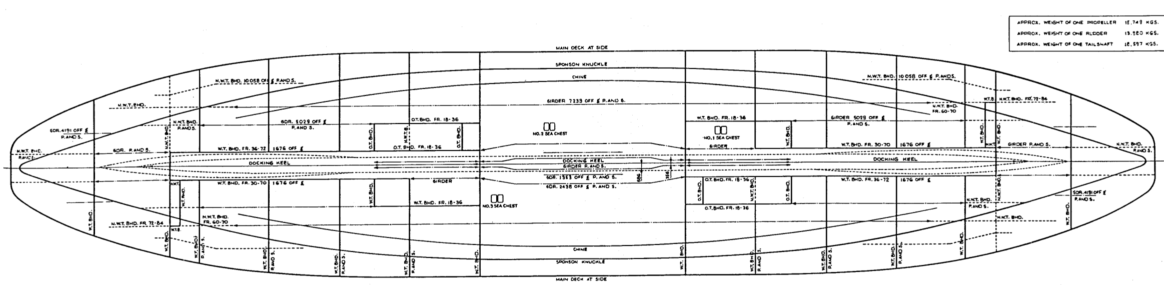
M.V. Queen of Oak Bay



Profile



Outboard Profile



Hold Plan

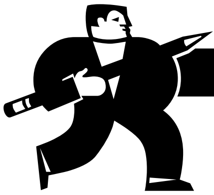
Student Activity: Spaghetti and Marshmallow Tower

Student Name: ______________________________ Date: ____________________

1. Sketch the tower of your dreams.

Student Activity: Spaghetti and Marshmallow Tower

2. Sketch the actual tower you built.



Student Activity: Spaghetti and Marshmallow Tower

3. In what ways do the actual results compare with the original plan?

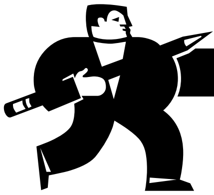
4. In what ways are the actual results different from the original plan?

5. What are some additional comments you have about this activity?

6. Action Plan
   a) Describe your construction project.

   b) Sketch what your finished project will look like.

   c) List the materials and equipment you need to complete your project.



Student Activity: Spaghetti and Marshmallow Tower

d) Project Schedule

Table 1

Project Plan Timeline

Date	Project Phase	Math Concepts	Check	Comments

Phase II: Elements of Construction

Objective:

Phase II is the concept development phase of this project. In this phase, students move from reality to representations of reality while they participate in a variety of different activities. The nine activities provide opportunities for students to:

- reinforce their understanding of the properties of plane figures;
- acquire a working understanding of the concept of 'locus of a point' in preparing the designs and constructing various curves;
- integrate the concepts related to forces, and relate them to the geometry of the triangle by constructing different models of trusses; and
- use co-ordinate geometry and geometric transformations to simplify construction.

In the first two activities, Arcs and Right Angles, students explore various construction techniques that involve polygons, perimeters and area. These techniques require an understanding of various mathematical concepts. The teacher should inform the students of the underlying mathematical ideas during the manipulations. During this exploration, students use concepts related to rational numbers and apply their skills and abilities regarding the use of pattern recognition.

The next three activities are Tangents, Tangent Circles, and Transformations and Logos. In these activities students explore various shapes involving curves and the loci of points in a plane. At the end of this part, students construct a set of curves and produce a design by applying coordinate geometry and pattern recognition. They also explore the effects of the basic geometric transformations.

In the following two activities, Forces and Levers, and Geodesic Domes students construct and analyse rigid structures. They determine how to evaluate the stress on a framework of rigid bars connected together to form a truss. Trusses are all around us in the form of bridges, buildings, boats and so on. In fact, they are in any structure that uses triangles.

The last two activities in Phase II are each closely related to model construction. In the first one, students review geometric solids. In the other activity, students learn more about templates and representing geometric solids.

Activity 1: Arcs

Objective:

Students review the properties of triangles and circles by constructing a coat hanger of given dimensions or a cardboard model of an arched structure.

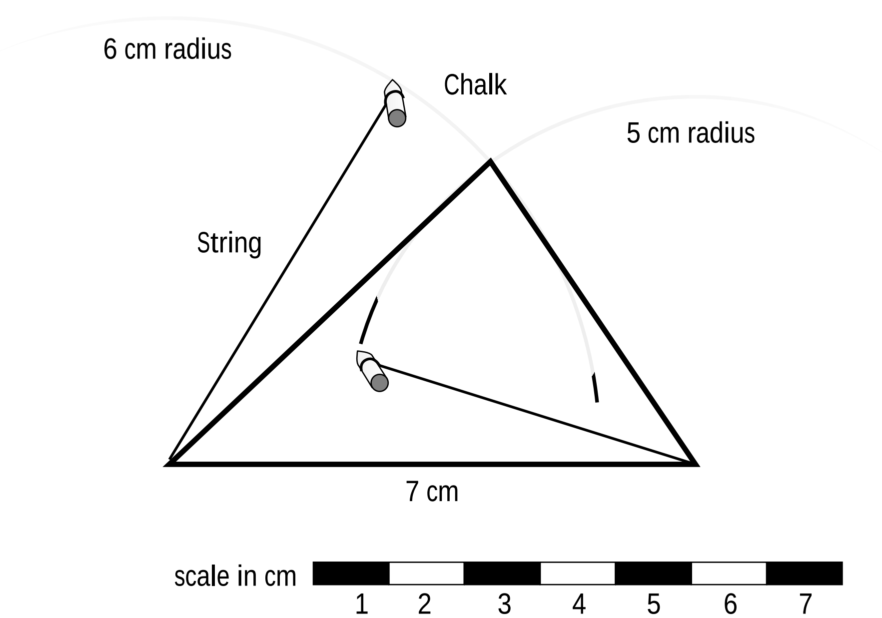
Materials:

- string
- two pieces of wood
- one clamp
- measuring tape
- piece of chalk
- piece of wire

Procedure:

Review, with the students, how to construct a triangle whose three sides are known by using a piece of string and chalk as shown in Figure 4: Drawing Arcs.

Figure 4: Drawing Arcs



Example of a triangle with sides 50 cm, 60 cm, and 70 cm.

- Draw the 70 cm side on the floor or chalkboard and indicate the points *A* and *B*.
- Take a piece of the string of length 60 cm and scribe the arc of a circle having *A* as its centre.
- Take a piece of string of length 50 cm and draw the arc of a circle having its centre at *B*.
- The intersection of the arcs of the circles is the third vertex of the triangle.

Have students comment on how the arc would change if they were to draw a triangle having sides 70 cm, 30 cm, and 20 cm.

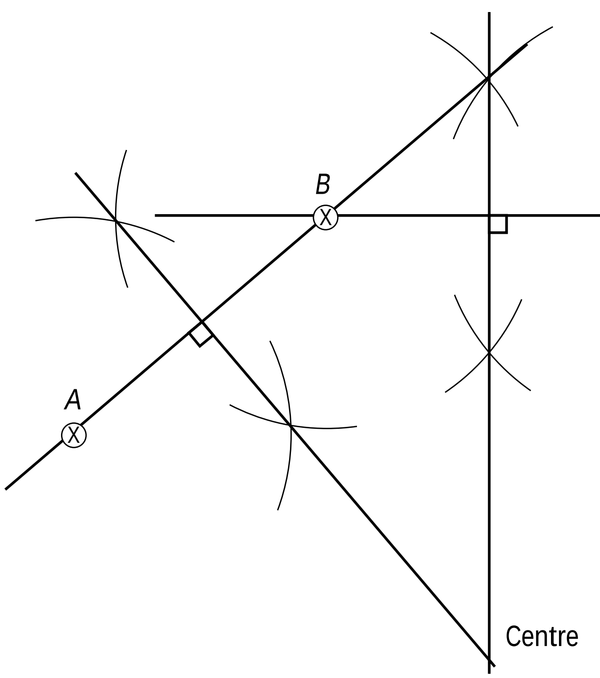
Have students find an illustration of a cathedral vault and an arched bridge in a Japanese garden. Ask them to determine a way of constructing an arc joining the points *A* and *B* while clearing a given elevation (point *C*).

Demonstrate how to draw the arc of a circle given three points as shown below in Figure 5: Vaults.

- Draw three points (not on a straight line) on the floor (or chalkboard).
- Draw the lines between *A* and *B* and between *B* and *C*.
- Construct the perpendicular bisector of each segment and indicate the point where they intersect. This is the centre of the circle upon which points *A*, *B* and *C* lie.
- Trace the arc passing through *A*, *B*, and *C* with the string at the centre.

Have students comment on the length of the radius and the curvature of the circle.

Figure 5: Vaults



1. Join *AB* and *BC*.
2. Construct the perpendicular bisectors of *AB* and *BC*.
3. The intersection of the two perpendicular bisectors is the centre of the circle.

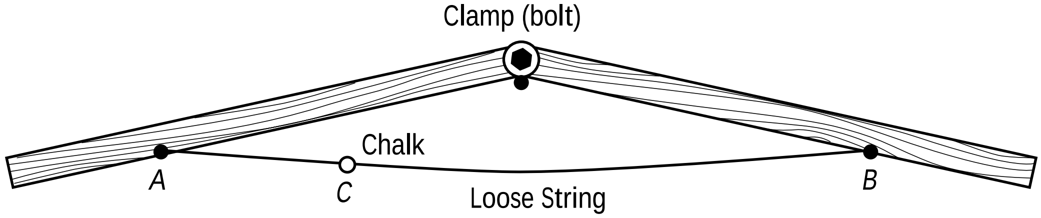
Ask students to propose a way to draw an arc. If the radius is small, they can use the construction procedure described above, but if the radius is large, the following procedure is more appropriate.

First, firmly attach two pieces of wood with a clamp as shown below in Figure 6: Coat Hanger. Loosely attach a piece of string at points *A* and *B* on each arm. Attach a piece of chalk *C*, anywhere along *AB*. As you move the arms, the piece of chalk moves along with *AB*, and as it does so it draws an arc. The arc or curve is the locus of *C*.

Wherever you are on the curve, and as long as you are drawing the arc, then:

$AC + CB$ = constant

Figure 6: Coat Hanger



Point C moves along the curve.
AC + CB = constant wherever you are on the curve as long as you are drawing the arc.

Have students comment on the procedure and justify their comments by using mathematical arguments.

Students use Student Activity: Scale Plans and answer questions.

Construction:

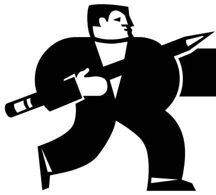
- Have students draw the plans (scale factor of $\frac{1}{2}$) of a coat hanger and then bend a piece of wire to reproduce the coat hanger.
- Have students draw a scale plan (factor of $\frac{1}{10}$) of an arc 2 metres wide (the distance between the two arms *AB*), and a height 0.20 metres at its centre. Then have them construct a scale model with cardboard.

Student Activity: Scale Plans

Student Name: ____________________________ Date: ___________________

Explain why the construction technique illustrated in Figure 5: Vaults does not apply easily in situations like the one in Figure 6: Coat Hanger

Draw the scale plan (factor of $\frac{1}{2}$) of a coat hanger in the space below. The dimensions are those of a normal coat hanger.



Student Activity: Scale Plans

Draw the scale plan (factor of $\frac{1}{10}$) of an arched bridge 2 metres wide with a height of 0.20 metres at its centre.

Student Activity: Scale Plans

Solve the following construction problem and show all the steps you need to use in the solution. Provide a sketch of the problem.

What length of garland would you need to decorate a 2 metre long mantle?

- You have a mantle above a fireplace. The mantle is 2 m long.
- You want to make a two-colour, twisted paper garland to decorate the mantle.
- The garland is glued or taped at both ends to ensure that the twisting does not come undone.
- Each end of the garland is attached to the ends of the mantle, allowing the garland to hang below the mantle at the 1 metre point.

What are some comments you have about this activity?

Activity 2: Right Angles

Objective:

The objective of this activity is for students to realize that the accuracy of angle measurement is important in construction. Students apply the properties of triangles and review the trigonometric ratios and measurement strategies.

Materials:

- string
- measuring tape
- piece of chalk

Procedure:

Students review the concepts related to polygons, and apply their knowledge of patterns and using rational numbers in a construction context.

Briefly review the Pythagorean Relation.

Ask students to determine the accuracy of a room's dimensions, and how to determine the dimensions and angles of a rectangular room.

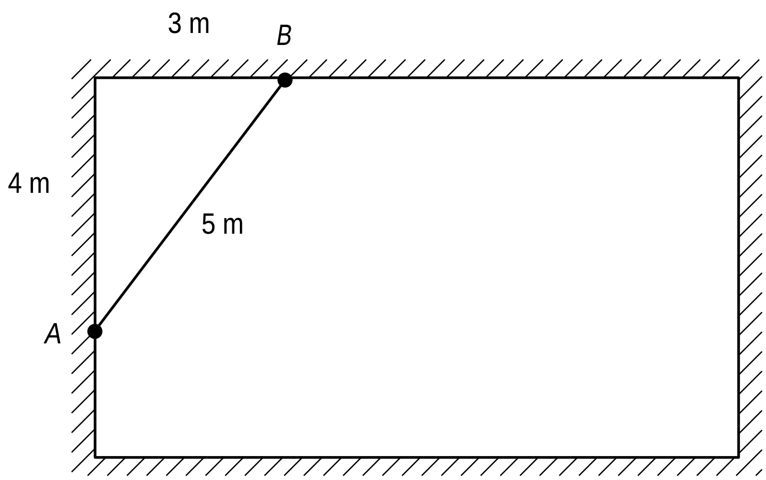
Have students choose a corner of the classroom or gym and ask them to find a practical way of determining how close the corner walls are to forming a right angle.

Ask students to first imagine situations where they will need to evaluate the accuracy of a right angle, and then to imagine some consequences of a lack of precision. Ask them to comment on the accuracy of using a carpenter's square in such a situation and to propose strategies for determining a right angle.

Propose the following procedure that is illustrated below in Figure 7: Right Angles:

- students measure horizontally the lengths of two adjacent walls of a room.
- they measure 4 m (or 40 cm) horizontally on one wall (this locates point *A*), and 3 m (or 30 cm) on the other wall, (this locates point *B*).
- they measure the distance *AB* and record the measurement.
- students reason that if *AB* = 5 m (or 50 cm), the angle is a right angle. If *AB* is larger than 5 m, the angle is larger than a right angle. If *AB* is smaller than 5 m, the angle is less than a right angle.
- students comment on this method and propose other methods of determining a right angle.

Figure 7: Right Angles



Steps

1. Measure 3 m on a wall to locate *B*.
2. Measure 4 m on the other wall to locate *A*.
3. Draw the line *AB*.
4. Measure *AB*.

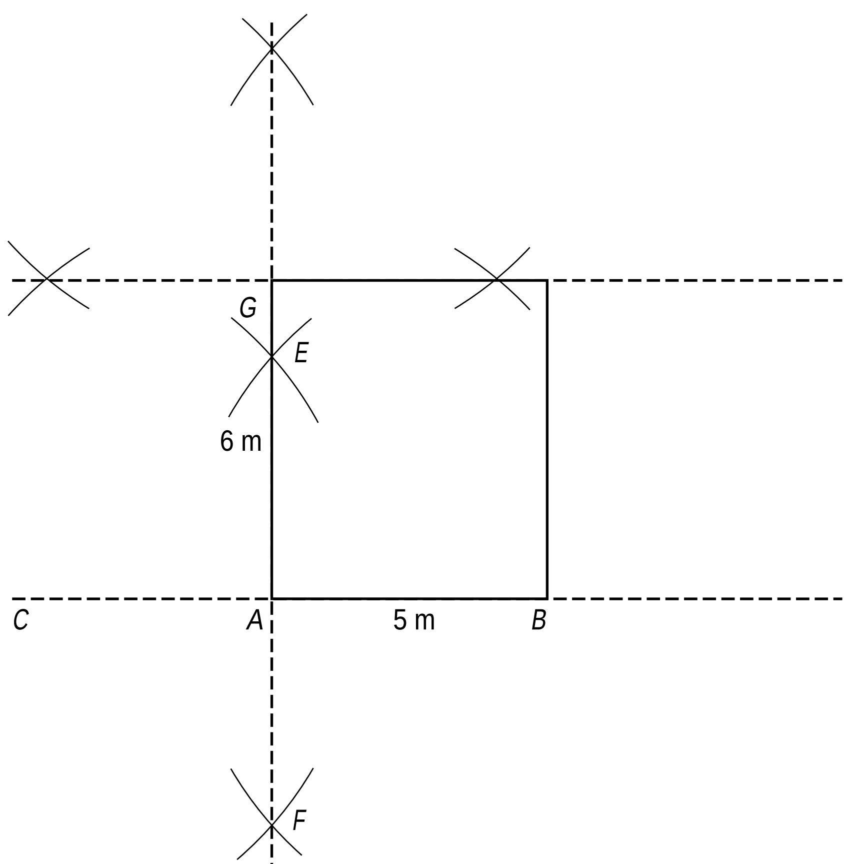
Construction:

Have students construct a rectangle knowing one side and the area by using only a string and a measuring tape. This activity can be done on the classroom floor, gym floor or parking lot.

Ask students to propose and defend a strategy for drawing such a rectangle. Comment on their proposals before giving them the following instructions. Refer to Figure 8: Constructing a Rectangle.

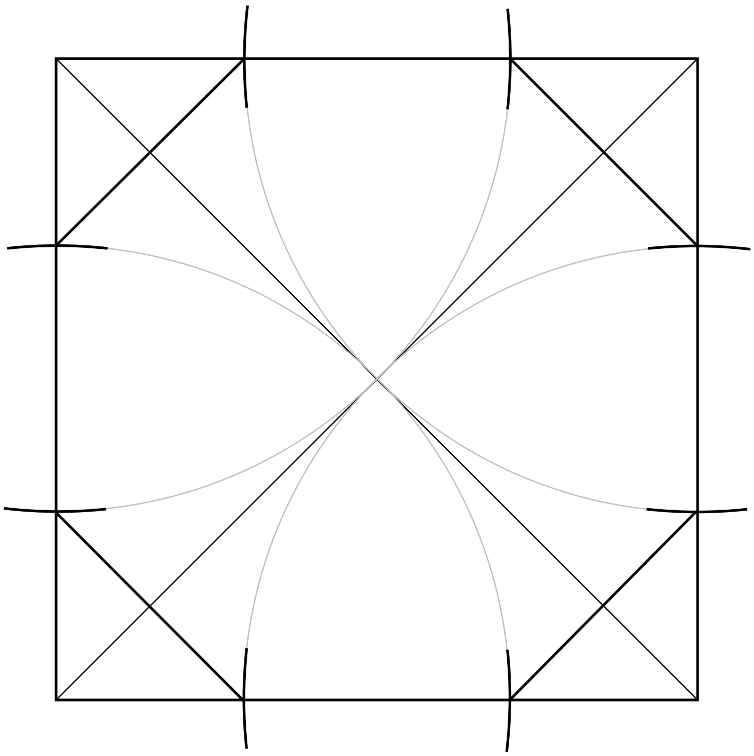
- Determine the length of the second side of the rectangle by using the formula of the area (if a = 5 m and the area is 30 m^2, then the second side of the rectangle is 6 metres).
- Trace one side on the floor and extend the side to the left.
- Use the string to draw a circle having its centre at *A* in such a way that it intersects the side at *B* and *C*.
- From *B* and *C* draw arcs of the same radius (larger than the distance *AB*). Points *E* and *F* are the intersection points of the two arcs.
- Join *E* and *F* and record the length of the second side along *AE* to obtain the point *G*.
- Proceed the same way from *G* and continue to draw the entire rectangle.
- Verify that each of the angles is a right angle.

