MATHEMATICS GRADE 7

Integrated Resource Package 2007

Many people contributed their expertise to this document. The Project Co-ordinator was Mr. Richard DeMerchant of the Ministry of Education, working with other ministry personnel and our partners in education. We would like to thank all who participated in this process with a special thank you to Western and Northern Canadian Protocol (WNCP) partners in education for creation of the WNCP Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics from which this IRP is based.

MATHEMATICS K TO 7 IRP DEVELOPMENT TEAM

Lori Boychuk	School District No. 91 (Nechako Lakes)
Rosamar Garcia	School District No. 38 (Richmond)
Glen Gough	School District No. 81 (Fort Nelson)
Linda Jensen	School District No. 35 (Langley)
Carollee Norris	School District No. 60 (Peace River North)
Barb Wagner	School District No. 60 (Peace River North)
Joan Wilson	School District No. 46 (Sunshine Coast)
Donna Wong	School District No. 36 (Surrey)

MATHEMATICS K TO 7 IRP DEVELOPMENT TEAM SUPPORT PROVIDED BY

Liliane Gauthier	Saskatchewan Learning
Pamela Hagen	School District 43 (Coquitlam), University of British Columbia
Jack Kinakin	School District 20 (Kootney-Columbia)
Heather Morin	British Columbia Ministry of Education
Janice Novakowski	School District 38 (Richmond), University of British Columbia
GT Publishing Services Ltd.	Project co-ordination, writing, and editing

This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Mathematics K to 7. Once fully implemented, this document will supersede Mathematics K to 7 (1995).

> The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

INTRODUCTION

The Introduction provides general information about Mathematics K to 7, including special features and requirements.

Included in this section are
- a rationale for teaching Mathematics K to 7 in BC schools
- goals for Mathematics K to 7
- descriptions of the curriculum organizers – groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each grade
- a graphic overview of the curriculum content from K to 7
- additional information that sets the context for teaching Mathematics K to 7

CONSIDERATIONS FOR PROGRAM DELIVERY

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

PRESCRIBED LEARNING OUTCOMES

This section contains the *prescribed learning outcomes.* Prescribed learning outcomes are the legally required content standards for the provincial education system. They define the required attitudes, skills, and knowledge for each subject. The learning outcomes are statements of what students are expected to know and be able to do by the end of the grade.

The prescribed learning outcomes for the Mathematics K to 7 IRP are based on the Learning Outcomes contained within the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for K to 9 Mathematics available at www.wncp.ca.

STUDENT ACHIEVEMENT

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist teachers in assessing how well their students achieve the prescribed learning outcomes.

The achievement indicators for the Mathematics K to 7 IRP are based on the achievement indicators contained within the WNCP Common Curriculum Framework for K to 9 Mathematics.

> The WNCP CCF for K to 9 Mathematics is available online at www.wncp.ca

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

CLASSROOM ASSESSMENT MODEL

This section contains a series of classroom units that address the learning outcomes. The units have been developed by BC teachers, and are provided to support classroom assessment. These units are suggestions only – teachers may use or modify the units to assist them as they plan for the implementation of this curriculum.

Each unit includes the prescribed learning outcomes and suggested achievement indicators, a suggested timeframe, a sequence of suggested assessment activities, and sample assessment instruments.

LEARNING RESOURCES

This section contains general information on learning resources, providing a link to titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections. [*Note: Grade Collections for Mathematics K to 7 will be updated as new resources matching the IRP are authorized.*]

GLOSSARY

The glossary section provides a link to an online glossary that contains definitions for selected terms used in this Integrated Resource Package

INTRODUCTION

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Mathematics K to 7. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The Mathematics K to 7 IRP is based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics (May 2006). A complete list of references used to inform the revisions of the WNCP CCF for K to 9 Mathematics as well as this IRP can be found at the end of this section of the IRP.

Mathematics K to 7, in draft form, was available for public review and response from September to November, 2006. Input from educators, students, parents, and other educational partners informed the development of this document.

RATIONALE

The aim of Mathematics K to 7 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers 1998)

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of a variety of manipulatives and pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics. Information gathered from these discussions can be used for formative assessment to guide instruction.

As facilitators of learning educators are encouraged to highlight mathematics concepts as they occur within the K to 7 school environment and within home environments. Mathematics concepts are present within every school's subjects and drawing students' attention to these concepts as they occur can help to provide the "teachable moment."

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary. Positive learning experiences build self-confidence and develop attitudes that value learning mathematics.

ABORIGINAL PERSPECTIVE

Aboriginal students in British Columbia come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. Depending on their learning styles, students may look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. *The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks 1993).*

AFFECTIVE DOMAIN

Bloom's taxonomy of learning behaviours identified three domains of educational activities, affective (growth in feelings or emotional areas – attitude), cognitive (mental skills – knowledge), and psychomotor (manual or physical skills – skills). The affective domain involves the way in which we perceive and respond to things emotionally, such as feelings, values, appreciation, enthusiasms, motivations, and attitudes.

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Research has shown that students who are more engaged with school and with mathematics are far

more likely to be successful in school and in learning mathematics. (Nardi & Steward 2003). Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Substantial progress has been made in research in the last decade that has examined the importance and use of the affective domain as part of the learning process. In addition there has been a parallel increase in specific research involving the affective domain and its' relationship to the learning of mathematics which has provided powerful evidence of the importance of this area to the learning of mathematics (McLeod 1988, 1992 & 1994; Hannula 2002 & 2006; Malmivuori 2001 & 2006). Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Students who are feeling more comfortable with a subject, demonstrate more confidence and have the opportunity for greater academic achievement (Denton & McKinney 2004; Hannula 2006; Smith et al. 1998). Educators can include opportunities for active and co-operative learning in their mathematics lessons which has been shown in research to promote greater conceptual understanding, more positive attitudes and subsequently improved academic achievement from students (Denton & McKinney 2004). By allowing the sharing and discussion of answers and strategies used in mathematics, educators are providing rich opportunities for students mathematical development. Educators can foster greater conceptual understanding in students by having students practice certain topics and concepts in mathematics in a meaningful and engaging manner.

It is important for educators, students, and parents to recognize the relationship between the affective and cognitive domains and attempt to nurture those aspects of the affective domain that contribute to positive attitudes and success in learning.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that are integral to the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty. These components are woven throughout this curriculum.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:
- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms.

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics.

These skills contribute to students' interaction with and understanding of their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

GOALS FOR MATHEMATICS K TO 7

Mathematics K to 7 represents the first formal steps that students make towards becoming life-long learners of mathematics.

GOALS FOR MATHEMATICS K TO 7

The Mathematics K-7 curriculum is meant to start students toward achieving the main goals of mathematics education:

- using mathematics confidently to solve problems
- using mathematics to better understand the world around us
- communicating and reasoning mathematically
- appreciating and valuing mathematics
- making connections between mathematics and its applications
- committing themselves to lifelong learning
- becoming mathematically literate and using mathematics to participate in, and contribute to, society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- be able to use mathematics to make and justify decisions about the world around us
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

CURRICULUM ORGANIZERS

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Mathematics K to 7 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers:

Curriculum Organizers and Suborganizers
MATHEMATICS K-7
NUMBER
PATTERNS AND RELATIONS • Patterns • Variables and Equations
SHAPE AND SPACE • Measurement • 3-D Objects and 2-D Shapes • Transformations
STATISTICS AND PROBABILITY • Data Analysis • Chance and Uncertainty

These curriculum organizers reflect the main areas of mathematics that students are expected to address. The ordering of organizers, suborganizers, and outcomes in the Mathematics K to 7 curriculum does not imply an order of instruction. The order in which various topics are addressed is left to the professional judgment of teachers. Mathematics teachers are encouraged to integrate topics throughout the curriculum and within other subject areas to emphasize the connections between mathematics concepts.

Number

Students develop their concept of the number system and relationships between numbers. Concrete, pictorial and symbolic representations are used to help students develop their number sense. Computational fluency, the ability to connect understanding of the concepts with accurate, efficient and flexible computation strategies for multiple purposes, is stressed throughout the number organizer with an emphasis on the development of personal strategies, mental mathematics and estimation strategies.

The Number organizer does not contain any suborganizers.

Patterns and Relations

Students develop their ability to recognize, extend, create, and use numerical and non- numerical patterns to better understand the world around them as well as the world of mathematics. This organizer provides opportunities for students to look for relationships in the environment and to describe the relationships. These relationships should be examined in multiple sensory forms.

The Patterns and Relations organizer includes the following suborganizers:
• Patterns
• Variables and Equations

Shape and Space

Students develop their understanding of objects and shapes in the environment around them. This includes recognition of attributes that can be measured, measurement of these attributes, description of these attributes, the identification and use of referents, and positional change of 3-D objects and 2-D shapes on the environment and on the Cartesian plane.

The Shape and Space organizer includes the following suborganizers:
• Measurement
• 3-D Objects and 2-D Shapes
• Transformations

Statistics and Probability

Students collect, interpret and present data sets in relevant contexts to make decisions. The development of the concepts involving probability is also presented as a means to make decisions. The Shape and Space organizer includes the following suborganizers:
• Data Analysis
• Chance and Uncertainty

KEY CONCEPTS: OVERVIEW OF MATHEMATICS K TO 7 TOPICS

	Kindergarten	Grade 1	Grade 2	Grade 3
NUMBER	• number sequence to 10 • familiar number arrangements up to 5 objects • one-to-one correspondence • numbers in-depth to 10	• skip counting starting at 0 to 100 • arrangements up to 10 objects • numbers in-depth to 20 • addition & subtraction to 20 • mental math strategies to 18	• skip counting at starting points other than 0 to 100 • numbers in-depth to 100 • even, odd & ordinal numbers • addition & subtraction to 100 • mental math strategies to 18	• skip counting at starting points other than 0 to 1000 • numbers in-depth to 1000 • addition & subtraction to 1000 • mental math strategies for 2-digit numerals • multiplication up to 5 × 5 • representation of fractions
PATTERNS & RELATIONS *Patterns*	• repeating patterns of two or three elements	• repeating patterns of two to four elements • representation of pattern	• repeating patterns of three to five elements • increasing patterns	• increasing patterns • decreasing patterns
PATTERNS & RELATIONS *Variables & Equations*		• equalities & inequalities • symbol for equality	• equality & inequality • symbols for equality & inequality	• one-step addition and subtraction equations
SHAPE & SPACE *Measurement*	• direct comparison for length, mass & volume	• process of measurement using comparison	• days, weeks, months, & years • non-standard units of measure for length, height distance around, mass (weight)	• non-standard & standard units of time • measurements of length (cm, m) & mass (g, kg) • perimeter of regular & irregular shapes
SHAPE & SPACE *3-D Objects & 2-D Shapes*	• single attribute of 3-D objects	• one attribute of 3-D objects & 2-D shapes • composite 2-D shapes & 3-D objects • 2-D shapes in the environment	• two attributes of 3-D objects & 2-D shapes • cubes, spheres, cones, cylinders, pyramids • triangles, squares, rectangles, circles • 2-D shapes in the environment	• faces, edges & vertices of 3-D objects • triangles, quadrilaterals, pentagons, hexagons, octagons
SHAPE & SPACE *Transformations*				
STATISTICS & PROBABILITY *Data Analysis*			• data about self and others • concrete graphs and pictographs	• first-hand data • bar graphs
STATISTICS & PROBABILITY *Chance & Uncertainty*				

