Project E: Building a Scale Model for a Teen Recreation Centre

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Project E: Building a Scale Model for a Teen Recreation Centre

Orientation

Time:

6 to 7 hours

Instructional Strategies:

- Learning Centres
- Math Lab
- Direct Instruction
- Group Work
- Independent Study

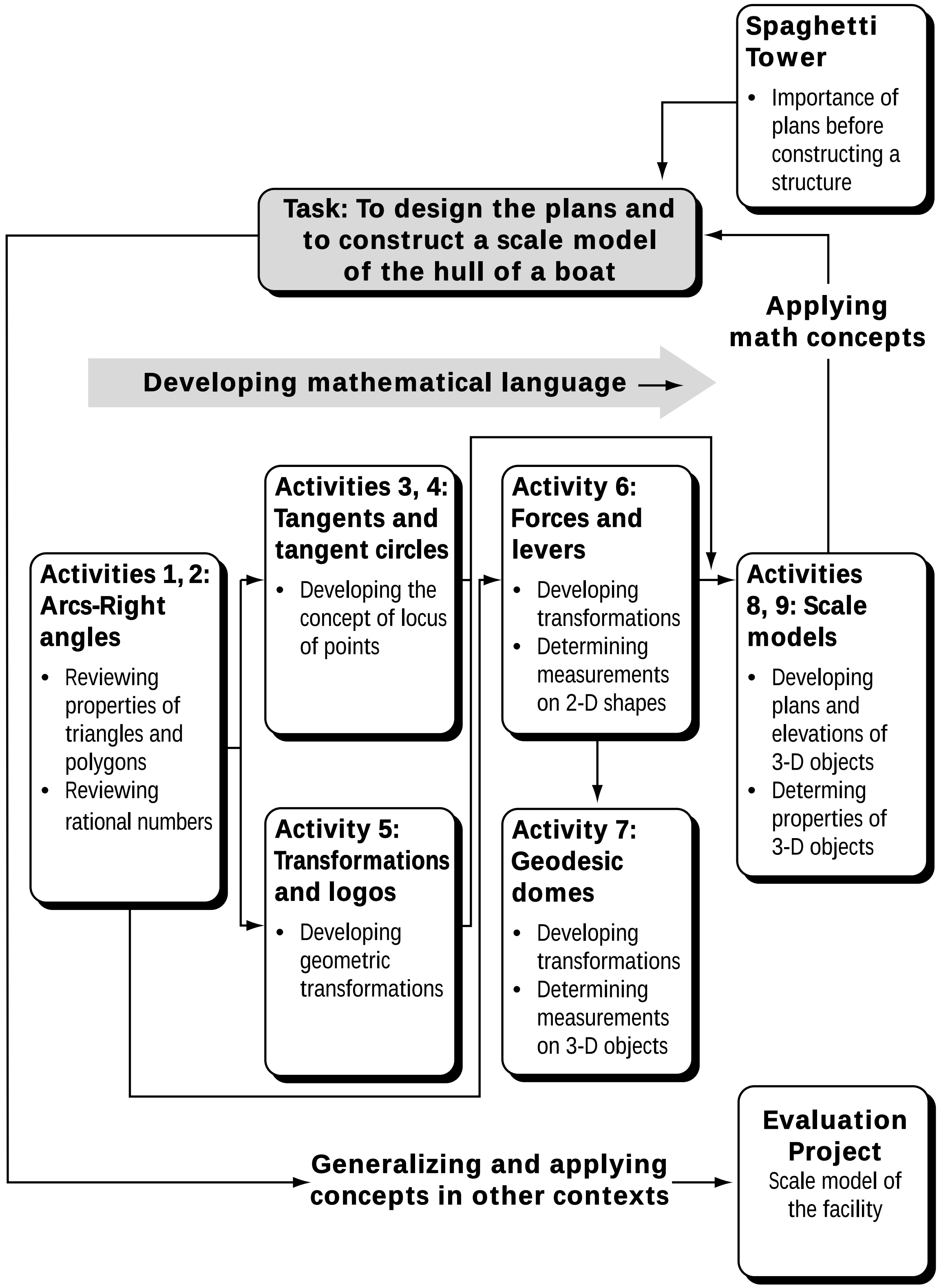
Real-World Applications:

Some real world applications of the skills and concepts learned in this project include:
- maintaining accuracy and precision in construction
- employing accuracy and precision in scale drawings and maps
- constructing a model of the stage sets required for a school play
- building a model railway or doll house to scale

Project Overview:

Figure 1 on the following page is a schematic to help you conceptualize Project E: Building a Scale Model for a Teen Recreation Centre.

Figure 1: Project E: Building a Scale Model for a Teen Recreation Centre



Project Synopsis

This project aims to familiarize the students with the design of precise and accurate scale plans of three dimensional objects by using their abilities with spatial relations.

An additional objective of the project is to provide the students with some mathematical tools related to geometric properties of solids and geometric transformations. These mathematical tools help students simplify the task of constructing 3-D objects from a set of plans and vice versa.

In Phase I, students construct a tower. They draw a scale plan, construct the tower, then compare their plan to reality. This process increases student awareness about some of the elements required in construction.

Phase II is the concept development phase of this project. In this phase, students move from reality to representations of reality while they:
- reinforce their understanding of the properties of plane figures;
- acquire a working understanding of the concept of locus of a point in preparing designs and constructing various curves;
- integrate the concepts related to forces, and relate them to the geometry of the triangle by constructing different models of trusses; and
- use co-ordinate geometry and geometric transformations to simplify construction.

In Phase III, the application part, students move from representations of reality back to reality while they:
- prepare a set of scale plans; and
- use scale plans to construct scale models.

Students apply concepts to prepare a scale plan of a prescribed device. That is, they go from reality to a representation of reality. Then the students construct a device from plans, that is, they go from a representation to reality.

Understanding and designing the plans of a 3-D object, or building a project from a set of plans are skills students will use throughout their lives. In this project, students prepare a scale plan of a support structure (e.g., sawhorse, trestle, or truss) or a lifting structure (e.g., crane or cantilever bridge).

Have students research and bring images of devices similar to those identified above. Using a set of plans created by another group of students, students construct a scale model of a full scale structure able to lift a given weight to a given height or to support a given weight. The plans should be complete and precise enough to permit the construction without any misunderstanding between designers and builders.

Divide the class into an even number of student groups. Initially, groups prepare the scale plans of the device. Then, after having exchanged the plans, students either build scale models of the device or a full scale structure from the plans submitted by another group.

The weight that the device is supposed to lift or support, the height to which the device is supposed to lift the weight, and the particular characteristics of the structures have been determined by the planners and are clearly indicated on the blueprints.

This project can be viewed either as a self-contained project or as a step in the production of a scale model of a Teen Recreation Centre by combining this project with Projects C and D. In the first case, students are asked to produce a report at the end of the project on the construction of the lifting or supporting structure. Later, students may specialize in the production of the scale plans of a building or a structure of their choice. They complete a cardboard scale model as part of the class project of developing a proposal for a recreation facility.

Phase I: Introduction

Objective:

The objective of this phase is for students to appreciate and understand the importance of having a design and plan in hand before building a structure. Before constructing a bird feeder, stereo cabinet, dog house, or even renovating a room, it is wise to design a scale plan of the project.

A scale plan needs to be as precise as possible and flexible enough to allow room for changes during the construction phase. In any event, it is important to have a plan and the materials and equipment you need.

Sometimes it is worth constructing a scale model from the plans in order to have an idea of the final product. Students may experience some surprises if they do not make a scale plan and/or scale model. It could happen that if they decided to rearrange their room and they made only a rough sketch of what they wanted, they might end up with something completely different and unexpected (a desk may not fit, for example).

Activity 1: Constructing a Tower

Objective:

The objective of this activity is for students to develop a scale plan prior to starting a construction project. By constructing a tower of spaghetti and marshmallows, students learn first-hand the importance of scale plans.

Materials:

- one bag of marshmallows
- one package of spaghetti
- measuring tape

Procedure:

This activity is best done by groups of students working together.
Have students construct a tower using spaghetti and marshmallows. The structure should be able to support a certain weight (for example, the minimum requirements might be that it is to support the weight of a tennis ball and be higher than 15 cm). See Figure 2: Spaghetti and Marshmallow Tower below.

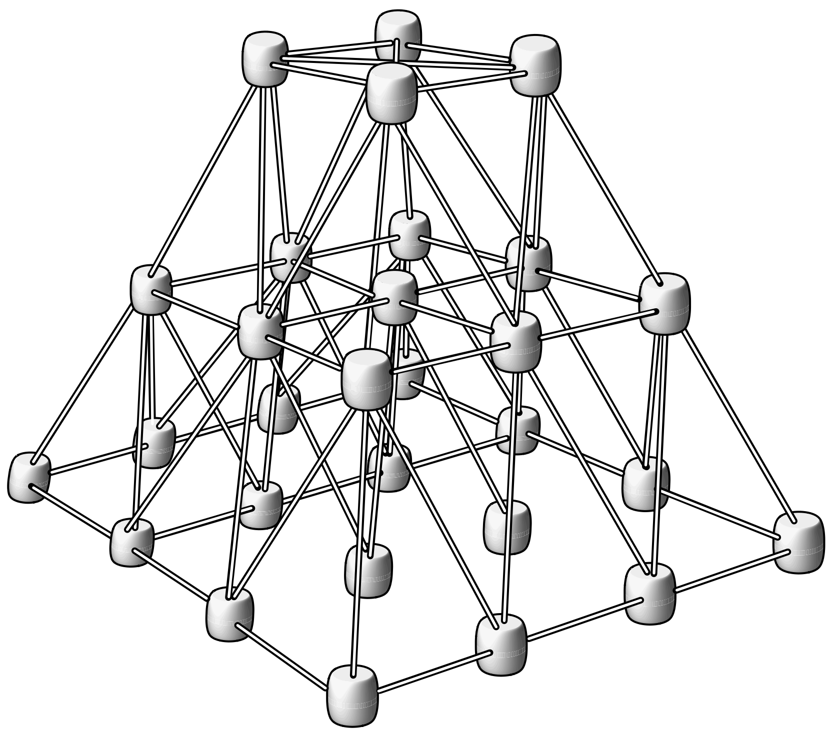
This activity can take the form of a class contest whose rules have been established previously by the students. For instance, one point is awarded for each extra centimetre over 15 cm, or 5 points if the structure can support a basketball.

This kind of activity can be used to stress the importance of having a proper plan before starting any construction project.

Have students draw a rough sketch of what they want to construct and ask them to follow their plan precisely.

After they have completed the activity, discuss how the construction has taken place. Students use the Student Activity: Spaghetti and Marshmallow Tower to record their findings.

Figure 2: Spaghetti and Marshmallow Tower



After the contest, have students relate the tower projects to projects they completed in the past. These may include: room renovation, shelf, bird feeder or dog house construction. Ask them to comment upon some of the difficulties they experienced while constructing their projects.

Tell students what they need to produce at the end of this project and the parameters they need to consider.

Help students establish evaluation criteria for the final product. These criteria should consider the level of their understanding and their ability to apply relevant mathematical concepts.

If necessary, briefly review measurement techniques for triangles and polygons. Students should have a working knowledge of how to measure the perimeter, area, and volume of regular polygons and simple 3-D objects. They should also be able to use and apply reduction and enlargement.

Outline the different concepts students are going to explore and relate them to the project. Establish an action plan with the students by identifying the steps necessary both to produce an accurate scale plan of a complex 3-D object and to construct a 3-D object from the plans provided by another group.

With the entire class, decide upon the characteristics of the devices. Have students then establish the requirements of their own device. For example, what has to be lifted, to what height, or what object the structure needs to be able to support.

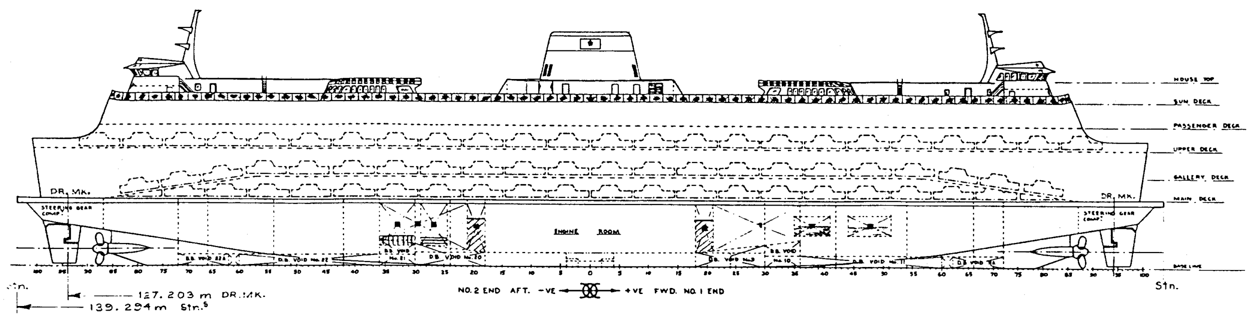
Divide the class into an even number of groups of two or three students and assign the tasks.

To introduce the concepts for this project you may:

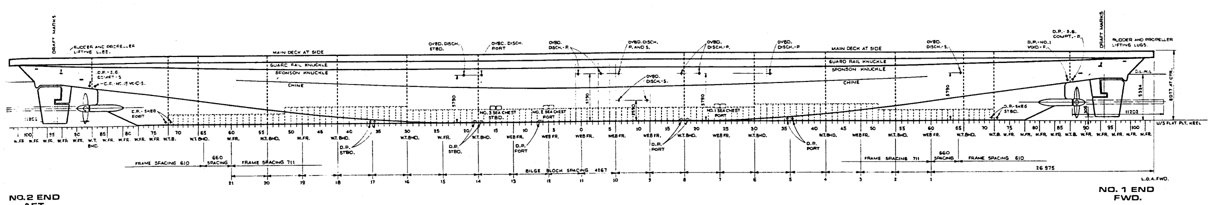
- present a sample of a scale plan. This may be obtained from an architect or a boat builder. With the students, discuss the features of the plans: why it is important for the person in charge to have a plan, and the elevations of the structure.
- ask an architect or a contractor to make a brief presentation to the class about how they make and use plans, and have the students prepare a series of questions.
- obtain website pictures of lifting devices used in construction of historical structures like the pyramids in Egypt, temples in Ancient Greece, and cathedrals of the Middle Ages.

Figure 3: Plans for a Ferry

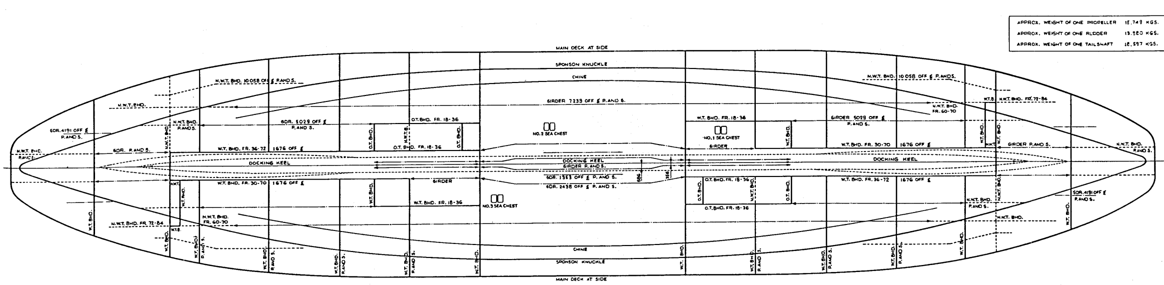
M.V. Queen of Oak Bay



Profile



Outboard Profile



Hold Plan

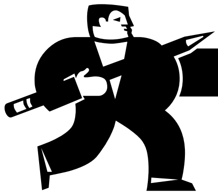
Student Activity: Spaghetti and Marshmallow Tower

Student Name: ____________________________ Date: ___________________

1. Sketch the tower of your dreams.

Student Activity: Spaghetti and Marshmallow Tower

2. Sketch the actual tower you built.



Student Activity: Spaghetti and Marshmallow Tower

3. In what ways do the actual results compare with the original plan?

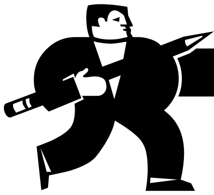
4. In what ways are the actual results different from the original plan?

5. What are some additional comments you have about this activity?

6. Action Plan
   a) Describe your construction project.

   b) Sketch what your finished project will look like.

   c) List the materials and equipment you need to complete your project.



Student Activity: Spaghetti and Marshmallow Tower

d) Project Schedule

Table 1

Project Plan Timeline

Date	Project Phase	Math Concepts	Check	Comments

Phase II: Elements of Construction

Objective:

Phase II is the concept development phase of this project. In this phase, students move from reality to representations of reality while they participate in a variety of different activities. The nine activities provide opportunities for students to:

- reinforce their understanding of the properties of plane figures;
- acquire a working understanding of the concept of 'locus of a point' in preparing the designs and constructing various curves;
- integrate the concepts related to forces, and relate them to the geometry of the triangle by constructing different models of trusses; and
- use co-ordinate geometry and geometric transformations to simplify construction.

In the first two activities, Arcs and Right Angles, students explore various construction techniques that involve polygons, perimeters and area. These techniques require an understanding of various mathematical concepts. The teacher should inform the students of the underlying mathematical ideas during the manipulations. During this exploration, students use concepts related to rational numbers and apply their skills and abilities regarding the use of pattern recognition.

The next three activities are Tangents, Tangent Circles, and Transformations and Logos. In these activities students explore various shapes involving curves and the loci of points in a plane. At the end of this part, students construct a set of curves and produce a design by applying coordinate geometry and pattern recognition. They also explore the effects of the basic geometric transformations.

In the following two activities, Forces and Levers, and Geodesic Domes students construct and analyse rigid structures. They determine how to evaluate the stress on a framework of rigid bars connected together to form a truss. Trusses are all around us in the form of bridges, buildings, boats and so on. In fact, they are in any structure that uses triangles.

The last two activities in Phase II are each closely related to model construction. In the first one, students review geometric solids. In the other activity, students learn more about templates and representing geometric solids.

Activity 1: Arcs

Objective:

Students review the properties of triangles and circles by constructing a coat hanger of given dimensions or a cardboard model of an arched structure.

