Mathematics Grade 6

Integrated Resource Package 2007

British Columbia
The Best Place on Earth

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This Integrated Resource Package (IRP) provides basic information teachers will require in order to implement Mathematics K to 7. Once fully implemented, this document will supersede Mathematics K to 7 (1995).

> The information contained in this document is also available on the Internet at www.bced.gov.bc.ca/irp/irp.htm

The following paragraphs provide brief descriptions of the components of the IRP.

Introduction

The Introduction provides general information about Mathematics K to 7, including special features and requirements.

Included in this section are

- a rationale for teaching Mathematics K to 7 in BC schools
- goals for Mathematics K to 7
- descriptions of the curriculum organizers – groupings for prescribed learning outcomes that share a common focus
- a suggested timeframe for each grade
- a graphic overview of the curriculum content from K to 7
- additional information that sets the context for teaching Mathematics K to 7

Considerations for Program delivery

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners.

Prescribed Learning Outcomes

This section contains the *prescribed learning outcomes.* Prescribed learning outcomes are the legally required content standards for the provincial education system. They define the required attitudes, skills, and knowledge for each subject. The learning outcomes are statements of what students are expected to know and be able to do by the end of the grade.

The prescribed learning outcomes for the Mathematics K to 7 IRP are based on the Learning Outcomes contained within the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for K to 9 Mathematics available at www.wncp.ca.

Student Achievement

This section of the IRP contains information about classroom assessment and measuring student achievement, including sets of specific achievement indicators for each prescribed learning outcome. Achievement indicators are statements that describe what students should be able to do in order to demonstrate that they fully meet the expectations set out by the prescribed learning outcomes. Achievement indicators are not mandatory; they are provided to assist teachers in assessing how well their students achieve the prescribed learning outcomes.

The achievement indicators for the Mathematics K to 7 IRP are based on the achievement indicators contained within the WNCP Common Curriculum Framework for K to 9 Mathematics.

> The WNCP CCF for K to 9 Mathematics is available online at www.wncp.ca

Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment Model

This section contains a series of classroom units that address the learning outcomes. The units have been developed by BC teachers, and are provided to support classroom assessment. These units are suggestions only – teachers may use or modify the units to assist them as they plan for the implementation of this curriculum.

Each unit includes the prescribed learning outcomes and suggested achievement indicators, a suggested timeframe, a sequence of suggested assessment activities, and sample assessment instruments.

Learning Resources

This section contains general information on learning resources, providing a link to titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections. [*Note: Grade Collections for Mathematics K to 7 will be updated as new resources matching the IRP are authorized.*]

Glossary

The glossary section provides a link to an online glossary that contains definitions for selected terms used in this Integrated Resource Package

INTRODUCTION

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for Mathematics K to 7. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student.
- People learn in a variety of ways and at different rates.
- Learning is both an individual and a group process.

In addition to these three principles, this document recognizes that British Columbia's schools include young people of varied backgrounds, interests, abilities, and needs. Wherever appropriate for this curriculum, ways to meet these needs and to ensure equity and access for all learners have been integrated as much as possible into the learning outcomes and achievement indicators.

The Mathematics K to 7 IRP is based on the Western and Northern Canadian Protocol (WNCP) Common Curriculum Framework (CCF) for Kindergarten to Grade 9 Mathematics (May 2006). A complete list of references used to inform the revisions of the WNCP CCF for K to 9 Mathematics as well as this IRP can be found at the end of this section of the IRP.

Mathematics K to 7, in draft form, was available for public review and response from September to November, 2006. Input from educators, students, parents, and other educational partners informed the development of this document.

Rationale

The aim of Mathematics K to 7 is to provide students with the opportunity to further their knowledge, skills, and attitudes related to mathematics.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers 1998)

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of a variety of manipulatives and pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial and symbolic representations of mathematics. Information gathered from these discussions can be used for formative assessment to guide instruction.

As facilitators of learning educators are encouraged to highlight mathematics concepts as they occur within the K to 7 school environment and within home environments. Mathematics concepts are present within every school's subjects and drawing students' attention to these concepts as they occur can help to provide the "teachable moment."

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary. Positive learning experiences build self-confidence and develop attitudes that value learning mathematics.

Aboriginal perspective

Aboriginal students in British Columbia come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings including urban, rural, and isolated communities. Teachers need to understand the diversity of cultures and experiences of students.

Aboriginal students come from cultures where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is also vital that teachers understand and respond to non-verbal cues so that student learning and mathematical understanding are optimized. Depending on their learning styles, students may look for connections in learning and learn best when mathematics is contextualized and not taught as discrete components.

A variety of teaching and assessment strategies is required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students. *The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks 1993).*

Affective Domain

Bloom's taxonomy of learning behaviours identified three domains of educational activities, affective (growth in feelings or emotional areas – attitude), cognitive (mental skills – knowledge), and psychomotor (manual or physical skills – skills). The affective domain involves the way in which we perceive and respond to things emotionally, such as feelings, values, appreciation, enthusiasms, motivations, and attitudes.

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Research has shown that students who are more engaged with school and with mathematics are far more likely to be successful in school and in learning mathematics. (Nardi & Steward 2003). Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

Substantial progress has been made in research in the last decade that has examined the importance and use of the affective domain as part of the learning process. In addition there has been a parallel increase in specific research involving the affective domain and its' relationship to the learning of mathematics which has provided powerful evidence of the importance of this area to the learning of mathematics (McLeod 1988, 1992 & 1994; Hannula 2002 & 2006; Malmivuori 2001 & 2006). Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Students who are feeling more comfortable with a subject, demonstrate more confidence and have the opportunity for greater academic achievement (Denton & McKinney 2004; Hannula 2006; Smith et al. 1998). Educators can include opportunities for active and co-operative learning in their mathematics lessons which has been shown in research to promote greater conceptual understanding, more positive attitudes and subsequently improved academic achievement from students (Denton & McKinney 2004). By allowing the sharing and discussion of answers and strategies used in mathematics, educators are providing rich opportunities for students mathematical development. Educators can foster greater conceptual understanding in students by having students practice certain topics and concepts in mathematics in a meaningful and engaging manner.

It is important for educators, students, and parents to recognize the relationship between the affective and cognitive domains and attempt to nurture those aspects of the affective domain that contribute to positive attitudes and success in learning.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that are integral to the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty. These components are woven throughout this curriculum.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, p. 184).

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (The Primary Program 2000, p. 146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms.

Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics.

These skills contribute to students' interaction with and understanding of their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions, and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example:

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Goals for Mathematics K to 7

Mathematics K to 7 represents the first formal steps that students make towards becoming life-long learners of mathematics.

Goals for Mathematics K to 7

The Mathematics K-7 curriculum is meant to start students toward achieving the main goals of mathematics education:

- using mathematics confidently to solve problems
- using mathematics to better understand the world around us
- communicating and reasoning mathematically
- appreciating and valuing mathematics
- making connections between mathematics and its applications
- committing themselves to lifelong learning
- becoming mathematically literate and using mathematics to participate in, and contribute to, society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- be able to use mathematics to make and justify decisions about the world around us
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

Curriculum Organizers

A curriculum organizer consists of a set of prescribed learning outcomes that share a common focus. The prescribed learning outcomes for Mathematics K to 7 progress in age-appropriate ways, and are grouped under the following curriculum organizers and suborganizers:

Curriculum Organizers and Suborganizers
Mathematics K-7
Number Patterns and Relations • Patterns • Variables and Equations Shape and Space • Measurement • 3-D Objects and 2-D Shapes • Transformations Statistics and Probability • Data Analysis • Chance and Uncertainty

These curriculum organizers reflect the main areas of mathematics that students are expected to address. The ordering of organizers, suborganizers, and outcomes in the Mathematics K to 7 curriculum does not imply an order of instruction. The order in which various topics are addressed is left to the professional judgment of teachers. Mathematics teachers are encouraged to integrate topics throughout the curriculum and within other subject areas to emphasize the connections between mathematics concepts.

Number

Students develop their concept of the number system and relationships between numbers. Concrete, pictorial and symbolic representations are used to help students develop their number sense. Computational fluency, the ability to connect understanding of the concepts with accurate, efficient and flexible computation strategies for multiple purposes, is stressed throughout the number organizer with an emphasis on the development of personal strategies, mental mathematics and estimation strategies.

The Number organizer does not contain any suborganizers.

Patterns and Relations

Students develop their ability to recognize, extend, create, and use numerical and non- numerical patterns to better understand the world around them as well as the world of mathematics. This organizer provides opportunities for students to look for relationships in the environment and to describe the relationships. These relationships should be examined in multiple sensory forms.

The Patterns and Relations organizer includes the following suborganizers:
- Patterns
- Variables and Equations

Shape and Space

Students develop their understanding of objects and shapes in the environment around them. This includes recognition of attributes that can be measured, measurement of these attributes, description of these attributes, the identification and use of referents, and positional change of 3-D objects and 2-D shapes on the environment and on the Cartesian plane.

The Shape and Space organizer includes the following suborganizers:
- Measurement
- 3-D Objects and 2-D Shapes
- Transformations

Statistics and Probability

Students collect, interpret and present data sets in relevant contexts to make decisions. The development of the concepts involving probability is also presented as a means to make decisions. The Shape and Space organizer includes the following suborganizers:
- Data Analysis
- Chance and Uncertainty

Key Concepts: Overview of Mathematics K to 7 Topics

	Kindergarten	Grade 1	Grade 2	Grade 3
Number	• number sequence to 10 • familiar number arrangements up to 5 objects • one-to-one correspondence • numbers in-depth to 10	• skip counting starting at 0 to 100 • arrangements up to 10 objects • numbers in-depth to 20 • addition & subtraction to 20 • mental math strategies to 18	• skip counting at starting points other than 0 to 100 • numbers in-depth to 100 • even, odd & ordinal numbers • addition & subtraction to 100 • mental math strategies to 18	• skip counting at starting points other than 0 to 1000 • numbers in-depth to 1000 • addition & subtraction to 1000 • mental math strategies for 2-digit numerals • multiplication up to 5 × 5 • representation of fractions
Patterns & Relations *Patterns*	• repeating patterns of two or three elements	• repeating patterns of two to four elements • representation of pattern	• repeating patterns of three to five elements • increasing patterns	• increasing patterns • decreasing patterns
Patterns & Relations *Variables & Equations*		• equalities & inequalities • symbol for equality	• equality & inequality • symbols for equality & inequality	• one-step addition and subtraction equations
Shape & Space *Measurement*	• direct comparison for length, mass & volume	• process of measurement using comparison	• days, weeks, months, & years • non-standard units of measure for length, height distance around, mass (weight)	• non-standard & standard units of time • measurements of length (cm, m) & mass (g, kg) • perimeter of regular & irregular shapes
Shape & Space *3-D Objects & 2-D Shapes*	• single attribute of 3-D objects	• one attribute of 3-D objects & 2-D shapes • composite 2-D shapes & 3-D objects • 2-D shapes in the environment	• two attributes of 3-D objects & 2-D shapes • cubes, spheres, cones, cylinders, pyramids • triangles, squares, rectangles, circles • 2-D shapes in the environment	• faces, edges & vertices of 3-D objects • triangles, quadrilaterals, pentagons, hexagons, octagons
Shape & Space *Transformations*				
Statistics & Probability *Data Analysis*			• data about self and others • concrete graphs and pictographs	• first-hand data • bar graphs
Statistics & Probability *Chance & Uncertainty*				