Figure 8: Constructing a Rectangle



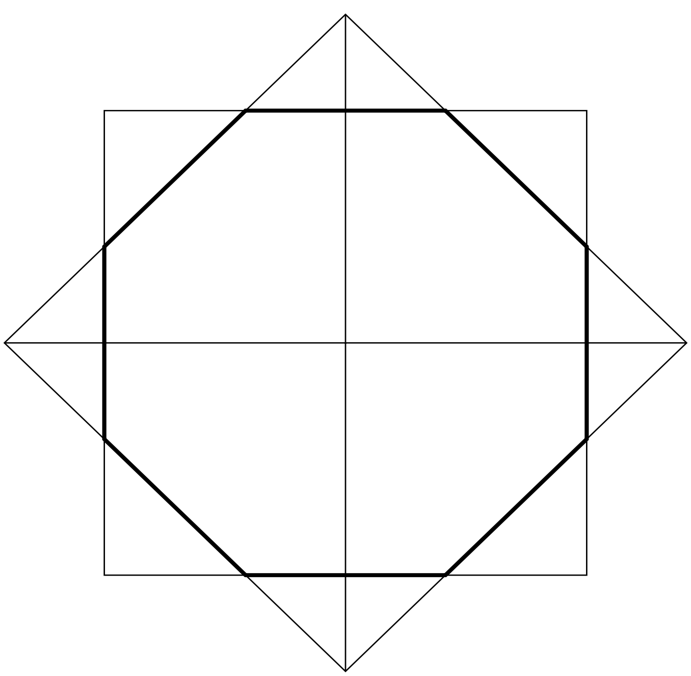
Have students use the floor to draw a square with 3 metre sides. When the square is complete, have students inscribe an octagon within the square. Refer to Figure 9: Constructing an Octagon #1.

Figure 9: Constructing an Octagon #1



Steps
1. Construct diagonals.
2. Draw circles (radius = ½ diagonal).
3. Join each of the 2 intersections to make an octagon.

Figure 10: Constructing an Octagon #2



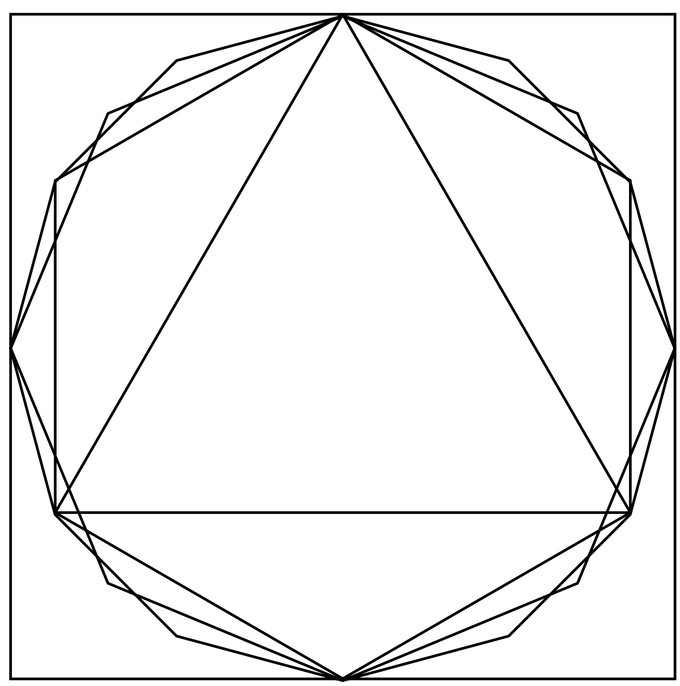
Steps
1. Draw a circle.
2. Draw parallels tangent to the circle.

Ask students to imagine a way to construct a regular hexagon on the floor whose sides are 2 metres and of transforming it into a dodecagon (12 sides), a square, an octagon and an equilateral triangle (Figure 11: Constructing a Regular Polygon). You may decide to give them the following hint: draw a circle with a 2 metre radius; this is the size of the regular hexagon. Have students measure the perimeter and the area for the five polygons. Use the table on Student Activity: Regular Polygons Inscribed in a Circle with a Radius of 2 Metres to record the results.

Have students draw a regular octagon on paper from a square having 78 mm sides.

Have students devise a way to draw a regular octagon with 50 cm sides.

Figure 11: Constructing a Regular Polygon

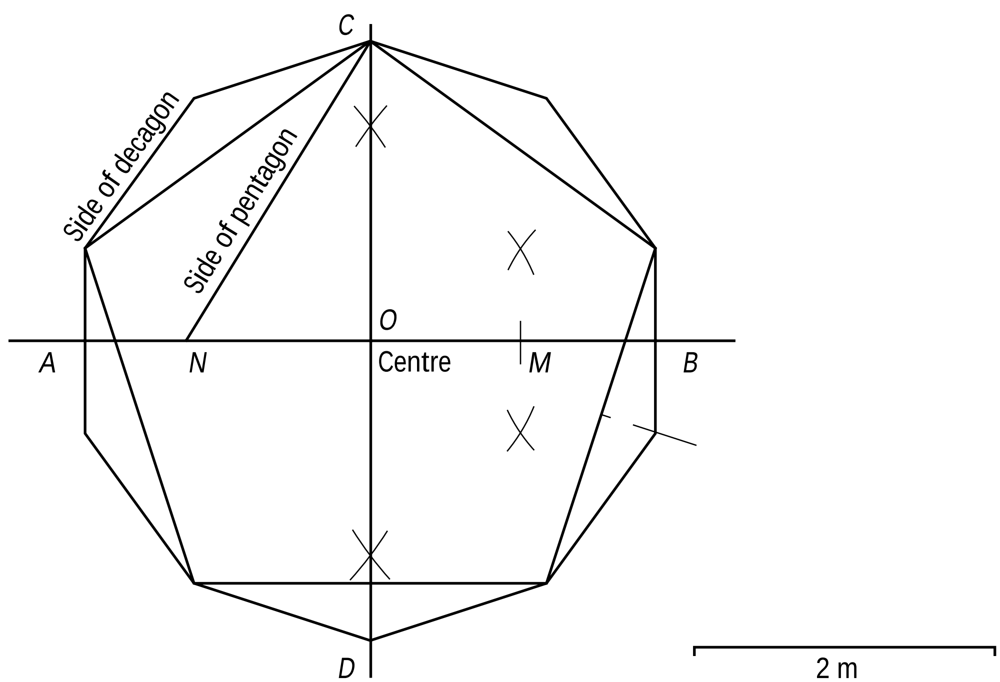


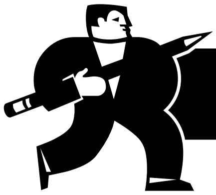
Extension:

Have students try to determine how to draw a regular pentagon and a decagon within a circle having a radius 2 metres. Refer to Figure 12: Constructing a Pentagon and Decagon.

- Draw a circle with a radius of 2 m and centre *O*. Indicate the diameter *AB* (any straight line through the centre).
- Draw the diameter *CD* perpendicular to *AB*.
- Determine the middle of *OB* with the string and call this point *M*.
- Draw a circle with a radius *CM* and centre *M*. Call N its intersection with the diameter *AB*.
- *CN* is the side length of the regular pentagon.
- Draw the length of *CN* five times on the circle and join the points.
- Take the middle of each side and use these points to form a decagon.
- Have students measure the perimeters and the area, and complete the table on Student Activity: Regular Polygons Inscribed in a Circle with a Radius of 2 Metres.

Figure 12: Constructing a Pentagon and Decagon





Student Activity: Tracing Regular Polygons for Construction Purposes

Student Name: ____________________________ Date: ___________________

In what ways does an inaccuracy in the construction of a right angle affect the overall dimensions of a rectangular design such as the foundation of a building or a football field?

Estimate the difference between the two sides of a building foundation 35 m long if an error has been made on a right angle. Complete Table 2: Results of Inaccurate Measurement below.

Table 2

Results of Inaccurate Measurement

Error (degrees)	Side *A* (metres)	Side *B* (metres)	Difference *A* - *B* (metres)

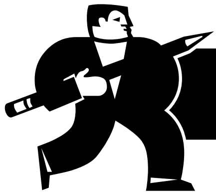
What are some of the consequences of this and similar errors?

Your task is to draw a rectangle whose dimensions are those of a soccer field. How do you proceed?

Student Activity: Tracing Regular Polygons for Construction Purposes

Illustrate the sequence of steps in the space below.

What are some additional comments you would like to make about this activity?

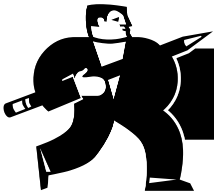


Student Activity: Tracing Regular Polygons For Construction Purposes

Your task is to carve a post having the section shaped as an octagon. You have a log with a diameter of 30 cm and a length of 1 m. How do you proceed?

Sketch the carved post and estimate its volume compared to the volume of the log.

Sketch:



Student Activity: Regular Polygons Inscribed in a Circle with a Radius of 2 Metres

Student Name: ___________________________ Date: __________________

Table 3

Polygon Measurements

# of Sides	Name of Polygon	Length of Side	Perimeter	Area	Ratio Area/ Perimeter
3					
4					
5					
6					
8					
10					
12					
7					
9					
11					
24					

Express in your own words what the pattern of ratios is and then express this in algebraic terms.

Activity 3: Tangents

Objective:

In this activity, students are provided with the opportunity to explore the properties of tangent lines to a circle in a design context. The abilities they develop are useful for identifying the symmetry of geometric figures and exploring the concept of the locus of a point.

Materials:

- pair of compasses
- ruler
- straight edge
- graph paper
- graphics software

Procedure:

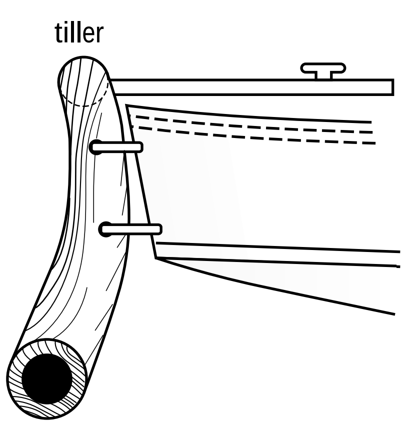
Ask students to identify situations where such designs are used (logo design, landscaping, painting, sculpture, stained glass, and home decoration among other examples).

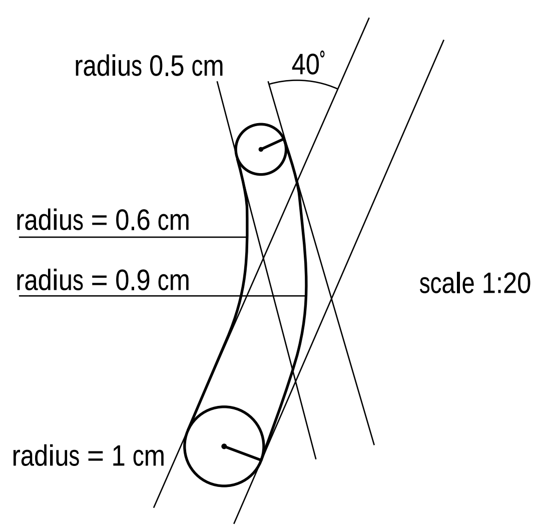
Ask students to either find some logos and reproduce them, or to sketch and create their own logo. Then ask students what skills they need to develop in order to draw a scale plan and build a cardboard model. Invite them to propose different alternatives to accomplish this task (e.g., computer graphics).

Have students find an example of a curve and ask them to determine how to reproduce a scale plan that includes all the details of the design.

Indicate that in most cases, one has to draw the tangent to a certain curve or draw a curve tangent to a certain line. For example, if students want to draw plans for the tiller of a small boat, as illustrated in Figure 13: Constructing a Tiller below, they need to construct lines tangent to the two circles.

Figure 13: Constructing a Tiller





Ask students to propose a way of drawing a line tangent to a circle at a given point on the circle, and of drawing a circle tangent to a given point on a line.

Ask students to identify the difficulties they would expect in such constructions.

Propose techniques for the following:
- Draw a tangent to a circle at some given point illustrated in Figure 14: Drawing a Tangent to a Circle.
- Draw an arc of a circle tangent to a line at a given point as illustrated in Figure 15: Drawing an Arc of a Circle Tangent to a Line.
- Draw a line tangent to a circle from an external point as illustrated in Figure 16: Drawing an Arc #1.

Figure 14: Drawing a Tangent to a Circle

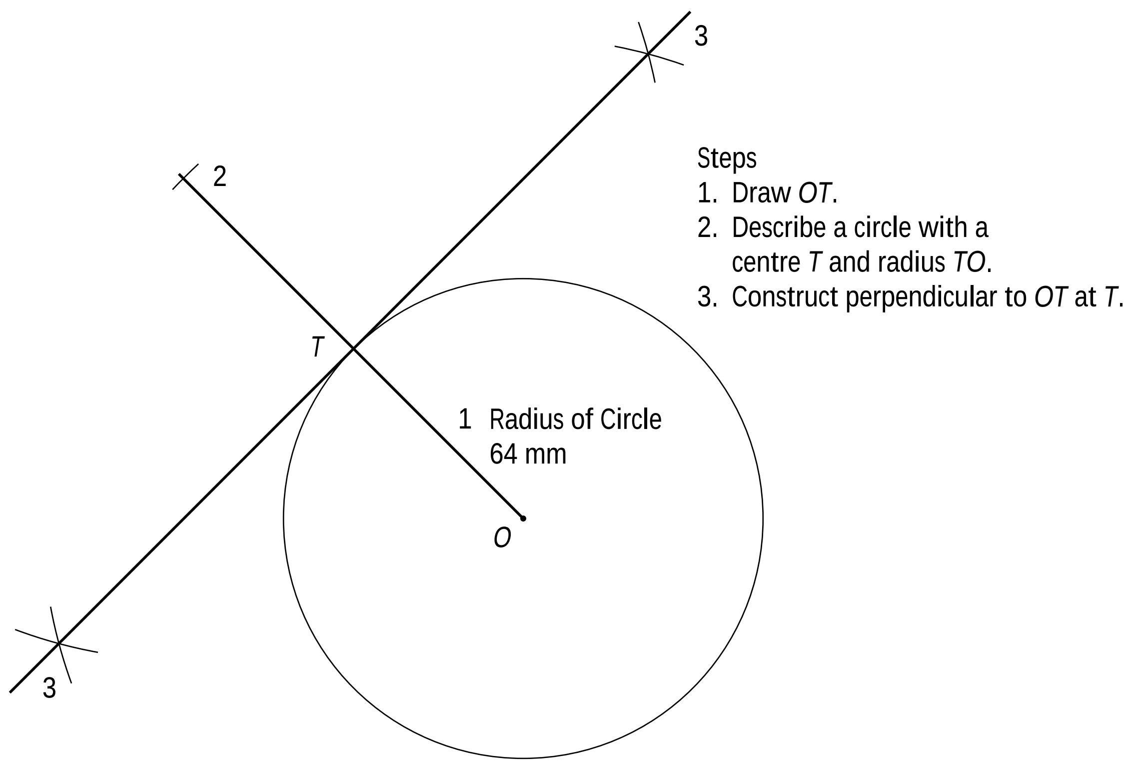
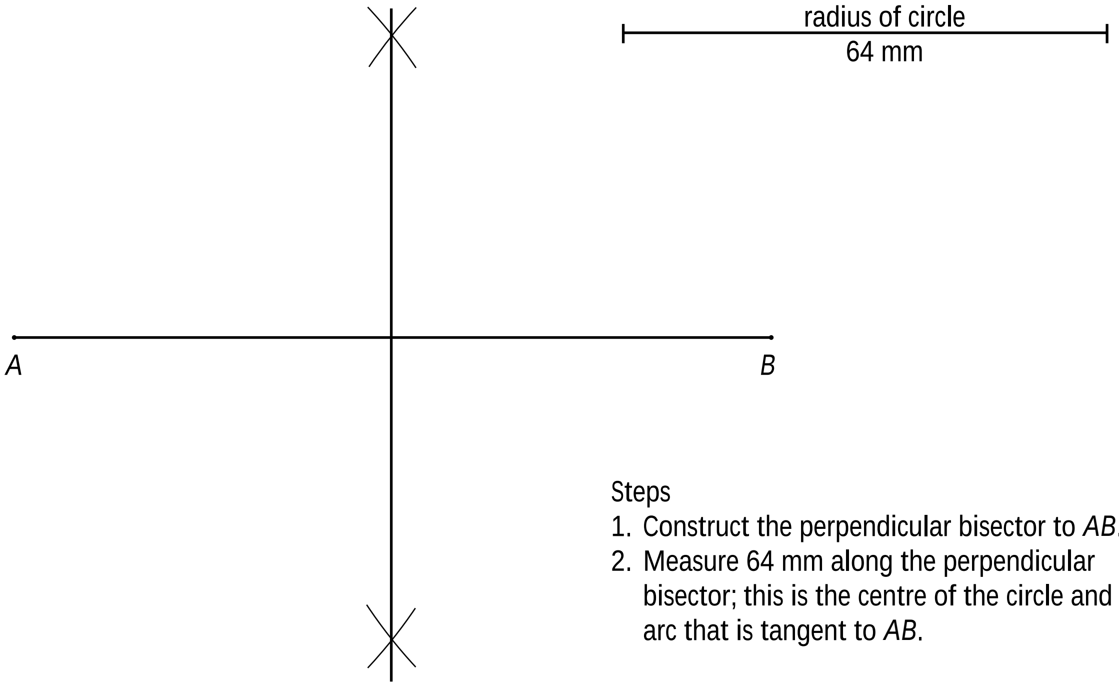
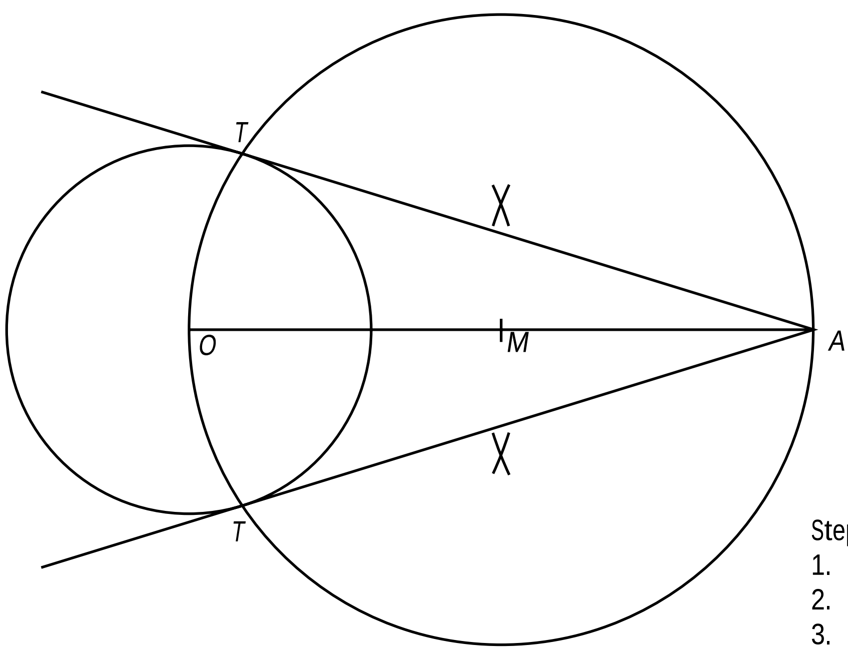


Figure 15: Drawing an Arc of a Circle Tangent to a Line



Steps
1. Construct the perpendicular bisector to *AB*.
2. Measure 64 mm along the perpendicular bisector; this is the centre of the circle and arc that is tangent to *AB*.