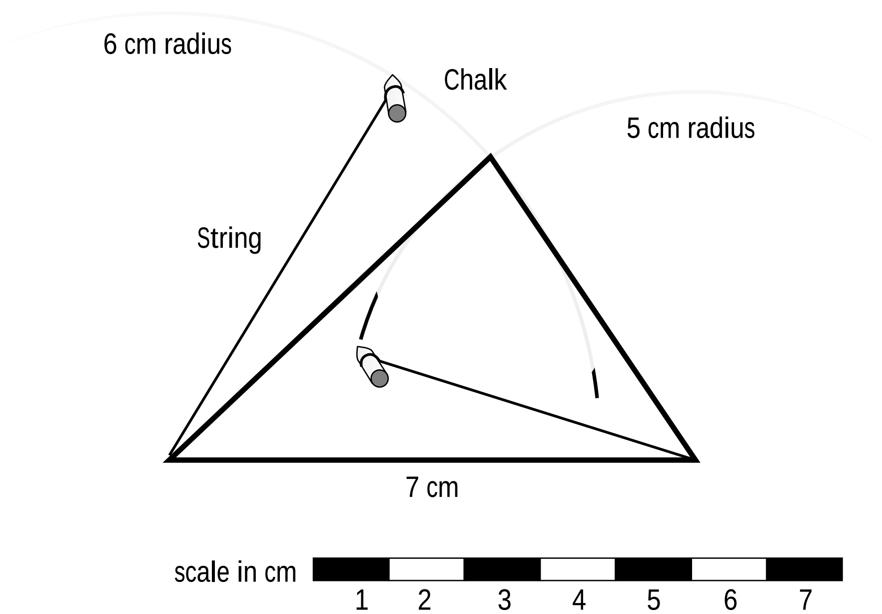
Materials:

- string
- two pieces of wood
- one clamp
- measuring tape
- piece of chalk
- piece of wire

Procedure:

Review, with the students, how to construct a triangle whose three sides are known by using a piece of string and chalk as shown in Figure 4: Drawing Arcs.

Figure 4: Drawing Arcs



Example of a triangle with sides 50 cm, 60 cm, and 70 cm.

- Draw the 70 cm side on the floor or chalkboard and indicate the points *A* and *B*.
- Take a piece of the string of length 60 cm and scribe the arc of a circle having *A* as its centre.
- Take a piece of string of length 50 cm and draw the arc of a circle having its centre at *B*.
- The intersection of the arcs of the circles is the third vertex of the triangle.

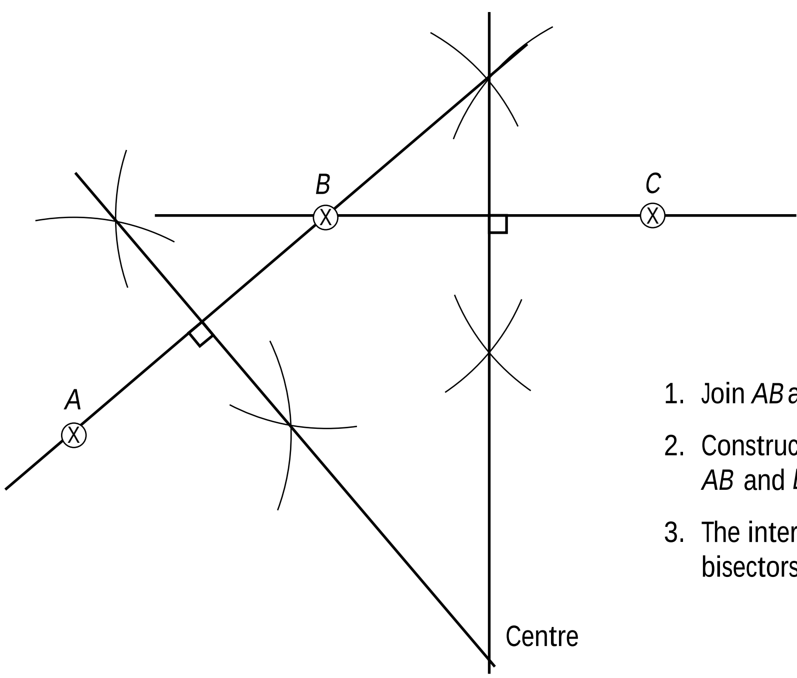
Have students comment on how the arc would change if they were to draw a triangle having sides 70 cm, 30 cm, and 20 cm.

Have students find an illustration of a cathedral vault and an arched bridge in a Japanese garden. Ask them to determine a way of constructing an arc joining the points *A* and *B* while clearing a given elevation (point *C*).

Demonstrate how to draw the arc of a circle given three points as shown below in Figure 5: Vaults.
- Draw three points (not on a straight line) on the floor (or chalkboard).
- Draw the lines between *A* and *B* and between *B* and *C*.
- Construct the perpendicular bisector of each segment and indicate the point where they intersect. This is the centre of the circle upon which points *A*, *B* and *C* lie.
- Trace the arc passing through *A*, *B*, and *C* with the string at the centre.

Have students comment on the length of the radius and the curvature of the circle.

Figure 5: Vaults



1. Join *AB* and *BC*.
2. Construct the perpendicular bisectors of *AB* and *BC*.
3. The intersection of the two perpendicular bisectors is the centre of the circle.

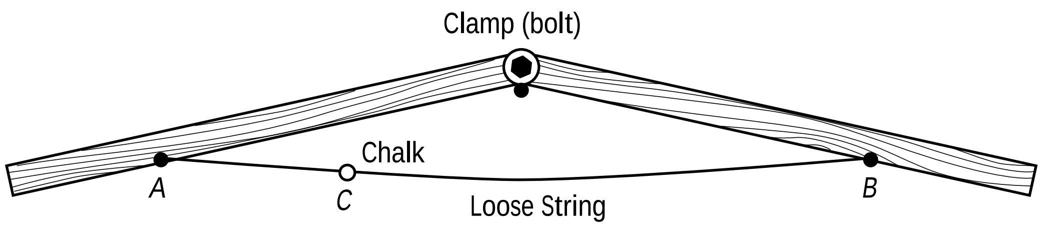
Ask students to propose a way to draw an arc. If the radius is small, they can use the construction procedure described above, but if the radius is large, the following procedure is more appropriate.

First, firmly attach two pieces of wood with a clamp as shown below in Figure 6: Coat Hanger. Loosely attach a piece of string at points *A* and *B* on each arm. Attach a piece of chalk *C*, anywhere along *AB*. As you move the arms, the piece of chalk moves along with *AB*, and as it does so it draws an arc. The arc or curve is the locus of *C*.

Wherever you are on the curve, and as long as you are drawing the arc, then:

$AC + CB$ = constant

Figure 6: Coat Hanger



Point C moves along the curve.
AC + CB = constant wherever you are on the curve as long as you are drawing the arc.

Have students comment on the procedure and justify their comments by using mathematical arguments.

Students use Student Activity: Scale Plans and answer questions.

Construction:

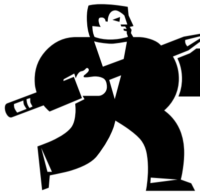
- Have students draw the plans (scale factor of $\frac{1}{2}$) of a coat hanger and then bend a piece of wire to reproduce the coat hanger.
- Have students draw a scale plan (factor of $\frac{1}{10}$) of an arc 2 metres wide (the distance between the two arms *AB*), and a height 0.20 metres at its centre. Then have them construct a scale model with cardboard.

Student Activity: Scale Plans

Student Name: ___________________________ Date: __________________

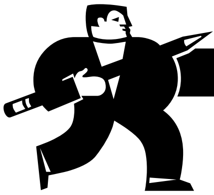
Explain why the construction technique illustrated in Figure 5: Vaults does not apply easily in situations like the one in Figure 6: Coat Hanger

Draw the scale plan (factor of $\frac{1}{2}$) of a coat hanger in the space below. The dimensions are those of a normal coat hanger.



Student Activity: Scale Plans

Draw the scale plan (factor of $\frac{1}{10}$) of an arched bridge 2 metres wide with a height of 0.20 metres at its centre.



Student Activity: Scale Plans

Solve the following construction problem and show all the steps you need to use in the solution. Provide a sketch of the problem.

What length of garland would you need to decorate a 2 metre long mantle?

- You have a mantle above a fireplace. The mantle is 2 m long.
- You want to make a two-colour, twisted paper garland to decorate the mantle.
- The garland is glued or taped at both ends to ensure that the twisting does not come undone.
- Each end of the garland is attached to the ends of the mantle, allowing the garland to hang below the mantle at the 1 metre point.

What are some comments you have about this activity?

Activity 2: Right Angles

Objective:

The objective of this activity is for students to realize that the accuracy of angle measurement is important in construction. Students apply the properties of triangles and review the trigonometric ratios and measurement strategies.

Materials:

- string
- measuring tape
- piece of chalk

Procedure:

Students review the concepts related to polygons, and apply their knowledge of patterns and using rational numbers in a construction context.

Briefly review the Pythagorean Relation.

Ask students to determine the accuracy of a room's dimensions, and how to determine the dimensions and angles of a rectangular room.

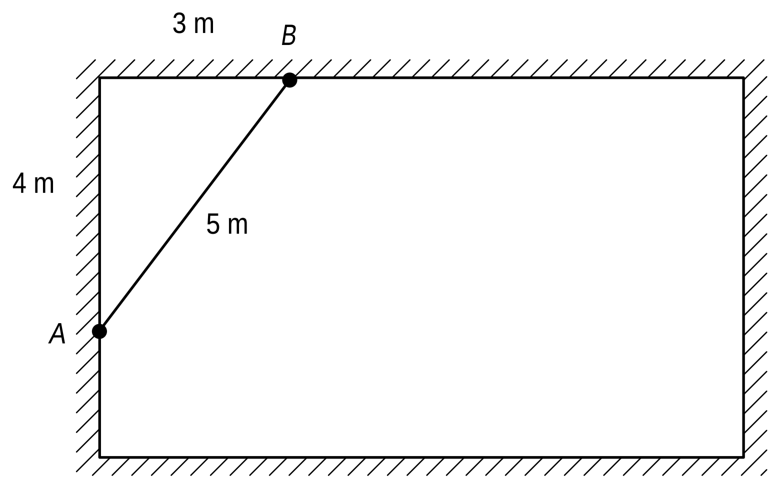
Have students choose a corner of the classroom or gym and ask them to find a practical way of determining how close the corner walls are to forming a right angle.

Ask students to first imagine situations where they will need to evaluate the accuracy of a right angle, and then to imagine some consequences of a lack of precision. Ask them to comment on the accuracy of using a carpenter's square in such a situation and to propose strategies for determining a right angle.

Propose the following procedure that is illustrated below in Figure 7: Right Angles:

- students measure horizontally the lengths of two adjacent walls of a room.
- they measure 4 m (or 40 cm) horizontally on one wall (this locates point *A*), and 3 m (or 30 cm) on the other wall, (this locates point *B*).
- they measure the distance *AB* and record the measurement.
- students reason that if *AB* = 5 m (or 50 cm), the angle is a right angle. If *AB* is larger than 5 m, the angle is larger than a right angle. If *AB* is smaller than 5 m, the angle is less than a right angle.
- students comment on this method and propose other methods of determining a right angle.

Figure 7: Right Angles



Steps
1. Measure 3 m on a wall to locate *B*.
2. Measure 4 m on the other wall to locate *A*.
3. Draw the line *AB*.
4. Measure *AB*.

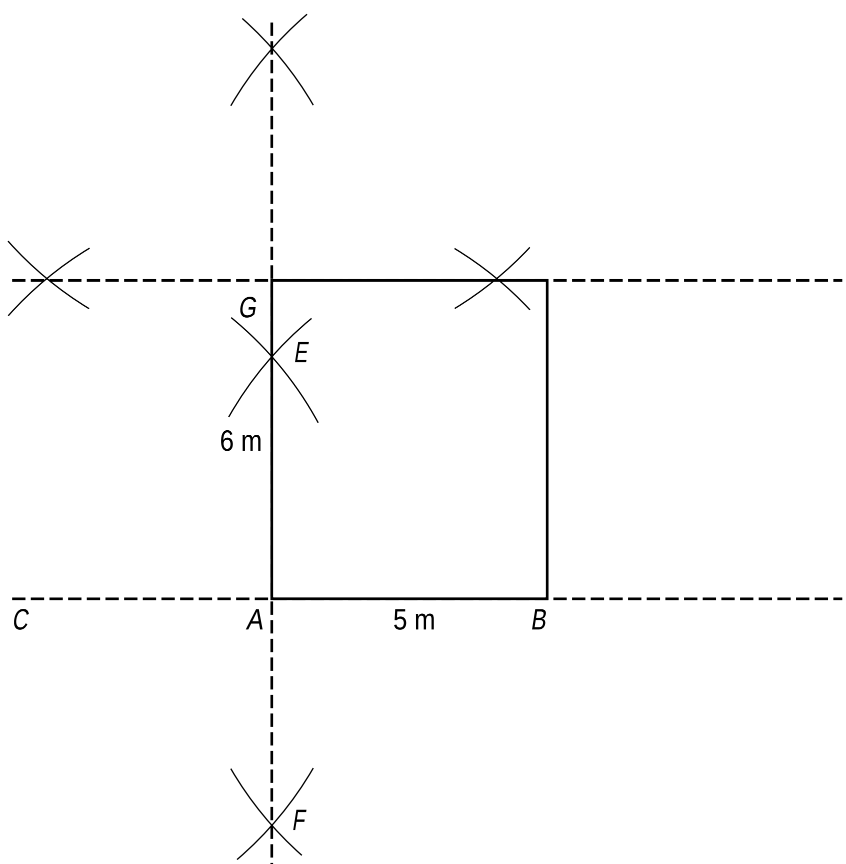
Construction:

Have students construct a rectangle knowing one side and the area by using only a string and a measuring tape. This activity can be done on the classroom floor, gym floor or parking lot.

Ask students to propose and defend a strategy for drawing such a rectangle. Comment on their proposals before giving them the following instructions. Refer to Figure 8: Constructing a Rectangle.

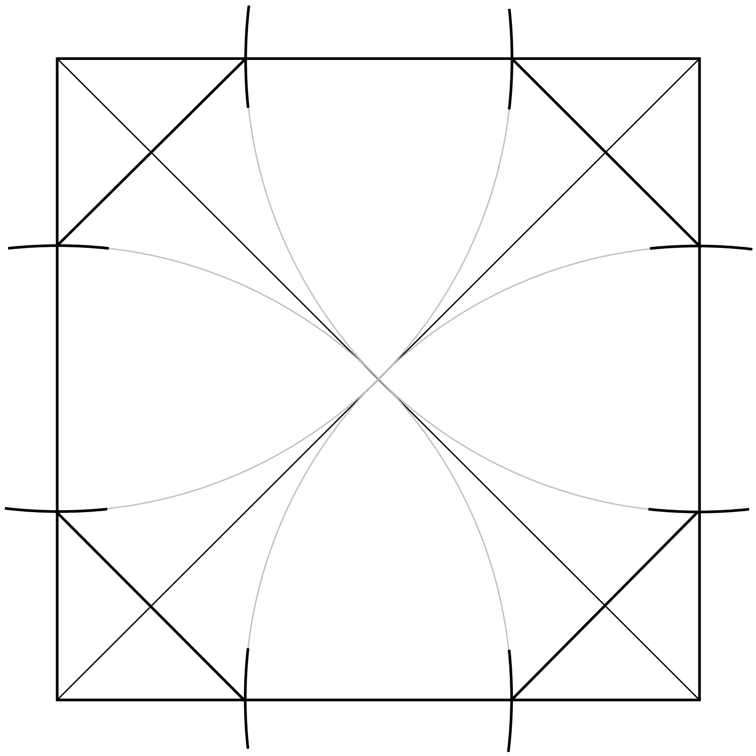
- Determine the length of the second side of the rectangle by using the formula of the area (if a = 5 m and the area is 30 m^2, then the second side of the rectangle is 6 metres).
- Trace one side on the floor and extend the side to the left.
- Use the string to draw a circle having its centre at *A* in such a way that it intersects the side at *B* and *C*.
- From *B* and *C* draw arcs of the same radius (larger than the distance *AB*). Points *E* and *F* are the intersection points of the two arcs.
- Join *E* and *F* and record the length of the second side along *AE* to obtain the point *G*.
- Proceed the same way from *G* and continue to draw the entire rectangle.
- Verify that each of the angles is a right angle.

Figure 8: Constructing a Rectangle



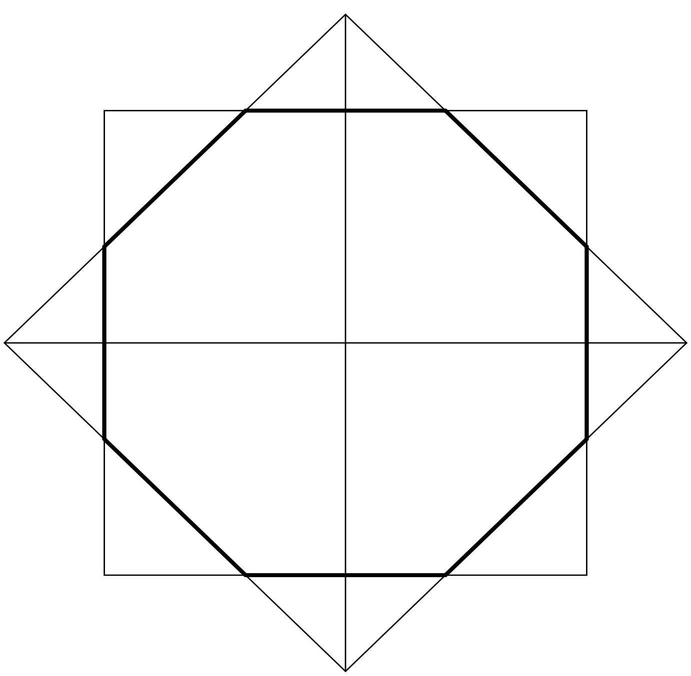
Have students use the floor to draw a square with 3 metre sides. When the square is complete, have students inscribe an octagon within the square. Refer to Figure 9: Constructing an Octagon #1.

Figure 9: Constructing an Octagon #1



Steps
1. Construct diagonals.
2. Draw circles (radius = ½ diagonal).
3. Join each of the 2 intersections to make an octagon.