Grade 4	Grade 5	Grade 6	Grade 7
• numbers in-depth to 10 000 • addition & subtraction to 10 000 • multiplication & division of numbers • fractions less than or equal to one • decimals to hundredths	• numbers in-depth to 1 000 000 • estimation strategies for calculations & problem solving • mental mathematics strategies for multiplication facts to 81 & corresponding division facts • mental mathematics for multiplication • multiplication for 2-digit by 2-digit & division for 3-digit by 1-digit • decimal & fraction comparison • addition & subtraction of decimals	• numbers in-depth greater than 1 000 000 & smaller than one thousandth • factors & multiples • improper fractions & mixed numbers • ratio & whole number percent • integers • multiplication & division of decimals • order of operations excluding exponents	• divisibility rules • addition, subtraction, multiplication, & division of numbers • percents from 1% to 100% • decimal & fraction relationships for repeating & terminating decimals • addition & subtraction of positive fractions & mixed numbers • addition & subtraction of integers
• patterns in tables & charts	• prediction using a pattern rule	• patterns & relationships in graphs & tables including tables of value	• table of values & graphs of linear relations
• symbols to represent unknowns • one-step equations	• single-variable, one-step equations with whole number coefficients & solutions	• letter variable representation of number relationships • preservation of equality	• preservation of equality • expressions & equations • one-step linear equations
• digital clocks, analog clocks, & calendar dates • area of regular & irregular 2-D shapes	• perimeter & area of rectangles • length, volume, & capacity	• perimeter & area of rectangles • length, volume, & capacity	• properties of circles • area of triangles, parallelograms, & circles
• rectangular & triangular prisms	• parallel, intersecting, perpendicular, vertical & horizontal edges & faces • rectangles, squares, trapezoids, parallelograms & rhombuses	• types of triangles • regular & irregular polygons	• geometric constructions
• line symmetry	• 2-D shape single transformation	• combinations of transformations • single transformation in the first quadrant of the Cartesian plane	• four quadrants of the Cartesian plane • transformations in the four quadrants of the Cartesian plane
• many-to-one correspondence including bar graphs & pictographs	• first-hand & second-hand data • double bar graphs	• line graphs • methods of data collection • graph data	• central tendency, outliers & range • circle graphs
	• likelihood of a single outcome	• experimental & theoretical probability	• ratios, fractions, & percents to express probabilities • two independent events • tree diagrams for two independent events

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Mathematics K to 7.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to:
- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K to 3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

SUGGESTED TIMEFRAME

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. For Mathematics K to 7, the Ministry of Education recommends a time allotment of 20% (approximately 95 hours in Kindergarten and 185 hours in Grades 1 to 7) of the total instructional time for each school year. In the primary years, teachers determine the time allotments for each required area of study and may choose to combine various curricula to enable students to integrate ideas and see the application of mathematics concepts across curricula.

The Mathematics K to 7 IRP for grades 1 to 7 is based on approximately 170 hours of instructional time to allow flexibility to address local needs. For Kindergarten, this estimate is approximately 75 hours. Based on these recommendations, teachers should be spending about 2 to 2.5 hours each week on Mathematics in Kindergarten and 4.5 to 5 hours of instructional time each week on Mathematics grades 1 to 7.

REFERENCES

The following references have been used to inform the revisions of the BC Mathematics K to 7 IRP as well as the WNCP CCF for K-9 Mathematics upon which the Prescribed Learning Outcomes and Achievement Indicators are based.

American Association for the Advancement of Science. *Benchmark for Science Literacy.* New York, NY: Oxford University Press, 1993.

Anderson, A.G. "Parents as Partners: Supporting Children's Mathematics Learning Prior to School." Teaching Children Mathematics, 4 (6), February 1998, pp. 331–337.

Armstrong, T. *Seven Kinds of Smart: Identifying and Developing Your Many Intelligences.* New York, NY: NAL-Dutton, 1993.

Ashlock, R. "Diagnosing Error Patterns in Computation." *Error Patterns in Computation.* Columbus, Ohio: Prentice Hall, 1998, pp. 9–42.

Banks, J.A. and C.A.M. Banks. *Multicultural Education: Issues and Perspectives.* Boston: Allyn and Bacon, 1993.

Becker, J.P. and S. Shimada. *The Open-Ended Approach: A New Proposal for Teaching Mathematics.* Reston, VA: The National Council of Teachers of Mathematics, 1997.

Ben-Chaim, D. et al. "Adolescents Ability to Communicate Spatial Information: Analyzing and Effecting Students' Performance." *Educational Studies Mathematics,* 20(2), May 1989, pp. 121–146.

Barton, M. and C. Heidema. *Teaching Reading in Mathematics (2nd ed.).* Aurora, CO: McRel, 2002.

Billmeyer, R. and M. Barton. *Teaching Reading in the Conent Areas: If Not Me Then Who? (2nd ed.).* Aurora, CO: McRel, 1998.

Bloom B. S. Taxonomy of Educational Objectives, Handbook I: The Cognitive Domain. New York: David McKay Co Inc., 1956.

Borasi, R. *Learning Mathematics through Inquiry.* Portsmouth, NH: Heinmann, 1992.

Borsai, R. *Reconceiving Mathematics Instruction: A Focus on Errors.* Norwood, NJ: Ablex, 1996.

Bright, George W. et al. *Navigating through Data Analysis in Grades 6−8.* Reston, VA: The National Council of Teachers of Mathematics, 2003.

British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*, Victoria BC: Queens Printer, 2000.

British Columbia Ministry of Education. *Mathematics K to 7 Integrated Resource Package (1995). Victoria BC: Queens Printer*, 1995.

British Columbia Ministry of Education. *Shared Learnings: Integrating BC Aboriginal Content K-10.* Victoria, BC. Queens Printer, 2006.

Burke, M.J. and F.R. Curcio. *Learning Mathematics for a New Century* (2000 yearbook). Reston, VA: National Council of Teachers of Mathematics, 2000.

Burke, M., D. Erickson, J. Lott, and M. Obert. *Navigating through Algebra in Grades 9−12.* Reston, VA: The National Council of Teachers of Mathematics, 2001.

Burns, M. *About Teaching Mathematics: A K-8 Resource.* Sausalto, CA: Math Solutions Publications, 2000.

Buschman, L. "Using Student Interviews to Guide Classroom Instruction: An Action Research Project." *Teaching Children Mathematics,* December 2001, pp. 222–227.

Caine, R. N. and G. Caine. *Making Connections: Teaching and the Human Brain.* Menlo Park, CA: Addison-Wesley Publishing Company, 1991.

Chambers, D.L., Editor. *Putting Research into Practice in the Elementary Grades.* Virginia: The National Council of Teachers of Mathematics, 2002.

Chapin, Suzanne et al. *Navigating through Data Analysis and Probability in Grades 3−5.* Reston VA: The National Council of Teachers of Mathematics, 2003.

Charles, Randall and Joanne Lobato. *Future Basics: Developing Numerical Power, a Monograph of the National Council of Supervisors of Mathematics.* Golden, CO: National Council of Supervisors of Mathematics, 1998.

Clements D.H . "Geometric and Spatial Thinking in Young Children." In J. Copley (ed.), *Mathematics in the Early Years.* Reston, VA: The National Council of Teachers of Mathematics, 1999, pp. 66–79.

Clements, D.H. "Subitizing: What is it? Why teach it?" *Teaching Children Mathematics,* March, 1999, pp. 400–405.

Colan, L., J. Pegis. *Elementary Mathematics in Canada: Research Summary and Classroom Implications.* Toronto, ON: Pearson Education Canada, 2003.

Confrey, J. "A Review of the Research on Student Conceptions in Mathematics, Science and Programming." In C. Cadzen (ed.), *Review of Research in Education,* 16. Washington, DC: American Educational Research Association, 1990, pp. 3–56.

Cuevas, G., K. Yeatt. *Navigating through Algebra in Grades 3−5.* Reston VA: The National Council of Teachers of Mathematics, 2001.

Dacey, Linda et al. *Navigating through Measurement in Prekindergarten − Grade 2.* Reston, VA: National Council of Teachers of Mathematics, 2003.

Davis, R.B. and C.M. Maher. "What Do We Do When We 'Do Mathematics'?" *Constructivist Views on the Teaching and Learning of Mathematics.* Reston, VA: The National Council of the Teachers of Mathematics, 1990, pp. 195–210.

Day, Roger et al. *Navigating through Geometry in Grades 9−12.* Reston VA: The National Council of Teachers of Mathematics, 2002.

Denton, L.F., McKinney, D., Affective Factors and Student Achievement: A Quantitative and Qualitative Study, Proceedings of the 34th ASEE/IEEE Conference on Frontiers in Education, Downloaded 13.12.06 www.cis.usouthal.edu/~mckinney/FIE2004-1447DentonMcKinney.pdf, 2004.

Egan, K. *The Educated Mind: How Cognitive Tools Shape our Understanding.* Chicago & London: University of Chicago Press, 1997.

Findell, C. et al. *Navigating through Geometry in Prekindergarten − Grade 2.* Reston, VA: The National Council of Teachers of Mathematics, 2001.

Friel, S., S. Rachlin and D. Doyle. *Navigating through Algebra in Grades 6−8.* Reston, VA: The National Council of Teachers of Mathematics, 2001.

Fuys, D., D. Geddes and R. Tischler. *The van Hiele Model of Thinking in Geometry Among Adolescents.* Reston, VA: The National Council of Teachers of Mathematics, 1998.

Gattegno, C. *The Common Sense of Teaching Mathematics.* New York, NY: Educational Solutions, 1974.

Gavin, M., Belkin, A. Spinelli and J. St. Marie. *Navigating through Geometry in Grades 3−5.* Reston, VA: The National Council of Teachers of Mathematics, 2001.

Gay, S. and M. Thomas. "Just Because They Got It Right, Does it Mean They Know It?" In N.L. Webb (ed.), *Assessment in the Mathematics Classroom.* Reston, VA: The National Council of Teachers of Mathematics, 1993, pp. 130–134.

Ginsburg, H.P. et al. "Happy Birthday to You: Early Mathematical Thinking of Asian, South American, and U.S. Children." In T. Nunes and P. Bryant (eds.), *Learning and Teaching Mathematics: An International Perspective.* Hove, East Sussex: Psychology Press, 1997, pp. 163–207.

Goldin, G.A., Problem Solving Heuristics, Affect and Discrete Mathematics, Zentralblatt fur Didaktik der Mathematik (International Reviews on Mathematical Education), 36, 2, 2004.

Goldin, G.A., Children's Visual Imagery: Aspects of Cognitive Representation in Solving Problems with Fractions. Mediterranean Journal for Research in Mathematics Education. 2, 1, 2003, pp. 1-42.

Goldin, G.A. Affective Pathways and Representation in Mathematical Problem Solving, Mathematical Thinking and Learning, 2, 3, 2000, pp. 209-219.

Greenes, C., M. et al. *Navigating through Algebra in Prekindergarten – Grade 2.* Reston, VA: The National Council of Teachers of Mathematics, 2001.

Greeno, J. Number sense as a situated knowing in a conceptual domain. *Journal for Research in Mathematics Education* 22 (3), 1991, pp. 170–218.

Griffin, S. *Teaching Number Sense.* ASCD Educational Leadership, February, 2004, pp. 39–42.

Griffin, L., Demoss, G. *Problem of the Week: A Fresh Approach to Problem-Solving.* Instructional Fair TS Denison, Grand Rapids, Michigan 1998.

Hannula, M.S. Motivation in Mathematics: Goals Reflected in Emotions, Educational Studies in Mathematics, Retrieved 17.10.06 from 10.1007/s10649-005-9019-8, 2006.

Hannula, M.S.,Attitude Towards Mathematics: Emotions, Expectations and Values, Educational Studies in Mathematics, 49, 200225-46.

Haylock, Derek and Anne Cockburn. *Understanding Mathematics in the Lower Primary Years.* Thousand Oaks, California: SAGE Publications Inc., 2003.