Grade 4	Grade 5	Grade 6	Grade 7
• numbers in-depth to 10 000 • addition & subtraction to 10 000 • multiplication & division of numbers • fractions less than or equal to one • decimals to hundredths	• numbers in-depth to 1 000 000 • estimation strategies for calculations & problem solving • mental mathematics strategies for multiplication facts to 81 & corresponding division facts • mental mathematics for multiplication • multiplication for 2-digit by 2-digit & division for 3-digit by 1-digit • decimal & fraction comparison • addition & subtraction of decimals	• numbers in-depth greater than 1 000 000 & smaller than one thousandth • factors & multiples • improper fractions & mixed numbers • ratio & whole number percent • integers • multiplication & division of decimals • order of operations excluding exponents	• divisibility rules • addition, subtraction, multiplication, & division of numbers • percents from 1% to 100% • decimal & fraction relationships for repeating & terminating decimals • addition & subtraction of positive fractions & mixed numbers • addition & subtraction of integers
• patterns in tables & charts	• prediction using a pattern rule	• patterns & relationships in graphs & tables including tables of value	• table of values & graphs of linear relations
• symbols to represent unknowns • one-step equations	• single-variable, one-step equations with whole number coefficients & solutions	• letter variable representation of number relationships • preservation of equality	• preservation of equality • expressions & equations • one-step linear equations
• digital clocks, analog clocks, & calendar dates • area of regular & irregular 2-D shapes	• perimeter & area of rectangles • length, volume, & capacity	• perimeter & area of rectangles • length, volume, & capacity	• properties of circles • area of triangles, parallelograms, & circles
• rectangular & triangular prisms	• parallel, intersecting, perpendicular, vertical & horizontal edges & faces • rectangles, squares, trapezoids, parallelograms & rhombuses	• types of triangles • regular & irregular polygons	• geometric constructions
• line symmetry	• 2-D shape single transformation	• combinations of transformations • single transformation in the first quadrant of the Cartesian plane	• four quadrants of the Cartesian plane • transformations in the four quadrants of the Cartesian plane
• many-to-one correspondence including bar graphs & pictographs	• first-hand & second-hand data • double bar graphs	• line graphs • methods of data collection • graph data	• central tendency, outliers & range • circle graphs
	• likelihood of a single outcome	• experimental & theoretical probability	• ratios, fractions, & percents to express probabilities • two independent events • tree diagrams for two independent events

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The following seven mathematical processes should be integrated within Mathematics K to 7.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students need to be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhances flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental mathematics (NCTM May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein 2001).

Mental mathematics "provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers" (Hope 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you...?" or "How could you...?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K to 3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong 1993, p. 10). The use of visualization in the study of mathematics provides students with the opportunity to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to decide when to measure, when to estimate and to know several estimation strategies (Shaw & Cliatt 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Suggested Timeframe

Provincial curricula are developed in accordance with the amount of instructional time recommended by the Ministry of Education for each subject area. For Mathematics K to 7, the Ministry of Education recommends a time allotment of 20% (approximately 95 hours in Kindergarten and 185 hours in Grades 1 to 7) of the total instructional time for each school year. In the primary years, teachers determine the time allotments for each required area of study and may choose to combine various curricula to enable students to integrate ideas and see the application of mathematics concepts across curricula.

The Mathematics K to 7 IRP for grades 1 to 7 is based on approximately 170 hours of instructional time to allow flexibility to address local needs. For Kindergarten, this estimate is approximately 75 hours. Based on these recommendations, teachers should be spending about 2 to 2.5 hours each week on Mathematics in Kindergarten and 4.5 to 5 hours of instructional time each week on Mathematics grades 1 to 7.

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The following references have been used to inform the revisions of the BC Mathematics K to 7 IRP as well as the WNCP CCF for K-9 Mathematics upon which the Prescribed Learning Outcomes and Achievement Indicators are based.

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Considerations for Program Delivery

This section of the IRP contains additional information to help educators develop their school practices and plan their program delivery to meet the needs of all learners. Included in this section is information about

- alternative delivery policy
- inclusion, equity, and accessibility for all learners
- working with the Aboriginal community
- information and communications technology
- copyright and responsibility
- fostering the development of positive attitudes
- instructional focus
- applying mathematics

Alternative Delivery Policy

The Alternative Delivery policy does not apply to the Mathematics K to 7 curriculum.

The Alternative Delivery policy outlines how students, and their parents or guardians, in consultation with their local school authority, may choose means other than instruction by a teacher within the regular classroom setting for addressing prescribed learning outcomes contained in the Health curriculum organizer of the following curriculum documents:

- Health and Career Education K to 7, and Personal Planning K to 7 Personal Development curriculum organizer (until September 2008)
- Health and Career Education 8 and 9
- Planning 10

The policy recognizes the family as the primary educator in the development of children's attitudes, standards, and values, but the policy still requires that all prescribed learning outcomes be addressed and assessed in the agreed-upon alternative manner of delivery.

It is important to note the significance of the term "alternative delivery" as it relates to the Alternative Delivery policy. The policy does not permit schools to omit addressing or assessing any of the prescribed learning outcomes within the health and career education curriculum. Neither does it allow students to be excused from meeting any learning outcomes related to health. It is expected that students who arrange for alternative delivery will address the health-related learning outcomes and will be able to demonstrate their understanding of these learning outcomes.

For more information about policy relating to alternative delivery, refer to www.bced.gov.bc.ca/policy/

Inclusion, Equity, and Accessibility for All Learners

British Columbia's schools include young people of varied backgrounds, interests, and abilities. The Kindergarten to Grade 12 school system focuses on meeting the needs of all students. When selecting specific topics, activities, and resources to support the implementation of Mathematics K to 7, teachers are encouraged to ensure that these choices support inclusion, equity, and accessibility for all students. In particular, teachers should ensure that classroom instruction, assessment, and resources reflect sensitivity to diversity and incorporate positive role portrayals, relevant issues, and themes such as inclusion, respect, and acceptance.

Government policy supports the principles of integration and inclusion of students who have English as a second language and of students with special needs. Most of the prescribed learning outcomes and suggested achievement indicators in this IRP can be met by all students, including those with special needs and/or ESL needs. Some strategies may require adaptations to ensure that those with special and/or ESL needs can successfully achieve the learning outcomes. Where necessary, modifications can be made to the prescribed learning outcomes for students with Individual Education Plans.

For more information about resources and support for students with special needs, refer to www.bced.gov.bc.ca/specialed/

For more information about resources and support for ESL students, refer to www.bced.gov.bc.ca/esl/

Working with the Aboriginal Community

The Ministry of Education is dedicated to ensuring that the cultures and contributions of Aboriginal peoples in BC are reflected in all provincial curricula. To address these topics in the classroom in a way that is accurate and that respectfully reflects Aboriginal concepts of teaching and learning, teachers are strongly encouraged to seek the advice and support of local Aboriginal communities. Aboriginal communities are diverse in terms of language, culture, and available resources, and each community will have its own unique protocol to gain support for integration of local knowledge and expertise. To begin discussion of possible instructional and assessment activities, teachers should first contact Aboriginal education co-ordinators, teachers, support workers, and counsellors in their district who will be able to facilitate the identification of local resources and contacts such as Elders, chiefs, tribal or band councils, Aboriginal cultural centres, Aboriginal Friendship Centres, and Métis or Inuit organizations.

In addition, teachers may wish to consult the various Ministry of Education publications available, including the "Planning Your Program" section of the resource, *Shared Learnings* (2006). This resource was developed to help all teachers provide students with knowledge of, and opportunities to share experiences with, Aboriginal peoples in BC.

> For more information about these documents, consult the Aboriginal Education web site: www.bced.gov.bc.ca/abed/welcome.htm

Information and Communications Technology

The study of information and communications technology is increasingly important in our society. Students need to be able to acquire and analyze information, to reason and communicate, to make informed decisions, and to understand and use information and communications technology for a variety of purposes. Development of these skills is important for students in their education, their future careers, and their everyday lives.

Literacy in the area of information and communications technology can be defined as the ability to obtain and share knowledge through investigation, study, instruction, or transmission of information by means of media technology. Becoming literate in this area involves finding, gathering, assessing, and communicating information using electronic means, as well as developing the knowledge and skills to use and solve problems effectively with the technology. Literacy also involves a critical examination and understanding of the ethical and social issues related to the use of information and communications technology. Mathematics K to 7 provides opportunities for students to develop literacy in relation to information and communications technology sources, and to reflect critically on the role of these technologies in society.

Copyright and Responsibility

Copyright is the legal protection of literary, dramatic, artistic, and musical works; sound recordings; performances; and communications signals. Copyright provides creators with the legal right to be paid for their work and the right to say how their work is to be used. There are some exceptions in the law (i.e., specific things permitted) for schools but these are very limited, such as copying for private study or research. The copyright law determines how resources can be used in the classroom and by students at home

In order to respect copyright it is necessary to understand the law. It is unlawful to do the following, unless permission has been given by a copyright owner:

- photocopy copyrighted material to avoid purchasing the original resource for any reason
- photocopy or perform copyrighted material beyond a very small part – in some cases the copyright law considers it "fair" to copy whole works, such as an article in a journal or a photograph, for purposes of research and private study, criticism, and review
- show recorded television or radio programs to students in the classroom unless these are cleared for copyright for educational use (there are exceptions such as for news and news commentary taped within one year of broadcast that by law have record-keeping requirements – see the web site at the end of this section for more details)
- photocopy print music, workbooks, instructional materials, instruction manuals, teacher guides, and commercially available tests and examinations

- show video recordings at schools that are not cleared for public performance
- perform music or do performances of copyrighted material for entertainment (i.e., for purposes other than a specific educational objective)
- copy work from the Internet without an express message that the work can be copied

Permission from or on behalf of the copyright owner must be given in writing. Permission may also be given to copy or use all or some portion of copyrighted work through a licence or agreement. Many creators, publishers, and producers have formed groups or "collectives" to negotiate royalty payments and copying conditions for educational institutions. It is important to know what licences are in place and how these affect the activities schools are involved in. Some licences may also require royalty payments that are determined by the quantity of photocopying or the length of performances. In these cases, it is important to assess the educational value and merits of copying or performing certain works to protect the school's financial exposure (i.e., only copy or use that portion that is absolutely necessary to meet an educational objective).

It is important for education professionals, parents, and students to respect the value of original thinking and the importance of not plagiarizing the work of others. The works of others should not be used without their permission.

For more information about copyright, refer to www.cmec.ca/copyright/indexe.stm

Fostering the Development of Positive Attitudes in Mathematics

A positive attitude toward mathematics is often a result of a learning environment in the classroom that encourages students' own mathematical thinking and contributions to classroom activities and discussions. Teachers should provide a variety of instructional approaches used in the classroom in order to reach a variety of learning styles and dispositions. These include experiences that encourage students to

- enjoy and value mathematics
- develop mathematical habits of mind
- explore
- take risks
- exhibit curiosity
- make and correct errors
- persevere
- experience mathematics in non-threatening, engaging ways
- understand and appreciate the role of mathematics in human affairs

These learning opportunities enable students to gain confidence in their abilities to solve complex problems.