Figure 10: Constructing an Octagon #2



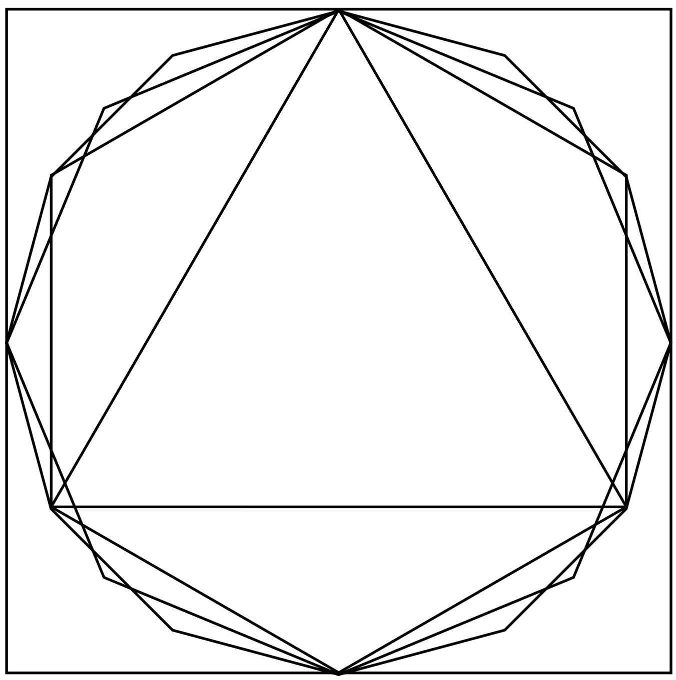
Steps
1. Draw a circle.
2. Draw parallels tangent to the circle.

Ask students to imagine a way to construct a regular hexagon on the floor whose sides are 2 metres and of transforming it into a dodecagon (12 sides), a square, an octagon and an equilateral triangle (Figure 11: Constructing a Regular Polygon). You may decide to give them the following hint: draw a circle with a 2 metre radius; this is the size of the regular hexagon. Have students measure the perimeter and the area for the five polygons. Use the table on Student Activity: Regular Polygons Inscribed in a Circle with a Radius of 2 Metres to record the results.

Have students draw a regular octagon on paper from a square having 78 mm sides.

Have students devise a way to draw a regular octagon with 50 cm sides.

Figure 11: Constructing a Regular Polygon

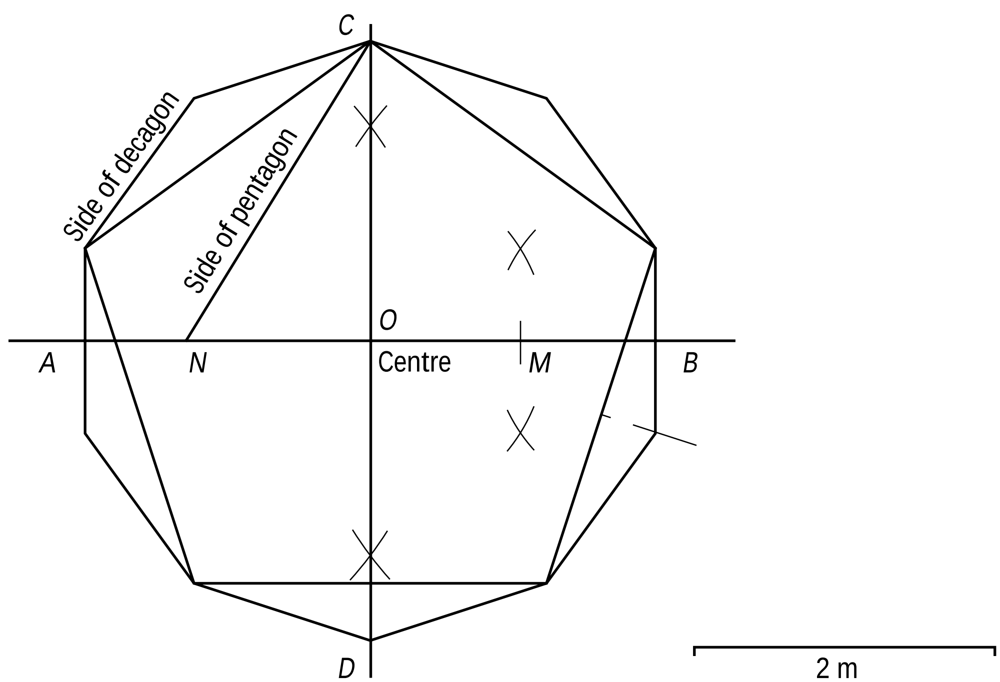


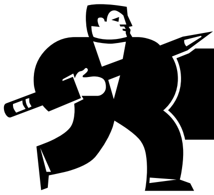
Extension:

Have students try to determine how to draw a regular pentagon and a decagon within a circle having a radius 2 metres. Refer to Figure 12: Constructing a Pentagon and Decagon.

- Draw a circle with a radius of 2 m and centre *O*. Indicate the diameter *AB* (any straight line through the centre).
- Draw the diameter *CD* perpendicular to *AB*.
- Determine the middle of *OB* with the string and call this point *M*.
- Draw a circle with a radius *CM* and centre *M*. Call N its intersection with the diameter *AB*.
- *CN* is the side length of the regular pentagon.
- Draw the length of *CN* five times on the circle and join the points.
- Take the middle of each side and use these points to form a decagon.
- Have students measure the perimeters and the area, and complete the table on Student Activity: Regular Polygons Inscribed in a Circle with a Radius of 2 Metres.

Figure 12: Constructing a Pentagon and Decagon





Student Activity: Tracing Regular Polygons for Construction Purposes

Student Name: ____________________________ Date: ___________________

In what ways does an inaccuracy in the construction of a right angle affect the overall dimensions of a rectangular design such as the foundation of a building or a football field?

Estimate the difference between the two sides of a building foundation 35 m long if an error has been made on a right angle. Complete Table 2: Results of Inaccurate Measurement below.

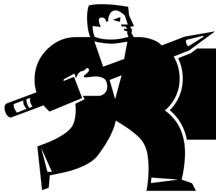
Table 2

Results of Inaccurate Measurement

Error (degrees)	Side *A* (metres)	Side *B* (metres)	Difference *A* - *B* (metres)

What are some of the consequences of this and similar errors?

Your task is to draw a rectangle whose dimensions are those of a soccer field. How do you proceed?



Student Activity: Tracing Regular Polygons for Construction Purposes

Illustrate the sequence of steps in the space below.

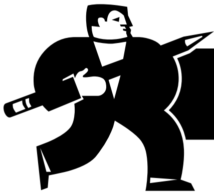
What are some additional comments you would like to make about this activity?

Student Activity: Tracing Regular Polygons For Construction Purposes

Your task is to carve a post having the section shaped as an octagon. You have a log with a diameter of 30 cm and a length of 1 m. How do you proceed?

Sketch the carved post and estimate its volume compared to the volume of the log.

Sketch:



Student Activity: Regular Polygons Inscribed in a Circle with a Radius of 2 Metres

Student Name: ____________________________ Date: ___________________

Table 3

Polygon Measurements

# of Sides	Name of Polygon	Length of Side	Perimeter	Area	Ratio Area/ Perimeter
3					
4					
5					
6					
8					
10					
12					
7					
9					
11					
24					

Express in your own words what the pattern of ratios is and then express this in algebraic terms.

Activity 3: Tangents

Objective:

In this activity, students are provided with the opportunity to explore the properties of tangent lines to a circle in a design context. The abilities they develop are useful for identifying the symmetry of geometric figures and exploring the concept of the locus of a point.

Materials:

- pair of compasses
- ruler
- straight edge
- graph paper
- graphics software

Procedure:

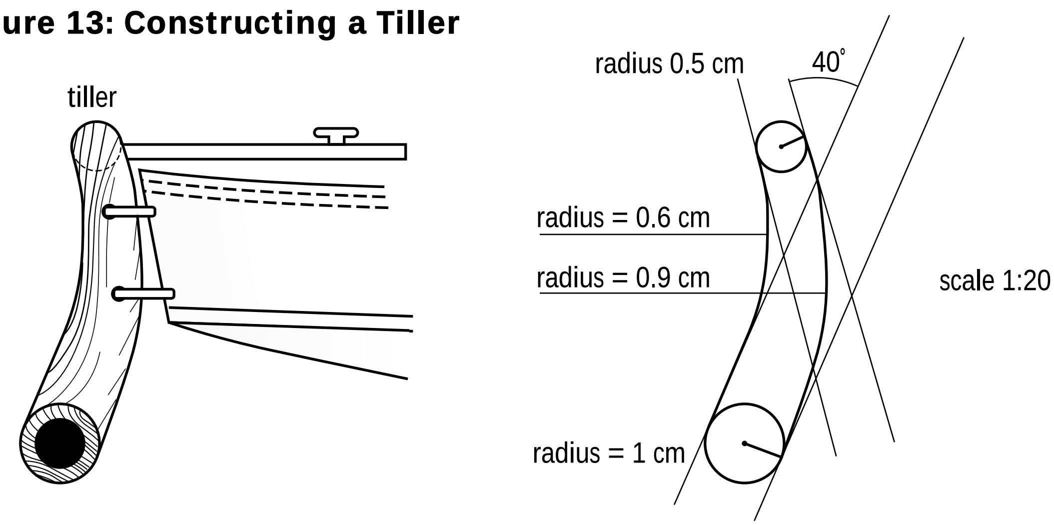
Ask students to identify situations where such designs are used (logo design, landscaping, painting, sculpture, stained glass, and home decoration among other examples).

Ask students to either find some logos and reproduce them, or to sketch and create their own logo. Then ask students what skills they need to develop in order to draw a scale plan and build a cardboard model. Invite them to propose different alternatives to accomplish this task (e.g., computer graphics).

Have students find an example of a curve and ask them to determine how to reproduce a scale plan that includes all the details of the design.

Indicate that in most cases, one has to draw the tangent to a certain curve or draw a curve tangent to a certain line. For example, if students want to draw plans for the tiller of a small boat, as illustrated in Figure 13: Constructing a Tiller below, they need to construct lines tangent to the two circles.

Figure 13: Constructing a Tiller



Ask students to propose a way of drawing a line tangent to a circle at a given point on the circle, and of drawing a circle tangent to a given point on a line.

Ask students to identify the difficulties they would expect in such constructions.

Propose techniques for the following:
- Draw a tangent to a circle at some given point illustrated in Figure 14: Drawing a Tangent to a Circle.
- Draw an arc of a circle tangent to a line at a given point as illustrated in Figure 15: Drawing an Arc of a Circle Tangent to a Line.
- Draw a line tangent to a circle from an external point as illustrated in Figure 16: Drawing an Arc #1.

Figure 14: Drawing a Tangent to a Circle

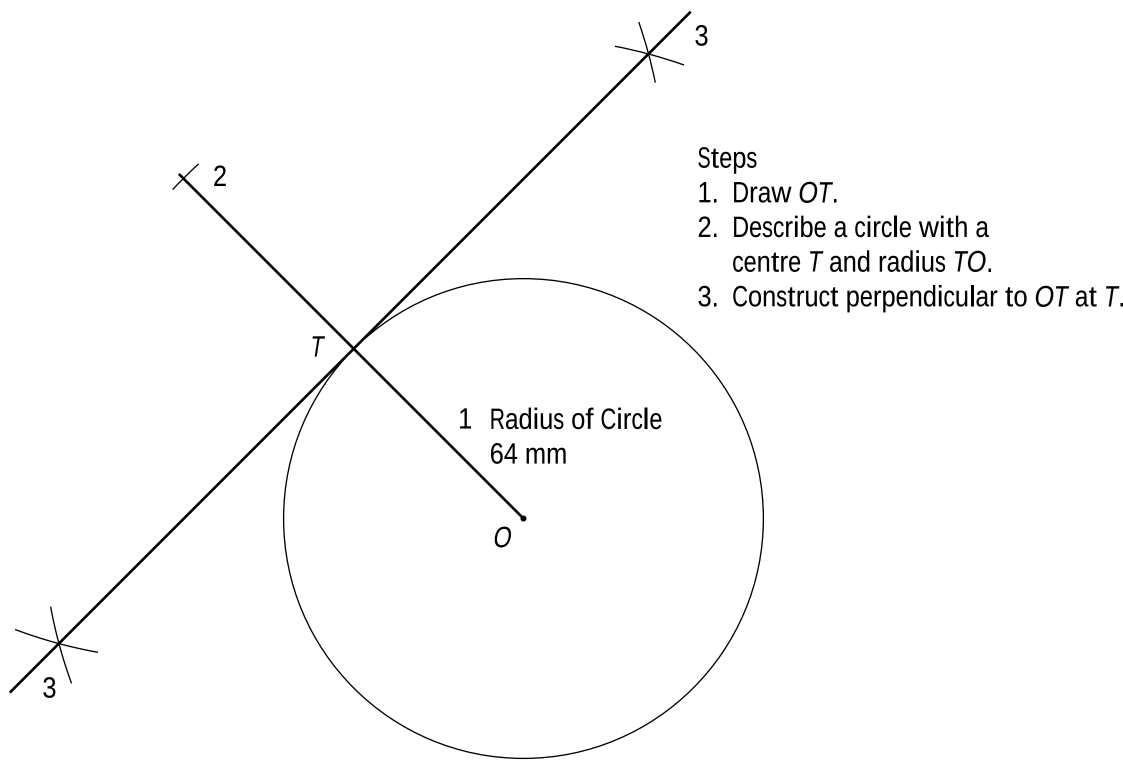
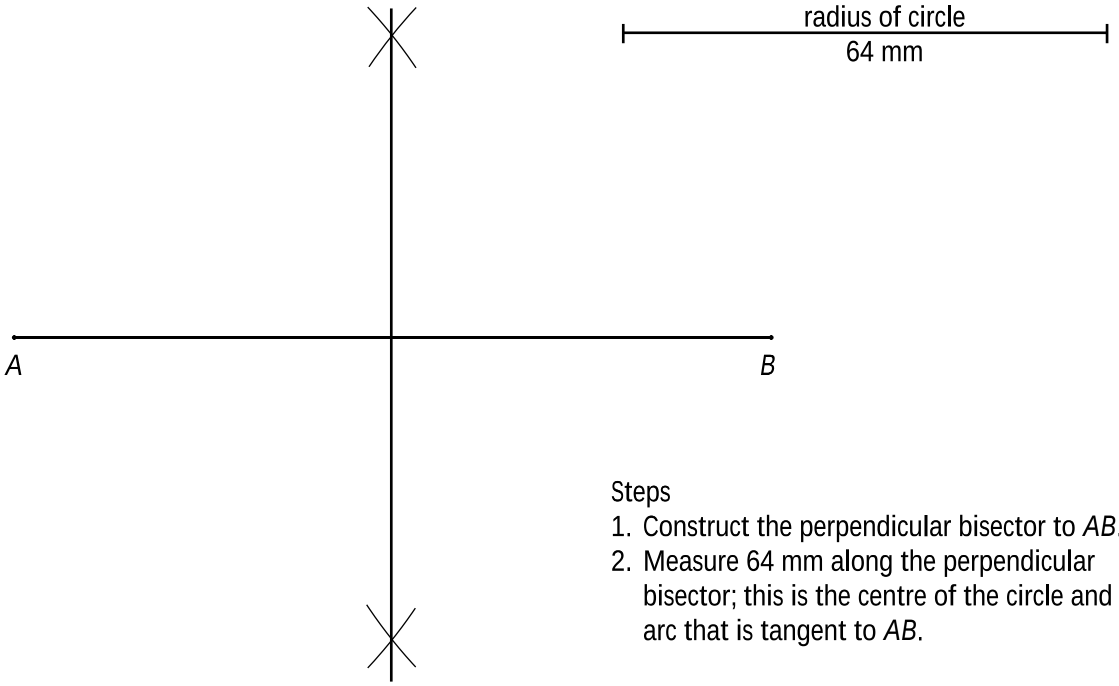
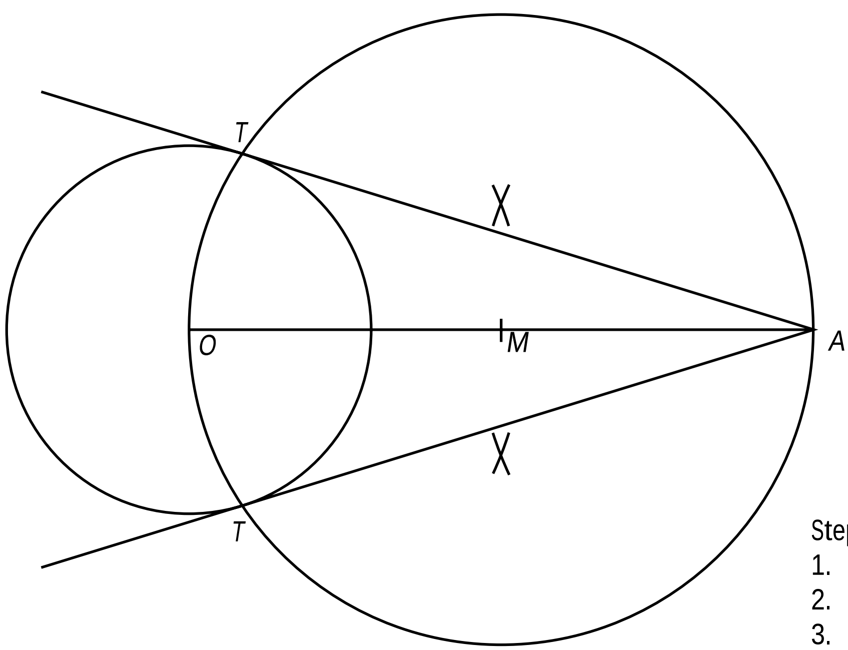


Figure 15: Drawing an Arc of a Circle Tangent to a Line



Steps
1. Construct the perpendicular bisector to *AB*.
2. Measure 64 mm along the perpendicular bisector; this is the centre of the circle and arc that is tangent to *AB*.

Figure 16: Drawing an Arc #1



Steps
1. Draw line *AO.*
2. Find centre of *AO* (*M*).
3. Draw a circle with a centre *M* and radius *MA.*
4. Draw tangent line *AT.*

Have students join two lines by an arc of a circle so that the connections are as smooth as possible. Refer to Figure 17: Drawing an Arc #2-Connecting Two Perpendicular Lines and to Figure 18: Drawing an Arc #3-Connecting Two Parallel Lines).