Heaton, R.M. *Teaching Mathematics to the New Standards: Relearning the Dance.* New York, NY: Teachers College Press, 2001.

Hiebert, J. et al. *Making Sense: Teaching and Learning Mathematics with Understanding.* Portsmouth NH: Heinemann, 1997.

Hiebert, J. et al. Rejoiner: Making mathematics problematic: A rejoiner to Pratwat and Smith. *Educational Researcher* 26 (2), 1997, pp. 24-26.

Hiebert, J. et al. Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher* 25 (4), 1996, pp. 12-21.

Hope, Jack A. et al. *Mental Math in the Primary Grades* (p. v). Dale Seymour Publications, 1988.

Hope, Jack A. et al. *Mental Math in Junior High* (p. v). Dale Seymour Publications, 1988.

Hopkins, Ros (ed.). *Early Numeracy in the Classroom.* Melbourne, Australia: State of Victoria, 2001.

Howden, H. Teaching Number Sense. *Arithmetic Teacher,* 36 (6), 1989, pp. 6–11.

Howe R. "Knowing and Teaching Elementary Mathematics: *Journal of Research in Mathematics Education,* 1999. 30(5), pp. 556–558.

Hunting, R. P. "Clinical Interview Methods in Mathematics Education Research and Practice." *Journal of Mathematical Behavior,* 1997, 16(2), pp. 145–165.

Identifying the van Hiele Levels of Geometry Thinking in Seventh-Grade Students through the Use of Journal Writing. Doctoral dissertation. University of Massachusetts, 1993, Dissertation Abstracts International, 54 (02), 464A.

Kamii, C. *Multidigit Division – Two Teachers Using Piaget's Theory.* Colchester, VT: Teachers College Press, 1990.

Kamii, C. and A. Dominick. "To Teach or Not to Teach Algorithms." *Journal of Mathematical Behavior,* 1997, 16(1), pp. 51–61.

Kelly, A.G. "Why Can't I See the Tree? A Study of Perspective." *Teaching Children Mathematics,* October 2002, 9(3), pp. 158–161.

Kersaint, G. "Raking Leaves – The Thinking of Students." *Mathematics Teaching in the Middle School,* November 2002, 9(3), pp. 158–161.

Kilpatrick, J., J. Swafford and B. Findell (eds.). *Adding it Up: Helping Children Learn Mathematics.* Washington, DC: National Academy Press, 2001.

Kilpatrick, J., W.G. Martin, and D. Schifter (eds.). *A Research Companion to Principles and Standards for School Mathematics,* Virginia: The National Council of Teachers of Mathematics, 2003.

King, J. *The Art of Mathematics.* New York: Fawcett Columbine, 1992.

Krathwohl, D. R., Bloom, B. S., & Bertram, B. M., *Taxonomy of Educational Objectives, the Classification of Educational Goals. Handbook II: Affective Domain.* New York: David McKay Co., Inc., 1973.

Lakoff, G. and R.E. Nunez. *Where Mathematics Comes From – How the Embodied Mind Brings Mathematics into Being.* New York, NY: Basic Books, 2000.

Lampert, M. *Teaching Problems and the Problems of Teaching*. New Haven & London: Yale University Press, 2001.

Ma, L. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum, 1999.

Malmivuori, M., Affect and Self-Regulation, Educational Studies in Mathematics, Educational Studies in Mathematics, Retrieved 17.10.06 from Springer Link 10.1007/s10649-006-9022-8, 2006.

Malmivuori, M-L., The dynamics of affect, cognition, and social environment in the regulation of personal learning processes: The case of mathematics, Research report 172, http:// ethesis.helsinki.fi/julkaisut/kas/kasva/vk/malmivuori/, University of Helsinki, Helsinki., 2001.

Mann, R. *Balancing Act: The Truth Behind the Equals Sign. Teaching Children Mathematics,* September 2004, pp. 65–69.

Martine, S.L. and J. Bay-Williams. "Investigating Students' Conceptual Understanding of Decimal Fractions." *Mathematics Teaching in the Middle School,* January 2003, 8(5), pp. 244–247.

McAskill, B. et al. *WNCP Mathematics Research Project: Final Report.* Victoria, BC: Holdfast Consultants Inc., 2004.

McAskill, B., G. Holmes, L. Francis-Pelton. *Consultation Draft for the Common Curriculum Framework Kindergarten to Grade 9 Mathematics.* Victoria, BC: Holdfast Consultants Inc., 2005.

McLeod, D.B., Research on Affect and Mathematics Learning in the JRME: 1970 to the Present, Journal for Research in Mathematics Education, 25, 6,1994, p. 637 – 647.

McLeod, D.B. Research on affect in mathematics education: A Reconceptualization. In D.A. Grouws (Ed.), Handbook of research on mathematics teaching and learning, 575 – 596, Old Tappan, NJ: Macmillan, 2002.

McLeod, D.B. 1988, Affective Issues in Mathematical Problem Solving: Some Theoretical Considerations, Journal for Research in Mathematics Education, 19, 2, 1988, p. 134 – 141.

National Council of Teachers of Mathematics, *Computation, Calculators, and Common Sense.* May 2005, NCTM Position Statement.

Nardi, E. & Steward, S., Attitude and Achievement of the disengaged pupil in the mathematics Classroom, Downloaded 20.6.06 from www.standards.dfes.gov.uk, 2003.

Nardi, E. & Steward, S., Is Mathematics T.I.R.E.D? A profile of Quiet Disaffection in the Secondary Mathematics Classroom, British Educational Research Journal, 29, 3, 2003, pp. 4-9.

Nardi, E. & Steward, S., I Could be the Best Mathematician in the World…If I Actually Enjoyed It – Part 1. Mathematics Teaching, 179, 2002, pp. 41-45.

Nardi, E. & Steward, S., 2002, I Could be the Best Mathematician in the World…If I Actually Enjoyed It – Part 2. Mathematics Teaching, 180, 4-9, 2002.

Nelson-Thomson. *Mathematics Education: A Summary of Research, Theories, and Practice.* Scarborough, ON: Nelson, 2002.

Pape, S. J. and M.A Tchshanov. "The Role of Representation(s) in Developing Mathematical Understanding." *Theory into Practice,* Spring 2001, 40(2), pp. 118–127.

Paulos, J. *Innumeracy: Mathematical Illiteracy and its Consequences.* Vintage Books, New York, 1998.

Peck, D., S. Jencks and M. Connell. "Improving Instruction through Brief Interviews." *Arithmetic Teacher,* 1989, 37(3), 15–17.

Pepper, K.L. and R.P. Hunting. "Preschoolers' Counting and Sharing." *Journal for Research in Mathematics Education,* March 1998, 28(2), pp. 164–183.

Peressini D. and J. Bassett. "Mathematical Communication in Students' Responses to a Performance-Assessment Task." In P.C. Elliot, *Communication in Mathematics K–12 and Beyond.* Reston, VA: The National Council of Teachers of Mathematics, 1996, pp. 146–158.

Perry, J.A. and S.L. Atkins. "It's Not Just Notation: Valuing Children's Representations." *Teaching Children Mathematics.* September 2002, 9(1), pp. 196–201.

Polya, G. *G. How to Solve It 2nd ed.*, Princeton, NJ. Princeton University Press, 1957.

Pugalee, D. et al. *Navigating Through Geometry in Grades 6–8.* Reston, VA: The National Council of Teachers of Mathematics, 2002.

Rasokas, P. et al. *Harcourt Math Assessment: Measuring Student Performance (K – 8 Series).* Toronto, ON: 2001

Rigby-Heinemann. *First Steps in Mathematics: Number.* Sydney, AU: Regby-Heinemann, 2004.

Robitaille, D., G. Orpwood, and A. Taylor. *The TIMSS-Canada Report, Vol. 2–G4.* Vancouver, BC: Dept. of CUST – UBC, 1997.

Robitaille, D., Beaton, A.E., Plomp, T., 2000, The Impact of TIMSS on the Teaching and Learning of Mathematics and Science, Vancouver, BC: Pacific Education Press.

Robitaille, D.F, Taylor, A.R. & Orpwood, G., The Third International Mathematics & Science Study TIMMSS-Canada Report Vol.1: Grade 8, Dept. of Curriculum Studies, Faculty of Education, UBC, Vancouver: BC, 1996.

Romagnano, L. *Wrestling with Change – The Dilemmas of Teaching Mathematics.* Portsmouth, NH: Heinemann, 1994.

Rubenstein, R. N. *Mental Mathematics beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.

Sakshaug, L., M. Olson, and J. Olson. *Children are mathematical problem solvers.* Reston, VA: The National Council of Teachers of Mathematics, 2002, pp. 17–20.

Sawyer, W.W. *Mathematician's Delight.* New York: Penguin Books, 1943. Cited in Moran, G.J.W., 1993.

Schuster, L. and N. Canavan Anderson. Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades 5–8. Sausalto, CA: Math Solutions Publications, 2005.

Seymour, D. *Mental Math in the Primary Grades.* Palo Alto, CA: Dale Seymour Publications, 1998.

Sakshaug, L. E., *Children Are Mathematical Problem Solvers.* Reston, VA: National Council of Teachers of Mathematics: 2002

Shaw, J.M. and M.F.P Cliatt. (1989). "Developing Measurement Sense." In P.R. Trafton (Ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: National Council of Teachers of Mathematics.

Sheffield, L. J. et al. *Navigating through Data Analysis and Probability in Prekindergarten – Grade 2.* Reston, VA: The National Council of Teachers of Mathematics, 2002.

Small, M. *PRIME: Patterns and Algebra.* Toronto, ON: Nelson Publishing, 2005.

Small, M. *PRIME: Number and Operations.* Toronto, ON: Nelson Publishing, 2005.

Smith, W.J., Butler-Kisber, L., LaRoque, L., Portelli, J., Shields, C., Sturge Sparkes, C., & Vilbert, A., Student Engagement in Learning and School Life: National Project Report, Montreal. Quebec: Ed-Lex., 1998.

Solomon, P. G. *The Math We Need to "Know" and "Do."* Thousand Oaks, California: Sage Publications, 2001.

Steen, L.A. (ed.). *On the Shoulders of Giants – New Approaches to Numeracy.* Washington, DC: National Research Council, 1990.

Stiff, L. *Constructivist Mathematics and Unicorns* (President's Message). In NCTM News Bulletin July/August 2001, 3.

Sullivan, P., Lilburn P. Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades K–6. Sausalto, CA: Math Solutions Publications, 2002.

Swarthout, M. "Average Days of Spring – Problem Solvers." *Teaching Children Mathematics,* March 2002, 8(7), pp. 404–406.

Tang, E.P., H.P. Ginsburg. "Young Children's Mathematical Reasoning – A Psychological View." In Stiff, L. and F. Curcio, *Developing Mathematical Reasoning in Grades K–12.* Reston, VA: The National Council of Teachers of Mathematics, 1999, pp. 45–61.

Teppo, Anne R. *Reflecting on NCTM's Principles and Standards in Elementary and Middle School Mathematics.* Preston, VA: The National Council of Teachers of Mathematics, 2002.

Van de Walle, J. and A. L. Lovin, *Teaching Student-Centered Mathematics Grades K-3.* Boston, MA: Pearson Education, Inc., 2006.

Van de Walle, J. and A. L. Lovin, *Teaching Student-Centered Mathematics Grades 3-5.* Boston, MA: Pearson Education, Inc., 2006.

Van de Walle, J. and A. L. Lovin, *Teaching Student-Centered Mathematics Grades 5-8.* Boston, MA: Pearson Education, Inc., 2006.

Van de Walle, J. A. *Elementary and Middle School Mathematics: Teaching Developmentally.* 5th ed. Boston, MA: Pearson Education, Inc., 2004.