The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

It is important for teachers to consider their role in developing a positive attitude in mathematics. Teachers and parents are role models from which students begin to develop their disposition toward mathematics. Teachers need to model these attitudes in order to help students develop them (Burns 2000). In this manner teachers need to "present themselves as problem solvers, as active learners who are seekers, willing to plunge into new situations, not always knowing the answer or what the outcome will be" (p. 29).

Instructional Focus

The Mathematics K to 7 courses are arranged into a number of organizers with mathematical processes integrated throughout. Students learn in different ways and at different rates. As in other subject areas, it is essential when teaching mathematics, that concepts are introduced to students in a variety of ways. Students should hear explanations, watch demonstrations, draw to represent their thinking, engage in experiences with concrete materials and be encouraged to visualize and discuss their understanding of concepts. Most students need a range of concrete or representational experiences with mathematics concepts before they develop symbolic or abstract understanding. The development of conceptual understanding should be emphasized throughout the curriculum as a means to develop students to become mathematical problem solvers.

Teaching through Problem Solving

Problem solving should be an integral part of all mathematics classrooms. Teachers are encouraged to weave problem solving throughout all curriculum organizers in the K to 7 mathematics curriculum on a regular basis. Problem solving provides a way of helping students learn mathematics.

Hiebert et al. (1996) encourage teachers to make mathematics problematic. A problem can be defined as any task or activity for which the students have not memorized a method or rule, nor is there an assumption by the students that there is only one correct way to solve the problem (Hiebert et al. 1997). Van de Walle (2006) notes that "a problem for learning mathematics also has these features:

- The problem must begin where the students are.
- The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.
- The problem must require justifications and explanations for answers and methods. (p. 11)

Why teach through problem solving?

- The math makes more sense. When using real world math problems, students are able to make the connections between what math is and how they can apply it.
- Problems are more motivating when they are challenging. Although some students are anxious when they are not directed by the teacher, most enjoy a challenge they can be successful in solving.
- Problem solving builds confidence. It maximizes the potential for understanding as each child makes his own sense out of the problem and allows for individual strategies.
- Problem solving builds perseverance. Because an answer is not instantaneous, many children think they are unable to do the math. Through the experience of problem solving they learn to apply themselves for longer periods of time and not give up.
- Problems can provide practice with concepts and skills. Good problems enable students to learn and apply the concepts in a meaningful way and an opportunity to practice the skills.
- Problem solving provides students with insight into the world of mathematics. Mathematicians struggle to find solutions to many problems and often need to go down more than one path to arrive at a solution. This is a creative process that is difficult to understand if one has never had to struggle.
- Problem solving provides the teacher with insight into a student's mathematical thinking. As students choose strategies and solve problems, the teacher has evidence of their thinking and can inform instruction based on this.
- Students need to practice problem solving. If we are expecting students to confront new situations involving mathematics, they need practice to become independent problem solvers (Small 2005).

Polya (1957) characterized a general method which can be used to solve problems, and to describe how problem-solving should be taught and learned. He advocated for the following steps in solving a mathematical problem:

- Understand the problem – What is unknown? What is known? Is enough information provided to determine the solution? Can a figure or model be used to represent the situation?
- Make a plan – Is there a similar problem that has been solved before? Can the problem be restated so it makes more sense?
- Carry out the plan – Have all of the steps been completed correctly?
- Look back – Do the results look correct? Is there another way to solve the problem that would verify the results?

While a number of variations of the problem solving model proposed by Polya (Van de Walle 2006; Small 2006; Burns 2000) they all have similar characteristics. The incorporation of a wide variety of strategies to solve problems is essential to developing students' ability to be flexible problem solvers.

The Mathematics K to 7 (1995) IRP provides a number of useful strategies that students can use to increase their flexibility in solving problems. These include:

- look for a pattern
- construct a table
- make an organized list
- act it out
- draw a picture
- use objects
- guess and check
- work backward
- write an equation
- solve a simpler (or similar) problem
- make a model (BC Ministry of Education 1995)

During problem-solving experiences, students are encouraged to solve problems using ways that make sense to them. As students share different ways of solving problems they can learn strategies from each other. Teachers are encouraged to facilitate this process to in an open and non-threatening environment. I this manner, students can develop a repertoire of strategies from which to draw upon when mathematical problems are presented to them.

Problem solving requires a shift in student attitudes and how teachers model these attitudes in the classroom. In order to be successful, students must develop, and teachers model, the following characteristics:

- interest in finding solutions to problems
- confidence to try various strategies
- willingness to take risks
- ability to accept frustration when not knowing
- understanding the difference between not knowing the answer and not having found it yet (Burns 2000)

Problems are not just simple computations embedded in a story nor are they contrived, that is, they do not exist outside the math classroom. Students will be engaged if the problems relate to their lives; their culture, interests, families, current events. They are tasks that are rich and open-ended so there is more than one way of arriving at a solution, or multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. The students should not know the answer immediately. Problem solving takes time and effort on the part of the student and the teacher. Teaching thought problem solving is one of the ways that teachers can bring increased depth to the Mathematics K to 7 curriculum.

Instruction should provide an emphasis on mental mathematics and estimation to check the reasonableness of paper and pencil exercises, and the solutions to problems which are determined through the use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

Applying Mathematics

For students to view mathematics as relevant and useful, they must see how it can be applied in a variety of contexts. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives both within and outside of the school context.

Teachers are encouraged to incorporate, and make explicit, mathematics concepts which naturally occur across the subject areas. Possible situations where cross curricular integration may occur in K to 7 include the following:

Fine Arts

- pattern, line, and form
- fractions in rhythm and metre
- spatial awareness in dance, drama, and visual arts
- geometric shapes in visual arts, drama, and dance
- symmetry and unison
- transformations
- perspective and proportion in visual arts
- measuring and proportional reasoning for mixing and applying materials in visual arts

Health and Career Education

- creating schedules
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- using mathematics to develop a logical argument to support a position on a topic or issue

Language Arts

- reading literature with a mathematics theme
- creating a picture book or writing a story with mathematical content
- listening to stories to decode mathematical contexts
- examine the plot of a story from a mathematical perspective
- create graphic organizers provide an explanation, proof, or justification for an argument
- role-play or oral presentations of problems and solutions
- creating word walls, personal dictionaries, or glossaries of mathematics terms
- examine the roots of mathematical terms

Physical Education

- examining the benefits of various physical activity (e.g. burning calories)
- examining patterns in physical movement
- measuring distances
- estimate distances and other quantise using referents
- reading and recording dates and time

Science

- discussing the magnitude of numbers
- classifying and sorting objects
- examining patterns to make a hypothesis
- measuring quantities
- use of referents for measurement
- units and conversions between units
- reading and writing quantities in multiple formats (e.g., numerals, words)
- collecting, organizing and interpreting data charts, graphs, diagrams, and tables
- creating a logical argument to support a hypothesis
- mental mathematics for calculations

Social Studies

- discussing the magnitude of numbers and building referents for numbers
- using concepts of area, perimeter, and distances when mapping
- graphing using the Cartesian plane
- using circle concepts to explain latitude and longitude, time zones, great circle routes
- interpreting statistical data
- collecting, organizing, and interpreting data charts, graphs, diagrams, and tables
- reading and recording dates and time
- examining the history of mathematics in context of world events
- using mathematics to develop a logical argument to support a position on a topic or issue

Students can also be encouraged to identify and examine the mathematics around them. In this way, students will come to see that mathematics is present outside of the classroom. There are many aspects of students' daily lives where they may encounter mathematic such as

- making purchases
- reading bus schedules
- reading sports statistics
- interpreting newspaper and media sources
- following a recipe
- estimating time to complete tasks
- estimating quantities
- creating patterns when doodling

Making these connections explicit for students helps to solidify the importance of mathematics.

PRESCRIBED LEARNING OUTCOMES

Prescribed learning outcomes are content standards for the provincial education system; they are the prescribed curriculum. Clearly stated and expressed in measurable and observable terms, learning outcomes set out the required attitudes, skills, and knowledge – what students are expected to know and be able to do – by the end of the subject and grade.

Schools have the responsibility to ensure that all prescribed learning outcomes in this curriculum are met; however, schools have flexibility in determining how delivery of the curriculum can best take place.

It is expected that student achievement will vary in relation to the learning outcomes. Evaluation, reporting, and student placement with respect to these outcomes are dependent on the professional judgment and experience of teachers, guided by provincial policy.

Prescribed learning outcomes for Mathematics K to 7 are presented by grade and by curriculum organizer and suborganizer, and are coded alphanumerically for ease of reference; however, this arrangement is not intended to imply a required instructional sequence.

Wording of Prescribed Learning Outcomes

All learning outcomes complete the stem, "It is expected that students will …."

When used in a prescribed learning outcome, the word "including" indicates that any ensuing item **must be addressed**. Lists of items introduced by the word "including" represent a set of minimum requirements associated with the general requirement set out by the outcome. The lists are not necessarily exhaustive, however, and teachers may choose to address additional items that also fall under the general requirement set out by the outcome.

Domains of Learning

Prescribed learning outcomes in BC curricula identify required learning in relation to one or more of the three domains of learning: cognitive, psychomotor, and affective. The following definitions of the three domains are based on Bloom's taxonomy.

The **cognitive domain** deals with the recall or recognition of knowledge and the development of intellectual abilities. The cognitive domain can be further specified as including three cognitive levels: knowledge, understanding and application, and higher mental processes. These levels are determined by the verb used in the learning outcome, and illustrate how student learning develops over time.

- Knowledge includes those behaviours that emphasize the recognition or recall of ideas, material, or phenomena.
- Understanding and application represents a comprehension of the literal message contained in a communication, and the ability to apply an appropriate theory, principle, idea, or method to a new situation.
- Higher mental processes include analysis, synthesis, and evaluation. The higher mental processes level subsumes both the knowledge and the understanding and application levels.

The **affective domain** concerns attitudes, beliefs, and the spectrum of values and value systems.

The **psychomotor domain** includes those aspects of learning associated with movement and skill demonstration, and integrates the cognitive and affective consequences with physical performances.

Domains of learning and cognitive levels also form the basis of the Assessment Overview Tables provided for each grade in the Classroom Assessment Model. In addition, domains of learning and, particularly, cognitive levels, inform the design and development of the Grades 4 and 7 Foundation Skills Assessment (FSA).