Figure 17: Drawing an Arc #2 - Connecting Two Perpendicular Lines

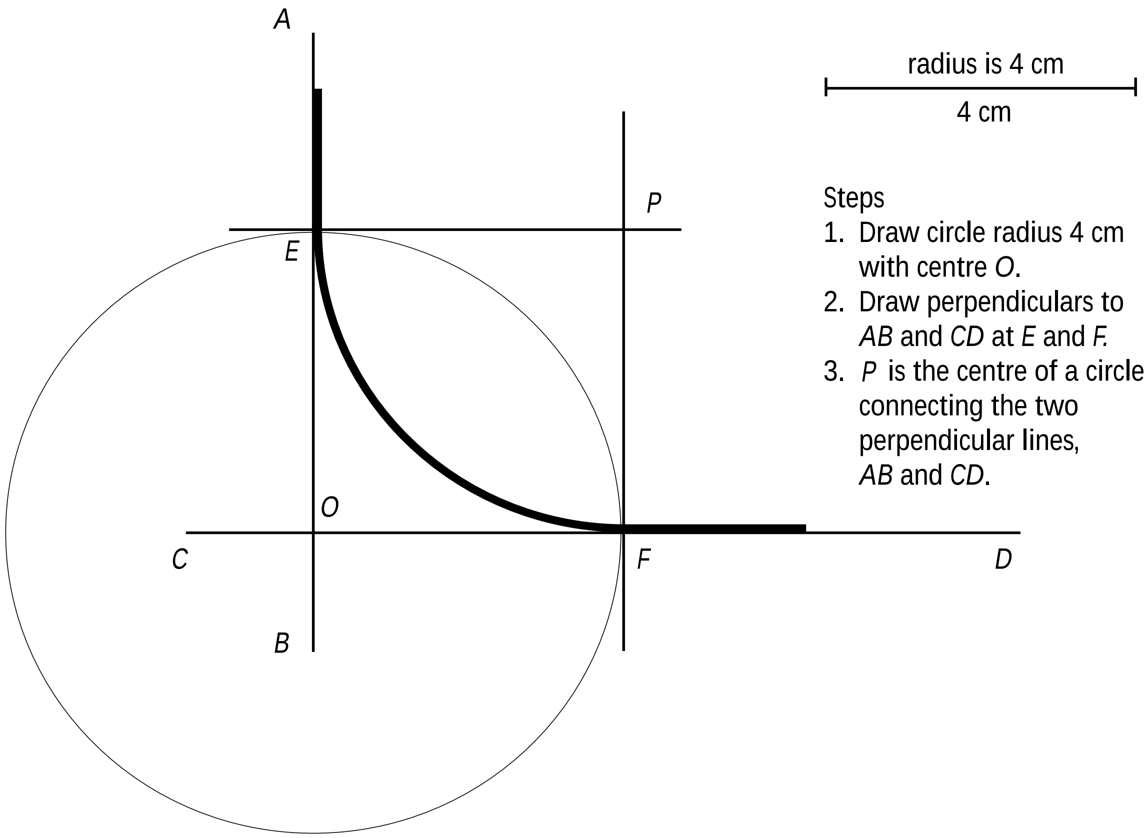


Figure 18: Drawing an Arc #3 - Connecting Two Parallel Lines

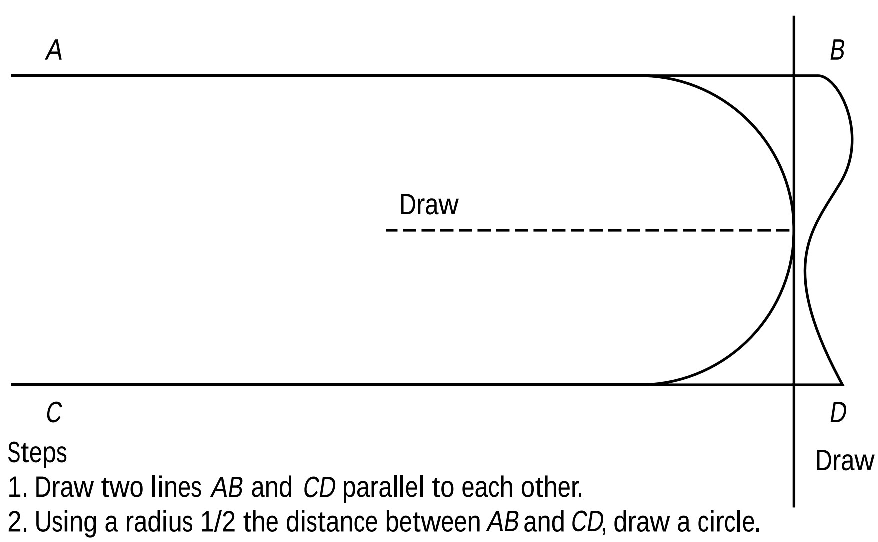
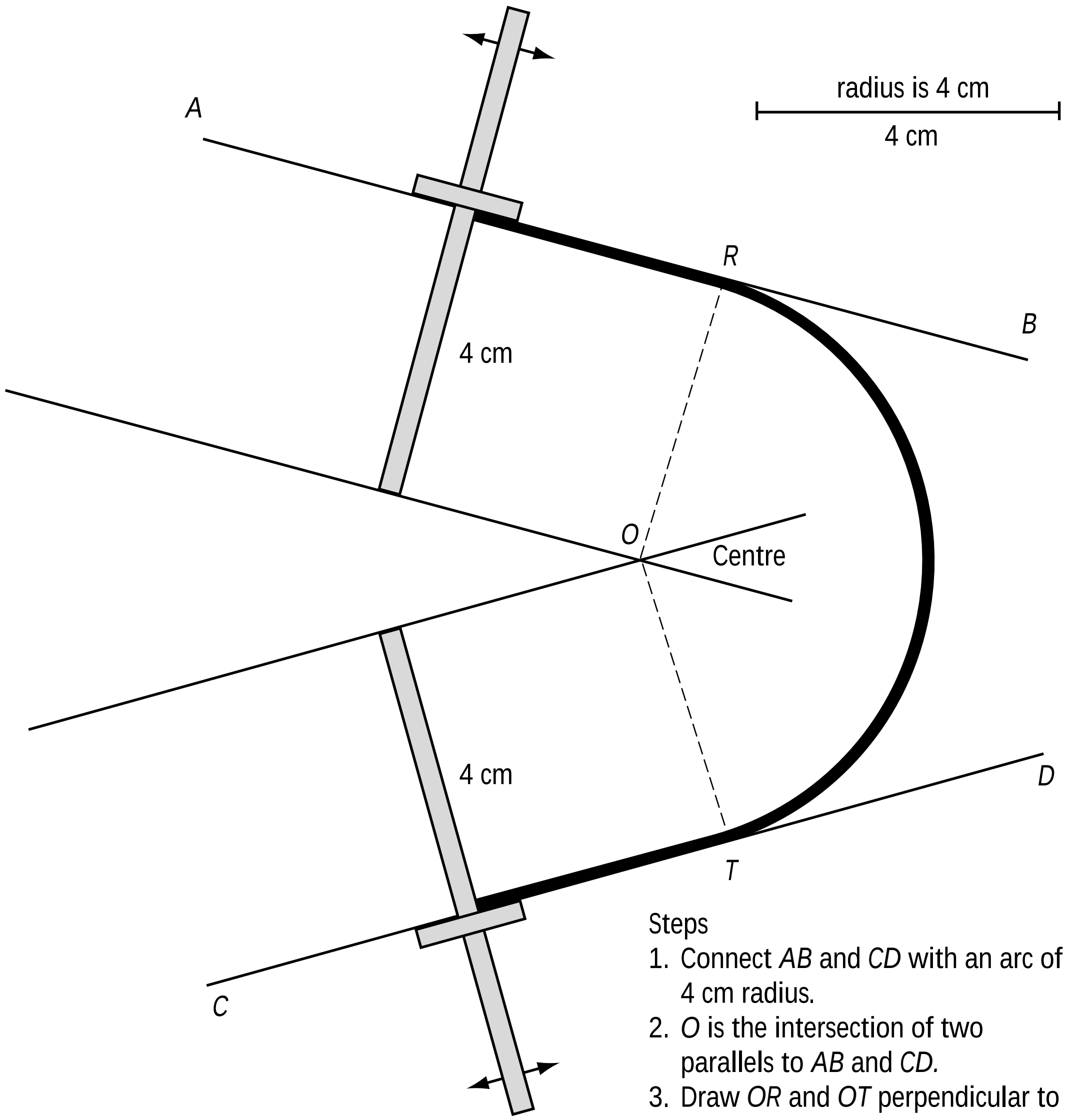


Figure 19: Drawing an Arc #4



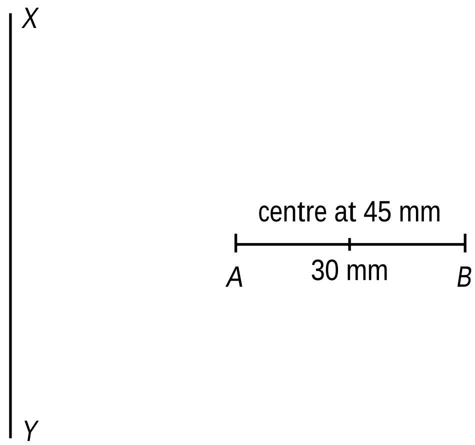
Steps

1. Connect *AB* and *CD* with an arc of 4 cm radius.
2. *O* is the intersection of two parallels to *AB* and *CD.*
3. Draw *OR* and *OT* perpendicular to *AB* and *CD.*
4. Draw the arc of a circle with centre *O.*

Propose the following situations as reinforcement:

- Draw as smooth a line as possible connecting segment *AB* to the arc of a circle of radius 30 mm such that the centre of the circle is situated 45 mm from a line *XY*. Refer to Figure 20: Situation #1.

Figure 20: Situation #1



- In Figure 21: Situation #2, and Figure 22: Situation #3 below, draw as smooth a line as possible connecting the two line segments with two arcs.

Figure 21: Situation #2

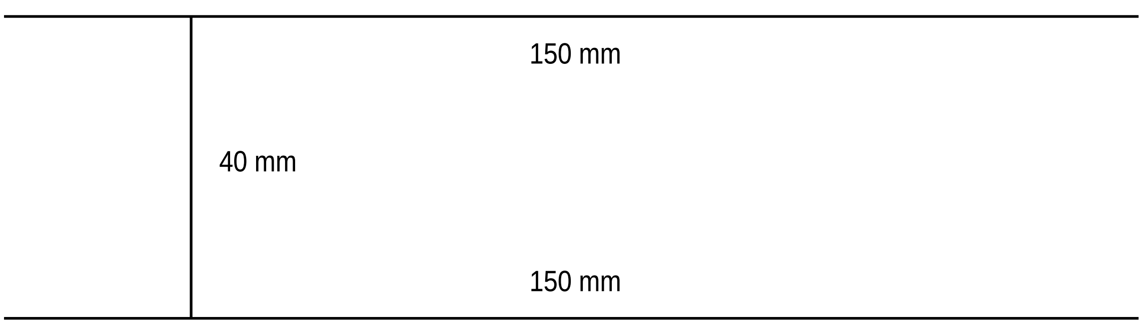
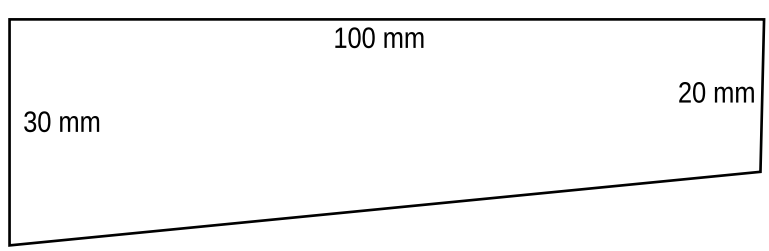


Figure 22: Situation #3



Activity 4: Tangent Circles

Objective:

In this activity, students apply their knowledge about the properties of circles and the locus of a point. Activity 4: Tangent Circles is a continuation of Activity 3: Tangents. Students explore basic geometric transformations and apply co-ordinate geometry to develop a more complex design.

Materials:

- pair of compasses
- ruler
- straight edge
- graph paper
- graphics software

Procedure:

Have students find an example of a curve that connects two circles like the one shown in Figure 23: Connecting Two Circles below. Ask them to determine how they can reproduce a scale plan that includes all the design details.

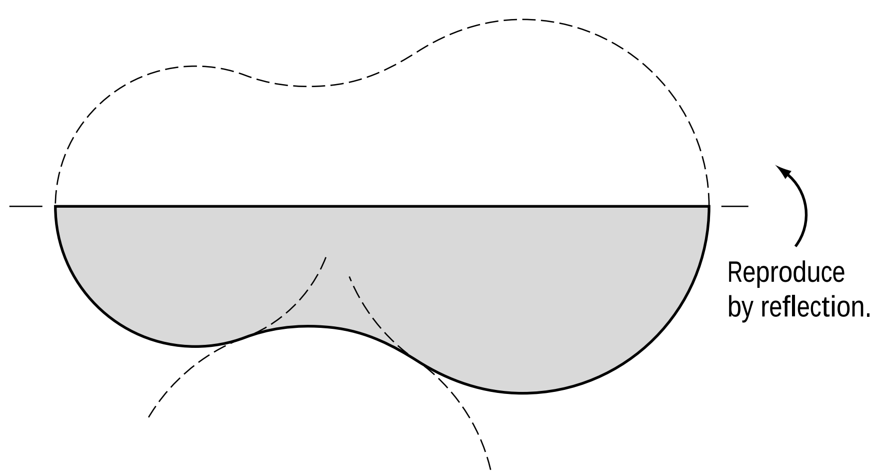
Comment on Figure 23: Connecting Two Circles and explain that in some cases it is necessary to connect two curves as smoothly as possible.

Ask students to create a design that uses connected curves and ask them to propose ways of drawing a scale plan of such a design.

Ask students to find an illustration of a propeller and comment on its design.

Distribute copies of Student Activity: Connecting Curves I to each student.

Figure 23: Connecting Two Circles

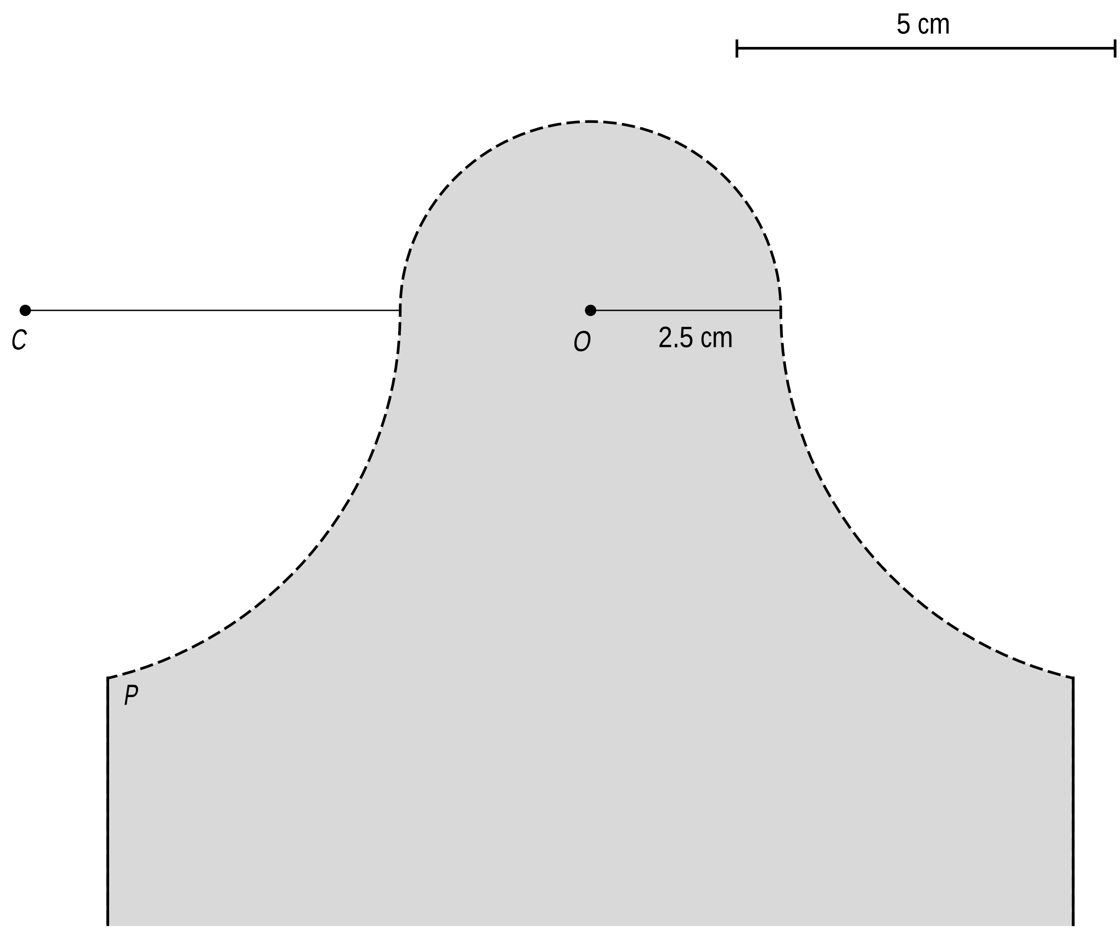


Student Activity: Connecting Curves

Student Name: ______________________ Date: ________________

1a. Draw an arc, tangent to another arc from a given point when the radius is known.