Van den Heuvel-Panhuizen, M. and Gravemejer (1991). "Tests Aren't All Bad – An Attempt to Change the Face of Written Tests in Primary School Mathematics Instruction." In Streefland, L., *Realistic Mathematics Education in Primary School: On the Occasion of the Opening of the Freudenthal Institute.* Utrecht, Netherlands: CD-B Press, 1991, pp. 54–64.

Van Hiele, P.M. *Structure and Insight: A Theory of Mathematics Education.* Orlando FL: Academic Press, 1986.

Vygotsky, L.S. *Thought and Language.* Cambridge, Mass: MIT Press, 1986.

Vygotsky, L.S. *Mind in Society: The Development of Higher Psychological Processes.* Cambridge, Mass: Harvard University Press, 1978.

Westley, J. (ed.) *Puddle Questions Assessing Mathematical Thinking (Grades 1 – 7 Series).* Chicago, IL: Creative Publications, 1995.

Willoughby, Steven. *Mathematics Education for a Changing World.* Alexandria, Virginia: Association of Supervision and Curriculum Development, 1990.

Wright, R.J. Martland, A.K. Stafford, G. Stanger. *Teaching Number,* London, England: Paul Chapman, 2002.

CONSIDERATIONS FOR PROGRAM DELIVERY

Thissection of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about
- alternative delivery policy
- inclusion, equity, and accessibility for all learners
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility
- fostering the development of positive attitudes
- instructional focus
- applying mathematics

ALTERNATIVE DELIVERY POLICY

The Alternative Delivery policy does not apply to the Mathematics K to 7 curriculum.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:
- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related

learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

> For more information about policy relating to alternative delivery, refer to
> www.bced.gov.bc.ca/policy/

INCLUSION, EQUITY, AND ACCESSIBILITY FOR ALL LEARNERS

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focuses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Mathematics K to 7, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students who have English as a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

> For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/
>
> For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

WORKING WITH THE ABORIGINAL COMMUNITY

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as Elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the "Planning Your Program" section of the resource, *Shared Learnings* (2006). This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

> For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

INFORMATION AND COMMUNICATIONS TECHNOLOGY

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyze information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. Mathematics K to 7 provides opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

COPYRIGHT AND RESPONSIBILITY

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. There are some exceptions in the law (i.e., specific things permitted) for schools but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:
- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations

- show video recordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

> For more information about copyright, refer to www.cmec.ca/copyright/indexe.stm

FOSTERING THE DEVELOPMENT OF POSITIVE ATTITUDES IN MATHEMATICS

A positive attitude toward mathematics is often a result of a learning environment in the classroom that encourages students' own mathematical thinking and contributions to classroom activities and discussions. Teachers should provide a variety of instructional approaches used in the classroom in order to reach a variety of learning styles and dispositions. These include experiences that encourage students to
- enjoy and value mathematics
- develop mathematical habits of mind

- explore
- take risks
- exhibit curiosity
- make and correct errors
- persevere
- experience mathematics in non-threatening, engaging ways
- understand and appreciate the role of mathematics in human affairs

These learning opportunities enable students to gain confidence in their abilities to solve complex problems.

The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

It is important for teachers to consider their role in developing a positive attitude in mathematics. Teachers and parents are role models from which students begin to develop their disposition toward mathematics. Teachers need to model these attitudes in order to help students develop them (Burns 2000). In this manner teachers need to "present themselves as problem solvers, as active learners who are seekers, willing to plunge into new situations, not always knowing the answer or what the outcome will be" (p. 29).

INSTRUCTIONAL FOCUS

The Mathematics K to 7 courses are arranged into a number of organizers with mathematical processes integrated throughout. Students learn in different ways and at different rates. As in other subject areas, it is essential when teaching mathematics, that concepts are introduced to students in a variety of ways. Students should hear explanations, watch demonstrations, draw to represent their thinking, engage in experiences with concrete materials and be encouraged to visualize and discuss their understanding of concepts. Most students need a range of concrete or representational experiences with mathematics concepts before they develop symbolic or abstract understanding. The development of conceptual understanding should be emphasized throughout the curriculum as a means to develop students to become mathematical problem solvers.

Teaching through Problem Solving

Problem solving should be an integral part of all mathematics classrooms. Teachers are encouraged to weave problem solving throughout all curriculum organizers in the K to 7 mathematics curriculum on a regular basis. Problem solving provides a way of helping students learn mathematics.

Hiebert et al. (1996) encourage teachers to make mathematics problematic. A problem can be defined as any task or activity for which the students have not memorized a method or rule, nor is there an assumption by the students that there is only one correct way to solve the problem (Hiebert et al. 1997). Van de Walle (2006) notes that "a problem for learning mathematics also has these features:

- The problem must begin where the students are.
- The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.
- The problem must require justifications and explanations for answers and methods. (p. 11)

Why teach through problem solving?

- The math makes more sense. When using real world math problems, students are able to make the connections between what math is and how they can apply it.
- Problems are more motivating when they are challenging. Although some students are anxious when they are not directed by the teacher, most enjoy a challenge they can be successful in solving.
- Problem solving builds confidence. It maximizes the potential for understanding as each child makes his own sense out of the problem and allows for individual strategies.
- Problem solving builds perseverance. Because an answer is not instantaneous, many children think they are unable to do the math. Through the experience of problem solving they learn to apply themselves for longer periods of time and not give up.
- Problems can provide practice with concepts and skills. Good problems enable students to learn and apply the concepts in a meaningful way and an opportunity to practice the skills.
- Problem solving provides students with insight into the world of mathematics. Mathematicians struggle to find solutions to many problems and often need to go down more than one path to arrive at a solution. This is a creative process that is difficult to understand if one has never had to struggle.
- Problem solving provides the teacher with insight into a student's mathematical thinking. As students choose strategies and solve problems, the teacher has evidence of their thinking and can inform instruction based on this.
- Students need to practice problem solving. If we are expecting students to confront new situations involving mathematics, they need practice to become independent problem solvers (Small 2005).

Polya (1957) characterized a general method which can be used to solve problems, and to describe how problem-solving should be taught and learned. He advocated for the following steps in solving a mathematical problem:

- Understand the problem – What is unknown? What is known? Is enough information provided to determine the solution? Can a figure or model be used to represent the situation?
- Make a plan – Is there a similar problem that has been solved before? Can the problem be restated so it makes more sense?
- Carry out the plan – Have all of the steps been completed correctly?
- Look back – Do the results look correct? Is there another way to solve the problem that would verify the results?

While a number of variations of the problem solving model proposed by Polya (Van de Walle 2006; Small 2006; Burns 2000) they all have similar characteristics. The incorporation of a wide variety of strategies to solve problems is essential to developing students' ability to be flexible problem solvers.

The Mathematics K to 7 (1995) IRP provides a number of useful strategies that students can use to increase their flexibility in solving problems. These include:

- look for a pattern
- construct a table
- make an organized list
- act it out
- draw a picture
- use objects
- guess and check
- work backward
- write an equation
- solve a simpler (or similar) problem
- make a model (BC Ministry of Education 1995)

During problem-solving experiences, students are encouraged to solve problems using ways that make sense to them. As students share different ways of solving problems they can learn strategies from each other. Teachers are encouraged to facilitate this process to in an open and non-threatening environment. I this manner, students can develop a repertoire of strategies from which to draw upon when mathematical problems are presented to them.

Problem solving requires a shift in student attitudes and how teachers model these attitudes in the classroom. In order to be successful, students must develop, and teachers model, the following characteristics:
- interest in finding solutions to problems
- confidence to try various strategies
- willingness to take risks
- ability to accept frustration when not knowing
- understanding the difference between not knowing the answer and not having found it yet (Burns 2000)

Problems are not just simple computations embedded in a story nor are they contrived, that is, they do not exist outside the math classroom. Students will be engaged if the problems relate to their lives; their culture, interests, families, current events. They are tasks that are rich and open-ended so there is more than one way of arriving at a solution, or multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. The students should not know the answer immediately. Problem solving takes time and effort on the part of the student and the teacher. Teaching thought problem solving is one of the ways that teachers can bring increased depth to the Mathematics K to 7 curriculum.

Instruction should provide an emphasis on mental mathematics and estimation to check the reasonableness of paper and pencil exercises, and the solutions to problems which are determined through the use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

APPLYING MATHEMATICS

For students to view mathematics as relevant and useful, they must see how it can be applied in a variety of contexts. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives both within and outside of the school context.

Teachers are encouraged to incorporate, and make explicit, mathematics concepts which naturally occur across the subject areas. Possible situations where cross curricular integration may occur in K to 7 include the following:

Fine Arts
- pattern, line, and form
- fractions in rhythm and metre
- spatial awareness in dance, drama, and visual arts
- geometric shapes in visual arts, drama, and dance
- symmetry and unison
- transformations
- perspective and proportion in visual arts
- measuring and proportional reasoning for mixing and applying materials in visual arts

Health and Career Education
- creating schedules
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- using mathematics to develop a logical argument to support a position on a topic or issue

Language Arts
- reading literature with a mathematics theme
- creating a picture book or writing a story with mathematical content
- listening to stories to decode mathematical contexts
- examine the plot of a story from a mathematical perspective
- create graphic organizers provide an explanation, proof, or justification for an argument
- role-play or oral presentations of problems and solutions
- creating word walls, personal dictionaries, or glossaries of mathematics terms
- examine the roots of mathematical terms

Physical Education

- examining the benefits of various physical activity (e.g. burning calories)
- examining patterns in physical movement
- measuring distances
- estimate distances and other quantise using referents
- reading and recording dates and time

Science

- discussing the magnitude of numbers
- classifying and sorting objects
- examining patterns to make a hypothesis
- measuring quantities
- use of referents for measurement
- units and conversions between units
- reading and writing quantities in multiple formats (e.g., numerals, words)
- collecting, organizing and interpreting data charts, graphs, diagrams, and tables
- creating a logical argument to support a hypothesis
- mental mathematics for calculations

Social Studies

- discussing the magnitude of numbers and building referents for numbers
- using concepts of area, perimeter, and distances when mapping

- graphing using the Cartesian plane
- using circle concepts to explain latitude and longitude, time zones, great circle routes
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- reading and recording dates and time
- examining the history of mathematics in context of world events
- using mathematics to develop a logical argument to support a position on a topic or issue

Students can also be encouraged to identify and examine the mathematics around them. In this way, students will come to see that mathematics is present outside of the classroom. There are many aspects of students' daily lives where they may encounter mathematic such as

- making purchases
- reading bus schedules
- reading sports statistics
- interpreting newspaper and media sources
- following a recipe
- estimating time to complete tasks
- estimating quantities
- creating patterns when doodling

Making these connections explicit for students helps to solidify the importance of mathematics.

PRESCRIBED LEARNING OUTCOMES

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required attitudes, skills, and knowledge – what students are expected to know and be able to do – by the end of the subject and grade.

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Mathematics K to 7 are presented by grade and by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

Wording of Prescribed Learning Outcomes

All learning outcomes complete the stem, "It is expected that students will …."

When used in a prescribed learning outcome, the word "including" indicates that any ensuing item **must be addressed**. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Domains of Learning

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.
- Knowledge includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- Understanding and application represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- Higher mental processes include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and cognitive levels also form the basis of the Assessment Overview Tables provided for each grade in the Classroom Assessment Model. In addition, domains of learning and, particularly, cognitive levels, inform the design and development of the Grades 4 and 7 Foundation Skills Assessment (FSA).