Prescribed Learning Outcomes

Grade 6

GRADE 6

It is expected that students will:

NUMBER

A1 demonstrate an understanding of place value for numbers
- greater than one million
- less than one thousandth [C, CN, R, T]

A2 solve problems involving large numbers, using technology [ME, PS, T]

A3 demonstrate an understanding of factors and multiples by
- determining multiples and factors of numbers less than 100
- identifying prime and composite numbers
- solving problems involving multiples [PS, R, V]

A4 relate improper fractions to mixed numbers [CN, ME, R, V]

A5 demonstrate an understanding of ratio, concretely, pictorially, and symbolically [C, CN, PS, R, V]

A6 demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically [C, CN, PS, R, V]

A7 demonstrate an understanding of integers, concretely, pictorially, and symbolically [C, CN, R, V]

A8 demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors) [C, CN, ME ,PS, R, V]

A9 explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers) [CN, ME, PS, T]

PATTERNS AND RELATIONS

Patterns

B1 demonstrate an understanding of the relationships within tables of values to solve problems [C, CN, PS, R]

B2 represent and describe patterns and relationships using graphs and tables [C, CN, ME, PS, R, V]

Variables and Equations

B3 represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]

B4 demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically [C, CN, PS, R, V]

GRADE 6

SHAPE AND SPACE

Measurement

C1 demonstrate an understanding of angles by
- identifying examples of angles in the environment
- classifying angles according to their measure
- estimating the measure of angles using 45°, 90°, and 180° as reference angles
- determining angle measures in degrees
- drawing and labelling angles when the measure is specified [C, CN, ME, V]

C2 demonstrate that the sum of interior angles is:
- 180° in a triangle
- 360° in a quadrilateral [C, R]

C3 develop and apply a formula for determining the
- perimeter of polygons
- area of rectangles
- volume of right rectangular prisms [C, CN, PS, R, V]

3-D Objects and 2-D Shapes

C4 construct and compare triangles, including
- scalene
- isosceles
- equilateral
- right
- obtuse
- acute

in different orientations [C, PS, R, V]

C5 describe and compare the sides and angles of regular and irregular polygons [C, PS, R, V]

Transformations

C6 perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image [C, CN, PS, T, V]

C7 perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations [C, CN, T, V]

C8 identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs [C, CN, V]

C9 perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices) [C, CN, PS, T, V]

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Grade 6

Statistics and Probability

Data Analysis

D1 create, label, and interpret line graphs to draw conclusions [C, CN, PS, R, V]

D2 select, justify, and use appropriate methods of collecting data, including
- questionnaires
- experiments
- databases
- electronic media [C, PS, T]

D3 graph collected data and analyze the graph to solve problems [C, CN, PS]

Chance and Uncertainty

D4 demonstrate an understanding of probability by
- identifying all possible outcomes of a probability experiment
- differentiating between experimental and theoretical probability
- determining the theoretical probability of outcomes in a probability experiment
- determining the experimental probability of outcomes in a probability experiment
- comparing experimental results with the theoretical probability for an experiment [C, ME, PS, T]

Student Achievement

This section of the IRP contains information about classroom assessment and student achievement, including specific achievement indicators that may be used to assess student performance in relation to each prescribed learning outcome. Also included in this section are key elements – descriptions of content that help determine the intended depth and breadth of prescribed learning outcomes.

Classroom Assessment and Evaluation

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment evidence can be collected using a wide variety of methods, such as

- observation
- student self-assessments and peer assessments
- quizzes and tests (written, oral, practical)
- samples of student work
- projects and presentations
- oral and written reports
- journals and learning logs
- performance reviews
- portfolio assessments

Assessment of student achievement is based on the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgments about student performance in relation to prescribed learning outcomes.

Three major types of assessment can be used in conjunction with each other to support student achievement.

- Assessment for learning is assessment for purposes of greater learning achievement.
- Assessment as learning is assessment as a process of developing and supporting students' active participation in their own learning.
- Assessment of learning is assessment for purposes of providing evidence of achievement for reporting.

Assessment for Learning

Classroom assessment for learning provides ways to engage and encourage students to become involved in their own day-to-day assessment – to acquire the skills of thoughtful self-assessment and to promote their own achievement.

This type of assessment serves to answer the following questions:

- What do students need to learn to be successful?
- What does the evidence of this learning look like?

Assessment for learning is criterion-referenced, in which a student's achievement is compared to established criteria rather than to the performance of other students. Criteria are based on prescribed learning outcomes, as well as on suggested achievement indicators or other learning expectations.

Students benefit most when assessment feedback is provided on a regular, ongoing basis. When assessment is seen as an opportunity to promote learning rather than as a final judgment, it shows students their strengths and suggests how they can develop further. Students can use this information to redirect their efforts, make plans, communicate with others (e.g., peers, teachers, parents) about their growth, and set future learning goals.

Assessment for learning also provides an opportunity for teachers to review what their students are learning and what areas need further attention. This information can be used to inform teaching and create a direct link between assessment and instruction. Using assessment as a way of obtaining feedback on instruction supports student achievement by informing teacher planning and classroom practice.

Assessment as Learning

Assessment as learning actively involves students in their own learning processes. With support and guidance from their teacher, students take responsibility for their own learning, constructing meaning for themselves. Through a process of continuous self-assessment, students develop the ability to take stock of what they have already learned, determine what they have not yet learned, and decide how they can best improve their own achievement.

Although assessment as learning is student-driven, teachers can play a key role in facilitating how this assessment takes place. By providing regular opportunities for reflection and self-assessment, teachers can help students develop, practise, and become comfortable with critical analysis of their own learning.

Assessment of Learning

Assessment of learning can be addressed through summative assessment, including large-scale assessments and teacher assessments. These summative assessments can occur at the end of the year or at periodic stages in the instructional process.

Large-scale assessments, such as Foundation Skills Assessment (FSA) and Graduation Program exams, gather information on student performance throughout the province and provide information

for the development and revision of curriculum. These assessments are used to make judgments about students' achievement in relation to provincial and national standards.

Assessment of learning is also used to inform formal reporting of student achievement.

For Ministry of Education reporting policy, refer to www.bced.gov.bc.ca/policy/policies/student_reporting.htm

Assessment *for* Learning	Assessment *as* Learning	Assessment *of* Learning
Formative assessment *ongoing in the classroom* • teacher assessment, student self-assessment, and/or student peer assessment • criterion-referenced criteria based on prescribed learning outcomes identified in the provincial curriculum, reflecting performance in relation to a specific learning task • involves both teacher and student in a process of continual reflection and review about progress • teachers adjust their plans and engage in corrective teaching in response to formative assessment	Formative assessment *ongoing in the classroom* • self-assessment • provides students with information on their own achievement and prompts them to consider how they can continue to improve their learning • student-determined criteria based on previous learning and personal learning goals • students use assessment information to make adaptations to their learning process and to develop new understandings	Summative assessment *occurs at end of year or at key stages* • teacher assessment • may be either criterion-referenced (based on prescribed learning outcomes) or norm-referenced (comparing student achievement to that of others) • information on student performance can be shared with parents/guardians, school and district staff, and other education professionals (e.g., for the purposes of curriculum development) • used to make judgments about students' performance in relation to provincial standards

For more information about assessment for, as, and of learning, refer to the following resource developed by the Western and Northern Canadian Protocol (WNCP): *Rethinking Assessment with Purpose in Mind.*

This resource is available online at www.wncp.ca

In addition, the BC Performance Standards describe levels of achievement in key areas of learning (reading, writing, numeracy, social responsibility, and information and communications technology integration) relevant to all subject areas. Teachers may wish to use the Performance Standards as resources to support ongoing formative assessment in mathematics.

BC Performance Standards are available at www.bced.gov.bc.ca/perf_stands/

Criterion-Referenced Assessment and Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather than to the performance of other students. Evaluation in relation to prescribed curriculum requires that criteria be established based on the learning outcomes.

Criteria are the basis for evaluating student progress. They identify, in specific terms, the critical aspects of a performance or a product that indicate how well the student is meeting the prescribed learning outcomes. For example, weighted criteria, rating scales, or scoring guides (reference sets) are ways that student performance can be evaluated using criteria.

Wherever possible, students should be involved in setting the assessment criteria. This helps students develop an understanding of what high-quality work or performance looks like.

Criterion-referenced assessment and evaluation may involve these steps:

Step 1	Identify the prescribed learning outcomes and suggested achievement indicators (as articulated in this IRP) that will be used as the basis for assessment.
Step 2	Establish criteria. When appropriate, involve students in establishing criteria.
Step 3	Plan learning activities that will help students gain the attitudes, skills, or knowledge outlined in the criteria.
Step 4	Prior to the learning activity, inform students of the criteria against which their work will be evaluated.
Step 5	Provide examples of the desired levels of performance.
Step 6	Conduct the learning activities.
Step 7	Use appropriate assessment instruments (e.g., rating scale, checklist, scoring guide) and methods (e.g., observation, collection, self-assessment) based on the particular assignment and student.
Step 8	Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.
Step 9	Where appropriate, provide feedback and/or a letter grade to indicate how well the criteria are met.
Step 10	Communicate the results of the assessment and evaluation to students and parents/guardians.

Key Elements

Key elements provide an overview of content in each curriculum organizer. They can be used to determine the expected depth and breadth of the prescribed learning outcomes.

Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for developmental learning.

Achievement Indicators

To support the assessment of provincially prescribed curricula, this IRP includes sets of achievement indicators in relation to each learning outcome.

Achievement indicators, taken together as a set, define the specific level of attitudes demonstrated, skills applied, or knowledge acquired by the student in relation to a corresponding prescribed learning outcome. They describe what evidence to look for to determine whether or not the student has fully met the intent of the learning outcome. Since each achievement indicator defines only one aspect of the corresponding learning outcome, the entire set of achievement indicators should be considered when determining whether students have fully met the learning outcome.

In some cases, achievement indicators may also include suggestions as to the type of task that would provide evidence of having met the learning outcome (e.g., a constructed response such as a list, comparison, or analysis; a product created and presented such as a report, poster, letter, or model; a particular skill demonstrated such as map making or critical thinking).

Achievement indicators support the principles of assessment for learning, assessment as learning, and assessment of learning. They provide teachers and parents with tools that can be used to reflect on what students are learning, as well as provide students with a means of self-assessment and ways of defining how they can improve their own achievement.

Achievement indicators are not mandatory; they are suggestions only, provided to assist in the assessment of how well students achieve the prescribed learning outcomes.

The following pages contain the suggested achievement indicators corresponding to each prescribed learning outcome for the Mathematics K to 7 curriculum. The achievement indicators are arranged by curriculum organizer for each grade; however, this order is not intended to imply a required sequence of instruction and assessment.

Student Achievement

Grade 6

KEY ELEMENTS: GRADE 6

MATHEMATICAL PROCESS (INTEGRATED)

The following mathematical processes have been integrated within the prescribed learning outcomes and achievement indicators for the grade: communication, connections, mental mathematics and estimation, problem solving, reasoning, technology, and visualization.