Figure 24: Connecting Curves #1



Given: Circle with centre *O* and radius = 2.5 cm
Point *P* on circle with radius 5 cm

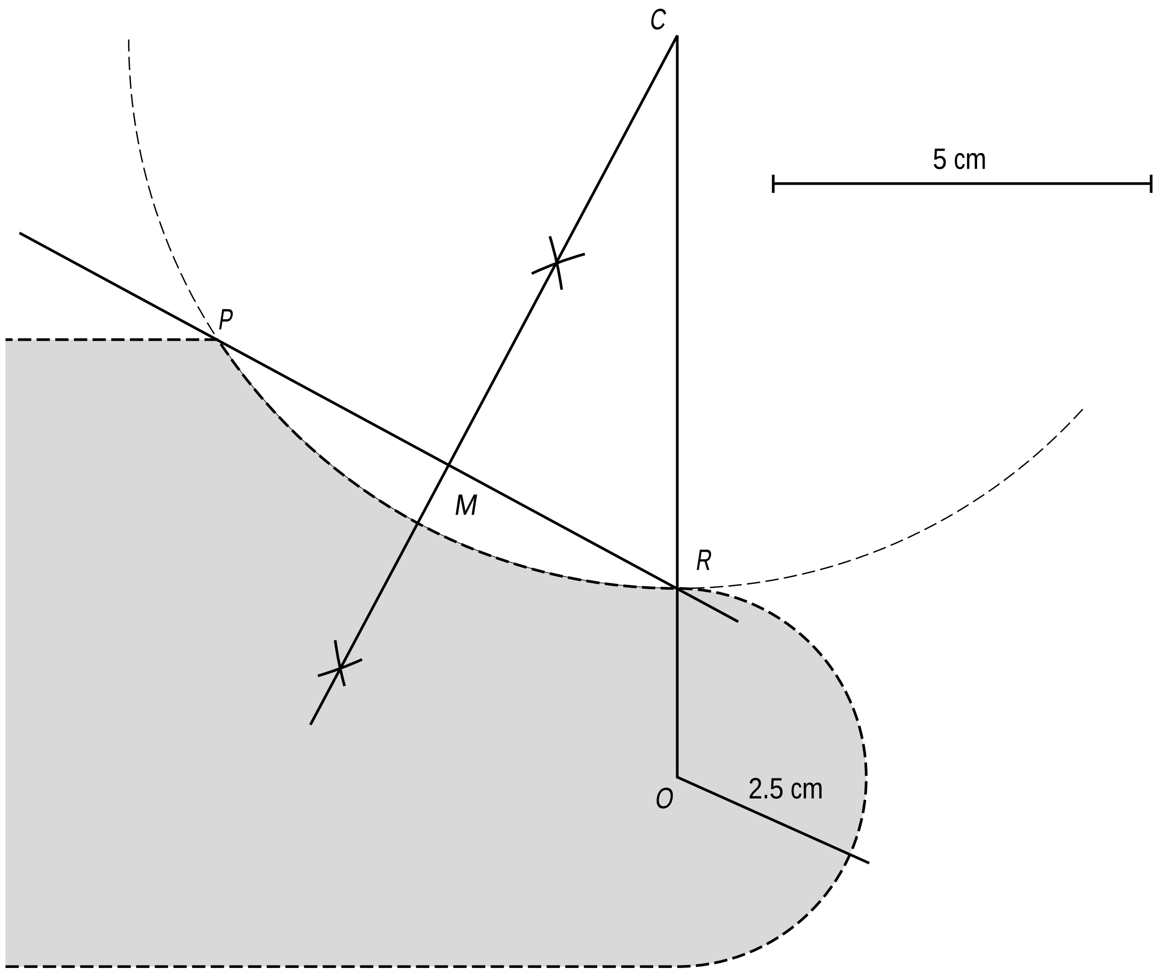
Steps

1. Draw an arc using *P* as the centre and a radius of 5 cm.
2. Draw an arc using *O* as the centre and a radius of 5 + 2.5 cm.
3. Draw an arc with a centre *C* (intersection of the two arcs from steps 1 and 2) and a radius of 5 cm.

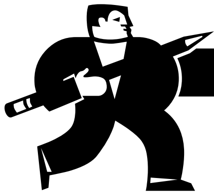
Student Activity: Connecting Curves

1b. Draw an arc, tangent to another arc from a given point when the point of contact is known.

Figure 25: Connecting Curves #2



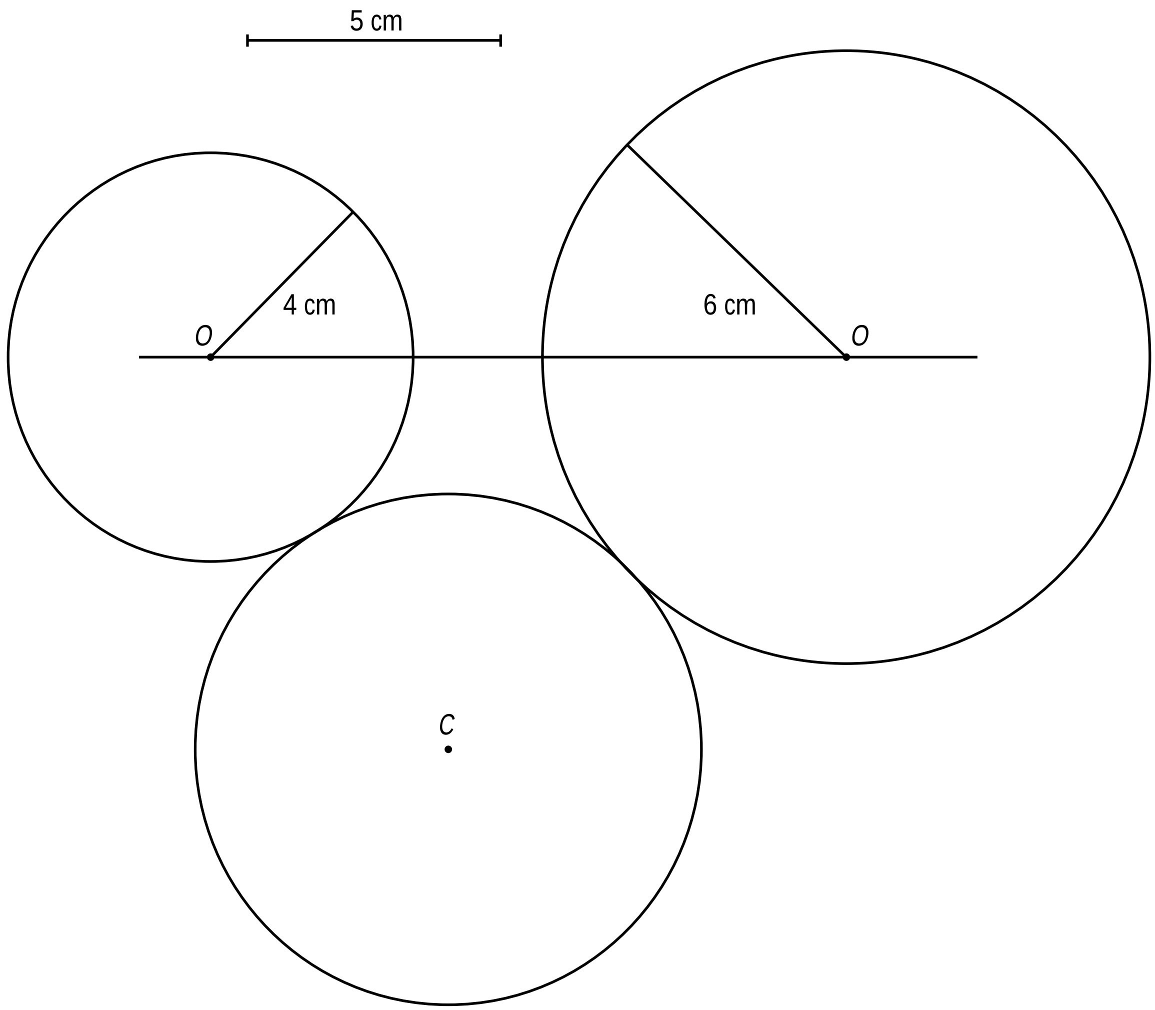
Steps
1. Draw a circle with a centre *O* and a radius = 2.5 cm.
2. *R* is the point of contact of two arcs.
3. Draw *OR* and *PR.*
4. Draw perpendicular bisector of *PR*.
5. Centre, *C,* is at the intersection of the perpendicular bisector and the line *OR.*



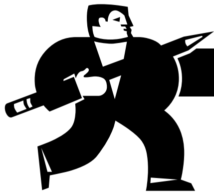
Student Activity: Connecting Curves

2. Connect two arcs with a given arc.

Figure 26: Connecting Two Arcs with a Given Arc



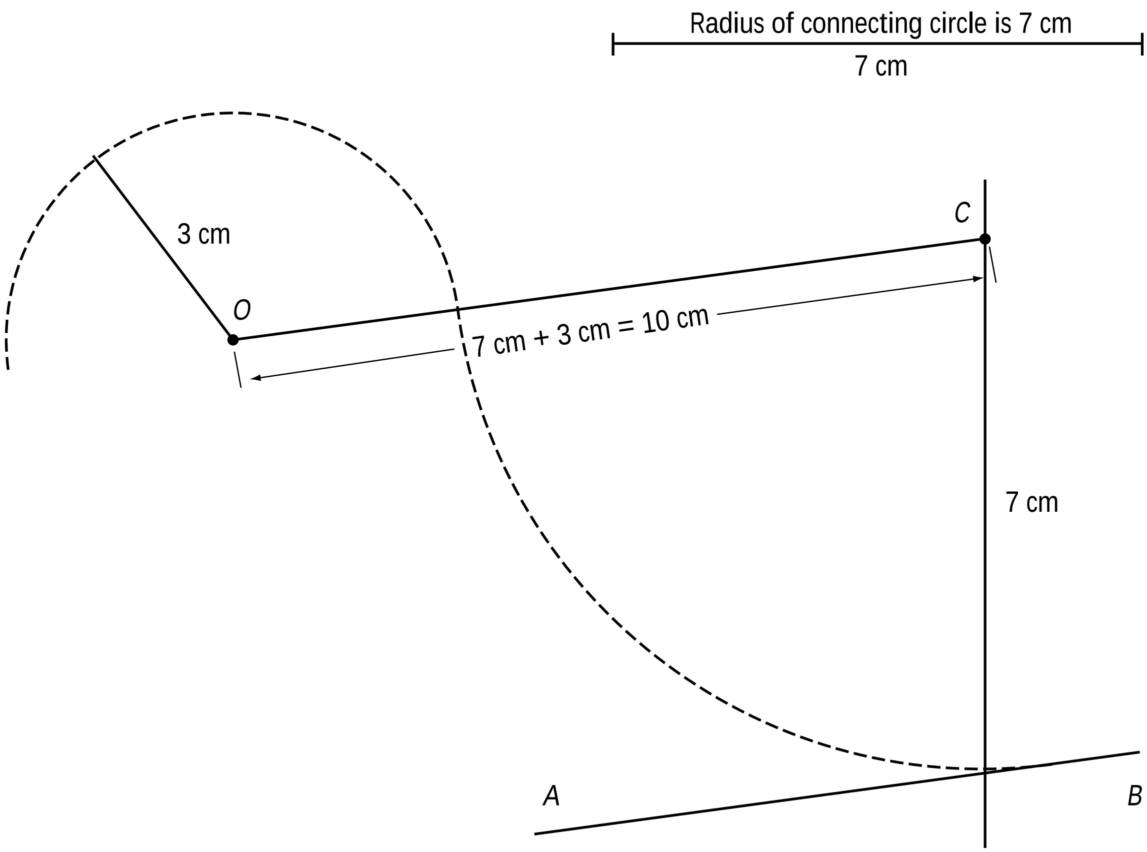
Steps
1. Draw an arc with centre *O*, radius 6 + 5 = 11 cm.
2. Draw a second arc so as to intersect the first arc, with centre *O*, radius 4 + 5= 9 cm.
3. *C* is the point of intersection of the two arcs above and the centre of the connecting curve (circle tangent to the two arcs).



Student Activity: Connecting Curves

3a. Connect an arc and a line when the radius is known.

Figure 27: Connecting an Arc and a Line When the Radius is Known



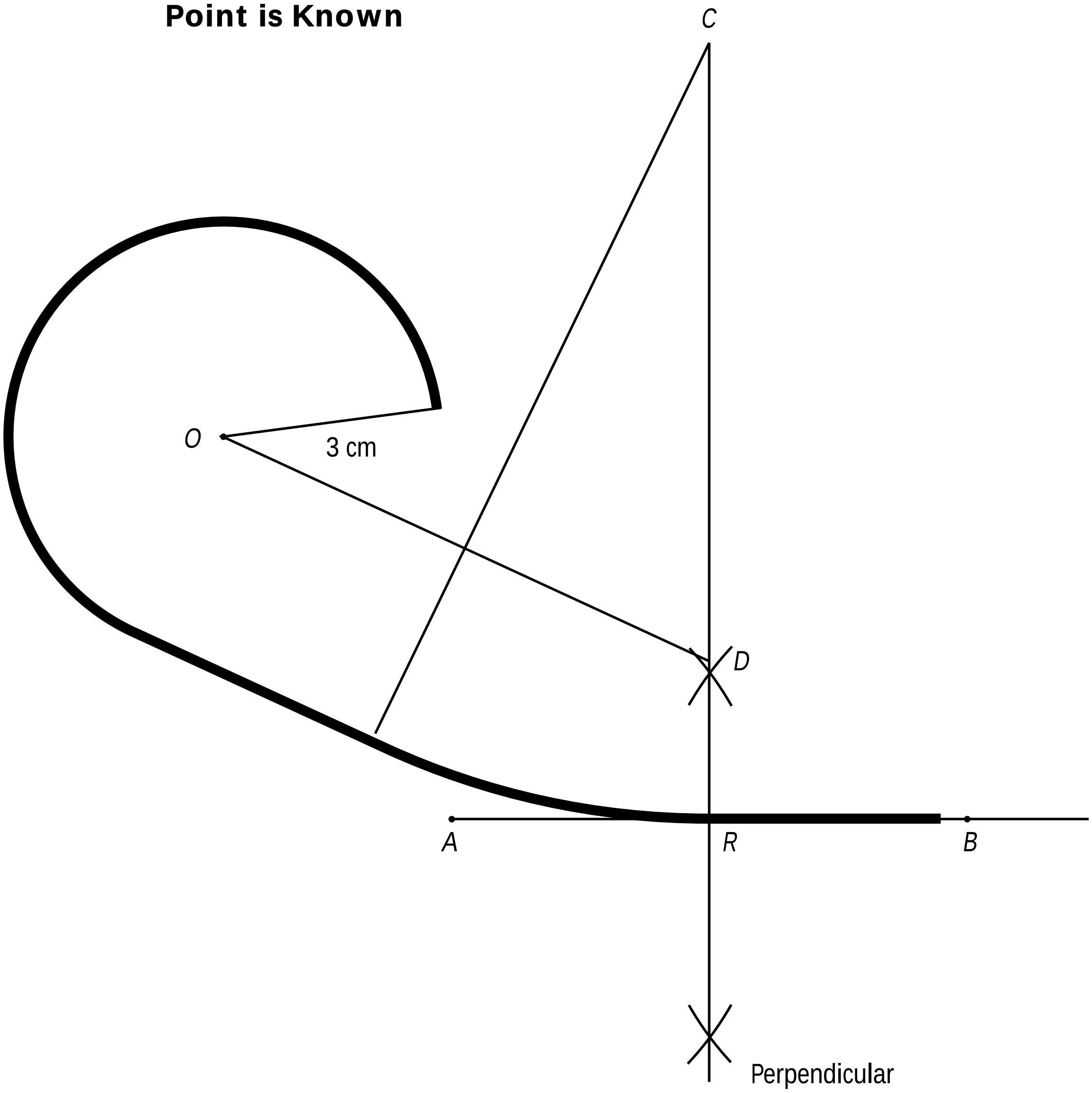
Given:
AB is the line, *O* is the centre of a circle.

Steps
1. Draw circles of radius 3 cm and 10 cm (7 cm + 3 cm).
2. Draw a line parallel to *AB* at 7 cm to obtain point *C*.
3. Draw an arc of a circle with centre *C* and a radius of 7 cm.

Student Activity: Connecting Curves

3b. Connect an arc and a line when the connecting point is known.

Figure 28: Connecting an Arc and a Line When the Connecting Point is Known



To connect the circle (centre *O* and radius 3 cm) to line *AB*, the connecting point *R* is given.

Steps

1. Draw a circle with a radius = 3 cm and centre *R.*
2. Point D is on the perpendicular to *AB* through *R*. *RD* =3 cm.
3. Connect *OD.*
4. Draw perpendicular bisector to *OD.*
5. C is centre of arc connecting circle with centre *O* to point *R* on *AB.*

Activity 5: Transformations and Logos

Objective:

In this activity, students construct a design of their choice, applying the knowledge and skills they acquired in the previous activities. In doing this activity, they realize the importance of geometric transformations. They also learn how to reduce the number of the steps needed to construct a model having symmetry.

Materials:

- pair of compasses
- ruler
- straight edge
- graph paper
- graphics software

Procedure:

Have students apply geometric transformations on the design they created in the previous activity. The geometric transformations are translation, rotation, reflection, and dilation.

Encourage students to place their cardboard model of the curve on graph paper and to construct different images by using co-ordinate geometry.

Have students accurately reproduce their design on a computer using graphics software. Encourage them to perform various transformations in order to create a complex design.

After completing each transformation, have students identify each of the geometric transformations and the image of the original design.

Some students may be interested in reproducing their design in the school parking lot, for example.

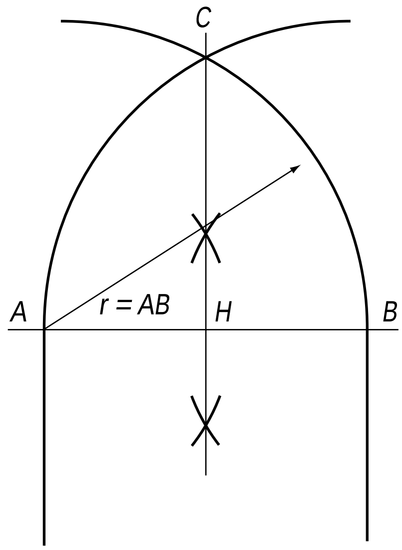
Exercises

- Draw an arc of radius 20 mm tangent to an arc of radius 30 mm. Show all the steps.
- Draw a vertical line and a circle whose center is at 25 mm from the vertical. Connect the line and the arc with an arc of 12 mm radius. Show all the steps.
- Pick a bow and a stern from Student Activity: Bow and Stern Design and connect them to form a boat. Draw the pattern on cardboard and cut it out. Students should work in pairs.

Extension: Curves that are not circles.

How to draw curves commonly used in designs.