PRESCRIBED LEARNING OUTCOMES
Grade 7

GRADE 7

It is expected that students will:

NUMBER

A1 determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10 and why a number cannot be divided by 0 [C, R]

A2 demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems [ME, PS, T]

A3 solve problems involving percents from 1% to 100% [C, CN, PS, R, T]

A4 demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions [C, CN, R, T]

A5 demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences) [C, CN, ME, PS, R, V]

A6 demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically [C, CN, PS, R, V]

A7 compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using
- benchmarks
- place value
- equivalent fractions and/or decimals [CN, R, V]

PATTERNS AND RELATIONS

Patterns

B1 demonstrate an understanding of oral and written patterns and their equivalent linear relations [C, CN, R]

B2 create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems [C, CN, R, V]

Variables and Equations

B3 demonstrate an understanding of preservation of equality by
- modelling preservation of equality concretely, pictorially, and symbolically
- applying preservation of equality to solve equations [C, CN, PS, R, V]

B4 explain the difference between an expression and an equation [C, CN]

B5 evaluate an expression given the value of the variable(s) [CN, R]

B6 model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers [CN, PS, R, V]

B7 model and solve problems that can be represented by linear equations of the form
- ax + b = c
- ax = b
- $\frac{x}{a} = b, a \neq 0$

concretely, pictorially, and symbolically, where *a, b,* and *c* are whole numbers [CN, PS, R, V]

GRADE 7

SHAPE AND SPACE

Measurement

C1   demonstrate an understanding of circles by
- describing the relationships among radius, diameter, and circumference of circles
- relating circumference to pi
- determining the sum of the central angles
- constructing circles with a given radius or diameter
- solving problems involving the radii, diameters, and circumferences of circles [C, CN, R, V]

C2   develop and apply a formula for determining the area of
- triangles
- parallelograms
- circles [CN, PS, R, V]

3-D Objects and 2-D Shapes

C3   perform geometric constructions, including
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors [CN, R, V]

Transformations

C4   identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs [C, CN, V]

C5   perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices) [CN, PS, T, V]

STATISTICS AND PROBABILITY

Data Analysis

D1   demonstrate an understanding of central tendency and range by
- determining the measures of central tendency (mean, median, mode) and range
- determining the most appropriate measures of central tendency to report findings [C, PS, R, T]

D2   determine the effect on the mean, median, and mode when an outlier is included in a data set [C, CN, PS, R]

D3   construct, label, and interpret circle graphs to solve problems [C, CN, PS, R, T, V]

Chance and Uncertainty

D4   express probabilities as ratios, fractions, and percents [C, CN, R, T, V]

D5   identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events [C, ME, PS]

D6   conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events [C, PS, R, T]

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

STUDENT ACHIEVEMENT

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators that may be used to assess student performance in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

CLASSROOM ASSESSMENT AND EVALUATION

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentations
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student achievement is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction with each other to support student achievement.
- Assessment for learning is assessment for purposes of greater learning achievement.
- Assessment as learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment of learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:
- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information

for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

> For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/ student_reporting.htm

Assessment *for* Learning	Assessment *as* Learning	Assessment *of* Learning
Formative assessment *ongoing in the classroom*	Formative assessment *ongoing in the classroom*	Summative assessment *occurs at end of year or at key stages*
• teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment	• self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings	• teacher assessment • may be either criterion-referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind*.

> This resource is available online at www.wncp.ca

In addition, the BC Performance Standards describe levels of achievement in key areas of learning (reading, writing, numeracy, social responsibility, and information and communications technology integration) relevant to all subject areas. Teachers may wish to use the Performance Standards as resources to support ongoing formative assessment in mathematics.

> BC Performance Standards are available at www.bced.gov.bc.ca/perf_stands/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:

Step 1   Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment.

Step 2   Establish criteria. When appropriate, involve students in establishing criteria.

Step 3   Plan learning activities that will help students gain the attitudes, skills, or knowledge outlined in the criteria.

Step 4   Prior to the learning activity, inform students of the criteria against which their work will be evaluated.

Step 5   Provide examples of the desired levels of performance.

Step 6   Conduct the learning activities.

Step 7   Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.

Step 8   Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.

Step 9   Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.

Step 10   Communicate the results of the assessment and evaluation to students and parents/guardians.

KEY ELEMENTS

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

ACHIEVEMENT INDICATORS

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of attitudes demonstrated, skills applied, or knowledge acquired by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not the student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., a constructed response such as a list, comparison, or analysis; a product created and presented such as a report, poster, letter, or model; a particular skill demonstrated such as map making or critical thinking).

Achievement indicators support the principles of assessment for learning, assessment as learning, and assessment of learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Mathematics K to 7 curriculum. The achievement indicators are arranged by curriculum organizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.

STUDENT ACHIEVEMENT
Grade 7

KEY ELEMENTS: GRADE 7

MATHEMATICAL PROCESS (INTEGRATED)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the grade: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization.

NUMBER – develop number sense
- divisibility rules
- addition, subtraction, multiplication and division of numbers
- percents from 1% to 100%
- decimal and fraction relationships for repeating and terminating decimals
- addition and subtraction of positive fractions and mixed numbers
- addition and subtraction of integers

PATTERNS AND RELATIONS – use patterns to describe the world and solve problems

Patterns
- table of values and graphs of linear relations

Variables and Equations
- preservation of equality
- expressions and equations
- one-step linear equations

SHAPE AND SPACE – use direct and indirect measurement to solve problems

Measurement
- properties of circles
- area of triangles, parallelograms, and circles

3-D Objects and 2-D Shapes
- geometric constructions

Transformations
- four quadrants of the Cartesian plane
- transformations in the four quadrants of the Cartesian plane

STATISTICS AND PROBABILITY – collect, display and analyze data to solve problems

Data Analysis
- central tendency, outliers and range
- circle graphs

Chance and Uncertainty
- ratios, fractions and percents to express probabilities
- two independent events
- tree diagrams for two independent events

NUMBER

General Outcome: Develop number sense.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
A1 determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10 and why a number cannot be divided by 0 [C, R]	❏ determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10 and explain why ❏ sort a given set of numbers based upon their divisibility using organizers, such as Venn and Carroll diagrams ❏ determine the factors of a given number using the divisibility rules ❏ explain, using an example, why numbers cannot be divided by 0
A2 demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems [ME, PS, T]	❏ solve a given problem involving the addition of two or more decimal numbers ❏ solve a given problem involving the subtraction of decimal numbers ❏ solve a given problem involving the multiplication of decimal numbers ❏ solve a given problem involving the multiplication or division of decimal numbers with 2-digit multipliers or 1-digit divisors (whole numbers or decimals) without the use of technology ❏ solve a given problem involving the multiplication or division of decimal numbers with more than a 2-digit multiplier or 1-digit divisor (whole number or decimal), with the use of technology ❏ place the decimal in a sum or difference using front-end estimation, (e.g., for 4.5 + 0.73 + 256.458, think 4 + 256, so the sum is greater than 260) ❏ place the decimal in a product using front-end estimation (e.g., for $12.33 × 2.4, think $12 × 2, so the product is greater than $24) ❏ place the decimal in a quotient using front-end estimation (e.g., for 51.50 m ÷ 2.1, think 50 m ÷ 2, so the quotient is approximately 25 m) ❏ check the reasonableness of solutions using estimation ❏ solve a given problem that involves operations on decimals (limited to thousandths) taking into consideration the order of operations
A3 solve problems involving percents from 1% to 100% [C, CN, PS, R, T]	❏ express a given percent as a decimal or fraction ❏ solve a given problem that involves finding a percent ❏ determine the answer to a given percent problem where the answer requires rounding and explain why an approximate answer is needed (e.g., total cost including taxes)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
A4 demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions [C, CN, R, T]	❏ predict the decimal representation of a given fraction using patterns (e.g. $\frac{1}{11} = 0.\overline{09}$, $\frac{2}{11} = 0.\overline{18}$, $\frac{3}{11} = ?$ …) ❏ match a given set of fractions to their decimal representations ❏ sort a given set of fractions as repeating or terminating decimals ❏ express a given fraction as a terminating or repeating decimal ❏ express a given repeating decimal as a fraction ❏ express a given terminating decimal as a fraction ❏ provide an example where the decimal representation of a fraction is an approximation of its exact value
A5 demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences) [C, CN, ME, PS, R, V]	❏ model addition and subtraction of a given positive fraction or a given mixed number using concrete representations, and record symbolically ❏ determine the sum of two given positive fractions or mixed numbers with like denominators ❏ determine the difference of two given positive fractions or mixed numbers with like denominators ❏ determine a common denominator for a given set of positive fractions or mixed numbers ❏ determine the sum of two given positive fractions or mixed numbers with unlike denominators ❏ determine the difference of two given positive fractions or mixed numbers with unlike denominators ❏ simplify a given positive fraction or mixed number by identifying the common factor between the numerator and denominator ❏ simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers ❏ solve a given problem involving the addition or subtraction of positive fractions or mixed numbers and determine if the solution is reasonable
A6 demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically [C, CN, PS, R, V]	❏ explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero ❏ illustrate, using a number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position) ❏ add two given integers using concrete materials or pictorial representations and record the process symbolically ❏ subtract two given integers using concrete materials or pictorial representations and record the process symbolically ❏ solve a given problem involving the addition and subtraction of integers

Prescribed Learning Outcomes	Suggested Achievement Indicators
A7 compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using - benchmarks - place value - equivalent fractions and/or decimals [CN, R, V]	❑ order the numbers of a given set that includes positive fractions, positive decimals and/or whole numbers in ascending or descending order, and verify the result using a variety of strategies ❑ identify a number that would be between two given numbers in an ordered sequence or on a number line ❑ identify incorrectly placed numbers in an ordered sequence or on a number line ❑ position fractions with like and unlike denominators from a given set on a number line and explain strategies used to determine order ❑ order the numbers of a given set by placing them on a number line that contains benchmarks, such as 0 and 1 or 0 and 5 ❑ position a given set of positive fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (PATTERNS)

General Outcome: Use patterns to describe the world and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
B1 demonstrate an understanding of oral and written patterns and their equivalent linear relations [C, CN, R]	❏ formulate a linear relation to represent the relationship in a given oral or written pattern ❏ provide a context for a given linear relation that represents a pattern ❏ represent a pattern in the environment using a linear relation
B2 create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems [C, CN, R, V]	❏ create a table of values for a given linear relation by substituting values for the variable ❏ create a table of values using a linear relation and graph the table of values (limited to discrete elements) ❏ sketch the graph from a table of values created for a given linear relation and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between n and $2n + 3$ ❏ describe the relationship shown on a graph using everyday language in spoken or written form to solve problems ❏ match a given set of linear relations to a given set of graphs ❏ match a given set of graphs to a given set of linear relations

PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

General Outcome: Represent algebraic expressions in multiple ways.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
B3 demonstrate an understanding of preservation of equality by - modelling preservation of equality concretely, pictorially, and symbolically - applying preservation of equality to solve equations [C, CN, PS, R, V]	❏ model the preservation of equality for each of the four operations using concrete materials or using pictorial representations, explain the process orally and record it symbolically ❏ solve a given problem by applying preservation of equality
B4 explain the difference between an expression and an equation [C, CN]	❏ identify and provide an example of a constant term, a numerical coefficient and a variable in an expression and an equation ❏ explain what a variable is and how it is used in a given expression ❏ provide an example of an expression and an equation, and explain how they are similar and different
B5 evaluate an expression given the value of the variable(s) [CN, R]	❏ substitute a value for an unknown in a given expression and evaluate the expression
B6 model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers [CN, PS, R, V]	❏ represent a given problem with a linear equation and solve the equation using concrete models (e.g., counters, integer tiles) ❏ draw a visual representation of the steps required to solve a given linear equation ❏ solve a given problem using a linear equation ❏ verify the solution to a given linear equation using concrete materials and diagrams ❏ substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality
B7 model and solve problems that can be represented by linear equations of the form - $ax + b = c$ - $ax = b$ - $\frac{x}{a} = b, a \neq 0$ concretely, pictorially, and symbolically, where *a, b,* and *c* are whole numbers [CN, PS, R, V]	❏ model a given problem with a linear equation and solve the equation using concrete models (e.g., counters, integer tiles ❏ draw a visual representation of the steps used to solve a given linear equation ❏ solve a given problem using a linear equation and record the process ❏ verify the solution to a given linear equation using concrete materials and diagrams ❏ substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving
[CN]	Connections			**[R]**	Reasoning