NUMBER – develop number sense

- numbers greater than 1 000 000 and smaller than one thousandth
- factors and multiples
- improper fractions and mixed numbers
- ratio and whole number percent
- integers
- multiplication and division of decimals
- order of operations excluding exponents

PATTERNS AND RELATIONS – use patterns to describe the world and solve problems

Patterns

- patterns and relationships in graphs and tables including a tables of value

Variables and Equations

- letter variable representation of number relationships
- preservation of equality

SHAPE AND SPACE – use direct and indirect measurement to solve problems

Measurement

- angle measure and construction
- sum of interior angles of a triangle and quadrilateral
- formulas for the perimeter of polygons, area of rectangles and volume of right rectangular prisms

3-D Objects and 2-D Shapes

- types of triangles
- regular and irregular polygons

Transformations

- combinations of transformations
- single transformation in the first quadrant of the Cartesian plane

STATISTICS AND PROBABILITY – collect, display and analyze data to solve problems

Data Analysis

- line graphs
- methods of data collection
- graph data

Chance and Uncertainty

- experimental and theoretical probability

NUMBER

General Outcome: Develop number sense.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
A1 demonstrate an understanding of place value for numbers - greater than one million - less than one thousandth [C, CN, R, T]	❑ explain how the pattern of the place value system (e.g., the repetition of ones, tens and hundreds) makes it possible to read and write numerals for numbers of any magnitude ❑ provide examples of where large numbers and small decimals are used (e.g., media, science, medicine, technology)
A2 solve problems involving large numbers, using technology [ME, PS, T]	❑ identify which operation is necessary to solve a given problem and solve it ❑ determine the reasonableness of an answer ❑ estimate the solution and solve a given problem.
A3 demonstrate an understanding of factors and multiples by - determining multiples and factors of numbers less than 100 - identifying prime and composite numbers - solving problems involving multiples [PS, R, V]	❑ identify multiples for a given number and explain the strategy used to identify them ❑ determine all the whole number factors of a given number using arrays ❑ identify the factors for a given number and explain the strategy used (e.g., concrete or visual representations, repeated division by prime numbers, or factor trees) ❑ provide an example of a prime number and explain why it is a prime number ❑ provide an example of a composite number and explain why it is a composite number ❑ sort a given set of numbers as prime and composite ❑ solve a given problem involving factors or multiples ❑ explain why 0 and 1 are neither prime nor composite
A4 relate improper fractions to mixed numbers [CN, ME, R, V]	❑ demonstrate using models that a given improper fraction represents a number greater than 1 ❑ express improper fractions as mixed numbers ❑ express mixed numbers as improper fractions ❑ place a given set of fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
A5 demonstrate an understanding of ratio, concretely, pictorially, and symbolically [C, CN, PS, R, V]	❑ provide a concrete or pictorial representation for a given ratio ❑ write a ratio from a given concrete or pictorial representation ❑ express a given ratio in multiple forms, such as 3:5, , or 3 to 5 ❑ identify and describe ratios from real-life contexts and record them symbolically ❑ explain the part/whole and part/part ratios of a set (e.g., for a group of 3 girls and 5 boys, explain the ratios 3:5, 3:8, and 5:8 ❑ solve a given problem involving ratio
A6 demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically [C, CN, PS, R, V]	❑ explain that "percent" means "out of 100." ❑ explain that percent is a ratio out of 100 ❑ use concrete materials and pictorial representations to illustrate a given percent ❑ record the percent displayed in a given concrete or pictorial representation ❑ express a given percent as a fraction and a decimal ❑ identify and describe percents from real-life contexts, and record them symbolically ❑ solve a given problem involving percents
A7 demonstrate an understanding of integers, concretely, pictorially, and symbolically [C, CN, R, V]	❑ extend a given number line by adding numbers less than zero and explain the pattern on each side of zero ❑ place given integers on a number line and explain how integers are ordered ❑ describe contexts in which integers are used (e.g., on a thermometer ❑ compare two integers, represent their relationship using the symbols <, >, and =, and verify using a number line ❑ order given integers in ascending or descending order.
A8 demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors) [C, CN, ME, PS, R, V]	❑ place the decimal point in a product using front-end estimation (e.g., for 15.205 m × 4, think 15 m × 4, so the product is greater than 60 m ❑ place the decimal point in a quotient using front-end estimation (e.g., for \$26.83 ÷ 4, think \$24 ÷ 4, so the quotient is greater than \$6 ❑ correct errors of decimal point placement in a given product or quotient without using paper and pencil ❑ predict products and quotients of decimals using estimation strategies ❑ solve a given problem that involves multiplication and division of decimals using multipliers from 0 to 9 and divisors from 1 to 9
A9 explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers) [CN, ME, PS, T]	❑ demonstrate and explain with examples why there is a need to have a standardized order of operations ❑ apply the order of operations to solve multi-step problems with or without technology (e.g., computer, calculator)

PATTERNS AND RELATIONS (PATTERNS)

General Outcome: Use patterns to describe the world and solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
B1 demonstrate an understanding of the relationships within tables of values to solve problems [C, CN, PS, R]	❑ generate values in one column of a table of values, given values in the other column and a pattern rule ❑ state, using mathematical language, the relationship in a given table of values ❑ create a concrete or pictorial representation of the relationship shown in a table of values ❑ predict the value of an unknown term using the relationship in a table of values and verify the prediction ❑ formulate a rule to describe the relationship between two columns of numbers in a table of values ❑ identify missing elements in a given table of values ❑ identify errors in a given table of values ❑ describe the pattern within each column of a given table of values ❑ create a table of values to record and reveal a pattern to solve a given problem
B2 represent and describe patterns and relationships using graphs and tables [C, CN, ME, PS, R, V]	❑ translate a pattern to a table of values and graph the table of values (limit to linear graphs with discrete elements) ❑ create a table of values from a given pattern or a given graph ❑ describe, using everyday language, orally or in writing, the relationship shown on a graph

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

General Outcome: Represent algebraic expressions in multiple ways.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
B3 represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]	❑ write and explain the formula for finding the perimeter of any given rectangle ❑ write and explain the formula for finding the area of any given rectangle ❑ develop and justify equations using letter variables that illustrate the commutative property of addition and multiplication (e.g., $a + b = b + a$ or $a \times b = b \times a$ ❑ describe the relationship in a given table using a mathematical expression ❑ represent a pattern rule using a simple mathematical expression, such as $4d$ or $2n + 1$
B4 demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically [C, CN, PS, R, V]	❑ model the preservation of equality for addition using concrete materials, such as a balance or using pictorial representations and orally explain the process ❑ model the preservation of equality for subtraction using concrete materials such as a balance or using pictorial representations and orally explain the process ❑ model the preservation of equality for multiplication using concrete materials, such as a balance or using pictorial representations and orally explain the process ❑ model the preservation of equality for division using concrete materials such as a balance or using pictorial representations and orally explain the process ❑ write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials (e.g., $3b = 12$ is the same as $3b + 5 = 12 + 5$ or $2r = 7$ is the same as $3(2r) = 3(7)$)

SHAPE AND SPACE (MEASUREMENT)

General Outcome: Use direct or indirect measurement to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C1 demonstrate an understanding of angles by - identifying examples of angles in the environment - classifying angles according to their measure - estimating the measure of angles using 45°, 90°, and 180° as reference angles - determining angle measures in degrees - drawing and labelling angles when the measure is specified [C, CN, ME, V]	❑ provide examples of angles found in the environment ❑ classify a given set of angles according to their measure (e.g., acute, right, obtuse, straight, reflex ❑ sketch 45°, 90° and 180° angles without the use of a protractor, and describe the relationship among them ❑ estimate the measure of an angle using 45°, 90°, and 180° as reference angles ❑ measure, using a protractor, given angles in various orientations ❑ draw and label a specified angle in various orientations using a protractor ❑ describe the measure of an angle as the measure of rotation of one of its sides ❑ describe the measure of angles as the measure of an interior angle of a polygon
C2 demonstrate that the sum of interior angles is: - 180° in a triangle - 360° in a quadrilateral [C, R]	❑ explain, using models, that the sum of the interior angles of a triangle is the same for all triangles ❑ explain, using models, that the sum of the interior angles of a quadrilateral is the same for all quadrilaterals
C3 develop and apply a formula for determining the - perimeter of polygons - area of rectangles - volume of right rectangular prisms [C, CN, PS, R, V]	❑ explain, using models, how the perimeter of any polygon can be determined ❑ generalize a rule (formula) for determining the perimeter of polygons, including rectangles and squares ❑ explain, using models, how the area of any rectangle can be determined ❑ generalize a rule (formula) for determining the area of rectangles ❑ explain, using models, how the volume of any right rectangular prism can be determined ❑ generalize a rule (formula) for determining the volume of right rectangular prisms ❑ solve a given problem involving the perimeter of polygons, the area of rectangles, and/or the volume of right rectangular prisms

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C4 construct and compare triangles, including - scalene - isosceles - equilateral - right - obtuse - acute in different orientations [C, PS, R, V]	❑ sort a given set of triangles according to the length of the sides ❑ sort a given set of triangles according to the measures of the interior angles ❑ identify the characteristics of a given set of triangles according to their sides and/or their interior angles ❑ sort a given set of triangles and explain the sorting rule ❑ draw a specified triangle (e.g., scalene) ❑ replicate a given triangle in a different orientation and show that the two are congruent
C5 describe and compare the sides and angles of regular and irregular polygons [C, PS, R, V]	❑ sort a given set of 2-D shapes into polygons and non-polygons, and explain the sorting rule ❑ demonstrate congruence (sides to sides and angles to angles) in a regular polygon by superimposing ❑ demonstrate congruence (sides to sides and angles to angles) in a regular polygon by measuring ❑ demonstrate that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure ❑ sort a given set of polygons as regular or irregular and justify the sorting ❑ identify and describe regular and irregular polygons in the environment

Shape and Space (Transformations)

General Outcome: Describe and analyze position and motion of objects and shapes.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
C6 perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image [C, CN, PS, T, V]	❑ demonstrate that a 2-D shape and its transformation image are congruent ❑ model a given set of successive translations, successive rotations or successive reflections of a 2-D shape ❑ model a given combination of two different types of transformations of a 2-D shape ❑ draw and describe a 2-D shape and its image, given a combination of transformations ❑ describe the transformations performed on a 2-D shape to produce a given image ❑ model a given set of successive transformations (translation, rotation, and/or reflection) of a 2-D shape ❑ perform and record one or more transformations of a 2-D shape that will result in a given image
C7 perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations [C, CN, T, V]	❑ analyze a given design created by transforming one or more 2-D shapes, and identify the original shape and the transformations used to create the design ❑ create a design using one or more 2-D shapes and describe the transformations used
C8 identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs [C, CN, V]	❑ label the axes of the first quadrant of a Cartesian plane and identify the origin ❑ plot a point in the first quadrant of a Cartesian plane, given its ordered pair ❑ match points in the first quadrant of a Cartesian plane with their corresponding ordered pair ❑ plot points in the first quadrant of a Cartesian plane with intervals of 1, 2, 5 or 10 on its axes, given whole number ordered pairs ❑ draw shapes or designs, given ordered pairs in the first quadrant of a Cartesian plane ❑ determine the distance between points along horizontal and vertical lines in the first quadrant of a Cartesian plane ❑ draw shapes or designs in the first quadrant of a Cartesian plane and identify the points used to produce them

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Prescribed Learning Outcomes	Suggested Achievement Indicators
C9 perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices) [C, CN, PS, T, V]	❑ identify the coordinates of the vertices of a given 2-D shape (limited to the first quadrant of a Cartesian plane) ❑ perform a transformation on a given 2-D shape and identify the coordinates of the vertices of the image (limited to the first quadrant) ❑ describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation (limited to first quadrant)

STATISTICS AND PROBABILITY (DATA ANALYSIS)