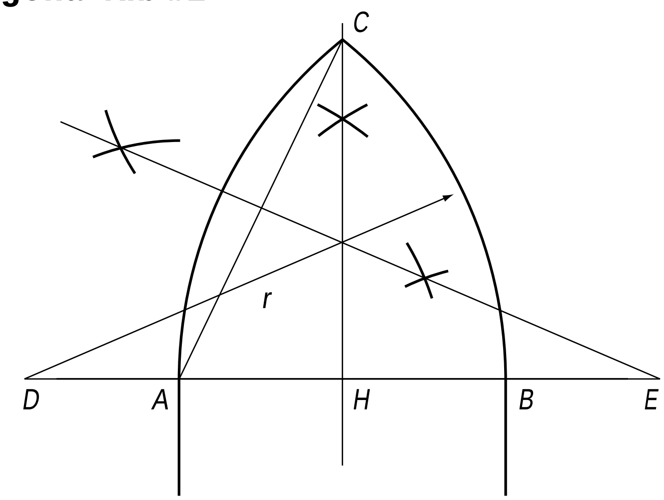
Figure 29: Diagonal Rib #1



AB

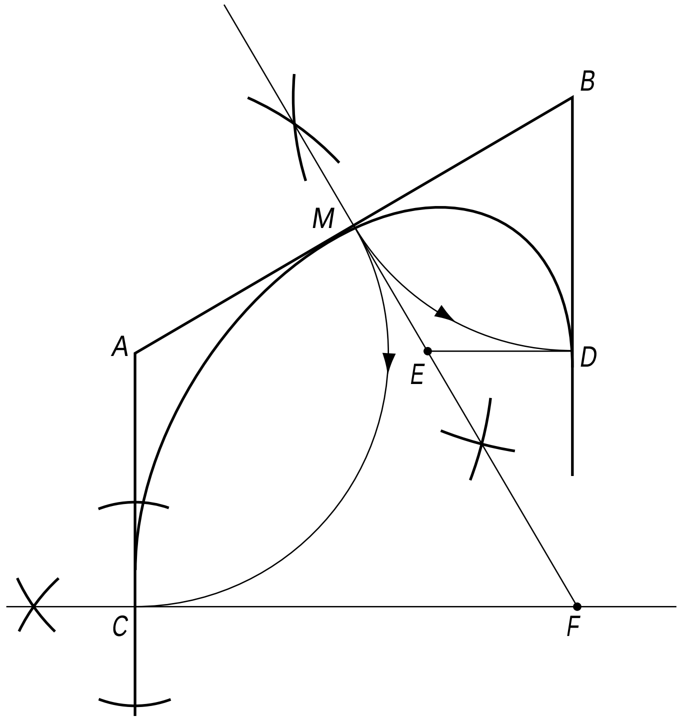
AB is the opening.
HC is the height.
B is the vertex.
From *A* and *B*, draw circles of radius *AB*.

Figure 30 Diagonal Rib #2



AB is the opening.
From *D* and *E* (*HD* = *HE*) draw circles of radius *r* = *DB*.

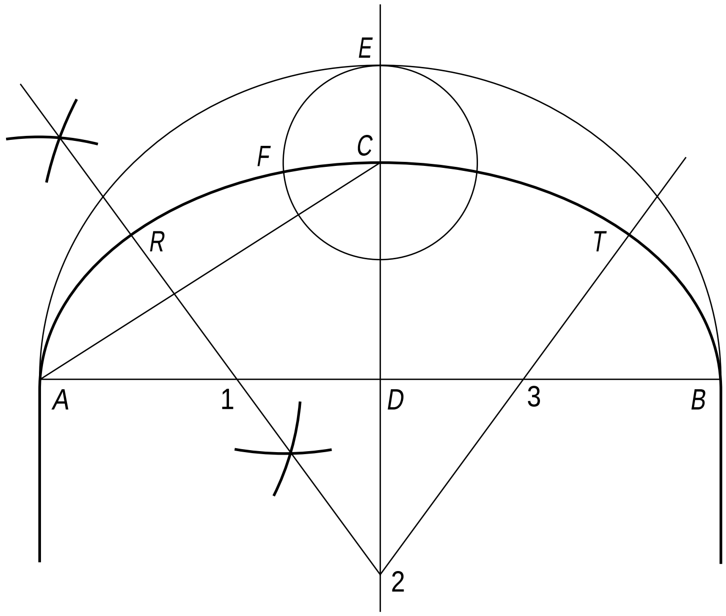
Figure 31: Ramping Arc



Given:
M is given and it is the connecting point of the two arcs with centres *A* and *B*.
Three lines are given.
BD and *AC* are parallel while *AB* is not parallel.

Steps:
1. Draw *AC* = *AM* and *BM* = *BD.*
2. Draw perpendiculars at *C, M* and *D.*
3. *E* and *F* are the centres of the two arcs tangent to the lines, *MC* and *MD.*

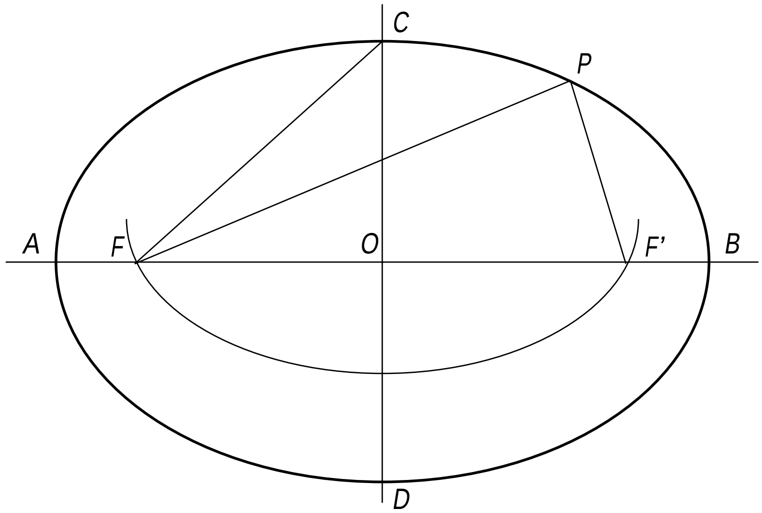
Figure 32: Basket Handle



Steps

1. *AB* is the opening and *CD* the height of the basket handle.
2. Draw a half-circle of radius *AD* with centre at *D*.
3. *CE* is the difference of half the opening minus the height.
4. Draw circle of radius *CE* and centre *C.*
5. Draw *AC.*
6. Draw perpendicular at middle of *AF* to give points 1 and 2.
7. Draw point 3: *D* - 3 = 1 - *D.*
8. Points 1, 2 and 3 are centres of the three arcs.
9. Draw *AR, RC, CT* and *TB.*

Figure 33: Ellipse



Given:
AO
AB = 14 cm
CD = 8 cm
AO is half the major axis

Steps
1. From centre *C* and radius *AO* = 7 cm, draw a circle to find F and F' (foci).
2. Using a rope of length *AB* (14 cm), fix ends at *F* and *F'*.
3. Draw the ellipse (keeping the rope tied).

Activity 6: Forces and Levers

Objective:

The objective of this activity is for students to relate the properties of triangles to forces acting on a structure. Tension and compression are the key forces that cause structures to stay up or fall down.

Materials:

(per pair of students)
- set of weights of 100 g, 500 g, 1 kg
- set of rigid bars of different lengths (wooden sticks 1 x 1 inch)
- graduated stick
- dynamometer
- string
- set of clamps

Procedure:

Tension within structures is a pulling force. It stretches the materials. We can see evidence of tension in rope bridges, telephone wires, tents, suspension bridges, steel cables supporting an elevator, and so on.

Compression is a pushing force. It squashes the materials. We can see evidence of compression in pyramids, telephone poles, arc bridges, tree trunks, and a squashed marshmallow.

Present Figure 34: Diagram of Forces #1.

Ask students to give examples of forces acting on pyramids, telephone poles, arc bridges, tree trunks, and a squashed marshmallow. Discuss these forces in terms of tension and compression.

Briefly review the concept of forces (direction, intensity, and point of application) by asking students to use the dynamometer and either lift some weights of different masses or pull weights from a hook attached to the wall.

Have students identify and name three important characteristics of forces (i.e., intensity, direction, and point of application) in various situations. Have students record their readings and indicate the direction in which the force is applied.

Review the Law of Levers by using the graduated stick and weights of various masses. Some suggestions for doing this are presented below in Figure 35: Diagram of Forces #2.

Figure 34: Diagram of Forces #1

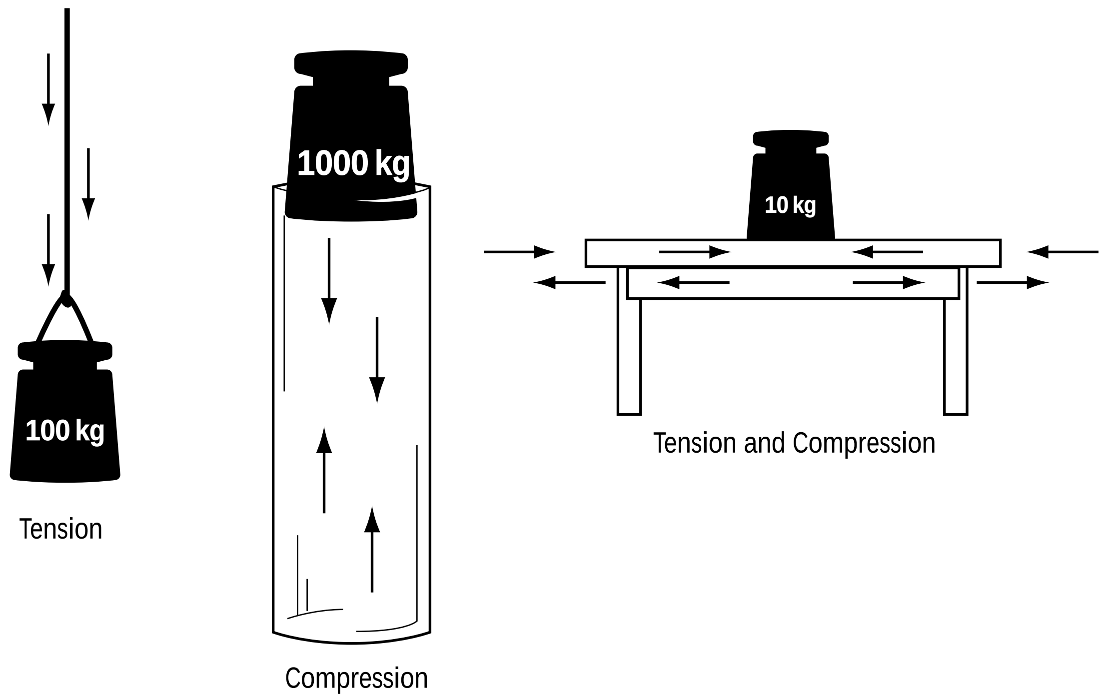
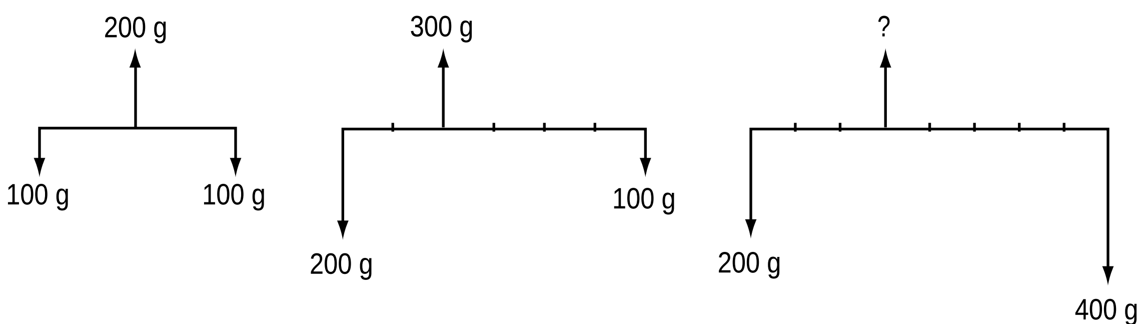


Figure 35: Diagram of Forces #2

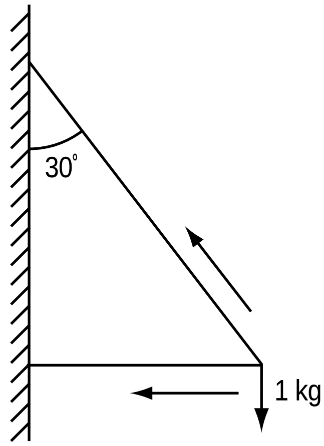


Discuss the principle of Action-Reaction with students. Ask them to identify the direction and the magnitude of the reaction once a force is applied at a particular point.

Ask students to draw a scale diagram to represent the action of a set of forces like those illustrated in Figure 35: Diagram of Forces #2. Students need to determine an appropriate scale for the diagram. For example, 100 g is represented by 1 cm on the scale diagram.

Ask students to apply the Pythagorean Relation and/or the tangent ratio to determine the components of the force applied on the wall bracket illustrated below in the Figure 36: Components of Force. Students may use the dynamometer to verify the information and then draw a scale diagram to represent the situation.

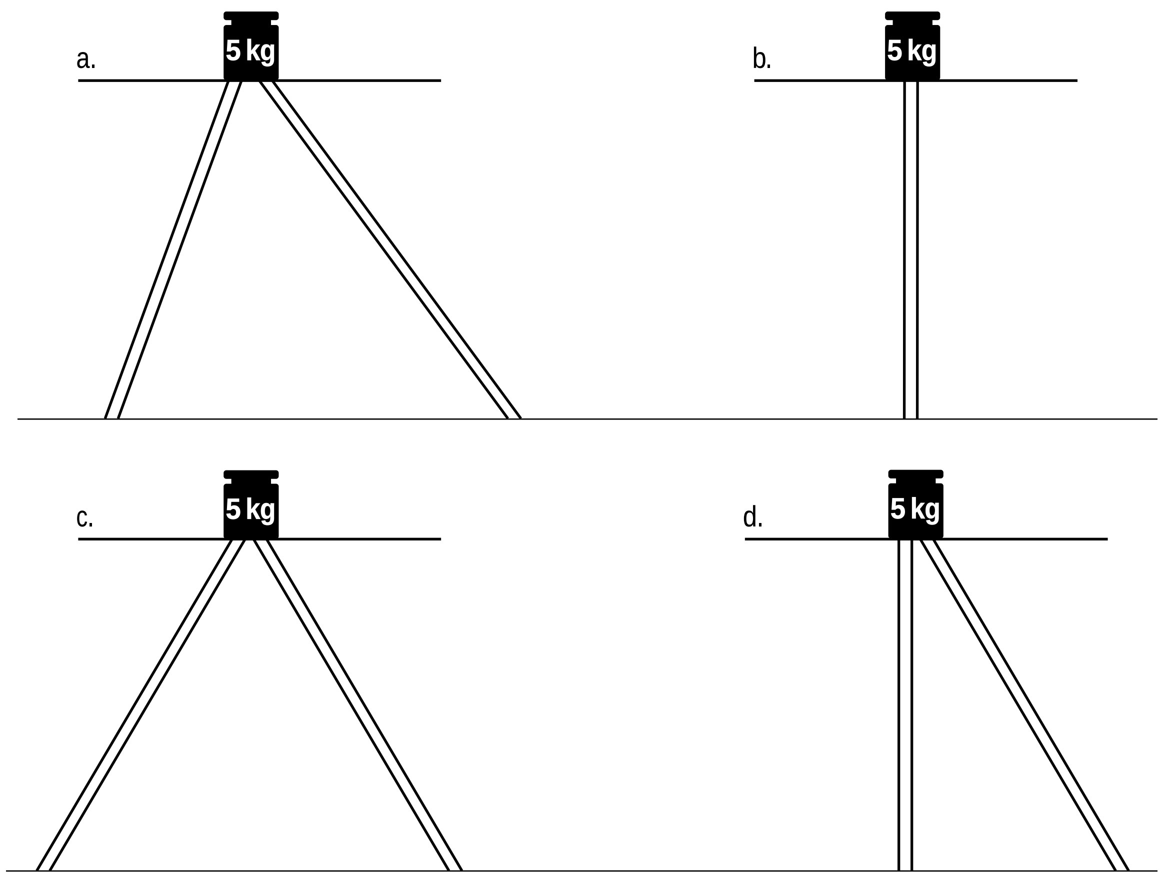
Figure 36: Components of Force



Illustrate the composition of a force with the following situation.

Use the wooden craft sticks to design the different models of structures that must support a mass of 5 kg. These are shown in Figure 37 : Structures Supporting 5 kg Mass (a, b, c, and d below).

Figure 37: Structures Supporting 5 kg Mass



Ask students to draw a scale diagram for each setting.

Ask students to decide which is the best structure by considering the resistance of the sticks to axial stress.

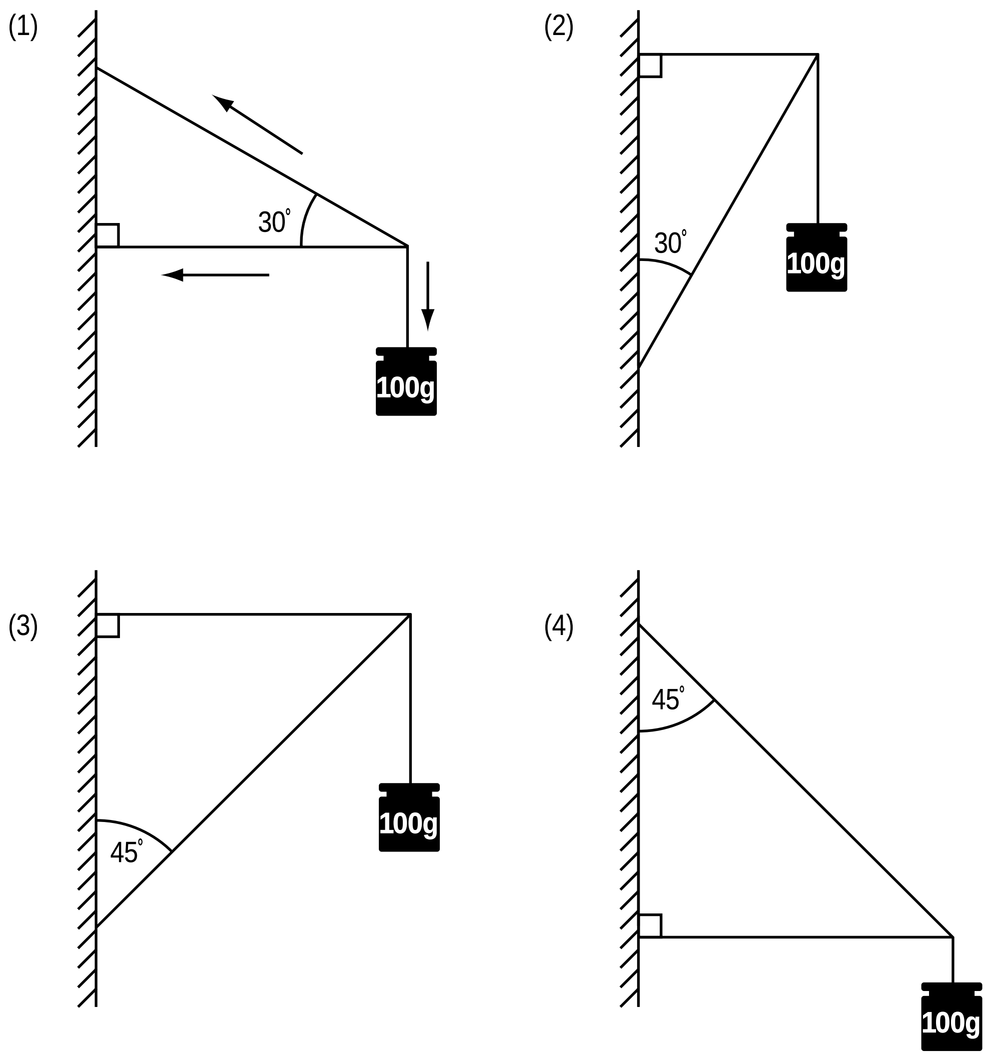
Students may verify their conclusions with strands of spaghetti.
Have students examine and comment on the situations presented in Student Activity: Representing Force. Ask them to draw a scale diagram for each example.