Figure 16: Drawing an Arc #1



Steps
1. Draw line *AO.*
2. Find centre of *AO* (*M*).
3. Draw a circle with a centre *M* and radius *MA.*
4. Draw tangent line *AT.*

Have students join two lines by an arc of a circle so that the connections are as smooth as possible. Refer to Figure 17: Drawing an Arc #2-Connecting Two Perpendicular Lines and to Figure 18: Drawing an Arc #3-Connecting Two Parallel Lines).

Figure 17: Drawing an Arc #2 - Connecting Two Perpendicular Lines

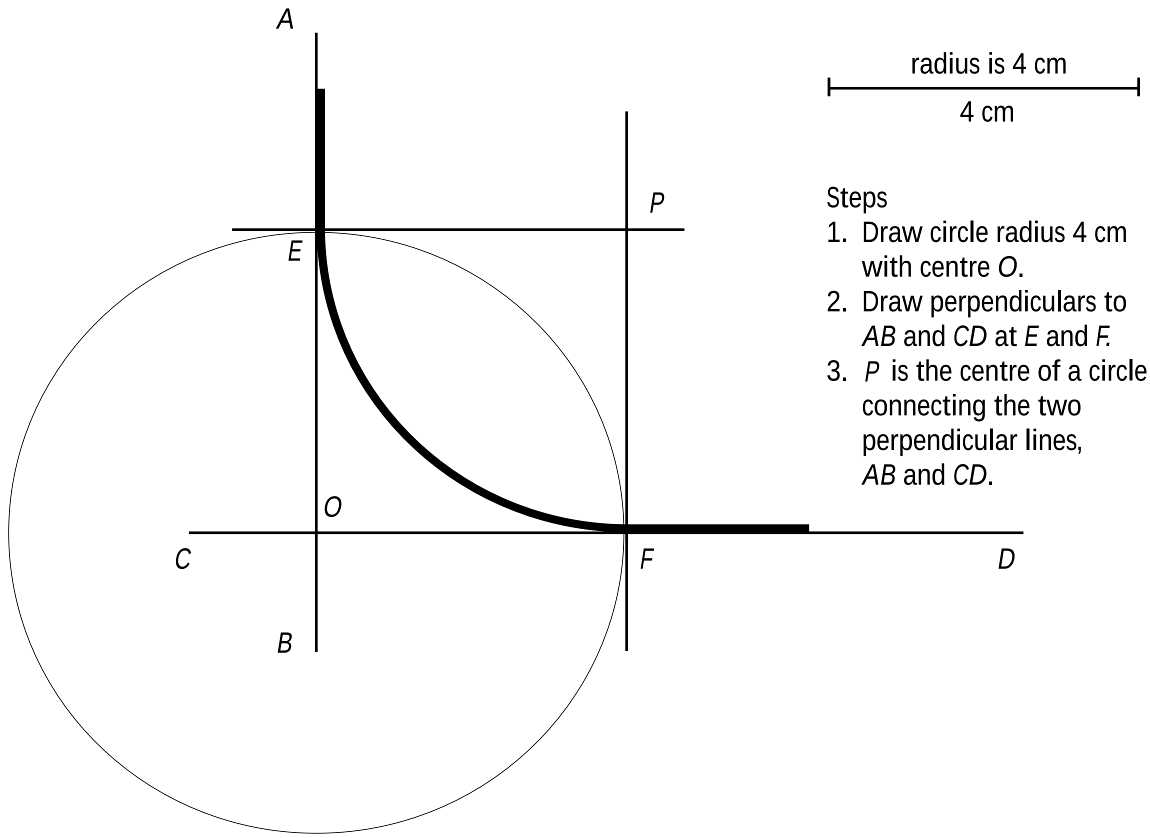
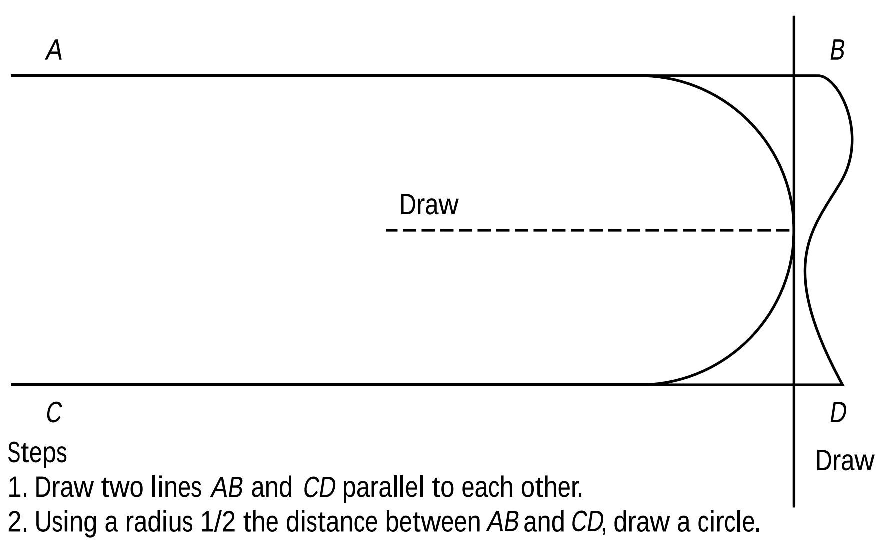
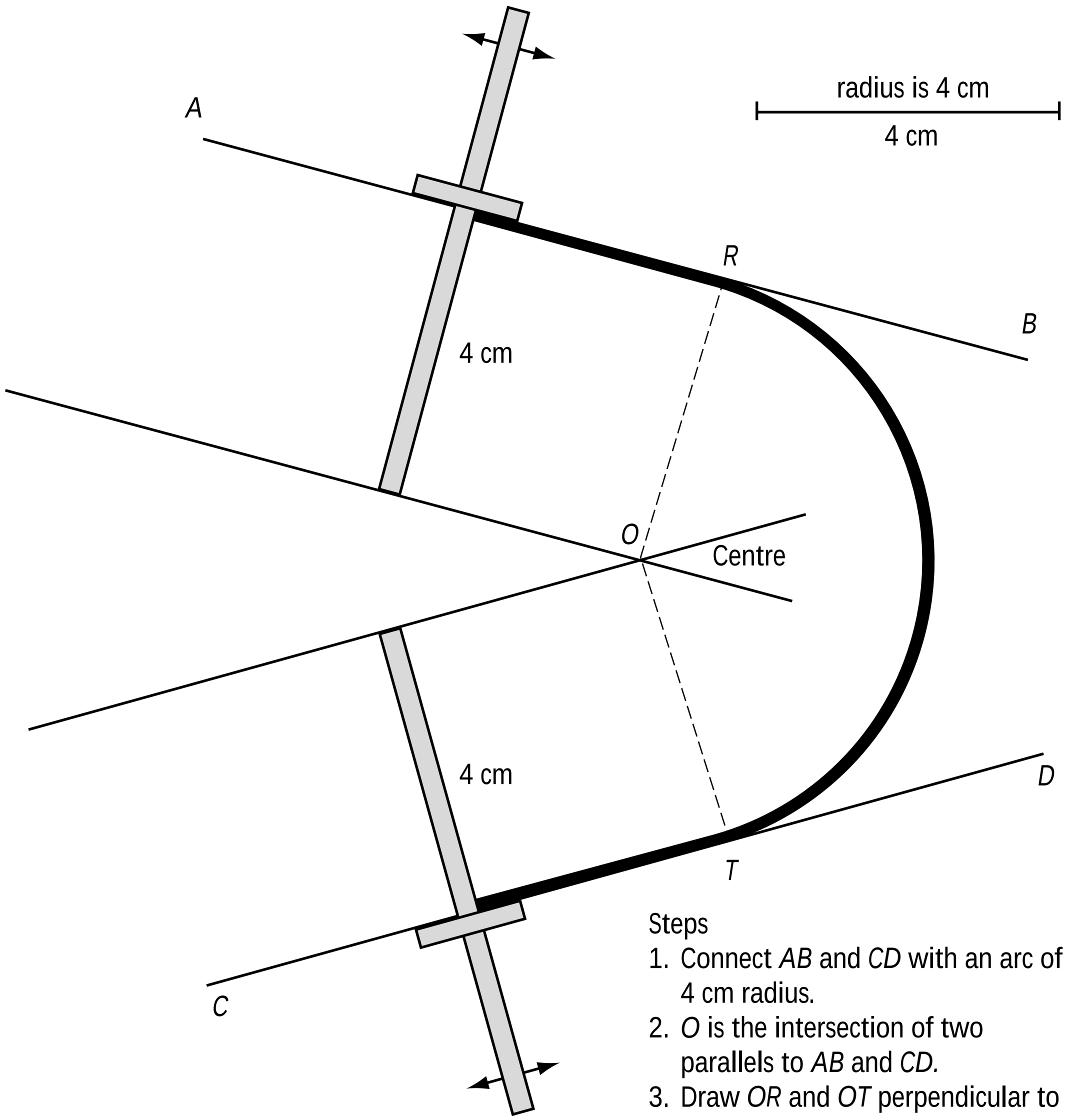


Figure 18: Drawing an Arc #3 - Connecting Two Parallel Lines



Steps
1. Draw two lines *AB* and *CD* parallel to each other.
2. Using a radius 1/2 the distance between *AB* and *CD*, draw a circle.

Figure 19: Drawing an Arc #4



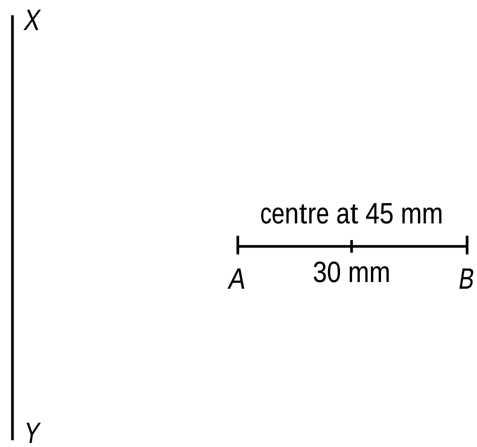
Steps

1. Connect *AB* and *CD* with an arc of 4 cm radius.
2. *O* is the intersection of two parallels to *AB* and *CD.*
3. Draw *OR* and *OT* perpendicular to *AB* and *CD.*
4. Draw the arc of a circle with centre *O.*

Propose the following situations as reinforcement:

- Draw as smooth a line as possible connecting segment *AB* to the arc of a circle of radius 30 mm such that the centre of the circle is situated 45 mm from a line *XY*. Refer to Figure 20: Situation #1.

Figure 20: Situation #1



- In Figure 21: Situation #2, and Figure 22: Situation #3 below, draw as smooth a line as possible connecting the two line segments with two arcs.

Figure 21: Situation #2

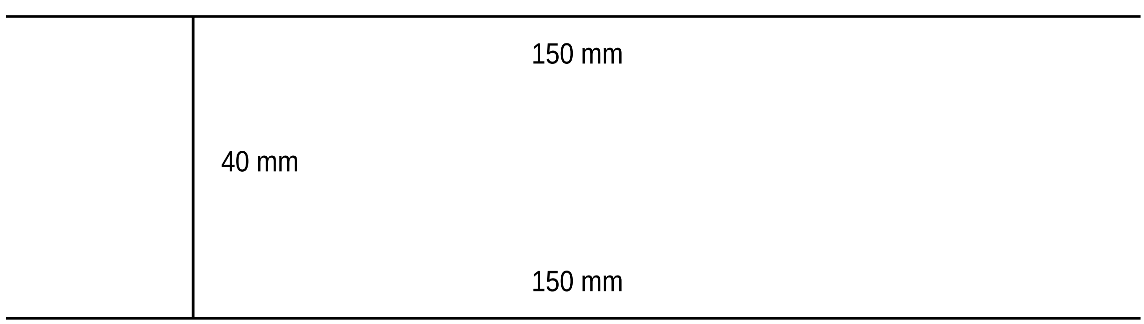
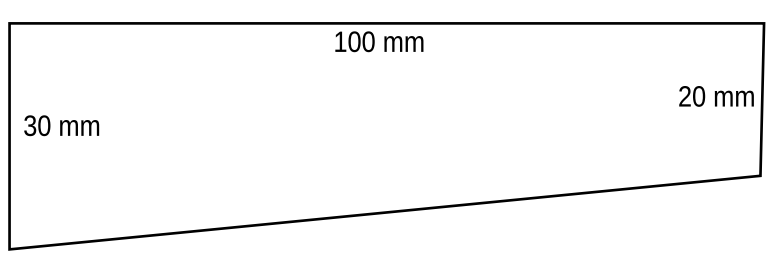


Figure 22: Situation #3



Activity 4: Tangent Circles

Objective:

In this activity, students apply their knowledge about the properties of circles and the locus of a point. Activity 4: Tangent Circles is a continuation of Activity 3: Tangents. Students explore basic geometric transformations and apply co-ordinate geometry to develop a more complex design.

Materials:

- pair of compasses
- ruler
- straight edge
- graph paper
- graphics software

Procedure:

Have students find an example of a curve that connects two circles like the one shown in Figure 23: Connecting Two Circles below. Ask them to determine how they can reproduce a scale plan that includes all the design details.

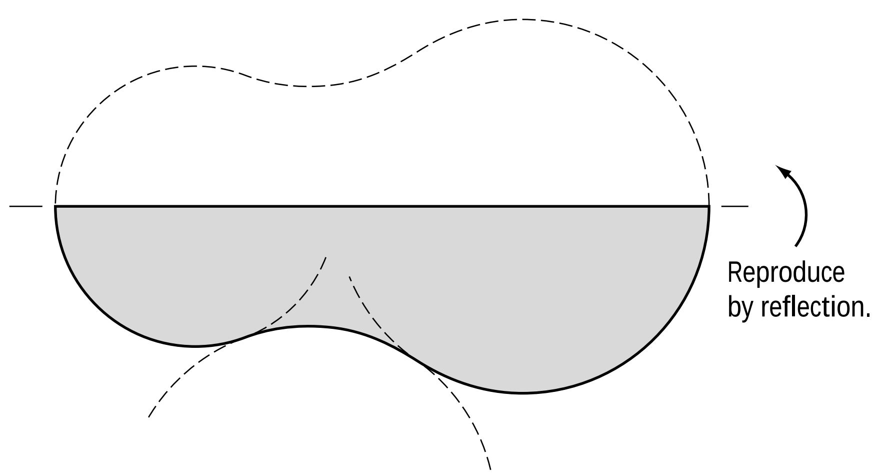
Comment on Figure 23: Connecting Two Circles and explain that in some cases it is necessary to connect two curves as smoothly as possible.

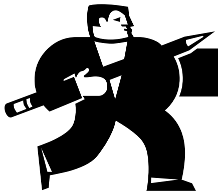
Ask students to create a design that uses connected curves and ask them to propose ways of drawing a scale plan of such a design.

Ask students to find an illustration of a propeller and comment on its design.

Distribute copies of Student Activity: Connecting Curves I to each student.

Figure 23: Connecting Two Circles



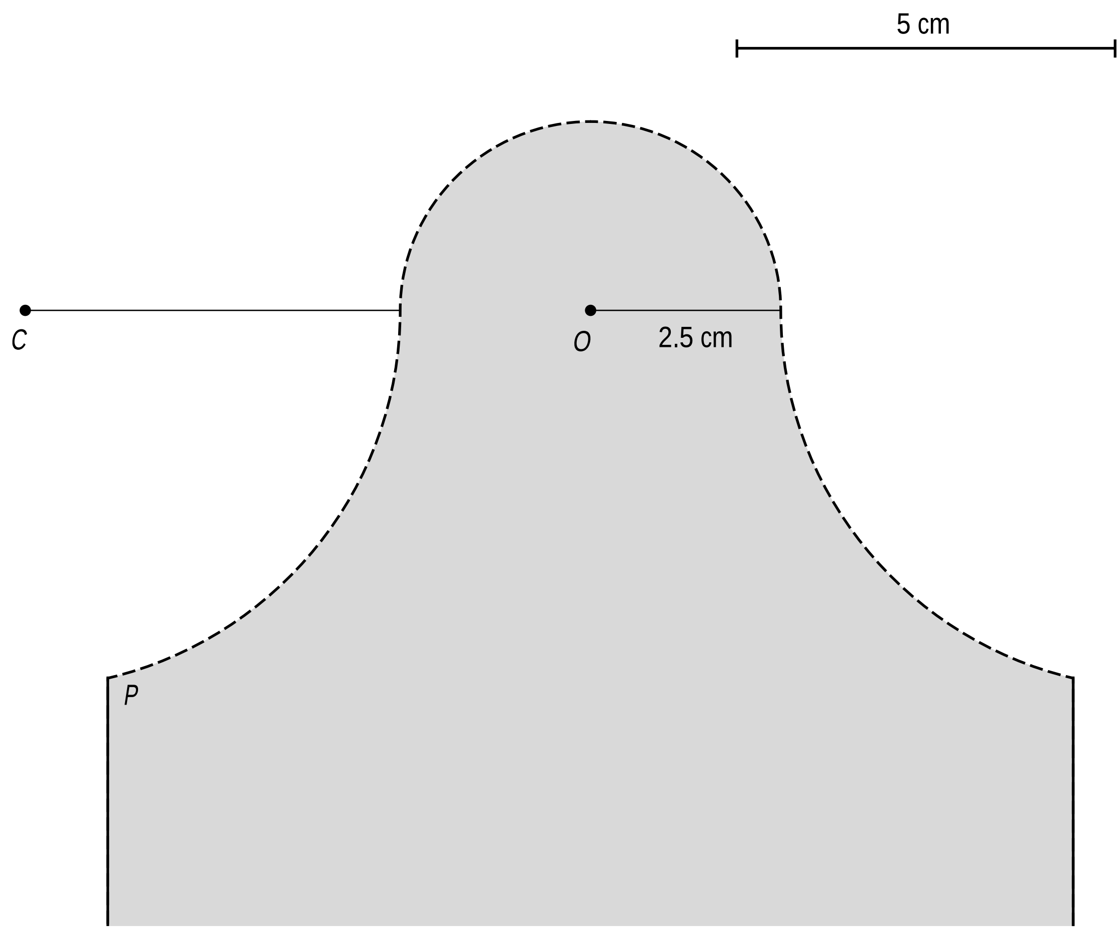


Student Activity: Connecting Curves

Student Name: ____________________ Date: ______________

1a. Draw an arc, tangent to another arc from a given point when the radius is known.