General Outcome: Collect, display and analyze data to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.* *Students who have fully met the prescribed learning outcome are able to:*
D1 create, label, and interpret line graphs to draw conclusions [C, CN, PS, R, V]	❑ determine the common attributes (title, axes and intervals) of line graphs by comparing a given set of line graphs ❑ determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why ❑ create a line graph from a given table of values or set of data ❑ interpret a given line graph to draw conclusions
D2 select, justify, and use appropriate methods of collecting data, including - questionnaires - experiments - databases - electronic media [C, PS, T]	❑ select a method for collecting data to answer a given question and justify the choice ❑ design and administer a questionnaire for collecting data to answer a given question, and record the results ❑ answer a given question by performing an experiment, recording the results, and drawing a conclusion ❑ explain when it is appropriate to use a database as a source of data ❑ gather data for a given question by using electronic media including selecting data from databases
D3 graph collected data and analyze the graph to solve problems [C, CN, PS]	❑ determine an appropriate type of graph for displaying a set of collected data and justify the choice of graph ❑ solve a given problem by graphing data and interpreting the resulting graph

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

STATISTICS AND PROBABILITY (CHANCE AND UNCERTAINTY)

General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Prescribed Learning Outcomes	Suggested Achievement Indicators
	The following set of indicators may be used to assess student achievement for each corresponding prescribed learning outcome.
It is expected that students will:	*Students who have fully met the prescribed learning outcome are able to:*
D4 demonstrate an understanding of probability by - identifying all possible outcomes of a probability experiment - differentiating between experimental and theoretical probability - determining the theoretical probability of outcomes in a probability experiment - determining the experimental probability of outcomes in a probability experiment - comparing experimental results with the theoretical probability for an experiment [C, ME, PS, T]	❑ list the possible outcomes of a probability experiment, such as - tossing a coin - rolling a die with a given number of sides - spinning a spinner with a given number of sectors ❑ determine the theoretical probability of an outcome occurring for a given probability experiment ❑ predict the probability of a given outcome occurring for a given probability experiment by using theoretical probability ❑ conduct a probability experiment, with or without technology, and compare the experimental results to the theoretical probability ❑ explain that as the number of trials in a probability experiment increases, the experimental probability approaches the theoretical probability of a particular outcome ❑ distinguish between theoretical probability and experimental probability, and explain the difference

Classroom Assessment Model

The Classroom Assessment Model outlines a series of assessment units for Mathematics K to 7.

These units have been structured by grade level and theme. Collectively the units address all of the prescribed learning outcomes for each grade, and provide one suggested means of organizing, ordering, and delivering the required content. This organization is not intended to prescribe a linear means of delivery. Teachers are encouraged to reorder the learning outcomes and to modify, organize, and expand on the units to meet the needs of their students, to respond to local requirements, and to incorporate relevant recommended learning resources as applicable. (See the Learning Resources section later in this IRP for information about the recommended learning resources for Mathematics K to 7). In addition, teachers are encouraged to consider ways to adapt assessment strategies from one grade to another.

Considerations for Instruction and Assessment in Mathematics K to 7

It is highly recommended that parents and guardians be kept informed about all aspects of Mathematics K to 7. Suggested strategies for involving parents and guardians are found in the Introduction to this IRP.

Teachers are responsible for setting a positive classroom climate in which students feel comfortable learning about and discussing topics in Mathematics K to 7. Guidelines that may help educators establish a positive climate that is open to free inquiry and respectful of various points of view can be found in the section on Establishing a Positive Classroom Climate in the Introduction to this IRP.

Teachers may also wish to consider the following:

- Involve students in establishing guidelines for group discussion and presentations. Guidelines might include using appropriate listening and speaking skills, respecting students who are reluctant to share personal information in group settings, and agreeing to maintain confidentiality if sharing of personal information occurs.
- Promote critical thinking and open-mindedness, and refrain from taking sides on one point of view.
- Develop and discuss procedures associated with recording and using personal information that may be collected as part of students' work for the purposes of instruction and/or assessment (e.g., why the information is being collected, what the information will be used for, where the information will be kept; who can access it – students, administrators, parents; how safely it will be kept).
- Ensure students are aware that if they disclose personal information that indicates they are at risk for harm, then that information cannot be kept confidential. For more information, see the section on Confidentiality in the Introduction to this IRP.

Classroom Assessment and Evaluation

Teachers should consider using a variety of assessment instruments and techniques to assess students' abilities to meet the prescribed learning outcomes. Tools and techniques for assessment in Mathematics K to 7 can include

- teacher assessment tools such as observation checklists, rating scales, and scoring guides
- self-assessment tools such as checklists, rating scales, and scoring guides
- peer assessment tools such as checklists, rating scales, and scoring guides
- journals or learning logs
- video (to record and critique student demonstration or performance)
- written tests, oral tests (true/false, multiple choice, short answer)
- questionnaires, worksheets
- portfolios
- student-teacher conferences

Assessment in Mathematics K to 7 can also occur while students are engaged in, and based on the product of, activities such as

- class and group discussions
- interviews and questioning
- sharing strategies
- object manipulation
- models and constructions
- charts, graphs, diagrams
- games
- experiments
- artwork, songs/stories, dramas
- centres/stations
- demonstrations and presentations
- performance tasks
- projects

For more information about student assessment, refer to the section on Student Achievement, as well as to the Assessment Overview Tables in each grade of the Classroom Assessment Model.

Information and Communications Technology

The Mathematics K to 7 curriculum requires students to be able to use and analyse the most current information to make informed decisions on a range of topics. This information is often found on the Internet as well as in other information and communications technology resources. When organizing for instruction and assessment, teachers should consider how students will best be able to access the relevant technology, and ensure that students are aware of school district policies on safe and responsible Internet and computer use.

Contents of the Model

Assessment Overview Tables

The Assessment Overview Tables provide teachers with suggestions and guidelines for assessment of each grade of the curriculum. These tables identify the domains of learning and cognitive levels of the learning outcomes, along with a listing of suggested assessment activities and a suggested weight for grading for each curriculum organizer.

Overview

Each grade includes an overview of the assessment units:

- Learning at Previous Grades, indicating any relevant learning based on prescribed learning outcomes from earlier grades of the same subject area. It is assumed that students will have already acquired this learning; if they have not, additional introductory instruction may need to take place before undertaking the suggested assessment outlined in the unit. Note that some topics appear at multiple grade levels in order to emphasize their importance and to allow for reinforcement and developmental learning.
- Curriculum Correlation – a table that shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model.

Prescribed Learning Outcomes

Each unit begins with a listing of the prescribed learning outcomes that are addressed by that unit. Collectively, the units address all the learning outcomes for that grade; some outcomes may appear in more than one unit. The units may not address all of the achievement indicators for each of the outcomes.

Suggested Assessment Activities

Assessment activities have been included for each set of prescribed learning outcomes and corresponding achievement indicators. Each assessment activity consists of two parts:

- Planning for Assessment – outlining the background information to explain the classroom context, opportunities for students to gain and practise learning, and suggestions for preparing the students for assessment
- Assessment Strategies – describing the assessment task, the method of gathering assessment information, and the assessment criteria as defined by the learning outcomes and achievement indicators.

A wide variety of activities have been included to address a variety of learning and teaching styles. The assessment activities describe a variety of tools and methods for gathering evidence of student performance. These assessment activities are also referenced in the Assessment Overview Tables, found at the beginning of each grade in the Model.

These strategies are suggestions only, designed to provide guidance for teachers in planning instruction and assessment to meet the prescribed learning outcomes.

Assessment Instruments

Sample assessment instruments have been included at the end of each grade where applicable, and are provided to help teachers determine the extent to which students are meeting the prescribed learning outcomes. These instruments contain criteria specifically keyed to one or more of the suggested assessment activities contained in the units. Ongoing formative assessment will be required throughout the year to guide instruction and provide evidence that students have met the breadth and depth of the prescribed learning outcomes.

Classroom Assessment Model

Grade 6

Grade 6: Assessment Overview Table

The purpose of this table is to provide teachers with suggestions and guidelines for formative and summative classroom-based assessment and grading of Grade 6 Mathematics.

Curriculum Organizers	Suggested Assessment Activities		Suggested Weight for Grading	Number of Outcomes	Number of Outcomes by Domain*		
					K	U&A	HMP
Number	• vocabulary • Frayer model maps • journals • discussions • observation	• finding errors • large number graphing • small group work • Venn diagrams	40-50%	9	2	7	0
Patterns and Relations	• posters • discussions • observations	• interviews • student demonstrations	5-15%	4	0	1	3
Shape and Space	• portfolio • posters • student presentations • observations	• artwork • Frayer model maps • problem solving • concrete manipulatives	30-40%	9	3	4	2
Statistics and Probability	• concrete materials • charts • group work • student work	• questionnaires • graphs • technology • observations	5-15%	4	0	2	2
		Totals	**100%**	**26**	**5**	**14**	**7**

* The following abbreviations are used to represent the three cognitive levels within the cognitive domain: K = Knowledge; U&A = Understanding and Application; HMP = Higher Mental Processes.

GRADE 6

OVERVIEW

Learning at Previous Grades

- whole numbers to 1 000 000
- estimation strategies for calculations and problem solving
- mental mathematics strategies for multiplication facts to 81 and corresponding division facts
- mental mathematics for multiplication
- multiplication for 2-digit by 2-digit and division for 3-digit by 1-digit
- decimal and fraction comparison
- addition and subtraction of decimals to thousandths
- prediction using a pattern rule
- single-variable, one-step equations with whole number coefficients and solutions
- perimeter and area of rectangles
- length, volume and capacity
- parallel, intersecting, perpendicular, vertical and horizontal edges and faces
- quadrilaterals including rectangles, squares, trapezoids, parallelograms and rhombuses
- 2-D shape single transformation
- first-hand and second-hand data
- double bar graphs
- likelihood of a single outcome

Curriculum Correlation

The following table shows which curriculum organizers and suborganizers are addressed by each unit in this grade of the Classroom Assessment Model. Note that some curriculum organizers/suborganizers are addressed in more than one unit.

	Order of Operations	Understanding Numbers I	Understanding Numbers II	'L' Pattern	Preservation of Equality	Decimal Error Correct	Geometry Portfolio	Boxes	Unfair Spinner	Survey
Number	X	X	X			X				X
Patterns and Relations *Patterns*				X						
Variables and Equations				X	X			X		
Space and Shape *Measurement*							X	X		
3-D Objects and 2-D Shapes							X			
Transformations				X			X			
Statistics and Probability *Data Analysis*				X						X
Chance and Uncertainty									X	

Order of Operations

Prescribed Learning Outcomes

It is expected that students will:

A9 explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers) [CN, ME, PS, T]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students create an order of operations question with a whole number answer. For this process students may use calculators. The criteria for the question is that the question - uses each operation at least once - uses parentheses in such a way that their removal changes the answer Exchange questions with a partner. Have students perform the necessary calculations without a calculator. Check for accuracy. Once students have created a question challenge students to create a story which involves the order of operations.	• Verify that students can explain, using their examples, why there is a need to have a standardized order of operations. Have students explain possible ways in which an incorrect solution may occur. Look for evidence that students can apply the order of operations to solve multi-step problems with and without technology. Discuss the ways that mental mathematics can be used to determine if the solution to the question or story is possible.