Student Activity: Representing Force

Student Name: ______________________________ Date: ____________________

Complete the following diagrams by representing the force diagram, that is, the composition of the force applied to the wall brackets.

Figure 38: Force Diagrams



Activity 7: Geodesic Domes

Objective:

The objective of this activity is for students to research properties and examples of geodesic structures. Web sites are a good source of information for these. Have students identify geodesic structures around them.

In this activity students construct a geodesic dome with rolled newspaper and explore the stresses experienced by the struts at the various joints. They analyse the advantages of geodesic domes compared with other structures.

Materials:

(per group of 2-3 students):

- 4-5 newspapers (large format)
- 1 broom handle
- stapler
- blue and red paint
- string
- chalk

Procedure:

Refer to Figure 39: Sequence of Steps for Constructing a Geodesic Dome, on the following pages to review the sequence of operations.

Step 1: Preparing the Struts
Have students roll newspaper around a broom handle (as shown in the illustration) to produce at least 75 struts of a 75 cm length. Then have them cut 35 struts, 71 cm long, and 30 struts of a 60 cm length.

Step 2: Drawing the Base
Students draw a circle on the floor so that they can inscribe a regular decagon with a 65 cm side inside the circle. Ask students to think of a reason for doing this.

Step 3: Constructing the Base
Students arrange the struts of the base as illustrated. Struts are attached using staples to make the joints.

Steps 4 to 9: Building the Structure With Trusses
Students follow the steps as indicated until the dome is complete.

By observing the behaviour of the staples at each of the joints, students should identify which struts are experiencing compression and which struts are experiencing tension. The overall force is the weight of the structure.

Have students paint the struts experiencing compression in blue and the struts experiencing tension in red.

Have students select a few joints and draw a scale diagram of the forces. For this task, remind them that no magnitude is required, only direction.

Ask students to attach a mass (500 g or 1 kg) at different joints and draw the scale diagrams. They can verify their prediction by observing the behaviour of the staples at the joint.

Ask students to compare the resistance of a geodesic dome compared to other structures.

What are some of the consequences of having smaller struts, that is, more triangles?

What would be the resistance of a structure of this kind composed of very small struts?

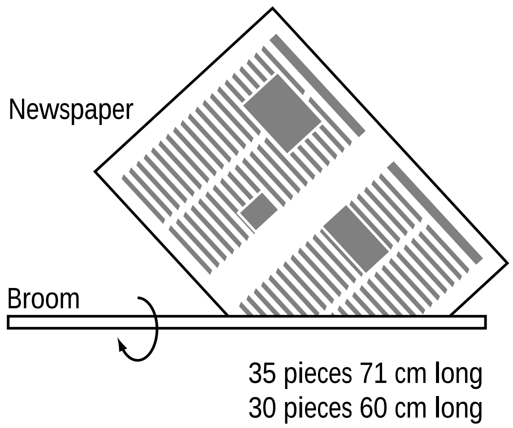
What are some reasons structures in nature, like egg shells, have a curved form?

What are some reasons geodesic structures are not as popular as they could be, considering their interesting features?

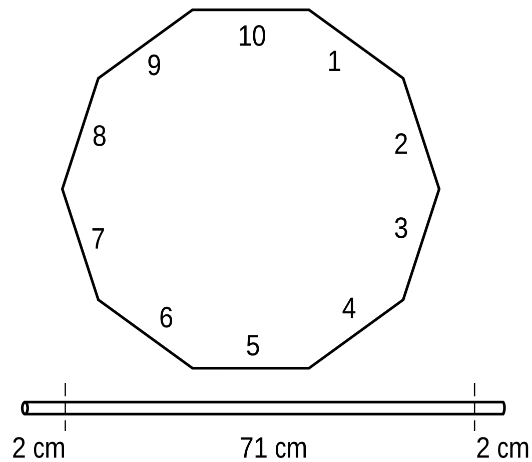
The following pages illustrate the sequence of construction steps for a geodesic dome.

Figure 39: Sequence of Steps for Constructing a Geodesic Dome

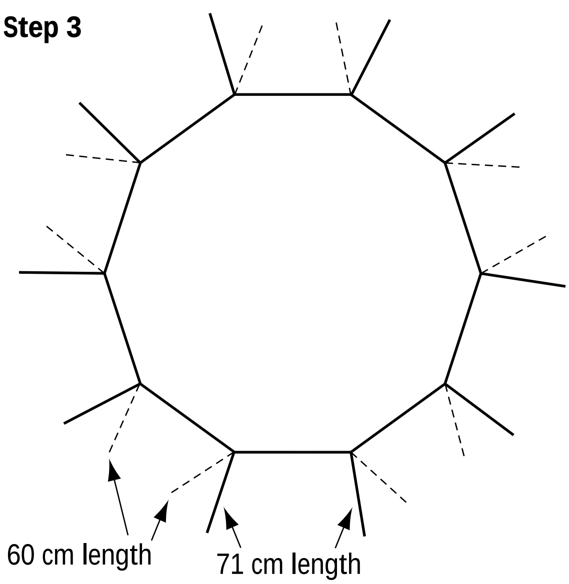
Step 1



Step 2 Place on the floor around the circle.



Step 3



Step 4

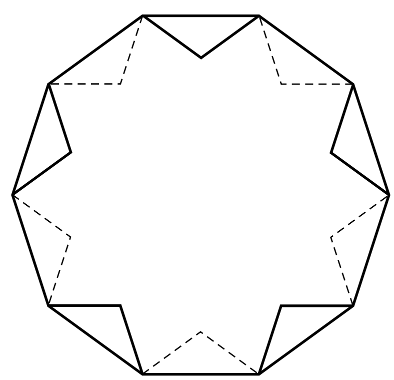
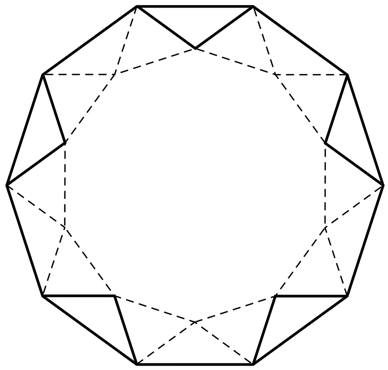
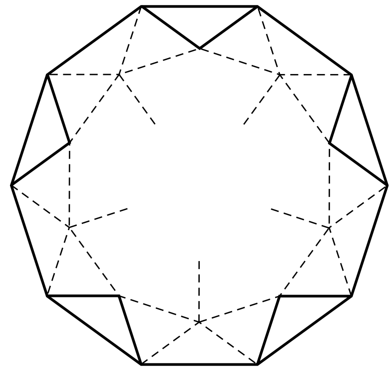


Figure 40: Sequence of Steps for Constructing a Geodesic Dome

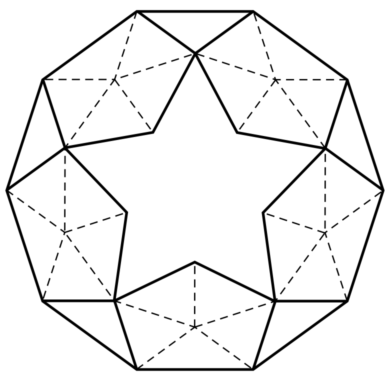
Step 5



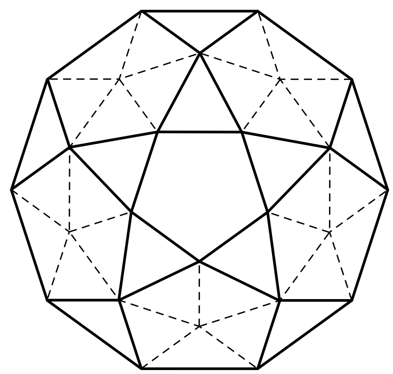
Step 6



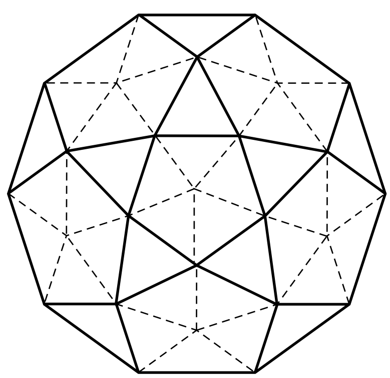
Step 7

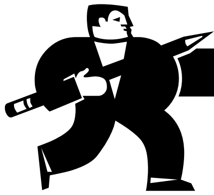


Step 8



Step 9





Student Activity: Report on Geodesic Dome Research

Student Name: ____________________________ Date: ___________________

Geodesic dome
(comments)

Activity 8: Reviewing Geometric Solids

Objective:

The objective of this review activity is to ensure that students understand the properties of geometric solids.

Materials:

(per pair of students)
- cardboard
- ruler
- scissors
- tape

Procedure:

If necessary, the teacher may ask the students to complete Student Activity: Two-Dimensional Shapes.

As a small project aimed at reviewing earlier concepts, students may evaluate the volume of a log, the volume of lumber of various dimensions, and cross-sections obtained from a log of given dimensions.

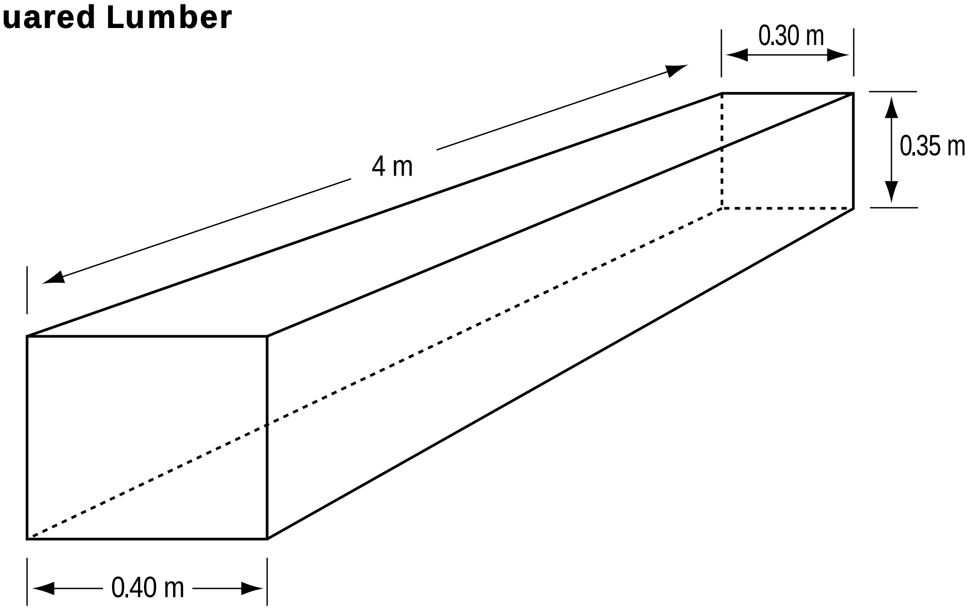
Students may research different guidelines used by foresters. Here are some examples teachers may use to review the concepts of the properties of geometric solids.

Determining the Volume of a Squared Log
Calculate the average width of the squared log (assuming there is a gradual taper). In Figure 41: Squared Lumber:

Average Width = $\left(\frac{0.40m + 0.30m}{2} = 0.35m\right)$

Volume will be: 0.35 m x 0.35 m x 4 m = 0.49 m^3
(w) (h) (l)
(h = log height)

Figure 39: Squared Lumber



Determining the Volume of a Short Log

Use the formula of the volume of a cylinder having the average circular area as its base. In Figure 42: Cylinder, below:

Volume will be: $\pi r^2 h = 3.14 \times \left(\frac{0.35m + 0.25m}{2}\right)^2 \times 5m$

$V = 3.14 \text{ x } (0.30m)^2 \text{ x } (5m)$

$V = 1.41 \ m^3$

or, since it is easier to measure the average circumference rather that the average radius,

$$\text{Volume} = \frac{1}{\pi} \times \left(\frac{\text{circumference}}{2}\right)^2 \times h$$
$$= 0.318 \times \left(\frac{\text{circumference}}{2}\right)^2 \times h$$

Determining the Volume of a Long Log

One usually divides the long log into smaller pieces (2, 3, or 5). See Figure 43: Dividing a Long Log.

Figure 42: Cylinder

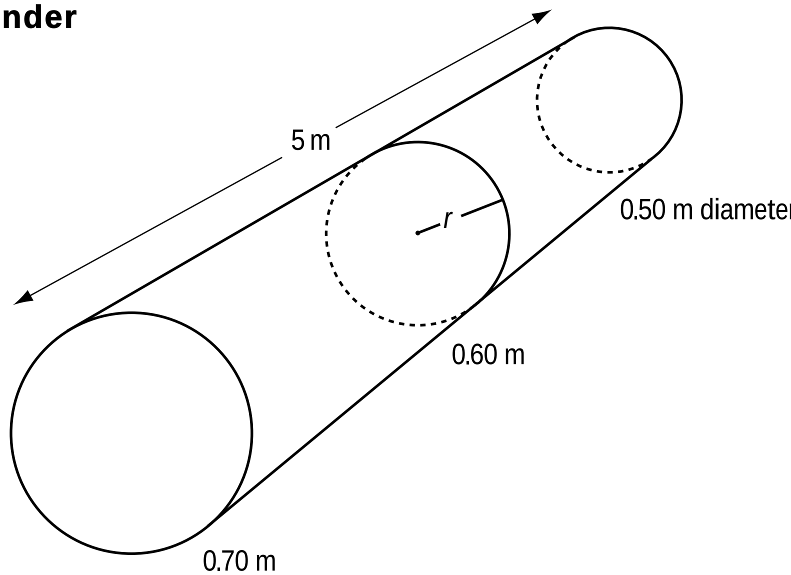
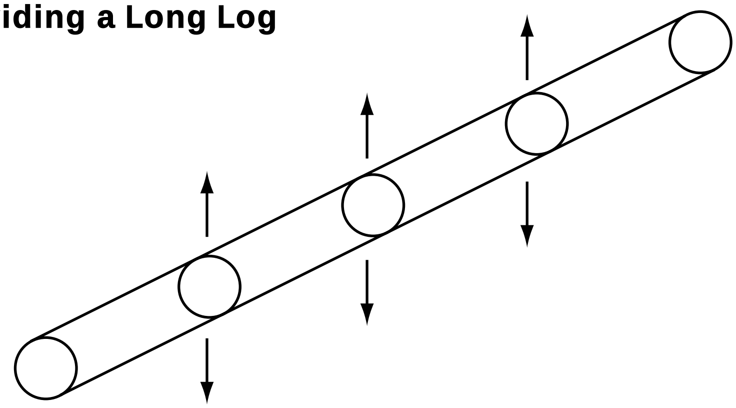


Figure 43: Dividing a Long Log



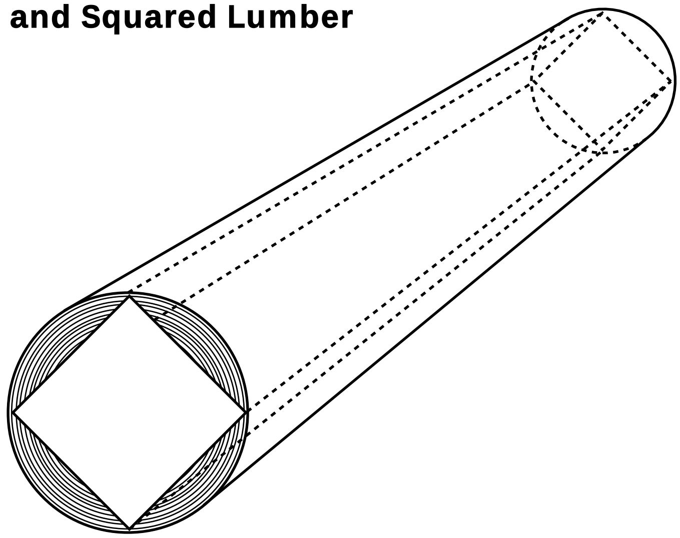
Estimating the Volume of a Squared Log

It is usually appropriate to use the formula,

$\text{Volume} = \left(\frac{C}{5}\right)^2 \times h,$ in which C is the average circumference of the log.

Ask students to estimate the decrease of volume from a log to a squared log (Figure 44: Log and Squared Lumber below).