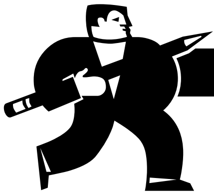
Figure 24: Connecting Curves #1



Given: Circle with centre *O* and radius = 2.5 cm
Point *P* on circle with radius 5 cm

Steps

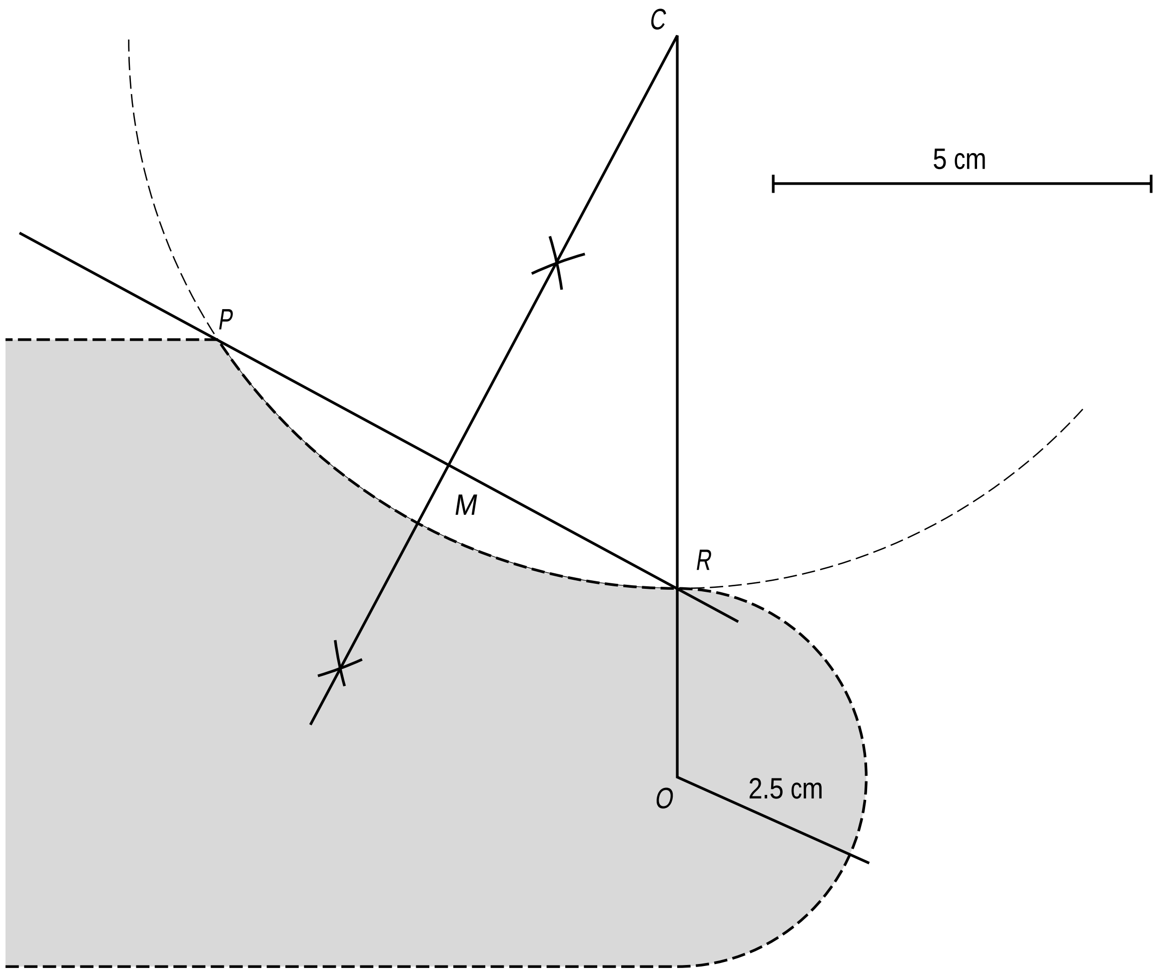
1. Draw an arc using *P* as the centre and a radius of 5 cm.
2. Draw an arc using *O* as the centre and a radius of 5 + 2.5 cm.
3. Draw an arc with a centre *C* (intersection of the two arcs from steps 1 and 2) and a radius of 5 cm.



Student Activity: Connecting Curves

1b. Draw an arc, tangent to another arc from a given point when the point of contact is known.

Figure 25: Connecting Curves #2

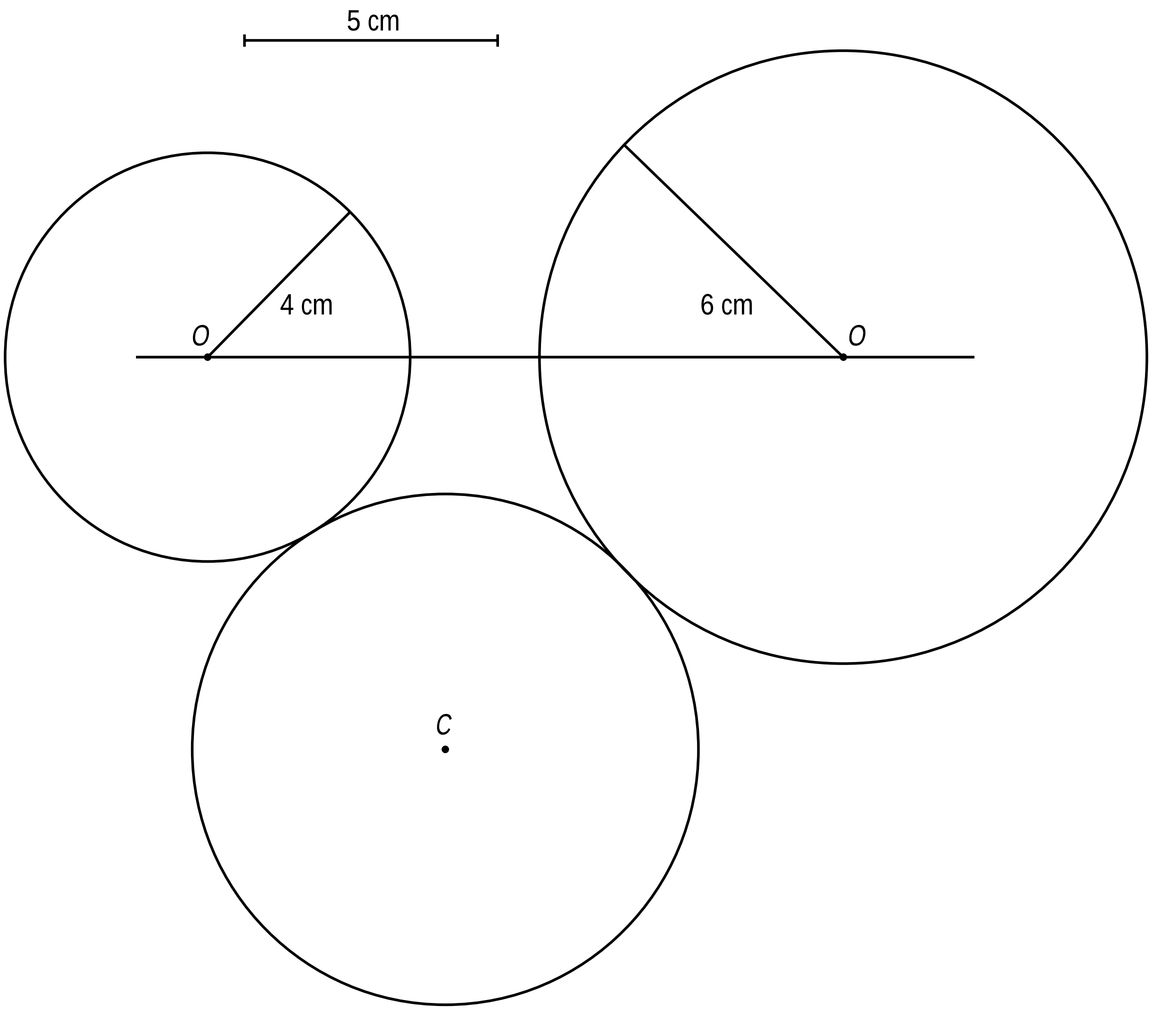


Steps
1. Draw a circle with a centre *O* and a radius = 2.5 cm.
2. *R* is the point of contact of two arcs.
3. Draw *OR* and *PR*.
4. Draw perpendicular bisector of *PR*.
5. Centre, *C*, is at the intersection of the perpendicular bisector and the line *OR*.

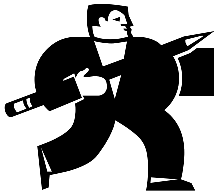
Student Activity: Connecting Curves

2. Connect two arcs with a given arc.

Figure 26: Connecting Two Arcs with a Given Arc



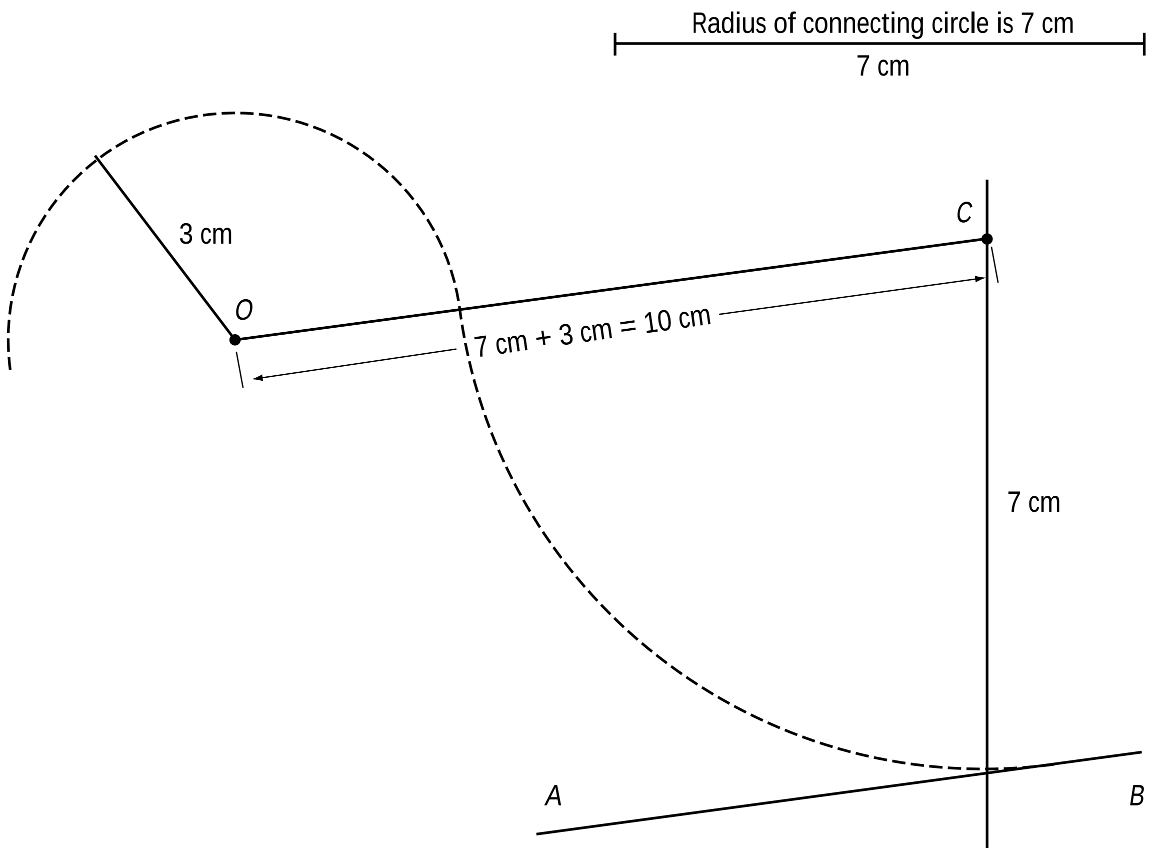
Steps
1. Draw an arc with centre *O*, radius 6 + 5 = 11 cm.
2. Draw a second arc so as to intersect the first arc, with centre *O*, radius 4 + 5= 9 cm.
3. *C* is the point of intersection of the two arcs above and the centre of the connecting curve (circle tangent to the two arcs).



Student Activity: Connecting Curves

3a. Connect an arc and a line when the radius is known.

Figure 27: Connecting an Arc and a Line When the Radius is Known



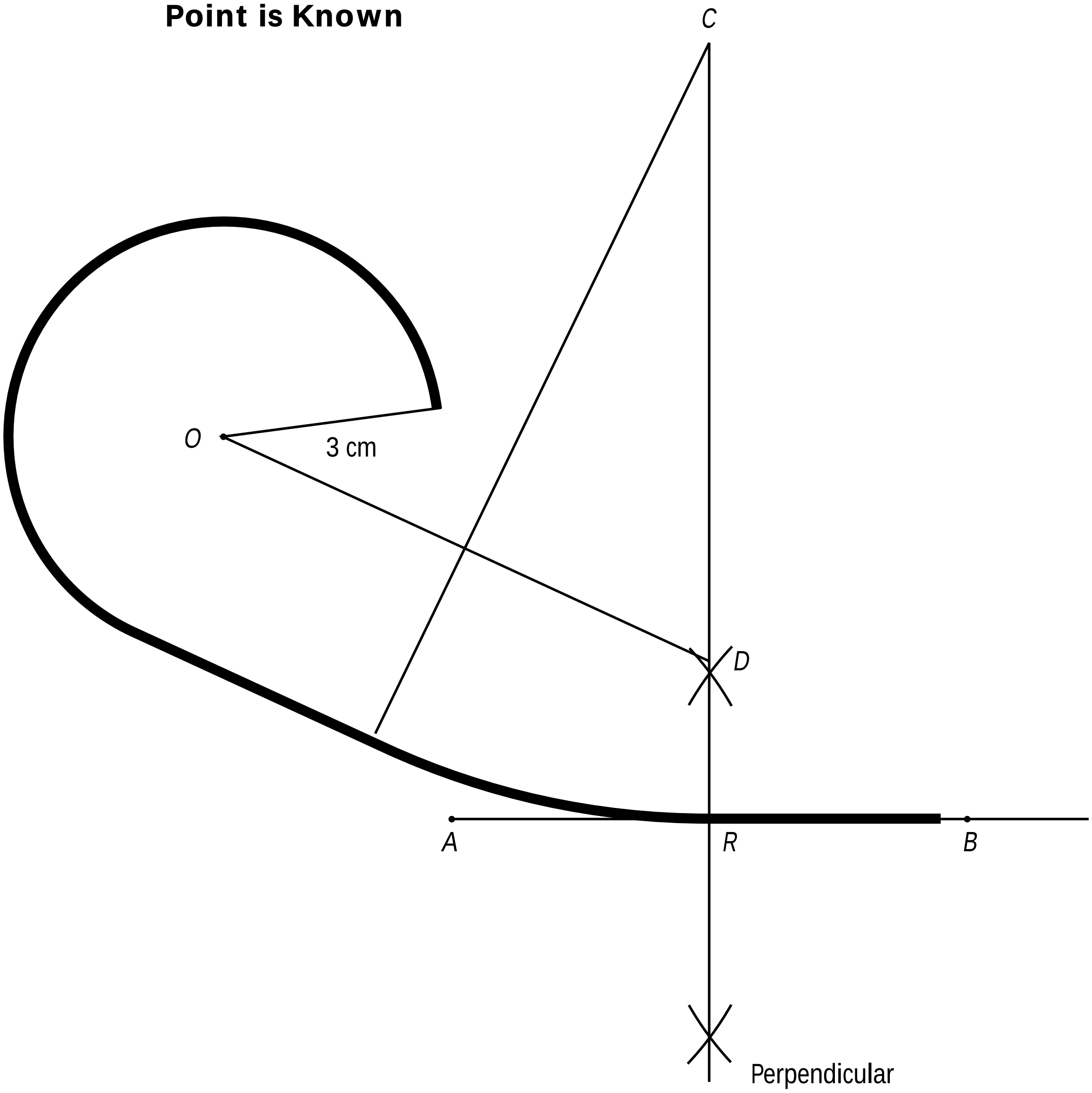
Given:
AB is the line, *O* is the centre of a circle.

Steps
1. Draw circles of radius 3 cm and 10 cm (7 cm + 3 cm).
2. Draw a line parallel to *AB* at 7 cm to obtain point *C*.
3. Draw an arc of a circle with centre *C* and a radius of 7 cm.

Student Activity: Connecting Curves

3b. Connect an arc and a line when the connecting point is known.

Figure 28: Connecting an Arc and a Line When the Connecting Point is Known



To connect the circle (centre *O* and radius 3 cm) to line *AB*, the connecting point *R* is given.

Steps

1. Draw a circle with a radius = 3 cm and centre *R.*
2. Point D is on the perpendicular to *AB* through *R*. *RD* =3 cm.
3. Connect *OD.*
4. Draw perpendicular bisector to *OD.*
5. C is centre of arc connecting circle with centre *O* to point *R* on *AB.*

Activity 5: Transformations and Logos

Objective:

In this activity, students construct a design of their choice, applying the knowledge and skills they acquired in the previous activities. In doing this activity, they realize the importance of geometric transformations. They also learn how to reduce the number of the steps needed to construct a model having symmetry.

Materials:

- pair of compasses
- ruler
- straight edge
- graph paper
- graphics software

Procedure:

Have students apply geometric transformations on the design they created in the previous activity. The geometric transformations are translation, rotation, reflection, and dilation.

Encourage students to place their cardboard model of the curve on graph paper and to construct different images by using co-ordinate geometry.

Have students accurately reproduce their design on a computer using graphics software. Encourage them to perform various transformations in order to create a complex design.