Understanding Numbers I

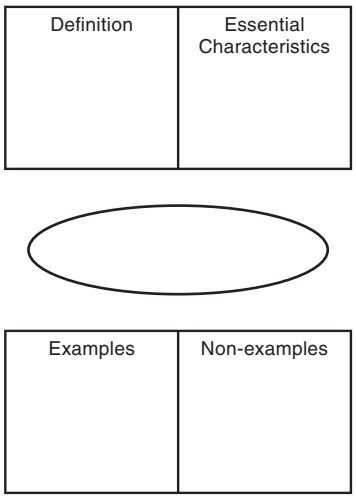
Prescribed Learning Outcomes

It is expected that students will:

A1 demonstrate an understanding of place value for numbers
- greater than one million
- less than one thousandth [C, CN, R, T]

A2 solve problems involving large numbers, using technology [ME, PS, T]

A3 demonstrate an understanding of factors and multiples by
- determining multiples and factors of numbers less than 100
- identifying prime and composite numbers
- solving problems involving multiples [PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Show students how to create a Frayer model:  	• Have students complete a Frayer model in their math journals for each of the following terms: factors, multiples, prime numbers and composite numbers. Verify that students have an accurate understanding of the terms.
• Give students the following tasks: - 2 numbers are multiplied to give 36 000 000. What might the 2 numbers be? Answer this question in as many ways as possible. - Identify each factor as being a prime number or a composite number. - Describe the patterns and/or strategies that helped you find your answers.	• Observe whether or not students know the patterns of the place value system and are able make use of them when solving the problem. Observe whether students have used multiplication patterns such as the following: 2 × 18 000 000 — 9 × 4 000 000 20 × 1 800 000 — 90 × 400 000 200 × 180 000 — 900 × 40 000 and can describe their thinking processes. Can they state why only 3 of the factors (2, 3, 5) are prime?
• Five friends all eat at the same restaurant. Amardeep eats there every day. Becky eats there every second day. Carlos eats there every third day. David eats there every fourth day. Elton eats there every fifth day. If they all eat together on Feb. 1st, when is the next time they will all eat together?	• Encourage students who use a calendar or chart to solve the problem to look for a more efficient method.

Understanding Numbers II

Prescribed Learning Outcomes

It is expected that students will:

A1 demonstrate an understanding of place value for numbers
- greater than one million
- less than one thousandth [C, CN, R, T]

A4 relate improper fractions to mixed numbers [CN, ME, R, V]

A6 demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially and symbolically [C, CN, PS, R, V]

A7 demonstrate an understanding of integers, concretely, pictorially and symbolically [C, CN, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students use a graphic organizer such as a Venn diagrams to compare the similarities and differences of the following sets of categories: - improper fractions and mixed numbers - decimals and mixed numbers - fractions, decimals, and percents - integers and whole numbers	• Verify that students have an accurate understanding of the terms and can discuss the similarities and differences.
• Give students a list of proper fractions, improper fractions, and percents and blank mini -hundred squares to represent the whole; students will give an illustration of each number.	• Verify that students can - explain "percent" as "out of 100" - explain percent is a ratio out of 100 - represent a given percent pictorially - represent a given improper fraction as being greater than 1
• Give students a list of mixed numbers and improper fraction. Have them change them to the other form and place them on a number line. Number cards could also be used to allow physical placement on the number line.	• Verify that students can - accurately change from one form to the other - explain the strategies they used to determine position on the number line

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• These number line activities can be done with the whole class using number cards and a number line on the floor. It could be also done in small groups, in a Math centre or as an individual task. • Have students construct a number line and place each number on their number line. Ask students to justify their placement decisions. $\frac{1}{4}$, $\frac{7}{10}$, 2.6, 40%, $\frac{7}{5}$, 75%, $\frac{247}{100}$, 0.05, $\frac{5000}{10000}$, $\frac{9999}{10000}$. Have students construct a second number line and place the following integers, again justifying their placement decisions. –7, 11, –5, –3, 0, 5, –6, –9, 2, 4 Using a number line with positive and negative intervals of one million, have students place these numbers and justify their reasoning: 0 6 456 902 2 989 098 –4 046 059 728 936 –3 489 562 –89 324 2 273 159 –4 872 396 5 231	• Monitor students' abilities to - use and/or label benchmarks - use common denominators to compare, where appropriate Note which students are using a less efficient method to place numbers (e.g., changing all fractions to decimals). Verify that students can explain how integers are ordered and can compare any 2 integers.

'L' Pattern

Prescribed Learning Outcomes

It is expected that students will:

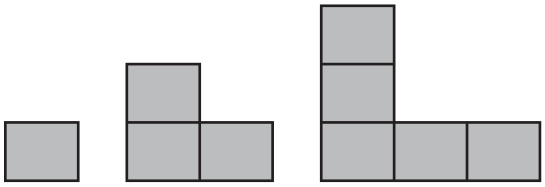
B1 demonstrate an understanding of the relationships within tables of values to solve problems [C, CN, PS, R]

B2 represent and describe patterns and relationships using graphs and tables [C, CN, ME, PS, R, V]

B3 represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]

C8 identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs [C, CN, V]

D1 create, label, and interpret line graphs to draw conclusions [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students look at a block pattern such as the following:  Have them - draw the next 2 patterns in the sequence - record the data on a table of values - complete the table to pattern 10 - record the data from the table as ordered pairs, then graph the data on coordinate grid paper - extend the line on the graph and predict the number of blocks in pattern 13 - give the rule to solve for any number of patterns (the n rule) Alternately, provide groups of students with different block patterns and have the groups present their findings on a poster.	• Verify that students can - state using mathematical language the relationship in a table of values - predict the value of an unknown term using the relationship in a table of values and verify the prediction - describe the pattern within each column of a table of values - formulate a rule to describe the relationship between 2 columns of numbers in a table of values - translate the pattern to a table of values and graph it - represent the pattern rule using a mathematical expression - label the axis of the first quadrant of a Cartesian plane and identify the origin - plot points in the first quadrant of a Cartesian plane given the ordered pairs
• Students are provided a graph of a linear equation. From this they are to provide the whole number table of values and the ordered pairs. Discuss with a partner and record in their journals the relationship shown by the graph.	• Verify that students can - create a table of values from the given graph - describe using everyday language, orally and in writing, the relationship shown on a graph

Preservation of Equality

Prescribed Learning Outcomes

It is expected that students will:

B4 demonstrate and explain the meaning of preservation of equality concretely, pictorially and symbolically [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Give students one-step equations such as the following, and have them represented concretely or pictorially, applying the preservation of equality to solve the equations and explaining the process. $x + 3 = 7$ $a - 3 = 5$ $3c = 12$ $\frac{n}{3} = 15$ Interview students and have them demonstrate the process to you.	• Verify that students can - model the preservation of equality for addition, subtraction, multiplication and division using concrete materials - model the preservation of equality for addition, subtraction, multiplication and division pictorially - explain orally the preservation of equality for addition, subtraction, multiplication and division

Decimal Error Correct

Prescribed Learning Outcomes

It is expected that students will:

A8 demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors) [C, CN, ME ,PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Give students problems (such as the following) and have them explain using words and/or pictures why the solutions are incorrect. Have them explain the misunderstandings of the person making these mistakes. $1.55 \times 4 = 62$ $3.4 \div 8 = 4.5$ Ask students to provide examples, such as consumer problems, to demonstrate the meaning of the correct and in correct solution. For example, is it likely that 4 tubes of toothpaste would cost \$62 if each tube was \$1.55? When multiplying a decimal number by a whole number, the answer can be either a whole number or a decimal number. Have students explain in pictures, numbers, or words why this it true. Students may use place value blocks to work out the answer, but have them transfer this to written work.	• Verify that students can - place the decimal point in a product using front end estimation - place the decimal point in a quotient using front end estimation - correct errors of decimal point placement in a given product or quotient without using paper and pencil - justify why a given solution is possible - predict products and quotients of decimals using estimation strategies

Geometry Portfolio

Prescribed Learning Outcomes

It is expected that students will:

C1 demonstrate an understanding of angles by
- identifying examples of angles in the environment
- classifying angles according to their measure
- estimating the measure of angles using 45°, 90°, and 180° as reference angles
- determining angle measures in degrees
- drawing and labelling angles when the measure is specified [C, CN, ME, V]

C2 demonstrate that the sum of interior angles is:
- 180° in a triangle
- 360° in a quadrilateral [C, R]

C4 construct and compare triangles, including
- scalene
- isosceles
- equilateral
- right
- obtuse
- acute

in different orientations [C, PS, R, V]

C5 describe and compare the sides and angles of regular and irregular polygons [C, PS, R, V]

C6 perform a combination of translation(s), rotation(s) and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image [C, CN, PS, T, V]

C7 perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations [C, CN, T, V]

C8 identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs [C, CN, V]

C9 perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices) [C, CN, PS, T, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students sketch examples of angles in the environment. Estimate the measure of each angle using referent angles 45°, 90°, and 180°. Classify as acute, right, obtuse, straight, or reflex.	• Verify that students can - provide examples of angles found in the environment - classify them correctly - estimate the measure of an angle using the referents Have students create a geometry portfolio to collect their work over the course of this unit. Use criteria such as those outlined in the **Geometry Portfolio** rubric provided at the end of this grade to assess students' portfolios.

Planning for Assessment	Assessment Strategies
• Give students worksheets each with a variety of examples of a particular shape including at least one regular shape and, where possible, one with a reflex angle. Polygons to be studied include: triangles, quadrilaterals, pentagons, hexagons, octagons and nonagons. Students measure the sides and angles of each shape. With the information groups of students make a master chart. Headings on the top of the chart are: regular, irregular, sum of angles, anomalies. Headings down the side are the names of the polygons. Students paste an example of a regular shape and an irregular shape on the chart. They report the sum of the interior angles and any anomalies they find on the chart. Each group presents their findings to the class. Discuss the chart and in particular resolve the anomalies found. Have the students create a table of values relating the number of sides of the polygon to the measure of the interior angles.	• Verify that students can - use a ruler and protractor accurately - describe the measure of an angle as the measure of an interior angle of a polygon - explain that within the classes of triangles that the sum of the interior angles is the same - explain that within the classes of quadrilaterals that the sum of the interior angles is the same - sort the set of polygons as regular and irregular and justify their sort - explain that the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure - create a table of values to record and reveal a pattern - explain the n rule for any number of sides on a polygon compared to the sum of the interior angles
• Have students construct a triangle comparison sheet. They are to label both the horizontal and vertical axes with the types of triangles (right, acute, obtuse, isosceles, scalene, and equilateral). On this sheet the students will be asked to compare kinds of triangles. Students indicate in each grid box whether or not it is possible for a triangle to meet both definitions. If the combination is possible they are to draw an example. Any 2 comparable drawings must be drawn in different orientations.	• Verify that students can - demonstrate and explain that orientation does not change the characteristics of the triangles. - draw a specified triangle - identify characteristics of a type of triangle
• Have students complete a Frayer model in their math journal for each of the terms, transformations, translation, reflection and rotation.	• Verify that students have an accurate understanding of the terms.
• On 3 separate coordinate grids have students plot these points: A (6,7); B (6,13); C (10,9). Have them draw the triangle and then - rotate the triangle CW 180° around Point F (4,3) and name the new vertices - reflect the triangle over $x = 5$ and name the new vertices - translate the triangle to the right 3 and up 1, then name the new vertices	• Verify that students can - draw a triangle given its vertices - name the vertices on the image - rotate a triangle around a point of rotation that is not a vertex of the triangle

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students draw a scalene triangle on heavy paper to use as a tracing template. Ask them to trace the shape on paper and mark it as the original position. Next, ask them to perform and label the following transformations in successive order: - reflection - rotation - translation Students could also create two identical scale triangles of different colours and show the position of the original and its transformed image. Have them draw the line of reflection, and mark the centre of rotation.	• Observe students as they work with reflections, rotations, and translations, and verify that they can draw a triangle and its image given a combination of successive transformations.
• Have students use their templates on another piece of paper to create a design using a combination of transformations. Ask them to explain the transformations they used to create their designs.	• Verify that students can create a design using a triangle and describe the transformations used in the design.