[T]	Technology
[V]	Visualization

SHAPE AND SPACE (MEASUREMENT)

General Outcome: Use direct or indirect measurement to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C1 demonstrate an understanding of circles by - describing the relationships among radius, diameter, and circumference of circles - relating circumference to pi - determining the sum of the central angles - constructing circles with a given radius or diameter - solving problems involving the radii, diameters, and circumferences of circles [C, CN, R, V]	❏ illustrate and explain that the diameter is twice the radius in a given circle ❏ illustrate and explain that the circumference is approximately three times the diameter in a given circle ❏ explain that, for all circles, pi is the ratio of the circumference to the diameter $\frac{c}{d}$, and its value is approximately 3.14 ❏ explain, using an illustration, that the sum of the central angles of a circle is 360° ❏ draw a circle with a given radius or diameter with and without a compass ❏ solve a given contextual problem involving circles
C2 develop and apply a formula for determining the area of - triangles - parallelograms - circles [CN, PS, R, V]	❏ illustrate and explain how the area of a rectangle can be used to determine the area of a triangle ❏ generalize a rule to create a formula for determining the area of triangles ❏ illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram ❏ generalize a rule to create a formula for determining the area of parallelograms ❏ illustrate and explain how to estimate the area of a circle without the use of a formula ❏ apply a formula for determining the area of a given circle ❏ solve a given problem involving the area of triangles, parallelograms, and/or circles

SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*

C3 perform geometric constructions, including
  - perpendicular line segments
  - parallel line segments
  - perpendicular bisectors
  - angle bisectors [CN, R, V]
	❏ describe examples of parallel line segments, perpendicular line segments, perpendicular bisectors and angle bisectors in the environment
	❏ identify line segments on a given diagram that are parallel or perpendicular
	❏ draw a line segment perpendicular to another line segment and explain why they are perpendicular
	❏ draw a line segment parallel to another line segment and explain why they are parallel
	❏ draw the bisector of a given angle using more than one method and verify that the resulting angles are equal
	❏ draw the perpendicular bisector of a line segment using more than one method and verify the construction

[C]	Communication	[ME]	Mental Mathematics and Estimation	[PS]	Problem Solving	[T]	Technology
[CN]	Connections			[R]	Reasoning	[V]	Visualization

SHAPE AND SPACE (TRANSFORMATIONS)

General Outcome: Describe and analyze position and motion of objects and shapes.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C4 identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs [C, CN, V]	❑ label the axes of a four quadrant Cartesian plane and identify the origin ❑ identify the location of a given point in any quadrant of a Cartesian plane using an integral ordered pair ❑ plot the point corresponding to a given integral ordered pair on a Cartesian plane with units of 1, 2, 5 or 10 on its axes ❑ draw shapes and designs, using given integral ordered pairs, in a Cartesian plane ❑ create shapes and designs, and identify the points used to produce the shapes and designs in any quadrant of a Cartesian plane
C5 perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices) [CN, PS, T, V]	(It is intended that the original shape and its image have vertices with integral coordinates.) ❑ identify the coordinates of the vertices of a given 2-D shape on a Cartesian plane ❑ describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane ❑ describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation or successive transformations on a Cartesian plane ❑ determine the distance between points along horizontal and vertical lines in a Cartesian plane ❑ perform a transformation or consecutive transformations on a given 2-D shape and identify coordinates of the vertices of the image ❑ describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or a combination of successive transformations ❑ describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by identifying the coordinates of the vertices of the image

STATISTICS AND PROBABILITY (DATA ANALYSIS)

General Outcome: Collect, display and analyze data to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
D1 demonstrate an understanding of central tendency and range by - determining the measures of central tendency (mean, median, mode) and range - determining the most appropriate measures of central tendency to report findings [C, PS, R, T]	❏ determine mean, median and mode for a given set of data, and explain why these values may be the same or different ❏ determine the range of given sets of data ❏ provide a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings ❏ solve a given problem involving the measures of central tendency
D2 determine the effect on the mean, median, and mode when an outlier is included in a data set [C, CN, PS, R]	❏ analyze a given set of data to identify any outliers ❏ explain the effect of outliers on the measures of central tendency for a given data set ❏ identify outliers in a given set of data and justify whether or not they are to be included in the reporting of the measures of central tendency ❏ provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency
D3 construct, label, and interpret circle graphs to solve problems [C, CN, PS, R, T, V]	❏ identify common attributes of circle graphs, such as - title, label or legend - the sum of the central angles is 360° - the data is reported as a percent of the total and the sum of the percents is equal to 100% ❏ create and label a circle graph, with and without technology, to display a given set of data ❏ find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines and the Internet ❏ translate percentages displayed in a circle graph into quantities to solve a given problem ❏ interpret a given circle graph to answer questions

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY (CHANCE AND UNCERTAINTY)

General Outcome: **Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
D4 express probabilities as ratios, fractions, and percents [C, CN, R, T, V]	❏ determine the probability of a given outcome occurring for a given probability experiment, and express it as a ratio, fraction and percent ❏ provide an example of an event with a probability of 0 or 0% (impossible) and an event with a probability of 1 or 100% (certain)
D5 identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events [C, ME, PS]	❏ provide an example of two independent events, such as - spinning a four section spinner and an eight-sided die - tossing a coin and rolling a twelve-sided die - tossing two coins - rolling two dice and explain why they are independent ❏ identify the sample space (all possible outcomes) for each of two independent events using a tree diagram, table, or another graphic organizer
D6 conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events [C, PS, R, T]	❏ determine the theoretical probability of a given outcome involving two independent events ❏ conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability to the theoretical probability ❏ solve a given probability problem involving two independent events

CLASSROOM ASSESSMENT MODEL

The Classroom Assessment Model outlines a series of assessment units for Mathematics K to 7.

These units have been structured by grade level and theme. Collectively the units address all of the prescribed learning outcomes for each grade, and provide one suggested means of organizing, ordering, and delivering the required content. This organization is not intended to prescribe a linear means of delivery. Teachers are encouraged to reorder the learning outcomes and to modify, organize, and expand on the units to meet the needs of their students, to respond to local requirements, and to incorporate relevant recommended learning resources as applicable. (See the Learning Resources section later in this IRP for information about the recommended learning resources for Mathematics K to 7). In addition, teachers are encouraged to consider ways to adapt assessment strategies from one grade to another.

CONSIDERATIONS FOR INSTRUCTION AND ASSESSMENT IN MATHEMATICS K TO 7

It is highly recommended that parents and guardians be kept informed about all aspects of Mathematics K to 7. Suggested strategies for involving parents and guardians are found in the Introduction to this IRP.

Teachers are responsible for setting a positive classroom climate in which students feel comfortable learning about and discussing topics in Mathematics K to 7. Guidelines that may help educators establish a positive climate that is open to free inquiry and respectful of various points of view can be found in the section on Establishing a Positive Classroom Climate in the Introduction to this IRP.

Teachers may also wish to consider the following:
- Involve students in establishing guidelines for group discussion and presentations. Guidelines might include using appropriate listening and speaking skills, respecting students who are reluctant to share personal information in group settings, and agreeing to maintain confidentiality if sharing of personal information occurs.
- Promote critical thinking and open-mindedness, and refrain from taking sides on one point of view.
- Develop and discuss procedures associated with recording and using personal information that may be collected as part of students' work for the purposes of instruction and/or assessment (e.g., why the information is being collected, what the information will be used for, where the information will be kept; who can access it – students, administrators, parents; how safely it will be kept).
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential. For more information, see the section on Confidentiality in the Introduction to this IRP.

Classroom Assessment and Evaluation

Teachers should consider using a variety of assessment instruments and techniques to assess students' abilities to meet the prescribed learning outcomes. Tools and techniques for assessment in Mathematics K to 7 can include
- teacher assessment tools such as observation checklists, rating scales, and scoring guides
- self-assessment tools such as checklists, rating scales, and scoring guides
- peer assessment tools such as checklists, rating scales, and scoring guides
- journals or learning logs
- video (to record and critique student demonstration or performance)
- written tests, oral tests (true/false, multiple choice, short answer)
- questionnaires, worksheets
- portfolios
- student-teacher conferences

Assessment in Mathematics K to 7 can also occur while students are engaged in, and based on the product of, activities such as
- class and group discussions
- interviews and questioning
- sharing strategies
- object manipulation
- models and constructions
- charts, graphs, diagrams
- games
- experiments
- artwork, songs/stories, dramas
- centres/stations
- demonstrations and presentations
- performance tasks
- projects

For more information about student assessment, refer to the section on Student Achievement, as well as to the Assessment Overview Tables in each grade of the Classroom Assessment Model.

Information and Communications Technology

The Mathematics K to 7 curriculum requires students to be able to use and analyse the most current information to make informed decisions on a range of topics. This information is often found on the Internet as well as in other information and communications technology resources. When organizing for instruction and assessment, teachers should consider how students will best be able to access the relevant technology, and ensure that students are aware of school district policies on safe and responsible Internet and computer use.

CONTENTS OF THE MODEL

Assessment Overview Tables

The Assessment Overview Tables provide teachers with suggestions and guidelines for assessment of each grade of the curriculum. These tables identify the domains of learning and cognitive levels of the learning outcomes, along with a listing of suggested assessment activities and a suggested weight for grading for each curriculum organizer.

Overview

Each grade includes an overview of the assessment units:

- Learning at Previous Grades, indicating any relevant learning based on prescribed learning outcomes from earlier grades of the same subject area. It is assumed that students will have already acquired this learning; if they have not, additional introductory instruction may need to take place before undertaking the suggested assessment outlined in the unit. Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for reinforcement and developmental learning.
- Curriculum Correlation – a table that shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model.

Prescribed Learning Outcomes

Each unit begins with a listing of the prescribed learning outcomes that are addressed by that unit. Collectively, the units address all the learning outcomes for that grade; some outcomes may appear in more than one unit. The units may not address all of the achievement indicators for each of the outcomes.

Suggested Assessment Activities

Assessment activities have been included for each set of prescribed learning outcomes and corresponding achievement indicators. Each assessment activity consists of two parts:
- Planning for Assessment – outlining the background information to explain the classroom context, opportunities for students to gain and practise learning, and suggestions for preparing the students for assessment
- Assessment Strategies – describing the assessment task, the method of gathering assessment information, and the assessment criteria as defined by the learning outcomes and achievement indicators.

A wide variety of activities have been included to address a variety of learning and teaching styles. The assessment activities describe a variety of tools and methods for gathering evidence of student performance. These assessment activities are also referenced in the Assessment Overview Tables, found at the beginning of each grade in the Model.

These strategies are suggestions only, designed to provide guidance for teachers in planning instruction and assessment to meet the prescribed learning outcomes.

Assessment Instruments

Sample assessment instruments have been included at the end of each grade where applicable, and are provided to help teachers determine the extent to which students are meeting the prescribed learning outcomes. These instruments contain criteria specifically keyed to one or more of the suggested assessment activities contained in the units. Ongoing formative assessment will be required throughout the year to guide instruction and provide evidence that students have met the breadth and depth of the prescribed learning outcomes.