After completing each transformation, have students identify each of the geometric transformations and the image of the original design.

Some students may be interested in reproducing their design in the school parking lot, for example.

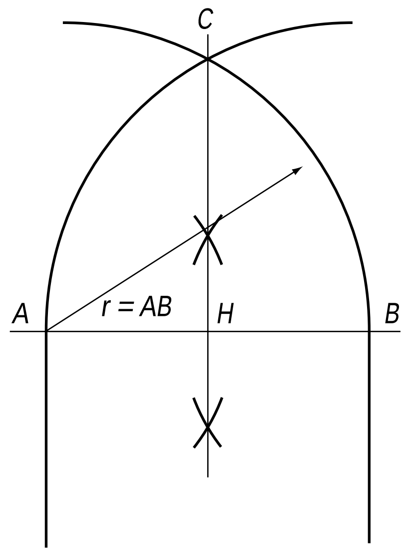
Exercises

- Draw an arc of radius 20 mm tangent to an arc of radius 30 mm. Show all the steps.
- Draw a vertical line and a circle whose center is at 25 mm from the vertical. Connect the line and the arc with an arc of 12 mm radius. Show all the steps.
- Pick a bow and a stern from Student Activity: Bow and Stern Design and connect them to form a boat. Draw the pattern on cardboard and cut it out. Students should work in pairs.

Extension: Curves that are not circles.

How to draw curves commonly used in designs.

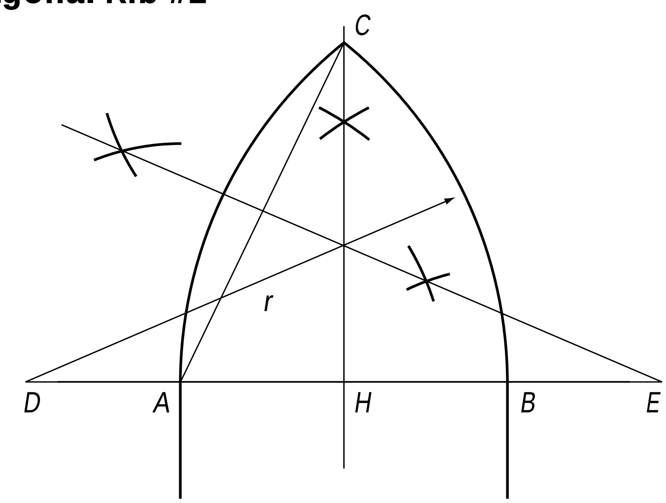
Figure 29: Diagonal Rib #1



AB

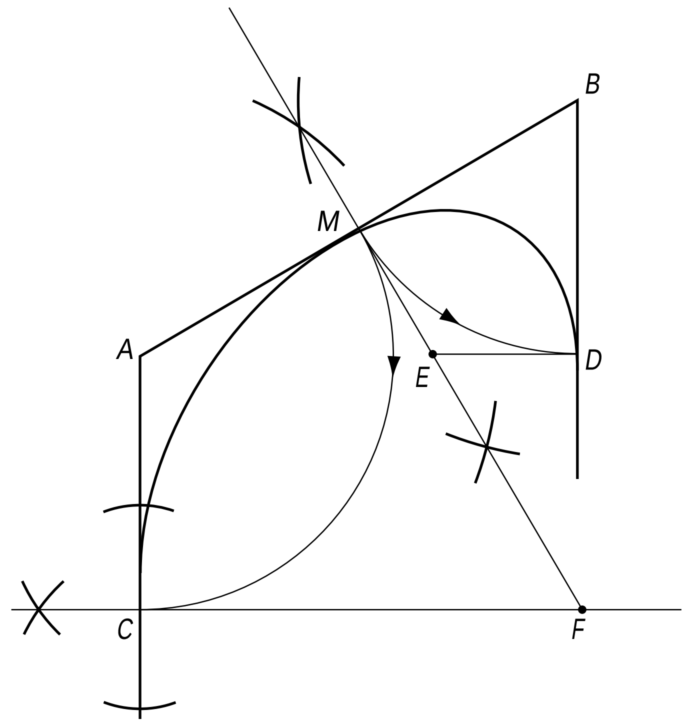
AB is the opening.
HC is the height.
B is the vertex.
From *A* and *B*, draw circles of radius *AB*.

Figure 30 Diagonal Rib #2



AB is the opening.
From *D* and *E* (*HD* = *HE*) draw circles of radius *r* = *DB*.

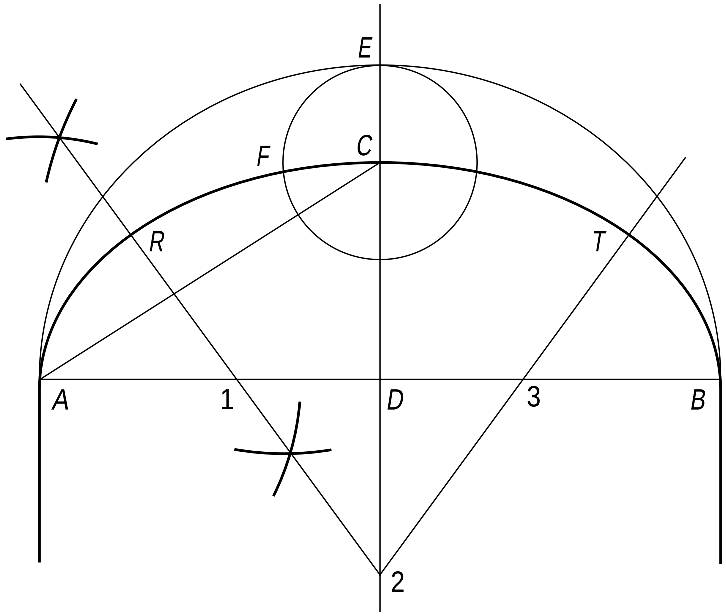
Figure 31: Ramping Arc



Given:
M is given and it is the connecting point of the two arcs with centres *A* and *B*.
Three lines are given.
BD and *AC* are parallel while *AB* is not parallel.

Steps:
1. Draw $AC = AM$ and $BM = BD$.
2. Draw perpendiculars at *C, M* and *D.*
3. *E* and *F* are the centres of the two arcs tangent to the lines, *MC* and *MD.*

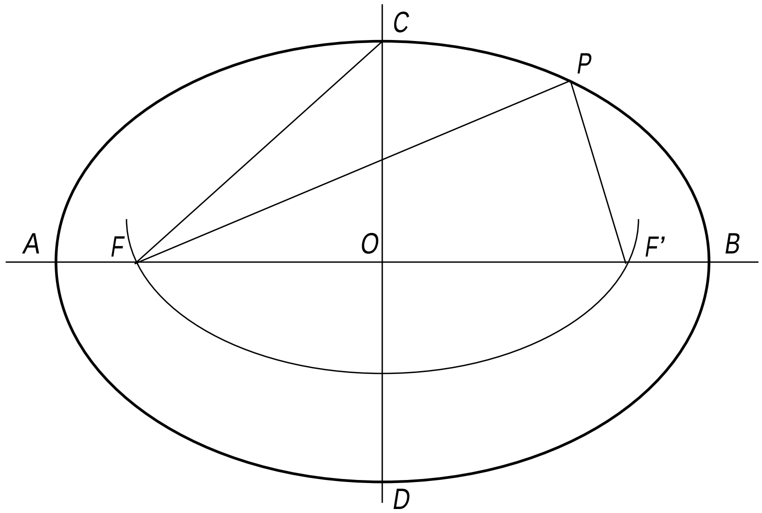
Figure 32: Basket Handle



Steps

1. *AB* is the opening and *CD* the height of the basket handle.
2. Draw a half-circle of radius *AD* with centre at *D*.
3. *CE* is the difference of half the opening minus the height.
4. Draw circle of radius *CE* and centre *C.*
5. Draw *AC.*
6. Draw perpendicular at middle of *AF* to give points 1 and 2.
7. Draw point 3: *D* - 3 = 1 - *D.*
8. Points 1, 2 and 3 are centres of the three arcs.
9. Draw *AR, RC, CT* and *TB.*

Figure 33: Ellipse



Given:
AO
AB = 14 cm
CD = 8 cm
AO is half the major axis

Steps
1. From centre *C* and radius *AO* = 7 cm, draw a circle to find F and F' (foci).
2. Using a rope of length *AB* (14 cm), fix ends at *F* and *F'*.
3. Draw the ellipse (keeping the rope tied).

Activity 6: Forces and Levers

Objective:

The objective of this activity is for students to relate the properties of triangles to forces acting on a structure. Tension and compression are the key forces that cause structures to stay up or fall down.

Materials:

(per pair of students)

- set of weights of 100 g, 500 g, 1 kg
- set of rigid bars of different lengths (wooden sticks 1 x 1 inch)
- graduated stick
- dynamometer
- string
- set of clamps

Procedure:

Tension within structures is a pulling force. It stretches the materials. We can see evidence of tension in rope bridges, telephone wires, tents, suspension bridges, steel cables supporting an elevator, and so on.

Compression is a pushing force. It squashes the materials. We can see evidence of compression in pyramids, telephone poles, arc bridges, tree trunks, and a squashed marshmallow.

Present Figure 34: Diagram of Forces #1.

Ask students to give examples of forces acting on pyramids, telephone poles, arc bridges, tree trunks, and a squashed marshmallow. Discuss these forces in terms of tension and compression.

Briefly review the concept of forces (direction, intensity, and point of application) by asking students to use the dynamometer and either lift some weights of different masses or pull weights from a hook attached to the wall.

Have students identify and name three important characteristics of forces (i.e., intensity, direction, and point of application) in various situations. Have students record their readings and indicate the direction in which the force is applied.

Review the Law of Levers by using the graduated stick and weights of various masses. Some suggestions for doing this are presented below in Figure 35: Diagram of Forces #2.

Figure 34: Diagram of Forces #1

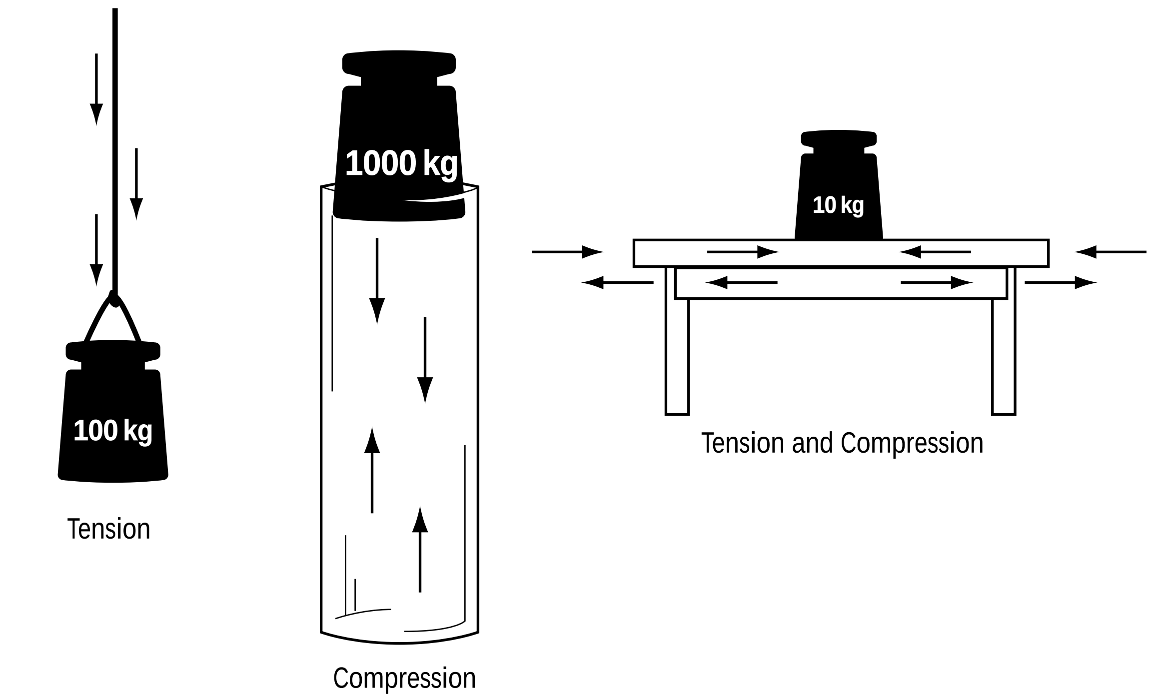
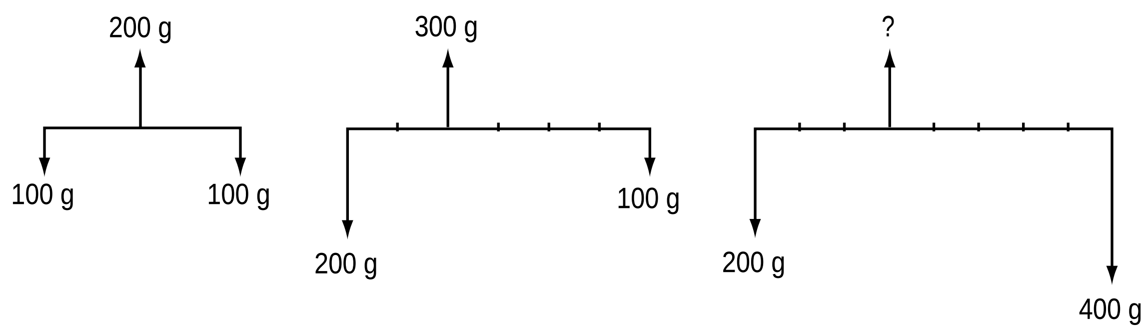


Figure 35: Diagram of Forces #2

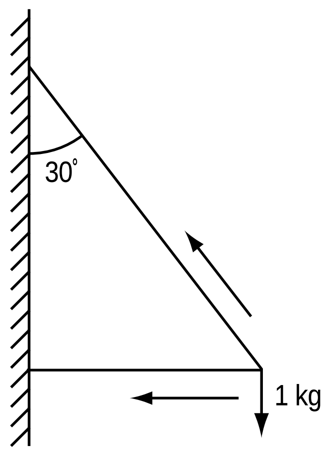


Discuss the principle of Action-Reaction with students. Ask them to identify the direction and the magnitude of the reaction once a force is applied at a particular point.

Ask students to draw a scale diagram to represent the action of a set of forces like those illustrated in Figure 35: Diagram of Forces #2. Students need to determine an appropriate scale for the diagram. For example, 100 g is represented by 1 cm on the scale diagram.

Ask students to apply the Pythagorean Relation and/or the tangent ratio to determine the **components** of the force applied on the wall bracket illustrated below in the Figure 36: Components of Force. Students may use the dynamometer to verify the information and then draw a scale diagram to represent the situation.

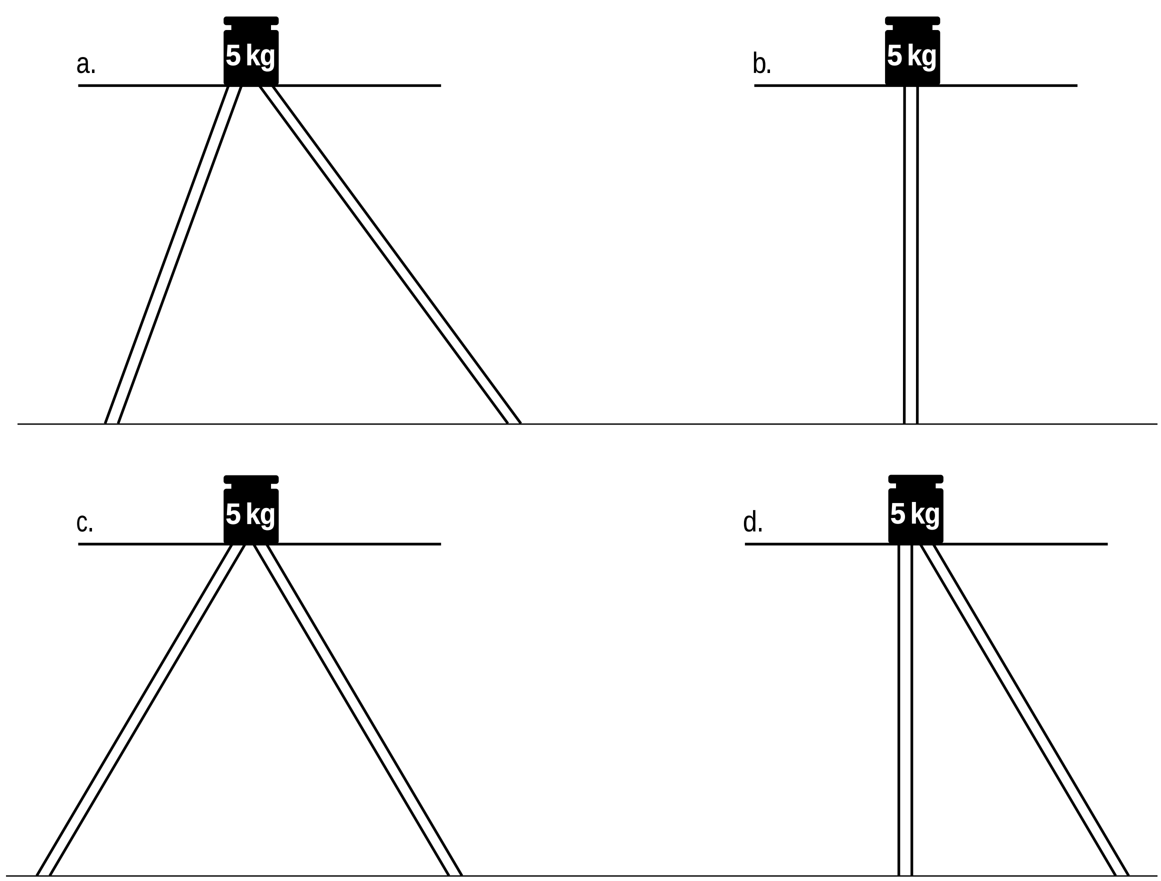
Figure 36: Components of Force



Illustrate the composition of a force with the following situation.

Use the wooden craft sticks to design the different models of structures that must support a mass of 5 kg. These are shown in Figure 37 : Structures Supporting 5 kg Mass (a, b, c, and d below).

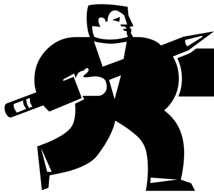
Figure 37: Structures Supporting 5 kg Mass



Ask students to draw a scale diagram for each setting.

Ask students to decide which is the best structure by considering the resistance of the sticks to axial stress.

Students may verify their conclusions with strands of spaghetti.
Have students examine and comment on the situations presented in Student Activity: Representing Force. Ask them to draw a scale diagram for each example.