Boxes

Prescribed Learning Outcomes

It is expected that students will:

B3 represent generalizations arising from number relationships using equations with letter variables. [C, CN, PS, R, V]

C3 develop and apply a formula for determining the
- perimeter of polygons
- area of rectangles
- volume of right rectangular prisms [C, CN, PS, R, V]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Have students in small groups use a set of 24 square tiles to construct as many rectangles as possible, each with an area of 24 square units. Each group should then - record its differing configurations on centimetre grid paper - determine the perimeters of their rectangles by adding the lengths of the sides - generalize from their findings to create a rule for calculating the perimeters - test their formula for rectangles built using only 20 tiles In a whole-class discussion, challenge students to identify other polygons with which they are familiar (e.g., triangle, pentagon, hexagon) and suggest a procedure and formula for calculating the perimeters of each of those).	• As students carry out the perimeter calculation exercise, monitor their work to ensure that they have - followed the instructions (e.g., used all 24 tiles for each attempt) - identified all the possible rectangles that can be formed (1×24, 2×12, 3×8, 4×6, 6×4, 8×3, 12×2, and 24×1) - correctly calculated perimeters using addition - recognized that there could be more than one way to write a formula for perimeter of a rectangle - derived a reliable formula involving recognition of equal values and multiplication - written their perimeter formulæ appropriately, taking account of the rules for order of operations During the follow-up discussion, monitor students' suggestions to ensure they - recognize the specific characteristics of the polygons they suggest and can distinguish between those whose sides are all of equal length and those that have sides of unequal lengths - are able to extend to other polygons the generalizations involved in creating a formula for determining the perimeter of a rectangle
• Using students' grid paper diagrams of the rectangles created using 24 square tiles as a point of departure, discuss the concept of square units and the relationship between the dimensions of rectangles and their areas.	• Have students use various methods to solve problems that involve calculating the areas of rectangles.

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Assume that each area model is the base of a box which will be filled with unit cubes to create 3 or 4 layers. Discuss the following relationships: - the area of the base to the volume of the box - the dimensions of the box and its volume Students could examine a variety of traditional Aboriginal storage containers such as cedar baskets, bentwood boxes, and quill baskets and approximate the capacity of these containers.	• Ask students to use their math journals to reflect on these area-volume relationships. What formula can be derived from the area to determine the volume? Is this the case for all objects?

Unfair Spinner

Prescribed Learning Outcomes

It is expected that students will:

D4 demonstrate an understanding of probability by
- identifying all possible outcomes of a probability experiment
- differentiating between experimental and theoretical probability
- determining the theoretical probability of outcomes in a probability experiment
- determining the experimental probability of outcomes in a probability experiment
- comparing experimental results with the theoretical probability for an experiment [C, ME, PS, T]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Give the students a spinner divided into 5 sectors in the following amounts but not labelled: - one sector of $\frac{3}{8}$ - three sectors of $\frac{1}{8}$ - one sector of $\frac{2}{8}$ Have students colour each sector a different colour. For example, colour the large sector red, the medium sector yellow, and each of the small sectors blue, green, and black. Have students identify the outcome they think is most likely, identify all the possible outcomes, conduct the experiment and determine the theoretical probability. Ask students to comment on their findings. Theoretical and experimental probability should be recorded as fractions. Have students keep track of their results to 48 spins. Compare the experimental results with the theoretical. Have groups combine their experimental results and compare them to the theoretical probability.	• Assess students' work to determine the extent to which they are able to - list the probable outcomes of a probability experiment - determine the theoretical probability of an outcome occurring for a probability experiment - conduct a probability experiment and compare the experimental results with the theoretical probability - explain that as the number of trials in a probability experiment increases, the experimental probability approaches theoretical probability of a particular outcome - distinguish between theoretical probability and experimental probability and can explain the differences

Survey

Prescribed Learning Outcomes

It is expected that students will:

A2 solve problems involving large numbers, using technology [ME, PS, T]
A5 demonstrate an understanding of ratio, concretely, pictorially, and symbolically [C, CN, PS, R, V]
A6 demonstrate an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically [C, CN, PS, R, V]
D1 create, label, and interpret line graphs to draw conclusions [C, CN, PS, R, V]
D2 select, justify, and use appropriate methods of collecting data, including
- questionnaires
- experiments
- databases
- electronic media [C, PS, T]

D3 graph collected data and analyze the graph to solve problems [C, CN, PS]

PLANNING FOR ASSESSMENT	ASSESSMENT STRATEGIES
• Give students a set of line graphs that represent continuous data such as the following: - the distance a bee is from the hive when gathering honey - the altitude of an aircraft from take off to landing - the filling of a bath tub to taking a bath to emptying the tub Have the students examine them for common attributes. Have them list other examples of continuous data that could be represented with a line graph. Have students create a line graph from hourly temperature readings for a twelve-hour period.	• Verify that students can do the following: - explain the difference between continuous data and discrete data - determine whether a given set of data can be represented by a line graph (continuous data) or a series of points (discrete data) and explain why - determine intervals appropriate for the data - interpret line graphs to draw conclusions
• Have students create a question such as: What is your favourite pasta dish (Macaroni and Cheese, Lasagne, Spaghetti, Tortellini, Fettuccine Alfredo or Other)? Students should have 4 to 6 categories on their questionnaire. Each student then collects data by asking at least 25 people. They are to construct an appropriate graph to represent their data. For each category, state their results as a fraction of the whole and a percentage. Additionally some categories should be compared as a ratio. Students will present their graph, the fractions, percentages, and the original data collection on a poster. Students will present their posters to the class. Included in their presentation will be a recommendation to an interested party. For example, a recommendation could be made to a local restaurant for their weekly pasta special.	• Verify that students can - design and administer a questionnaire for collecting data to answer a question and record the results - justify their choice of graph when representing data - correctly construct and label their graph - differentiate between fractions of the whole and ratios - interpret their graph to make a recommendation and can justify their reasoning

Planning for Assessment	Assessment Strategies
• Students create a question involving large numbers that must be researched using a database such as those compiled by Statistics Canada. For example, students could examine the change in Aboriginal population using the Aboriginal Peoples Survey. What information is included in this survey and also in the Statistics Canada data? What information is missing? Are there questions for which data from one source would be preferred over the other? Students can choose between a line graph or a bar graph to present their data.	• Verify that students can - select a method for collecting data to answer a question and justify the choice - explain when it is appropriate to use a database as a source of data - gather data for a question by using electronic media including selecting data from a databases - determine an appropriate type of graph for displaying a set of data and justify their choice - correctly construct and label their graph - interpret the graph to answer their question

GEOMETRY PORTFOLIO

Not Yet Within Expectations	Minimally Meets Expectations	Fully Meets Expectations	Exceeds Expectations
• The student is unable to meet basic requirements of the task without close, ongoing assistance.	• The work satisfies most basic requirements, but is flawed or incomplete. • Often needs some help.	• The work satisfies basic requirements of the task.	• The work is complete, accurate, and efficient.
• Transformation illustrations are often confusing with key information missing. • There are major errors in performing transformations.	• Transformation illustrations are clear though some information may be missing. • Some features of the transformations are inaccurate or incomplete.	• Transformation illustrations are generally clear and necessary information is included. • Some features of the transformations contain minor errors or flaws (e.g., slight misplacement of ordered pairs).	• Transformation illustrations are clear and all information is included. • The transformations are accurate and complete.

LEARNING RESOURCES

This section contains general information on learning resources, and provides a link to the titles, descriptions, and ordering information for the recommended learning resources in the Mathematics K to 7 Grade Collections.

What Are Recommended Learning Resources?

Recommended learning resources are resources that have undergone a provincial evaluation process using teacher evaluators and have Minister's Order granting them provincial recommended status. These resources may include print, video, software and CD-ROMs, games and manipulatives, and other multimedia formats. They are generally materials suitable for student use, but may also include information aimed primarily at teachers.

Information about the recommended resources is organized in the format of a Grade Collection. A Grade Collection can be regarded as a "starter set" of basic resources to deliver the curriculum. In many cases, the Grade Collection provides a choice of more than one resource to support curriculum organizers, enabling teachers to select resources that best suit different teaching and learning styles. Teachers may also wish to supplement Grade Collection resources with locally approved materials.

How Can Teachers Choose Learning Resources to Meet Their Classroom Needs?

Teachers must use either

- provincially recommended resources
  OR
- resources that have been evaluated through a local, board-approved process

Prior to selecting and purchasing new learning resources, an inventory of resources that are already available should be established through consultation with the school and district resource centres. The ministry also works with school districts to negotiate cost-effective access to various learning resources.

What Are the Criteria Used to Evaluate Learning Resources?

The Ministry of Education facilitates the evaluation of learning resources that support BC curricula, and that will be used by teachers and/or students for instructional and assessment purposes. Evaluation criteria focus on content, instructional design, technical considerations, and social considerations.

> Additional information concerning the review and selection of learning resources is available from the ministry publication, Evaluating, Selecting and Managing Learning Resources: A Guide (Revised 2002) www.bced.gov.bc.ca/irp/resdocs/esm_guide.pdf

What Funding is Available for Purchasing Learning Resources?

As part of the selection process, teachers should be aware of school and district funding policies and procedures to determine how much money is available for their needs. Funding for various purposes, including the purchase of learning resources, is provided to school districts. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

What Kinds of Resources Are Found in a Grade Collection?

The Grade Collection charts list the recommended learning resources by media format, showing links to the curriculum organizers. Each chart is followed by an annotated bibliography. Teachers should check with suppliers for complete and up-to-date ordering information. Most suppliers maintain web sites that are easy to access.

Mathematics K to 7 Grade Collections

The Grade Collections for Mathematics K to 7 include newly recommended learning resources as well as relevant resources previously recommended for prior versions of the Mathematics K to 7 curriculum. The ministry updates the Grade Collections on a regular basis as new resources are developed and evaluated.

> Please check the following ministry web site for the most current list of recommended learning resources in the Grade Collections for each IRP: www.bced.gov.bc.ca/irp_resources/lr/resource/gradcoll.htm

GLOSSARY

The British Columbia Ministry of Education recognizes the limitation of a glossary available only in print format. An online glossary has been developed by Alberta Education to support the implementation of their revised Kindergarten to Grade 9 Program of Studies. This glossary is based on the WNCP CCF for K-9 Mathematics and therefore also supports the British Columbia Mathematics K to 7 IRP.

This online glossary provides additional supports for teachers indlucing definitions, diagrams, pictures, and interactive applets that cannot be provided through a conventional print glossary. As a result, the Ministry of Education encourages educational stakeholders to access the glossary through a link which is provided on the British Columbia Ministry of Education website.

To access the glossary, follow the links for curriculum support material from the mathematics IRP main page at www.bced.gov.bc.ca/irp/irp_math.htm