CLASSROOM ASSESSMENT MODEL
Grade 7

GRADE 7: ASSESSMENT OVERVIEW TABLE

The purpose of this table is to provide teachers with suggestions and guidelines for formative and summative classroom-based assessment and grading of Grade 7 Mathematics.

Curriculum Organizers	Suggested Assessment Activities	Suggested Weight for Grading	Number of Outcomes	Number of Outcomes by Domain*		
				K	U&A	HMP
NUMBER	• journals • observations • benchmarks • Venn diagrams • Carroll diagrams • student interviews	40-50%	7	1	5	1
PATTERNS AND RELATIONS	• journals • Frayer model • sort and classify • interviews	10-20%	7	2	3	2
SHAPE AND SPACE	• portfolio • Frayer model • journals • charts	20-30%	5	2	1	2
STATISTICS AND PROBABILITY	• poster • oral report • partner work • observations	10-20%	6	2	3	1
Totals		100%	25	7	12	6

Suggested Assessment Activities (continued in cells):
- NUMBER: error correct, concrete materials, pictorial representations, technology, problem solving
- PATTERNS AND RELATIONS: student work, drawing, concrete manipulatives
- SHAPE AND SPACE: geometric constructions, student work, observations
- STATISTICS AND PROBABILITY: constructions, experiments, journals, Frayer model, problem solving

* The following abbreviations are used to represent the three cognitive levels within the cognitive domain: K = Knowledge; U&A = Understanding and Application; HMP = Higher Mental Processes.

GRADE 7

OVERVIEW

Learning at Previous Grades
- numbers greater than 1 000 000 and smaller than one thousandth
- factors and multiples
- improper fractions and mixed numbers
- ratio and whole number percent
- integers
- multiplication and division of decimals
- order of operations excluding exponents
- patterns and relationships in graphs and tables including a tables of value
- letter variables
- preservation of equality
- angle measure and construction
- sum of interior angles of a triangle and quadrilateral
- formulas for the perimeter of polygons, area of rectangles and volume of right rectangular prisms
- types of triangles
- regular and irregular polygons
- combinations of transformations
- single transformation in the first quadrant of the Cartesian plane
- line graphs
- methods of data collection
- graph data
- experimental and theoretical probability

Curriculum Correlation

The following table shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model. Note that some curriculum organizers/suborganizers are addressed in more than one unit.

	Geometry Portfolio	Transformations	Bag o' Marbles	Games of Chance	Fun with Statistics	Problems with Percent	Expressions and Equations	Variables and Equations	Decimal Operations	Divisibility	The Terminators	Fractions	Adding and Subtracting Integers	Ordering Numbers
Number			X			X			X	X	X	X	X	X
Patterns and Relations *Patterns*							X							
Variables and Equations							X	X						
Space and Shape *Measurement*	X													
3-D Objects and 2-D Shapes	X													
Transformations		X												
Statistics and Probability *Data Analysis*			X		X									
Chance and Uncertainty			X	X										

Geometry Portfolio

Prescribed Learning Outcomes

It is expected that students will:

C1 demonstrate an understanding of circles by
- describing the relationships among radius, diameter, and circumference of circles
- relating circumference to pi
- determining the sum of the central angles
- constructing circles with a given radius or diameter
- solving problems involving the radii, diameters, and circumferences of circles [C, CN, R, V]

C2 develop and apply a formula for determining the area of:
- triangles
- parallelograms
- circles [CN, PS, R, V]

C3 perform geometric constructions, including
- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors [CN, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Show students how to create Frayer models such as the following for geometry terms:	• Verify that students have a complete, accurate definition for each term. Check for misconceptions (e.g., students should know that the term circle refers to the curve and does not include the area within the curve), and students' ability to justify their entries in the Frayer model.

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Give students 2 or 3 parallelograms drawn on grid paper. For each parallelogram have the students change it into a rectangle of equal area by drawing or by cutting and pasting. Have them calculate the area. Students generalize the rule so they can use it to determine the area of any parallelogram. Repeat the activity using triangles. (Note that it is suggested to not use right triangles because they are an easier case.)	• Collect students' work and note their abilities to - illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram - generalize a rule to create a formula for determining the area of parallelograms - recognize that the height used to calculate the area of a parallelogram must be perpendicular to the base - calculate the area of parallelograms - illustrate and explain how the area of a rectangle or parallelogram can be used to determine the area of a triangle - generalize a rule to create a formula for determining the area of triangle - recognize that the height used to calculate the area of triangle must be perpendicular to the base
• Students will include in their portfolios their best sample of each geometric construction, circles, perpendicular line segments, parallel line segments, perpendicular bisectors and angle bisectors. For perpendicular bisectors and angle bisectors students include 2 different methods of construction.	• Verify that students are able to - identify each construction - explain why 2 line segments are parallel - explain why 2 line segments are perpendicular - draw the bisector of an angle using more than one method and verify that the resulting angles are equal - draw the perpendicular bisector of a line segment using more than one method and verify the construction
• Have students create a chart in which they show for a variety of circles these things: the measurements of the radius, diameter, circumference, the quotient of the circumference divided by the diameter and the quotient of diameter by the radius. Students record their observations concerning the relationships of these measurements.	• Collect students' work and note their abilities to - illustrate and explain that radius is half the diameter and diameter is double the radius - illustrate and explain that the circumference is approximately 3 times the diameter - explain that, for all circles, pi is the ratio of the circumference to the diameter and its value is approximately 3.14
• Give students a circle divided into 8 or more equal sized sectors. Have students cut these apart and paste them to approximate a parallelogram. (The more sectors there are, the more closely it resembles a parallelogram.) Using this, students should be able to derive the formula for area of a circle.	• Verify that students can illustrate and explain how rearranging circle segments can be used to develop the formula for the area of a circle.
• Give students several circles with multiple radii drawn. Have students measure the central angle of each sector and calculate the sum.	• Ask students to explain, using an illustration, that the sum of the central angles is 360°.

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

• Have students include in their portfolios their solutions to problems such as these:
  - draw and label the dimensions for possible triangles and parallelograms that have a total area of 36 square units
  - given a value for the radius of a circle, find the area
Have students create problems that involve any of the geometry used in this unit. They are to include the solutions to their problems.
	• Assess students abilities to
	  - explain that $C = \pi d = 2\pi r$
	  - explain and illustrate that half the circumference is πr
	  - explain the difference between r^2 and $2r$
	Monitor students work to ensure they can measure accurately using a protractor. Ask students to explain to each other how to use a protractor to measure an angle.

Transformations

Prescribed Learning Outcomes
It is expected that students will:
C4 identify and plot points in the four quadrants of a Cartesian plane using integral ordered pairs [C, CN, V]
C5 perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices) [CN, PS, T, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students create and play a version of the game Battleship using coordinate grid paper. (Note that, unlike Battleship, the game will use the intersection of the coordinates rather than the space between the coordinates.) The degree of difficulty may be lessened or increased depending on the spread of the integers on the x and y-axes. • Ask students to create a drawing using coordinates and then give the list of coordinates to a partner to recreate the drawing.	• Collect students' work and note their abilities to - label the axis of a four quadrant Cartesian plane and identify the origin - identify the location of a given point in any quadrant of a Cartesian plane using an integral ordered pair - plot the point corresponding to a given integral ordered pair - are not confusing pairs (e.g., 5,2 with 2,5)
• Students work with points and their knowledge of integers to create rules for determining distance between 2 points that are along vertical or horizontal lines.	• Assess students based on their abilities to - determine the distance between points along horizontal and vertical lines on a Cartesian plane. As an extension they can look for an operational rule. - explain that distance is a positive number
• Have students begin by drawing a scalene triangle on the Cartesian plane, and do the following: - identify the vertices of the triangle - perform and describe a translation (slide) - identify the new vertices Repeat these last 2 steps for a rotation (turn of 90°, 180°, or 270°) and a reflection (flip) over the x or y-axis. After checking the work, have students exchange the ordered pairs of their original vertices and the descriptions of the transformations. Each student performs the transformations of a classmate.	• Verify that students can - identify the coordinates of the vertices of a triangle on a Cartesian plane - describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane - describe the positional change of the vertices of a triangle to the corresponding vertices of its image as a result of a transformation - label the axis of a four quadrant Cartesian plane and identify the origin - perform a transformation on a triangle and identify coordinates of the vertices of the image

Bag o' Marbles

Prescribed Learning Outcomes	
It is expected that students will: A3 solve problems involving percents from 1% to 100% [C, CN, PS, R, T] D3 construct, label, and interpret circle graphs to solve problems [C, CN, PS, R, T, V] D4 express probabilities as ratios, fractions, and percents [C, CN, R, T, V]	
PLANNING FOR ASSESSMENT	**ASSESSMENT STRATEGIES**

• Give students the following problem to work on: There are four colours of marbles, red, yellow, green, and black in a bag in the following ratios; R:Y is 1:1; G:B is 5:1; G:Y is 5:3. There are at least 20 marbles in the bag. Draw a pictorial representation of the marbles that could be in the bag and justify your decision. Using the contents of your marble bag, write, as both a fraction and a percent, the probability of drawing a red marble from your bag. Repeat for the other 3 colours. Construct an accurate circle spinner representing the probability for each colour.	• Assess students based on their abilities to
	  - identify common attributes of circle graphs
	  - create and label a circle graph, with and without technology, to display a given set of data
	  - when given the ratio G:B = 5:1, can explain that there are 5 green marbles for every 1 black
	  - convert a fractional probability to a percent
	  - calculate the fractional portion of the 360° central angle of the circle
	  - construct angles accurately using a protractor

Games of Chance

Prescribed Learning Outcomes

It is expected that students will:

D5   identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events [C, ME, PS]

D6   conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or another graphic organizer) and experimental probability of two independent events [C, PS, R, T]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students work with a partner to create a probability experiment involving 2 independent events. Specify one of the events, such as by rolling a regular die, and then ask students to create a second event that has 6 or fewer outcomes (e.g., spinning a spinner, pulling a marble from a bag). Have students determine the theoretical probability for the outcomes using tables, tree diagrams or other graphics. Following this, the pair is to perform the 2 independent events and collect data. (You may wish to have all of the class set up their experiments and allow students to test each other's experiments.) Each pair should then state the results of the experimental data as percents. You may wish to have the students prepare a poster of their work and present their findings to the class.	• Observe student, noting the extent to which they are able to - describe and explain independent events - identify the sample space (all possible outcomes) for each of 2 independent events using a tree diagram, table or another graphic organizer - that theoretical probability is the mathematical model of a problem - determine the theoretical probability of a given outcome involving 2 independent events - conduct a probability experiment involving 2 independent events to compare the experimental probability with the theoretical probability

Fun With Statistics

Prescribed Learning Outcomes

It is expected that students will:

D1 demonstrate an understanding of central tendency and range by
  - determining the measures of central tendency (mean, median, mode) and range
  - determining the most appropriate measures of central tendency to report findings [C, PS, R, T]
D2 determine the effect on the mean, median, and mode when an outlier is included in a data set [C, CN, PS, R]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Students complete Frayer model in their math journal for the terms mean, median, mode, range, measures of central tendency, and outliers. For mean, median, and mode have the students describe a situation for which each is the most appropriate measure of central tendency. This can be added in the section characteristics. Provide students with problems such as the following, where part of the data set is missing: - The average of 3 numbers is 83. One of the numbers is 107. What might the other numbers be? Explain your choices. - The mean is 7, the median is 5, and the mode is 6. There are 13 scores. What might the numbers be? Explain how you arrived at your numbers. Given a set of data, have students calculate the mean, median, and mode. Have them exchange one of the numbers for a radically different value (an outlier) and explain the effect on the mean, median, and mode.	• Verify that students can - explain the meaning of the terms - determine mean, median and mode for a set of data, and explain why these values may be the same or different - determine the range of a set of data - provide a context in which the mean, median or mode is most appropriate measure of central tendency to use when reporting findings - solve problems involving the measures of central tendency