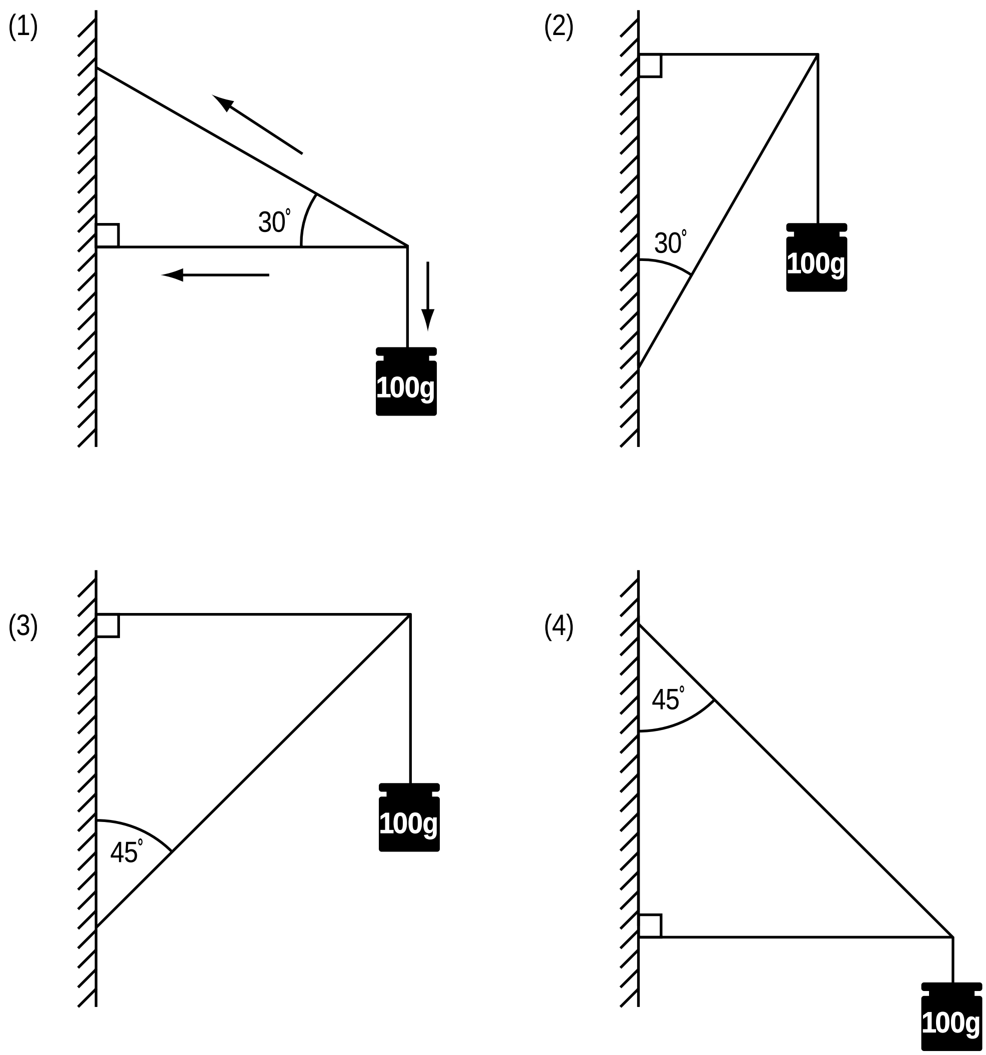


Student Activity: Representing Force

Student Name: ____________________________ Date: ___________________

Complete the following diagrams by representing the force diagram, that is, the composition of the force applied to the wall brackets.

Figure 38: Force Diagrams



Activity 7: Geodesic Domes

Objective:

The objective of this activity is for students to research properties and examples of geodesic structures. Web sites are a good source of information for these. Have students identify geodesic structures around them.

In this activity students construct a geodesic dome with rolled newspaper and explore the stresses experienced by the struts at the various joints. They analyse the advantages of geodesic domes compared with other structures.

Materials:

(per group of 2-3 students):
- 4-5 newspapers (large format)
- 1 broom handle
- stapler
- blue and red paint
- string
- chalk

Procedure:

Refer to Figure 39: Sequence of Steps for Constructing a Geodesic Dome, on the following pages to review the sequence of operations.

Step 1: Preparing the Struts
Have students roll newspaper around a broom handle (as shown in the illustration) to produce at least 75 struts of a 75 cm length. Then have them cut 35 struts, 71 cm long, and 30 struts of a 60 cm length.

Step 2: Drawing the Base
Students draw a circle on the floor so that they can inscribe a regular decagon with a 65 cm side inside the circle. Ask students to think of a reason for doing this.

Step 3: Constructing the Base
Students arrange the struts of the base as illustrated. Struts are attached using staples to make the joints.

Steps 4 to 9: Building the Structure With Trusses
Students follow the steps as indicated until the dome is complete.

By observing the behaviour of the staples at each of the joints, students should identify which struts are experiencing compression and which struts are experiencing tension. The overall force is the weight of the structure.

Have students paint the struts experiencing compression in blue and the struts experiencing tension in red.

Have students select a few joints and draw a scale diagram of the forces. For this task, remind them that no magnitude is required, only direction.

Ask students to attach a mass (500 g or 1 kg) at different joints and draw the scale diagrams. They can verify their prediction by observing the behaviour of the staples at the joint.

Ask students to compare the resistance of a geodesic dome compared to other structures.

What are some of the consequences of having smaller struts, that is, more triangles?

What would be the resistance of a structure of this kind composed of very small struts?

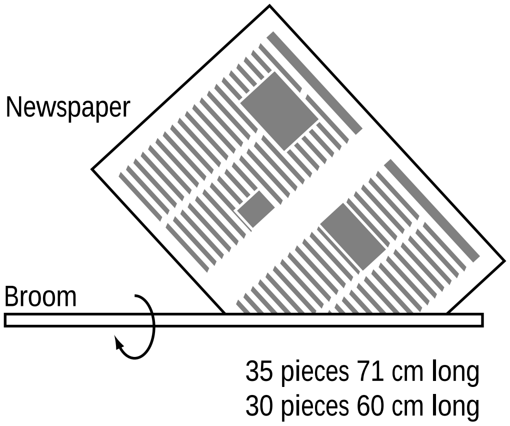
What are some reasons structures in nature, like egg shells, have a curved form?

What are some reasons geodesic structures are not as popular as they could be, considering their interesting features?

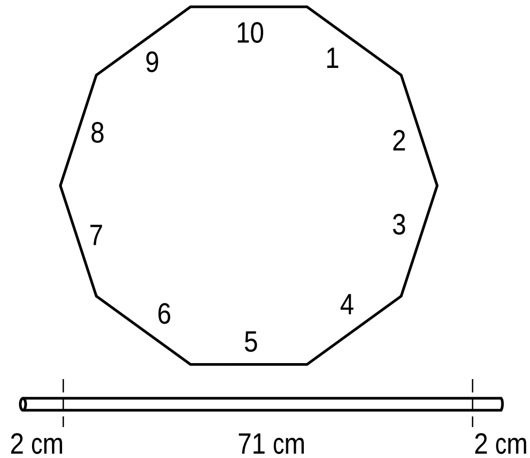
The following pages illustrate the sequence of construction steps for a geodesic dome.

Figure 39: Sequence of Steps for Constructing a Geodesic Dome

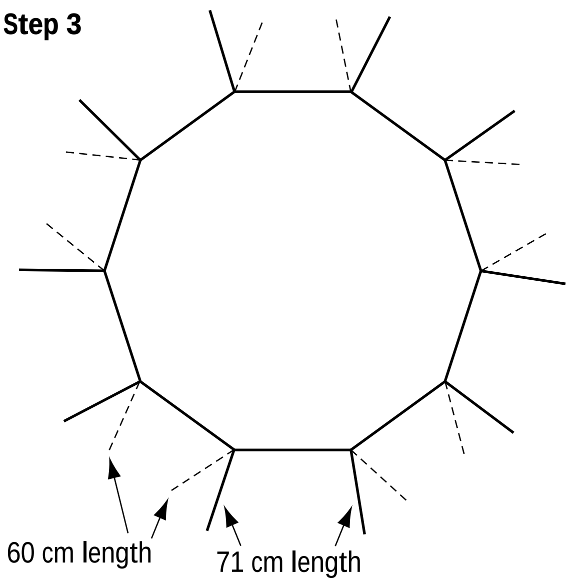
Step 1



Step 2 Place on the floor around the circle.



Step 3



Step 4

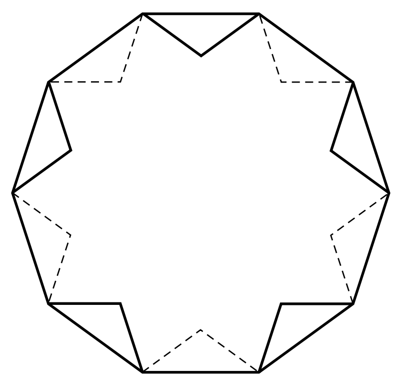
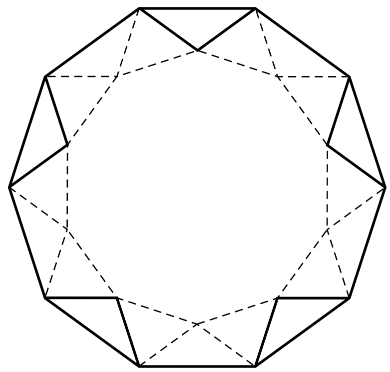
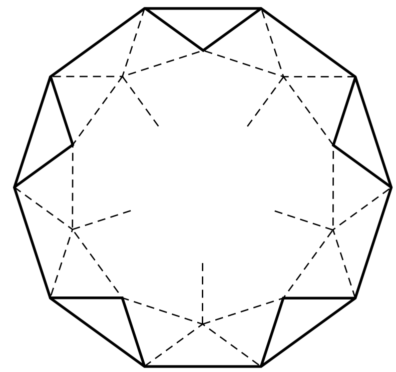


Figure 40: Sequence of Steps for Constructing a Geodesic Dome

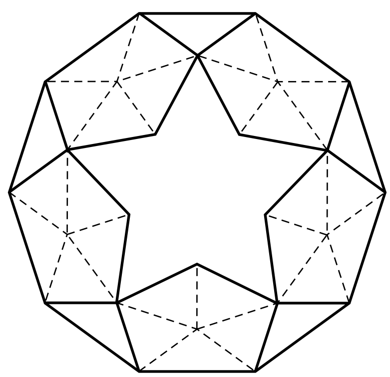
Step 5



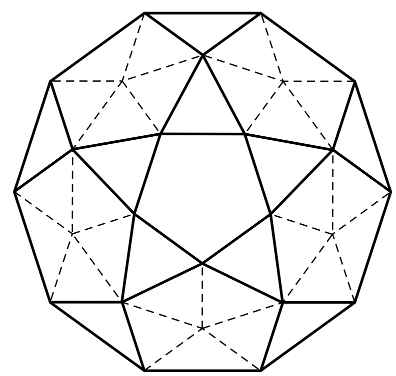
Step 6



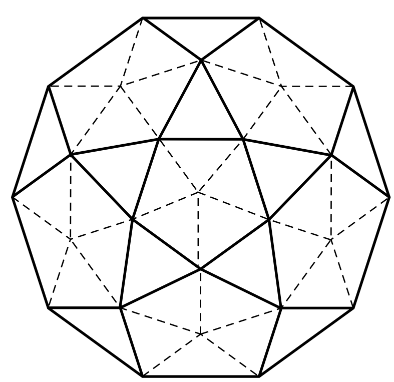
Step 7



Step 8



Step 9



Student Activity: Report on Geodesic Dome Research

Student Name: ______________________________ Date: ____________________

Geodesic dome
(comments)

Activity 8: Reviewing Geometric Solids

Objective:

The objective of this review activity is to ensure that students understand the properties of geometric solids.

Materials:

(per pair of students)
- cardboard
- ruler
- scissors
- tape

Procedure:

If necessary, the teacher may ask the students to complete Student Activity: Two-Dimensional Shapes.

As a small project aimed at reviewing earlier concepts, students may evaluate the volume of a log, the volume of lumber of various dimensions, and cross-sections obtained from a log of given dimensions.

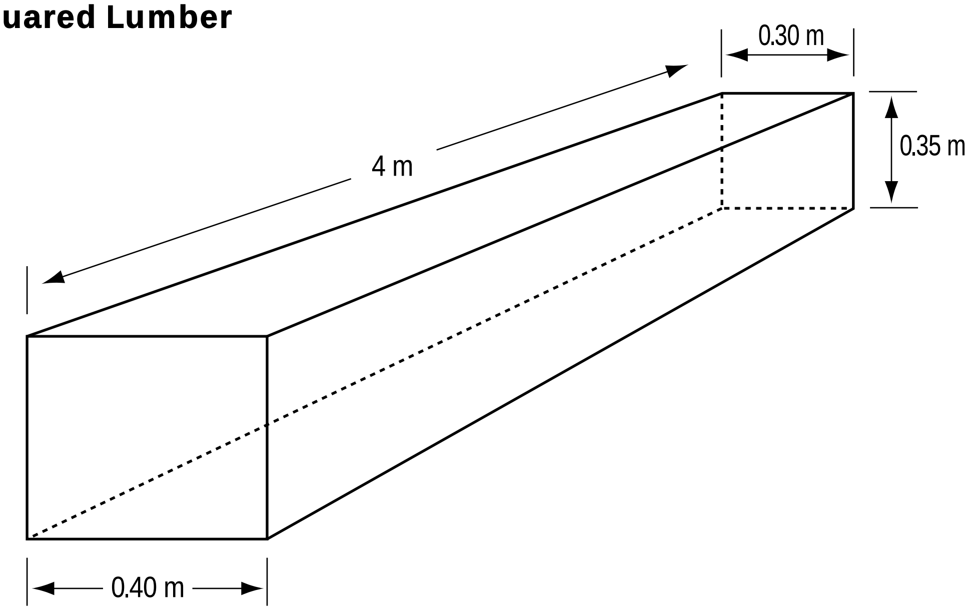
Students may research different guidelines used by foresters. Here are some examples teachers may use to review the concepts of the properties of geometric solids.

Determining the Volume of a Squared Log
Calculate the average width of the squared log (assuming there is a gradual taper). In Figure 41: Squared Lumber:

$$\text{Average Width} = \left(\frac{0.40m + 0.30m}{2} = 0.35m\right)$$

Volume will be: 0.35 m x 0.35 m x 4 m = 0.49 m^3
(w) (h) (l)
(h = log height)

Figure 39: Squared Lumber



Determining the Volume of a Short Log

Use the formula of the volume of a cylinder having the average circular area as its base. In Figure 42: Cylinder, below:

Volume will be: $\pi r^2 h = 3.14 \times \left(\frac{0.35m + 0.25m}{2}\right)^2 \times 5m$

$$V = 3.14 \times (0.30m)^2 \times (5m)$$

$$V = 1.41 \ m^3$$

or, since it is easier to measure the average circumference rather that the average radius,

$$\text{Volume} = \frac{1}{\pi} \times \left(\frac{\text{circumference}}{2}\right)^2 \times h$$

$$= 0.318 \times \left(\frac{\text{circumference}}{2}\right)^2 \times h$$

Determining the Volume of a Long Log

One usually divides the long log into smaller pieces (2, 3, or 5). See Figure 43: Dividing a Long Log.

Figure 42: Cylinder

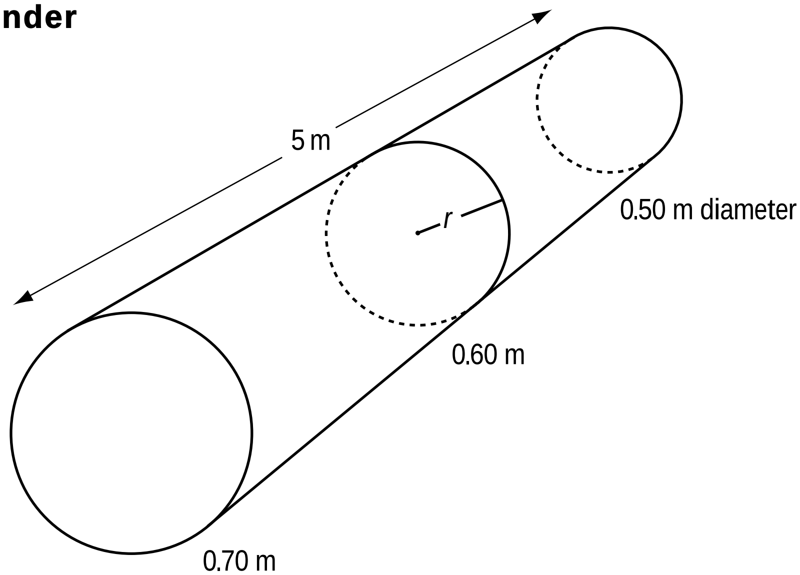
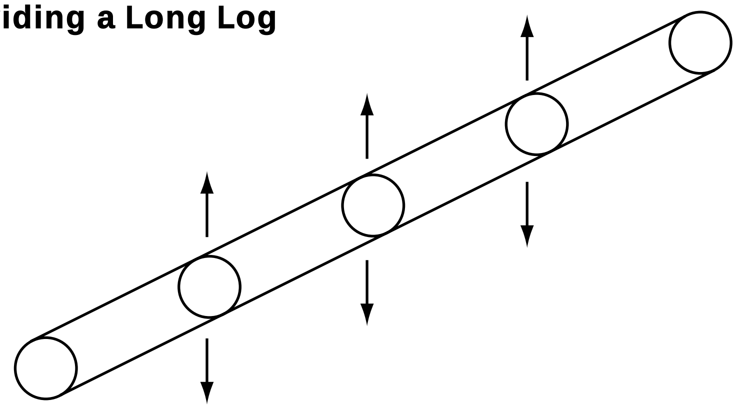


Figure 43: Dividing a Long Log



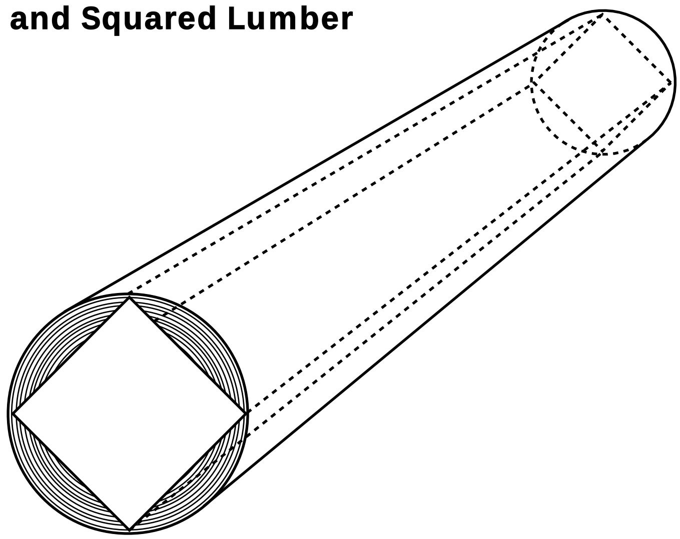
Estimating the Volume of a Squared Log

It is usually appropriate to use the formula,

$\text{Volume} = \left(\frac{C}{5}\right)^2 \times h,$ in which C is the average circumference of the log.