Problems with Percent

Prescribed Learning Outcomes
It is expected that students will A3   solve problems involving percents from 1% to 100% [C, CN, PS, R, T]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

• Pose these problems for the students:
  - At a minor hockey game one fourth of the people in the stands were children and the rest were parents. If the ratio of dads to moms is 1:4, what percent of the people attending the game were moms?
  - A pair of jeans priced between $50 and $100 was on sale for 25% off. When the original price (a whole dollar amount) was discounted, the sale price was also a whole number of dollars. What are the possible original prices for the jeans?
Have students explain in their math journals their thinking for each question.
	• Collect students' work and note their abilities to
	  - express a percent as a fraction or a decimal
	  - explain and illustrate how fractions relate to percent
	  - explain the difference between ratios and fractions
	  - determine the answer to a percent problem where the answer requires rounding and explain why an approximate answer is needed
	  - organize their data efficiently (e.g., spreadsheets)

Expressions and Equations

Prescribed Learning Outcomes

It is expected that students will:

B1   demonstrate an understanding of oral and written patterns and their equivalent linear relations [C, CN, R]

B2   create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems [C, CN, R, V]

B4   explain the difference between an expression and an equation [C, CN]

B5   evaluate an expression given the value of the variable(s) [CN, R]

B6   model and solve problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers [CN, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

• Students complete Frayer models in their math journals for the terms expression, equation, constant, coefficient, and variable.
 	
Give students a list of expressions and equations. They sort them into the 2 categories and identify the constants, coefficients, and variables in each set. Assign a value for each variable and have students evaluate the expressions.
 	
Students are given 2 linear equations. They create a table of values for each and graph each on the same coordinate graph. Discuss situations each line could represent.
 	
Give students problems of the form $x + a = b$, and have students represent the solution process using 2 different methods (concrete models, pictorial representation, or a symbolic representation). This is an excellent opportunity for using an interview with the students. They can demonstrate the process for you and give a verbal explanation of their thinking.
	• Assess students based on their abilities to
	  - explain and illustrate the meaning of all terms
	  - create a table of values from a pattern
	  - distinguish between an expression and an equation and compare and contrast them
	  - substitute a value for an unknown into an expression and correctly calculate the answer
	  - plot points on a coordinate graph
	  - represent a given problem with a linear equation and solve the equation using concrete model
	  - draw a visual representation of the steps required to solve a given linear equation

Variables and Equations

Prescribed Learning Outcomes

It is expected that students will:

B3   demonstrate an understanding of preservation of equality by
- modelling preservation of equality concretely, pictorially, and symbolically
- applying preservation of equality to solve equations [C, CN, PS, R, V]

B7   model and solve problems that can be represented by linear equations of the form
- $ax + b = c$
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$

concretely, pictorially, and symbolically, where a, b, and c are whole numbers [CN, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Represent equations such as the following concretely, pictorially, and symbolically. Have students apply the preservation of equality to solve the equations and explain the process. $2x + 5 = 11$ $x - 3 = 5$ $\frac{x}{3} = 15$	• Collect students' work and note their abilities to - model a given problem with a linear equation and solve the equation using concrete models - draw a visual representation of the steps used to solve a given linear equation - solve a given problem using a linear equation and record the process - verify the solution to a given linear equation using concrete materials and diagrams

Decimal Operations

Prescribed Learning Outcomes
It is expected that students will: A2 demonstrate an understanding of the addition, subtraction, multiplication, and division of decimals (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected) to solve problems [ME, PS, T]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

• Given a problem such as 4293 × 23 = 98 739, discuss how and why the answer changes when a decimal (or decimals) is added. Explain, using strategies other than counting decimal places, the placement of the decimal in the answer. Do this kind of problem for each of the operations.
 	
Set rich problems such as the following: At the video rental store you can buy a yearly membership for $30.00. Members can rent movies for $2.45 each while non-members must pay $3.12 each. How many movies would you have to rent that year before you are saving money with an annual membership? Students could also create a spreadsheet to examine "what if" situations.
	• Collect students' work and note their abilities to
	  - solve a given problem involving the addition, subtraction, multiplication and division of decimal numbers
	  - place the decimal in a sum, difference, product or quotient using front-end estimation
	  - check the reasonableness of solutions using estimation
	  - solve problems that involves operations on decimals taking into consideration the order of operations

Divisibility

Prescribed Learning Outcomes
It is expected that students will: A1 determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10 and why a number cannot be divided by 0 [C, R]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

• Give students a set of numbers to sort based upon their divisibility. Have students justify their sorting rule. Graphic organizers such as a 2-circle Venn diagram, a 3-circle Venn diagram, or a Carroll diagram may be used.
 	
Challenge students to answer the following questions using the number 14 897 2__6, which is missing the 10's place digit:
- What digit should be placed in the blank to make the number divisible by 4? by 6? by 8? by 3? by 9?
- Explain why the number cannot be divided by 0.
	• Assess students based on their abilities to
	  - determine if numbers are divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why
	  - explain, using an example, why a number cannot be divided by 0
	  - explain how patterns can be used to determine divisibility rules

The Terminators

Prescribed Learning Outcomes

It is expected that students will:

A4   demonstrate an understanding of the relationship between positive repeating decimals and positive fractions, and positive terminating decimals and positive fractions [C, CN, R, T]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

• Give the students a variety of fractions in lowest terms. Alternately, give the students fractions which must first be put into lowest terms. Have them write the prime factorization of each denominator. Then have them convert each fraction to a decimal. Ask the students to formulate a rule to predict when the fraction will become a terminating decimal and when it will become a repeating decimal. Explain why the rule works with our base-10 system. Have students create fractions they believe will be terminating decimals or repeating decimals. Have them test their predictions.
 	
Give students a set of fractions such as
$\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$ …
and have them calculate the decimal representation of the first few and then predict the subsequent decimal representations.
	• Look for evidence that students are able to
	  - predict the decimal representation of a given fraction using pattern
	  - sort a set of fractions as repeating or terminating decimals
	  - express a repeating decimal as a fraction
	  - express a terminating decimal as a fraction

Fractions

Prescribed Learning Outcomes

It is expected that students will:

A5 demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences) [C, CN, ME, PS, R, V]

A7 compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using
- benchmarks
- place value
- equivalent fractions and/or decimals [CN, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Give students a variety of problems involving the addition and subtraction of fractions and mixed numbers. Have students explain the process using concrete materials, pictures, or words. Students should practice explaining the process to each other. Conduct student interviews. Give the students a selection of problems, some of which have an error in them, such as these: $12\frac{1}{4} - 9\frac{2}{3} = 3\frac{5}{12}$ $\frac{7}{8} + \frac{1}{3} = \frac{8}{11}$ $2\frac{1}{5} - 1\frac{3}{5} = \frac{8}{5}$ Ask them to find the mistakes and explain the errors in thinking that were made and give the correct solution for each.	• Verify that students can - model addition and subtraction of a fraction or a mixed number using concrete representations, and record symbolically - solve a problem involving the addition or subtraction of positive fractions or mixed numbers and determine if the solution is reasonable - Look for students who use benchmarks to add and subtract fractions. Students should be able add simple fractions like $\frac{1}{2}$ and $\frac{1}{4}$ in their heads. They should also be able to estimate the answer to addition and subtraction questions and explain their estimation strategies. These strategies may often include the use of benchmarks. Notice which students are making errors when regrouping. Are they ignoring the regrouping (i.e., reversing the order of the numbers) or regrouping 10 (as in base 10 regrouping) rather than the equivalent to 1. Note which students are adding and/or subtracting both numerators and denominators.

Adding and Subtracting Integers

Prescribed Learning Outcomes
It is expected that students will:
A6 demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES

• Give students a variety of problems involving the addition and subtraction of integers. Have students explain the process using concrete materials, pictures, or words. Students should practice explaining the process to each other.
 	
Give the students a selection of problems, some of which have an error in them, such as these:
 	
    12 – (-5) = 7
    (-2) + (-9) = 11
    15 + (-6) = 21
 	
Ask them to find the mistakes and explain the errors in thinking that were made and give the correct solution for each.
	• In interviews, assess students on the basis of their abilities to
	  - explain, using concrete material, that the sum of opposite integers is zero
	  - add or subtract integers using concrete materials or pictorial representations and record the process symbolically

Ordering Numbers

Prescribed Learning Outcomes

It is expected that students will:

A7   compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using
- benchmarks
- place value
- equivalent fractions and/or decimals [CN, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students choose 2 fractions or 2 decimals that they think are close. Their task is to find five or more fractions or decimals that are between the 2 numbers. Have them explain the strategies they used to determine the new fractions and decimals.	• Verify that students can - order numbers of a given set in ascending or descending order, and verify the result using a variety of strategies - identify a number that would be between 2 given numbers in an ordered sequence - use benchmarks to help place numbers on a number line
• Give students a set of numbers that includes fractions, decimals, and/or integers to place in ascending or descending order, and verify the result using a variety of strategies.	• Notice when students have to change all fractions to common denominators or decimal equivalencies to correctly order the set of numbers.

INTERVIEW OBSERVATION

Name _____ Date _____

Task or Problem _____

The student:	Not Yet at Level	At a Minimal Level	At Expected Level	Beyond Expected Level
Concepts and Procedures				
• understands the math concepts and demonstrates correct procedures				
• selects and carries out appropriate strategies				
Reasoning				
• interprets and evaluates results by looking back at a solution				
• justifies a solution or a decision based on reasons				
Communication				
• expresses thoughts clearly and efficiently				
• uses correct mathematical terminology and in proper context to explain thinking				
• asks and answers questions that go beyond the scope of the original question posed				
Affective Domain				
• stays on task and perseveres				
• demonstrates a willingness to learn				
• confidently takes risks				
• appreciates a challenge				

LEARNING RESOURCES

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections.

What Are Recommended Learning Resources?

Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?

Teachers must use either
- provincially recommended resources
  OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?

The Ministry of Education facilitates the evaluation of learning resources that support BC curricula, and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

> Additional information concerning the review and selection of learning resources is available from the ministry publication, Evaluating, Selecting and Managing Learning Resources: A Guide (Revised 2002)
> www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?

As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?

The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

MATHEMATICS K TO 7 GRADE COLLECTIONS

The Grade Collections for Mathematics K to 7 include newly recommended learning resources as well as relevant resources previously recommended for prior versions of the Mathematics K to 7 curriculum. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

> Please check the following ministry web site for the most current list of recommended learning resources in the Grade Collections for each IRP: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

GLOSSARY

The British Columbia Ministry of Education recognizes the limitation of a glossary available only in print format. An online glossary has been developed by Alberta Education to support the implementation of their revised Kindergarten to Grade 9 Program of Studies. This glossary is based on the WNCP CCF for K-9 Mathematics and therefore also supports the British Columbia Mathematics K to 7 IRP.

This online glossary provides additional supports for teachers indlucing definitions, diagrams, pictures, and interactive applets that cannot be provided through a conventional print glossary. As a result, the Ministry of Education encourages educational stakeholders to access the glossary through a link which is provided on the British Columbia Ministry of Education website.

> To access the glossary, follow the links for curriculum support material from the mathematics IRP main page at www.bced.gov.bc.ca/irp/irp_math.htm