Ask students to estimate the decrease of volume from a log to a squared log (Figure 44: Log and Squared Lumber below).

Figure 44: Log and Squared Lumber

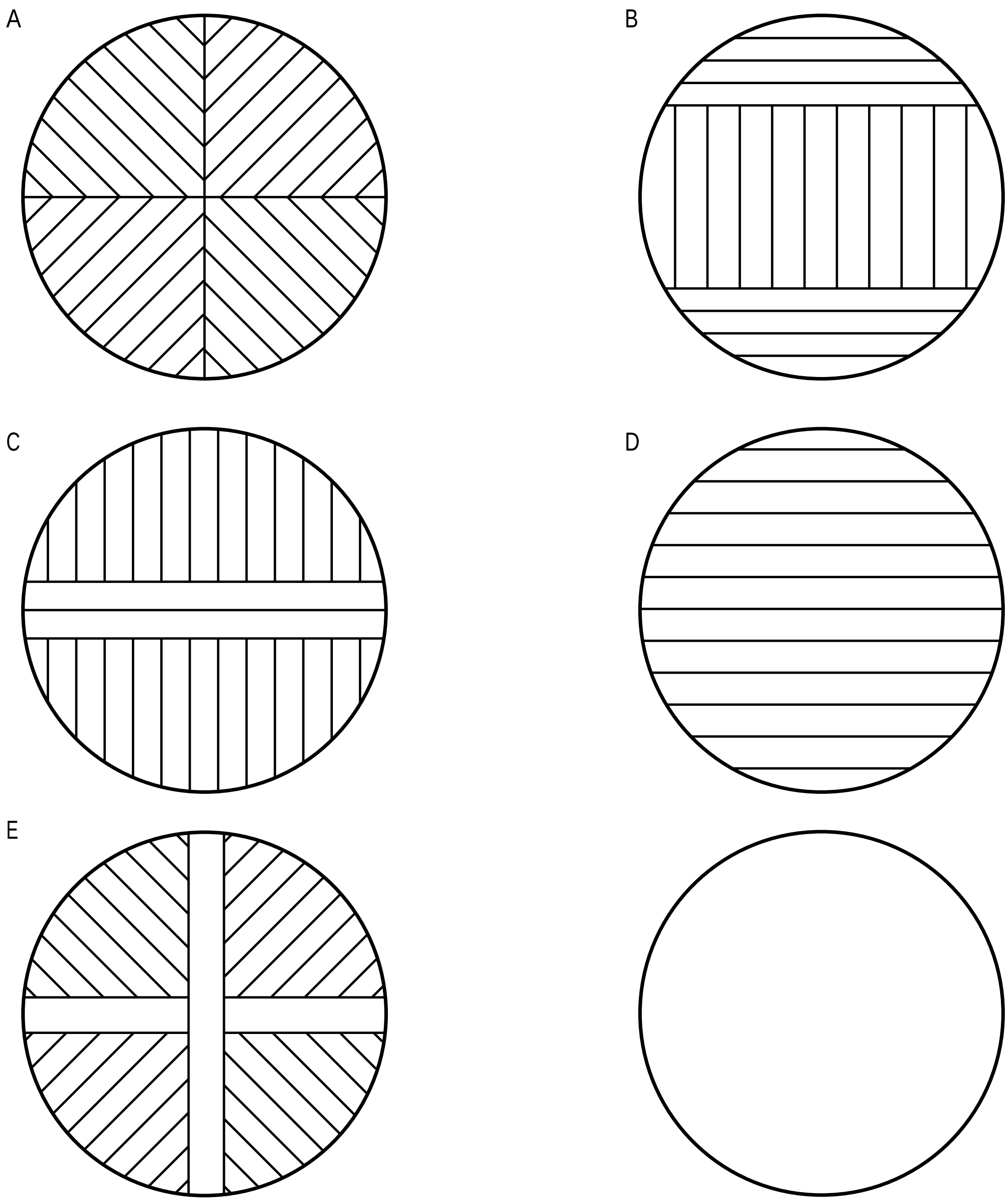


Distribute a copy of Student Activity: Lumber Cutting Patterns showing the most common ways of cutting lumber out of a log. Have students discuss the advantages and disadvantages of each type of cutting pattern.

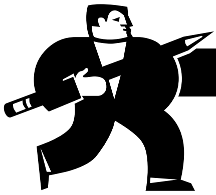
Student Activity: Lumber Cutting Patterns

Student Name: ______________________________ Date: ____________________

Figure 45: Lumber Cutting Patterns



Draw your own



Student Activity: Two- And Three- Dimensional Shapes And Objects

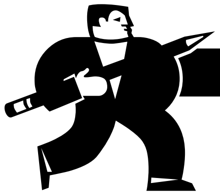
Student Name: ______________________ Date: ______________

Complete Table 4: Two-Dimensional Shapes.

Table 4

Two-Dimensional Shapes

Name of Shape	Area	Sketch
Square side *a*		
Rectangle sides *a* and *b*		
Parallelogram base *b* height *h*		
Triangle base *b* height *h*		
Trapezoid base *b* height *h*		
Circle radius *r*		
Ellipse axes *a* and *b*		



Student Activity: Two- And Three- Dimensional Shapes And Objects

Complete Table 5: Three-Dimensional Objects

Table 5

Three-Dimensional Objects

Name	Volume	Figure: Sketch
Cube side *a*		
Rectangular Prism length *a* width *w* height *h*		
Prism height *h* base area *b*		
Cylinder height *h* base circle radius *r*		
Pyramids height *h* square base rectangular base other		
Cone height *h* base circle radius *r*		
Sphere radius *r*		

Activity 9: Representing Geometric Solids

Objective:

In this activity students explore the projections of geometric solids onto the three planes of a trihedron in order to draw elevation views of the solids.

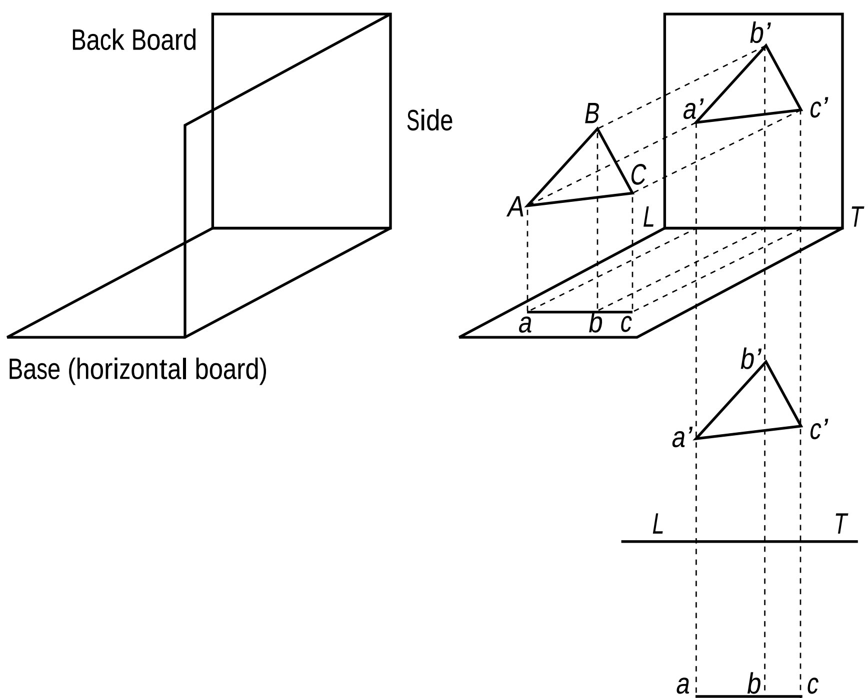
Materials:

(per pair of students)
- cardboard
- ruler
- scissors
- tape

Procedure:

Have students prepare several learning centres with different solids. For each learning centre, they should prepare a rectangular trihedron by taping three pieces of cardboard at right angles as shown in Figure 46: Preparation of a Regular Tetrahedron, below.

Figure 46: Preparation of a Regular Tetrahedron



Demonstrate how to observe the projections of a solid, for example a rectangular piece of lumber like a 2 x 4.

Have students cut out different shapes (e.g., triangle, square) of varying complexity. Ask one student to place the shapes inside the trihedron while her partner draws and labels the plane, vertical and face projections of the shape on the three faces of the trihedron. This is shown in Figure 47: Projection of Squared Lumber.

Confirm that students realize that the dimensions of the projections are not the same in reality (except for shapes that are placed parallel to one plane of the trihedron).

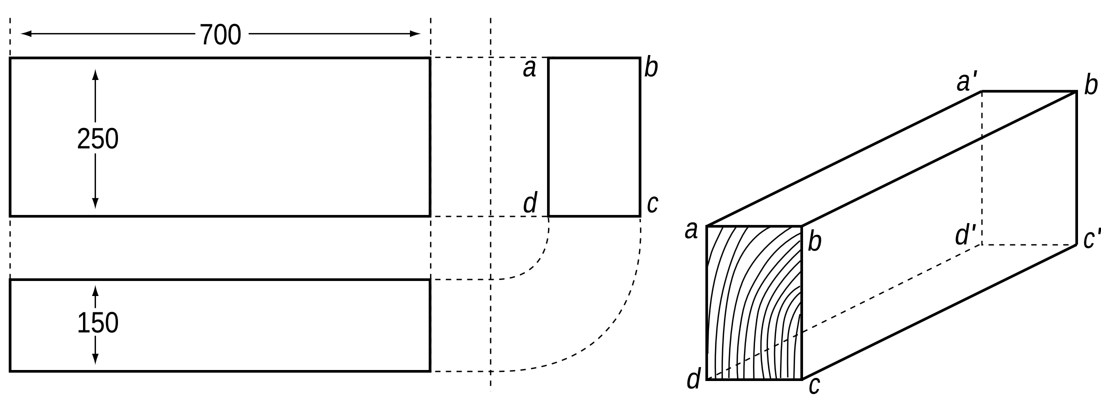
Have students change the orientation of the shape and decide which orientation has the simplest projections.

When students are comfortable with the projections of plane shapes, ask them to visit a few of the learning centres. Have them first sketch the solid and then sketch the projections of the solids onto the three planes.

Have students produce a scale plan of each of the solids (see Figure 47: Projection of Squared Lumber, below).

Provide a copy of Student Activity: Description and Drawing of Three-Dimensional Solids to each student and ask them to draw the scale plans of the solids of Student Activity: Two- and Three- Dimensional Shapes and Objects in the same way as the one illustrated in Figure 47: Projection of Squared Lumber.

Figure 47: Projection of Squared Lumber



Student Activity: Description and Drawing of Three-Dimensional Solids

Student Name: ____________________________ Date: ___________________

For the solids of Student Activity: Two- and Three-Dimensional Shapes and Objects, describe the position of the solids with respect to the planes of the trihedron and draw the scale plans in the space below. Use one page for each of the three-dimensional solids.

Phase III: Model Construction

Objective:

The purpose of this phase is to evaluate the spatial abilities of the students by having them construct a scale model of a simple hull of a boat from the plans and dimensions provided in Student Activity: Bow and Stern Design.

This phase incorporates most of the skills that students have developed during this project.

Materials:

(per group of 2 or 3 students)
- cardboard or thin plywood
- ruler and straight edge
- geometry kit
- scissors or hand jig-saw
- tape
- string
- rice paper or thin wrapping paper
- glue
- wooden craft sticks and straws

Procedure:

Provide students with a complete set of plans and a table of dimensions (Bow and Stern Design).

Distribute a copy of Figure 48: Flat Hull Plan and Profile, below and Figure 49: Diagram of V-Shape Hull, to have students familiarize themselves with the basic vocabulary used in naval construction, and to explain the different components of the plans. Show students a sample of the template by cutting off a piece of cardboard and explaining how it corresponds with the plans.

Explain to students what is expected from them and how their work will be evaluated. Discuss any additional criteria they might wish to add and provide them with an opportunity to give feedback about the evaluation process.

In the first part of this activity, students trace the templates on graph paper, draw them, and cut them out of a piece of cardboard.

Verify the precision of the students' work and record their level of understanding.

Students draw the base line and construct a square that is high enough (30 cm).

The templates are then assembled according to the plans and kept in place with tape.

In the second part of the activity, students attach the templates in position with pieces of straws.

Students determine a way to cover the hull using rice paper or wrapping paper cut in long strips and glued together.

Students remove the templates from the structure. Using straws or wooden craft sticks they make the deck structure stronger.

Figure 48: Flat Hull Plan and Profile

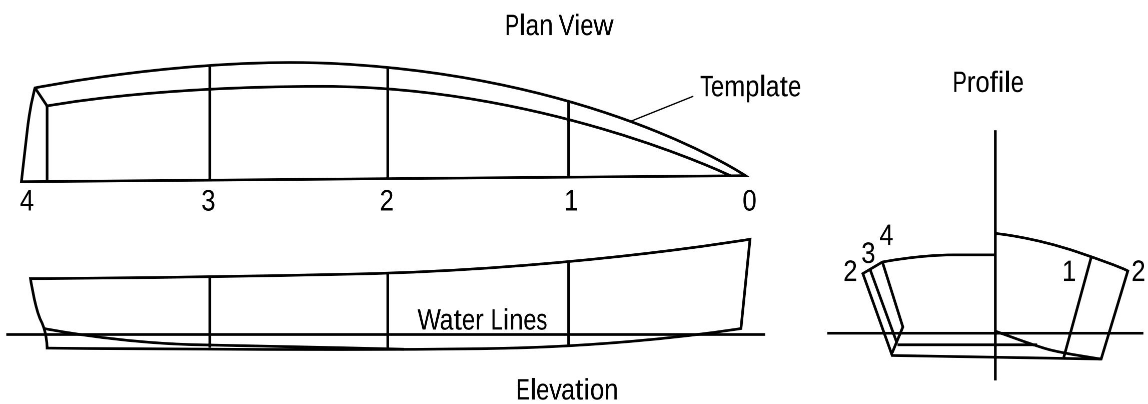
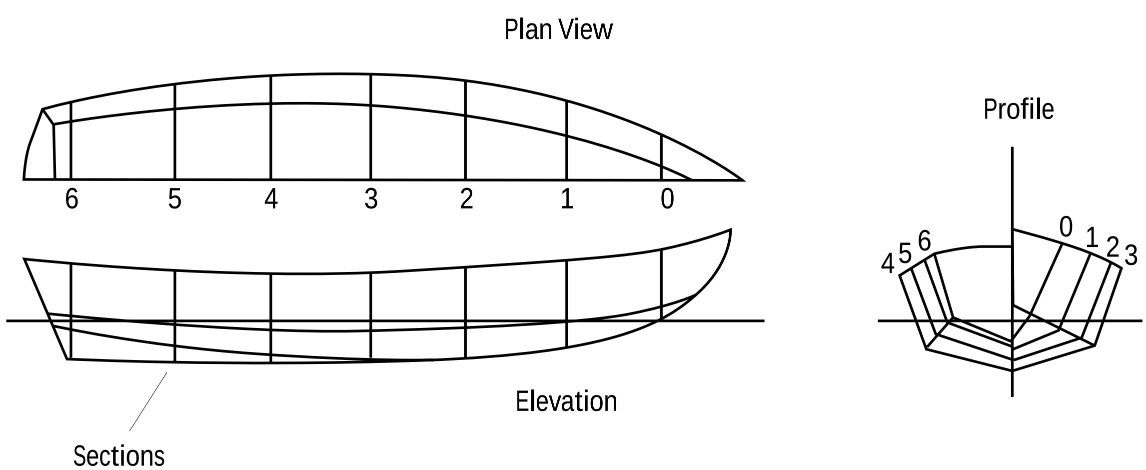


Figure 49: Diagram of V-Shape Hull



Note:
The evaluation component of this project may be building a scale model for a teen recreation centre based on Project C: Planning for a Teen Recreation Centre and Project D: Drawing a Scale Model of a Teen Recreation Centre.

Student Activity: Bow and Stern Design

Student Name: ____________________________ Date: ___________________

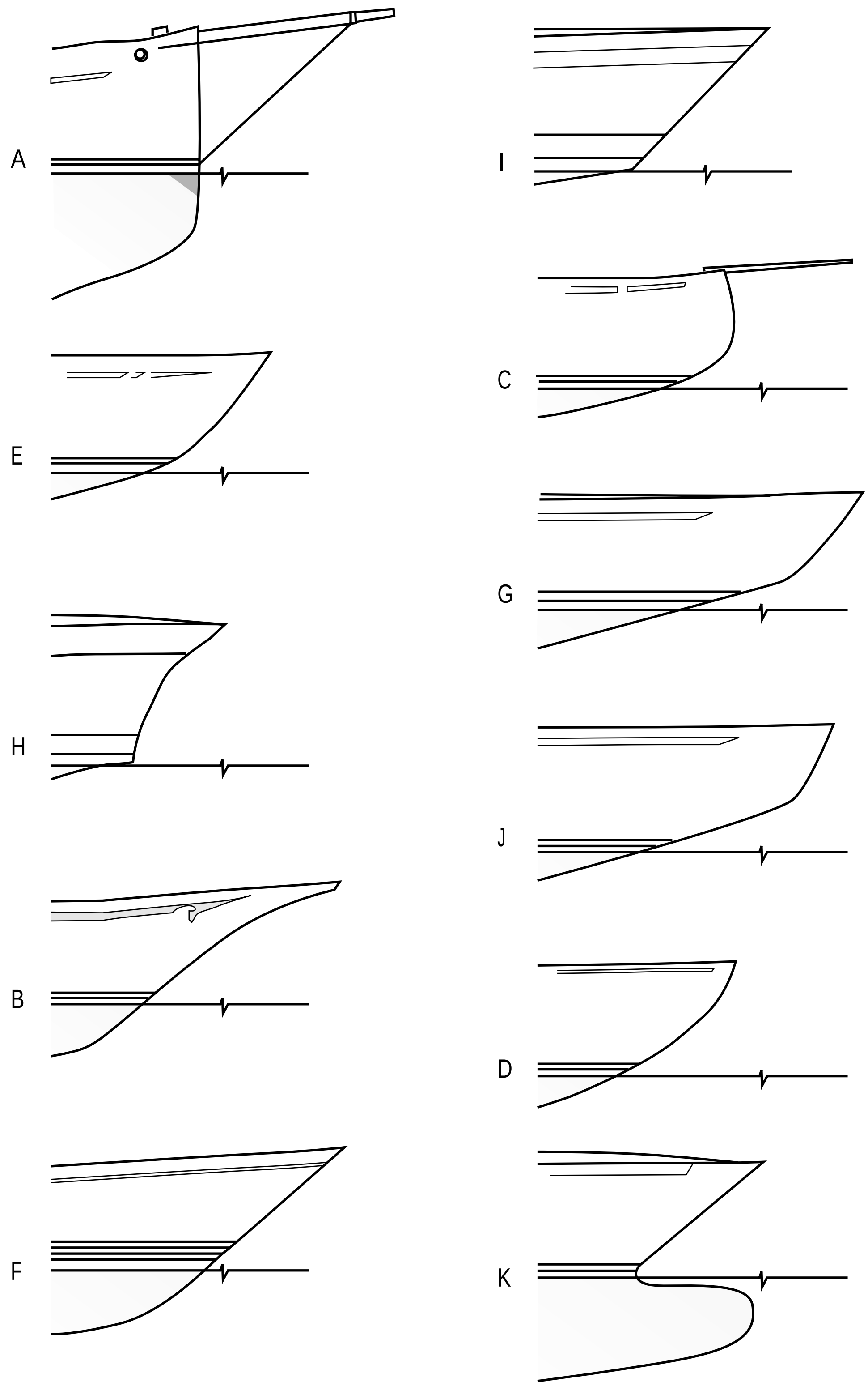
1. Choose one bow design and one stern design from those provided (Figure 50: Typical Bow Designs, and Figure 51: Typical Stern Designs) or from your own research.
2. Draw these designs on cardboard and cut them out.
3. You may wish to examine Figure 52: Hull Elevation and Plan, and Figure 53: Hull Profile, for additional information to add to your scale drawing.
4. Connect the bow and stern designs to produce your own hull design.
5. Comment on your design.

Hull Design:

Comments:

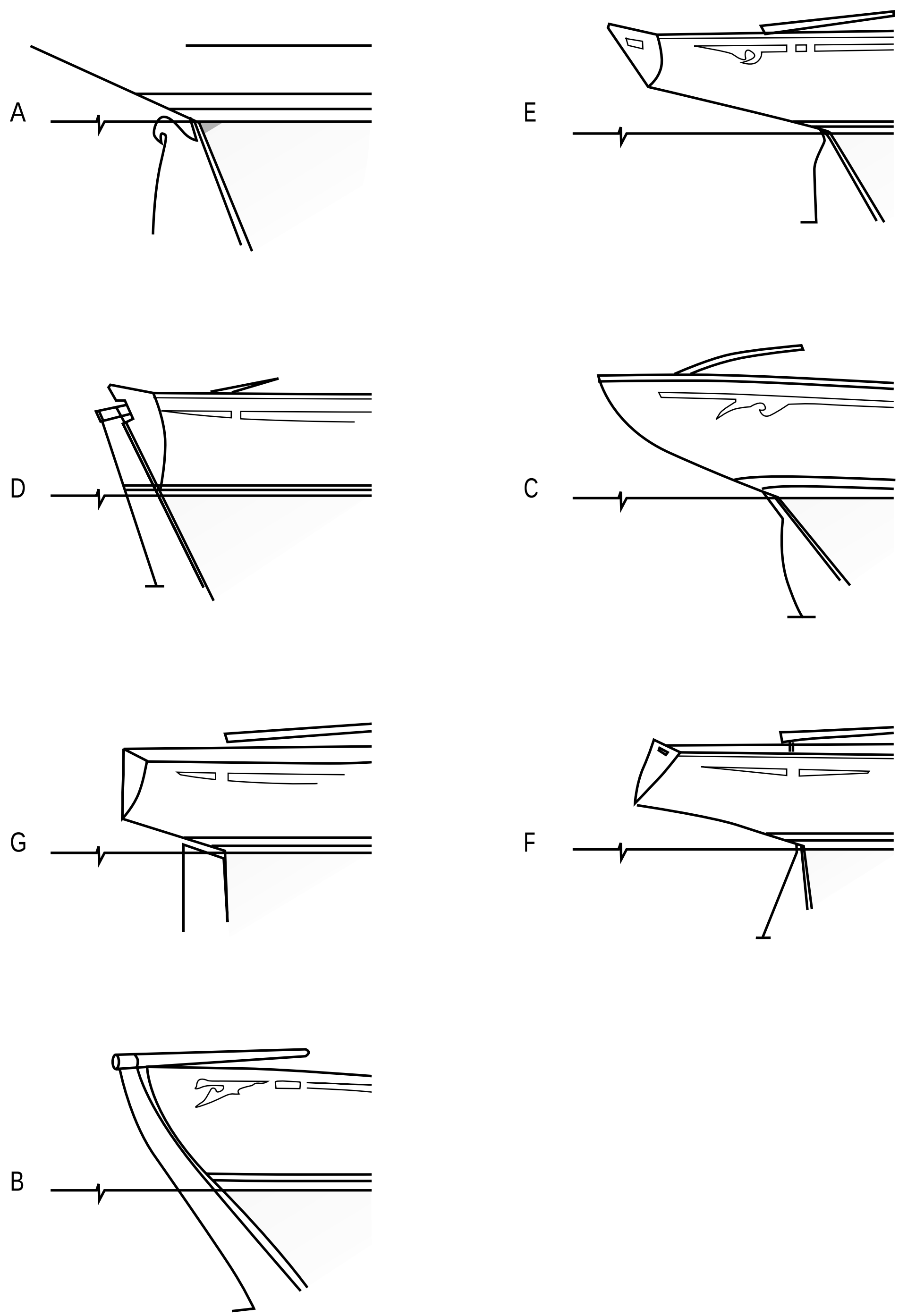
Student Activity: Bow and Stern Design

Figure 50: Typical Bow Designs



Student Activity: Bow and Stern Design

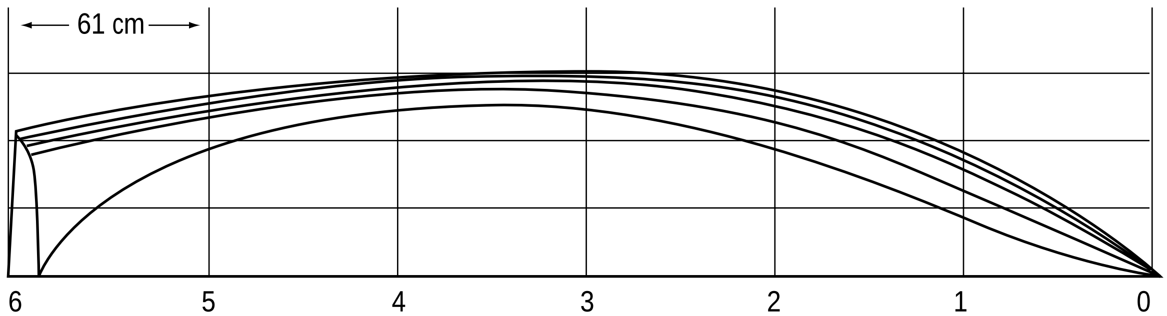
Figure 51: Typical Stern Designs



Student Activity: Bow and Stern Design

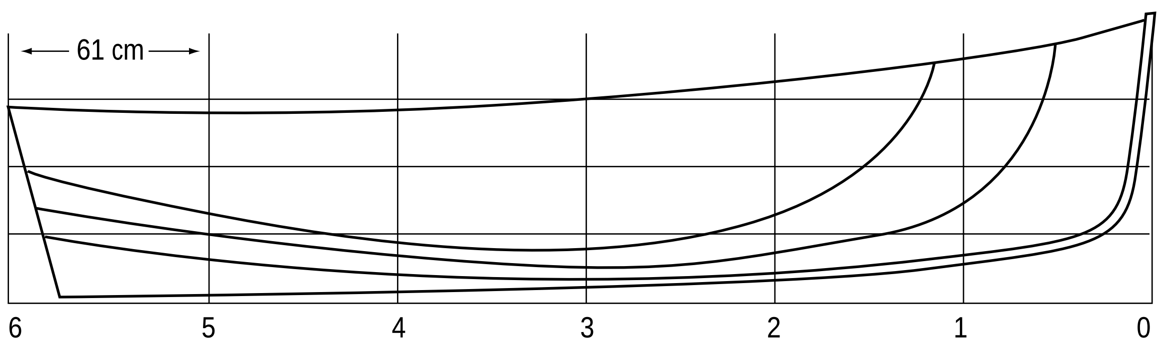
Figure 52: Hull Elevation and Plan

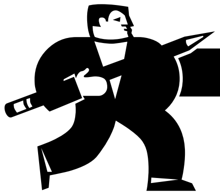
Elevation



Scale 1:20

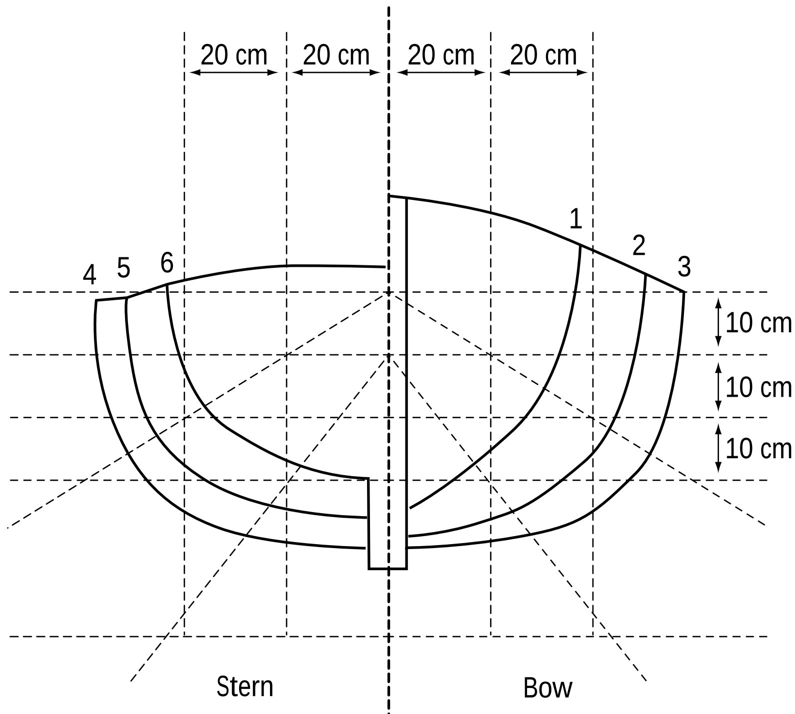
Plan





Student Activity: Bow and Stern Design

Figure 53: Hull Profile



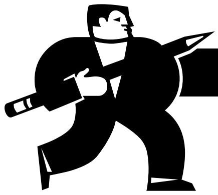
Boat designers draw hull plans in an unusual way. Because boat hulls are symmetrical, they often show only half of the bow and half of the stern. The boat builder simply folds the hull plan in half and draws its mirror image to get the full bow or stern plan.

The numbers 1, 2, 3, 4, 5, and 6 represent the templates for the bow and the stern.

The vertical dotted lines represent 20 cm distances, while the horizontal dotted lines represent 10 cm distances.

Student Activity: Bow and Stern Design

Draw your hull design:



Student Activity: Self-evaluation

Student Name: ___________________________ Date: __________________

1. Some things I learned about the properties of triangles include:

2. Some things I learned about real-world construction techniques include:

3. Some things I want to learn about planning a construction project include:

4. Some advice I would give to a friend who is planning a construction project is:

Appendix A: Correlation Between Projects and Mathematics 9 Learning Outcomes

Appendix A:
Correlation Between Projects and Mathematics 9 Learning Outcomes

PROBLEM SOLVING	Projects A B C D E
It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.	***It is expected that students will:*** • solve problems that involve a specific content area (e.g., geometry, algebra, trigonometry, statistics, probability, etc.) • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • use appropriate technology to assist in problem solving

NUMBER (Number Concepts) — Projects A B

It is expected that students will:

- **explain and illustrate the structure and the interrelationship of the sets of numbers within the rational number system**
- develop a number sense of powers with integral exponents and rational bases

It is expected that students will:

- give examples of numbers that satisfy the conditions for natural, whole, integral and rational numbers, and show that these numbers comprise the rational number system.
- **describe, orally and in writing, whether or not a number is rational.**
- give examples of situations where answers would involve the positive (principal) square root, or both positive and negative square roots of a number.
- **illustrate power, base, coefficient and exponent, using** rational numbers or **variables as bases or coefficients.**
- explain and apply the exponent laws for powers with integral exponents.

$$x^m \bullet x^n = x^{m+n}$$

$$x^m \div x^n = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^m = x^m y^m$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}, y \neq 0$$

$$x^0 = 1, x \neq 0$$

$$x^{-n} = \frac{1}{x^n}, x \neq 0$$

- determine the value of powers with integral exponents, using the exponent laws.

NUMBER (Number Operations)	Projects A B
It is expected that students will: • **use a scientific calculator or a computer to solve problems involving rational numbers** • explain how exponents can be used to bring meaning to large and small numbers, and use calculators or computers to perform calculations involving these numbers.	***It is expected that students will:*** • document and explain the calculator keying sequences used to perform calculations involving rational numbers. • **solve problems, using rational numbers in meaningful contexts.** • understand and use the exponent laws to simplify expressions with variable bases and evaluate expressions with numerical bases.

PATTERNS AND RELATIONS (Patterns)	Projects A B E
It is expected that students will generalize, design and justify mathematical procedures, using appropriate patterns, models and technology.	***It is expected that students will:*** • use logic and divergent thinking to present mathematical arguments in solving problems. • **model situations that can be represented by first -degree expressions.** • write equivalent forms of algebraic expressions, or equations, with rational coefficients.

PATTERNS AND RELATIONS (Variables and Equations) — Projects A B D

It is expected that students will:

- **evaluate, solve and verify linear equations** and inequalities **in one variable.**
- generalize arithmetic operations from the set of rational numbers to the set of polynomials.

It is expected that students will:

- Illustrate the solutions process for a first-degree, single-variable equation, using concrete materials or diagrams.
- **Solve and verify first-degree, single-variable equations of forms, such as:**

  $ax = b + cx$

  $a(x+b) = c$

  $ax + b = cx + d$

  $a(bx + c) = d(ex + f)$

  $\frac{a}{x} = b$

  where *a*, *b*, *c*, *d*, *e* and f are all rational numbers (with a focus on integers), and use equations of this type to model and solve problem situations.
- solve, algebraically, first-degree inequalities in one variable, display the solutions on a number line and test the solutions
- **identify constant terms, coefficients and variables in polynomial expressions**
- **evaluate polynomial expressions, given the value(s) of variables(s)**
- represent and justify the addition and subtraction of polynomial expressions, using concrete materials and diagrams
- perform the operations of addition and subtraction on polynomial expressions.
- represent multiplication, division and factoring of monomials, binomials, and trinomials of the form $x2 + bx + c$, using concrete materials and diagrams.
- find the product of two monomials, a monomial and a polynomial, and two binomials.
- determine equivalent forms of algebraic expressions by identifying common factors and factoring of the form $x^2 + bx + c$.

SHAPE AND SPACE (Measurement) — Projects D E

It is expected that students will:

- **use trigonometric ratios to solve problems involving a right triangle**
- **describe the effects of dimension changes in related 2-D shapes and 3-D objects in solving problems involving area, perimeter, surface area and volume.**

It is expected that students will:

- **explain the meaning of sine, cosine and tangent ratios in right triangles**
- **demonstrate the use of trigonometric ratios (sine, cosine and tangent) in solving right triangles**
- **calculate an unknown side or an unknown angle in a right triangle, using appropriate technology**
- **model and then solve given problem situations involving only one right triangle**
- relate expressions for volumes of pyramids to volumes of prisms, and volumes of cones to volumes of cylinders
- **calculate and apply the rate of volume to surface area to solve design problems in three dimensions**
- **calculate and apply the rate of area to perimeter to solve design problems in two dimensions**

SHAPE AND SPACE (3-D Objects and 2-D Shapes) — Projects D E

It is expected that the student will:

- **specify conditions under which triangles may be similar or congruent, and use these conditions to solve problems**
- **use spatial problem solving in building, describing and analyzing geometric shapes**

It is expected that students will:

- **draw the plan and elevation of a 3-D object from sketches and models**
- **sketch or build a 3-D object, given its plan and elevation views**
- recognize and draw the locus of points in solving practical problems
- **recognize when, and explain why, two triangles are similar, and use the properties of similar triangles to solve problems.**
- **recognize when, and explain why, two triangles are congruent, and use the properties of congruent triangles to solve problems.**
- relate congruence to similarity in the context of triangles

SHAPE AND SPACE (Transformation) — Projects A E

It is expected that students will apply coordinate geometry and pattern recognition to predict the effects of translations, rotations, reflections and dilatations on 1-D lines and 2-D shapes.

It is expected that students will:

- draw the image of a 2-D shape as a result of:
  - a single transformation
  - a dilatation
  - a combination of translation and/or reflections
- identify the single transformation that connects a shape with its image
- demonstrate that a triangle and its dilatation image are similar
- demonstrate the congruence of a triangle with its:
  - translation image
  - rotation image
  - reflection image

STATISTICS AND PROBABILITY (Data Analysis) — Projects A B C

It is expected that students will collect and analyze experimental results expressed in two variables, using technology, as required.

It is expected that students will:

- design, conduct and report on an experiment to investigate a relationship between two variables.
- create scatter plots for discrete and continuous variables.
- interpret a scatter plot to determine if there is an apparent relationship.
- determine the lines of best fit from a scatter plot for an apparent linear relationship by:
  - inspection
  - using technology (equations are not expected).
- draw and justify conclusions from the line of best fit.
- assess the strengths, weaknesses and biases of samples and data collection methods.
- critique ways in which statistical information and conclusions are presented by the media and other sources.

STATISTICS AND PROBABILITY (Chance and Uncertainty) — Projects A

It is expected that students will explain the use of probability and statistics in the solution of problems.

It is expected that students will:

- **recognize that decisions based on probability may be a combination of theoretical calculations, experimental results and subjective judgements.**
- **demonstrate an understanding of the role of probability and statistics in society**
- **solve problems involving the probability of independent events**

Appendix B: Blackline Masters

- Blackline Master 1: Field Log Book
- Blackline Master 2: Sketch
- Blackline Master 3: Strategy
- Blackline Master 4: Table of Measures
- Blackline Master 5: Scale Map
- Blackline Master 6: Constructions

Blackline Master 1

Field Log Book

Date:
Name:
Group:
Task assigned:

Description of the property:

Checklist:

Blackline Master 2

Sketch

Blackline Master 3

Strategy

Sequence	Description/Operation	Reason
Z		
U		
N		
V		
W		
T		
R		
M		

Blackline Master 4

Table of Measures

Fixed point:
Baseline:
Distance:
Direction:

Point	Distance to Fixed Point (*m*)	Direction (degrees)	Angle to Baseline (degrees)	Method of Measurement	Remarks

Blackline Master 5

Scale Map

Blackline Master 6

Constructions