Figure 44: Log and Squared Lumber

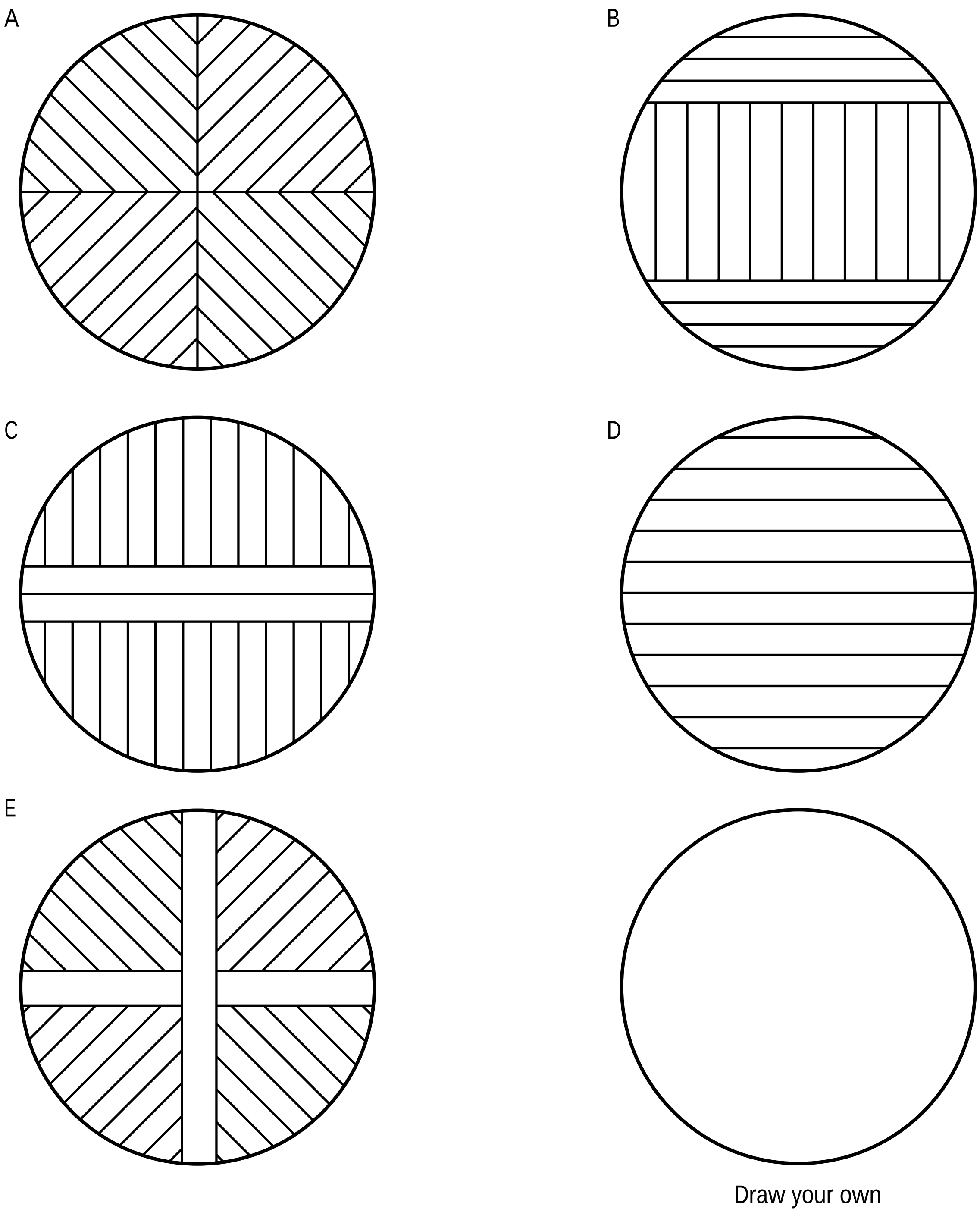


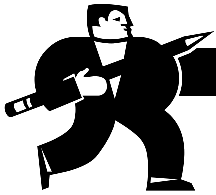
Distribute a copy of Student Activity: Lumber Cutting Patterns showing the most common ways of cutting lumber out of a log. Have students discuss the advantages and disadvantages of each type of cutting pattern.

Student Activity: Lumber Cutting Patterns

Student Name: ____________________ Date: ______________

Figure 45: Lumber Cutting Patterns





Student Activity: Two- And Three- Dimensional Shapes And Objects

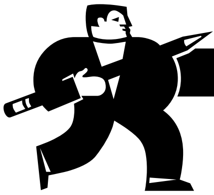
Student Name: ____________________________ Date: ___________________

Complete Table 4: Two-Dimensional Shapes.

Table 4

Two-Dimensional Shapes

Name of Shape	Area	Sketch
Square side *a*		
Rectangle sides *a* and *b*		
Parallelogram base *b* height *h*		
Triangle base *b* height *h*		
Trapezoid base *b* height *h*		
Circle radius *r*		
Ellipse axes *a* and *b*		



Student Activity: Two- And Three- Dimensional Shapes And Objects

Complete Table 5: Three-Dimensional Objects

Table 5

Three-Dimensional Objects

Name	Volume	Figure: Sketch
Cube side *a*		
Rectangular Prism length *a* width *w* height *h*		
Prism height *h* base area *b*		
Cylinder height *h* base circle radius *r*		
Pyramids height *h* square base rectangular base other		
Cone height *h* base circle radius *r*		
Sphere radius *r*		

Activity 9: Representing Geometric Solids

Objective:

In this activity students explore the projections of geometric solids onto the three planes of a trihedron in order to draw elevation views of the solids.

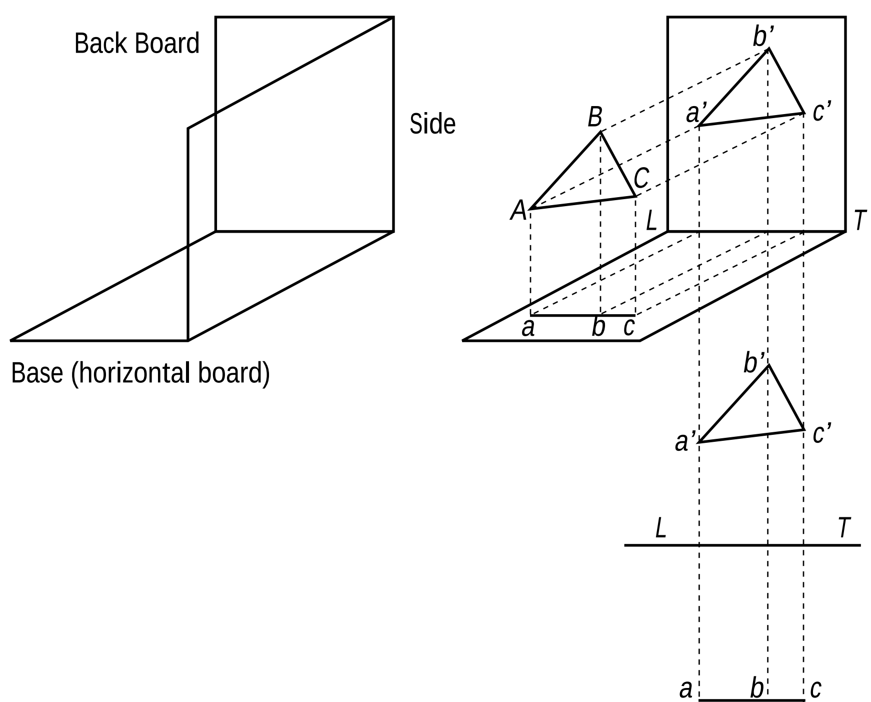
Materials:

(per pair of students)
- cardboard
- ruler
- scissors
- tape

Procedure:

Have students prepare several learning centres with different solids. For each learning centre, they should prepare a rectangular trihedron by taping three pieces of cardboard at right angles as shown in Figure 46: Preparation of a Regular Tetrahedron, below.

Figure 46: Preparation of a Regular Tetrahedron



Demonstrate how to observe the projections of a solid, for example a rectangular piece of lumber like a 2 x 4.

Have students cut out different shapes (e.g., triangle, square) of varying complexity. Ask one student to place the shapes inside the trihedron while her partner draws and labels the plane, vertical and face projections of the shape on the three faces of the trihedron. This is shown in Figure 47: Projection of Squared Lumber.

Confirm that students realize that the dimensions of the projections are not the same in reality (except for shapes that are placed parallel to one plane of the trihedron).

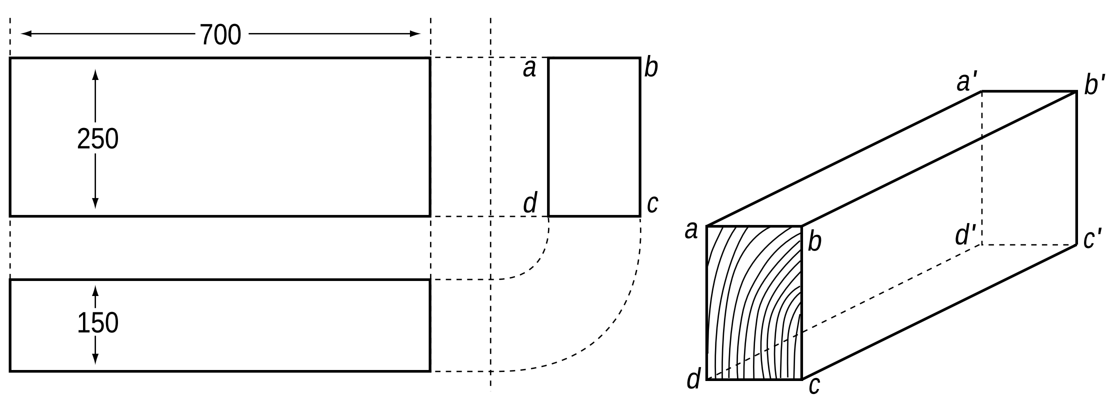
Have students change the orientation of the shape and decide which orientation has the simplest projections.

When students are comfortable with the projections of plane shapes, ask them to visit a few of the learning centres. Have them first sketch the solid and then sketch the projections of the solids onto the three planes.

Have students produce a scale plan of each of the solids (see Figure 47: Projection of Squared Lumber, below).

Provide a copy of Student Activity: Description and Drawing of Three-Dimensional Solids to each student and ask them to draw the scale plans of the solids of Student Activity: Two- and Three- Dimensional Shapes and Objects in the same way as the one illustrated in Figure 47: Projection of Squared Lumber.

Figure 47: Projection of Squared Lumber



Student Activity: Description and Drawing of Three-Dimensional Solids

Student Name: ____________________________ Date: ___________________

For the solids of Student Activity: Two- and Three-Dimensional Shapes and Objects, describe the position of the solids with respect to the planes of the trihedron and draw the scale plans in the space below. Use one page for each of the three-dimensional solids.

Phase III: Model Construction

Objective:

The purpose of this phase is to evaluate the spatial abilities of the students by having them construct a scale model of a simple hull of a boat from the plans and dimensions provided in Student Activity: Bow and Stern Design.

This phase incorporates most of the skills that students have developed during this project.

Materials:

(per group of 2 or 3 students)
- cardboard or thin plywood
- ruler and straight edge
- geometry kit
- scissors or hand jig-saw
- tape
- string
- rice paper or thin wrapping paper
- glue
- wooden craft sticks and straws

Procedure:

Provide students with a complete set of plans and a table of dimensions (Bow and Stern Design).

Distribute a copy of Figure 48: Flat Hull Plan and Profile, below and Figure 49: Diagram of V-Shape Hull, to have students familiarize themselves with the basic vocabulary used in naval construction, and to explain the different components of the plans. Show students a sample of the template by cutting off a piece of cardboard and explaining how it corresponds with the plans.

Explain to students what is expected from them and how their work will be evaluated. Discuss any additional criteria they might wish to add and provide them with an opportunity to give feedback about the evaluation process.

In the first part of this activity, students trace the templates on graph paper, draw them, and cut them out of a piece of cardboard.

Verify the precision of the students' work and record their level of understanding.

Students draw the base line and construct a square that is high enough (30 cm).

The templates are then assembled according to the plans and kept in place with tape.

In the second part of the activity, students attach the templates in position with pieces of straws.

Students determine a way to cover the hull using rice paper or wrapping paper cut in long strips and glued together.

Students remove the templates from the structure. Using straws or wooden craft sticks they make the deck structure stronger.

Figure 48: Flat Hull Plan and Profile

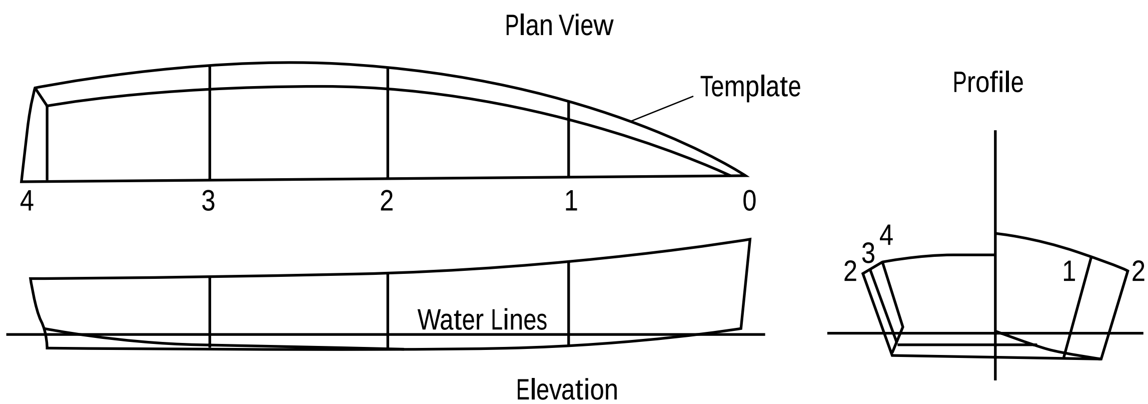
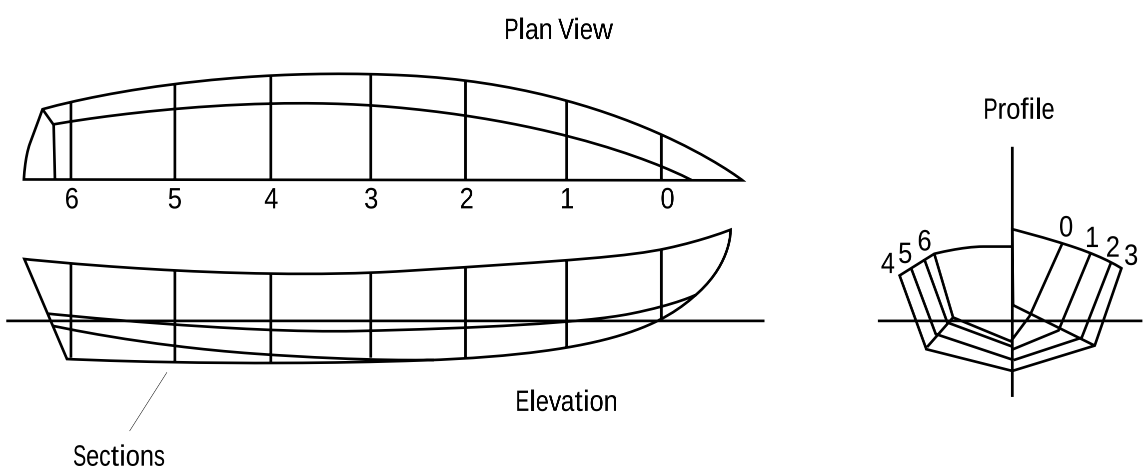
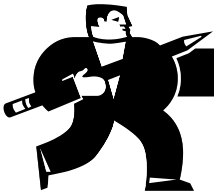


Figure 49: Diagram of V-Shape Hull



Note:
The evaluation component of this project may be building a scale model for a teen recreation centre based on Project C: Planning for a Teen Recreation Centre and Project D: Drawing a Scale Model of a Teen Recreation Centre.



Student Activity: Bow and Stern Design

Student Name: ____________________________ Date: ___________________

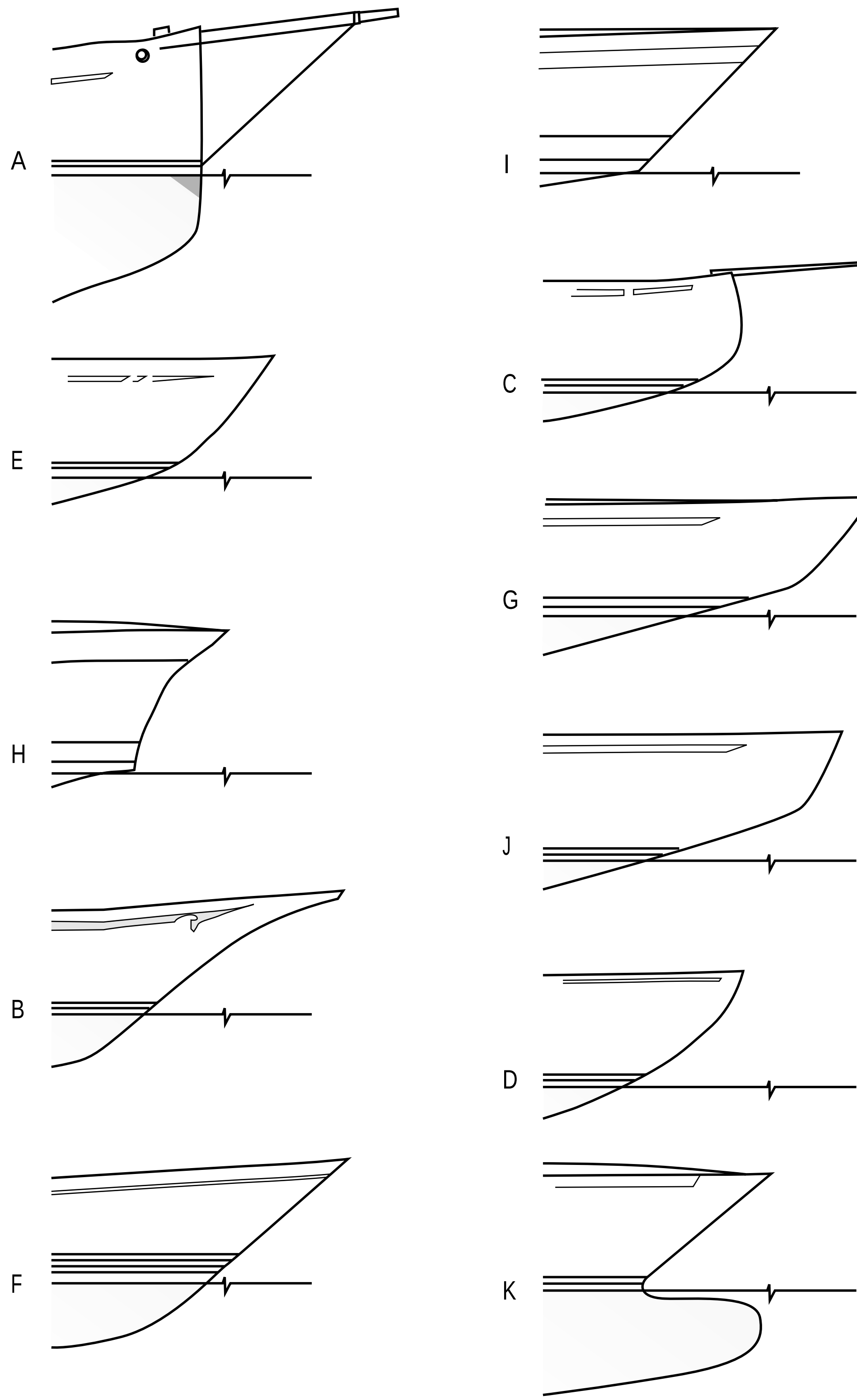
1. Choose one bow design and one stern design from those provided (Figure 50: Typical Bow Designs, and Figure 51: Typical Stern Designs) or from your own research.
2. Draw these designs on cardboard and cut them out.
3. You may wish to examine Figure 52: Hull Elevation and Plan, and Figure 53: Hull Profile, for additional information to add to your scale drawing.
4. Connect the bow and stern designs to produce your own hull design.
5. Comment on your design.

Hull Design:

Comments:

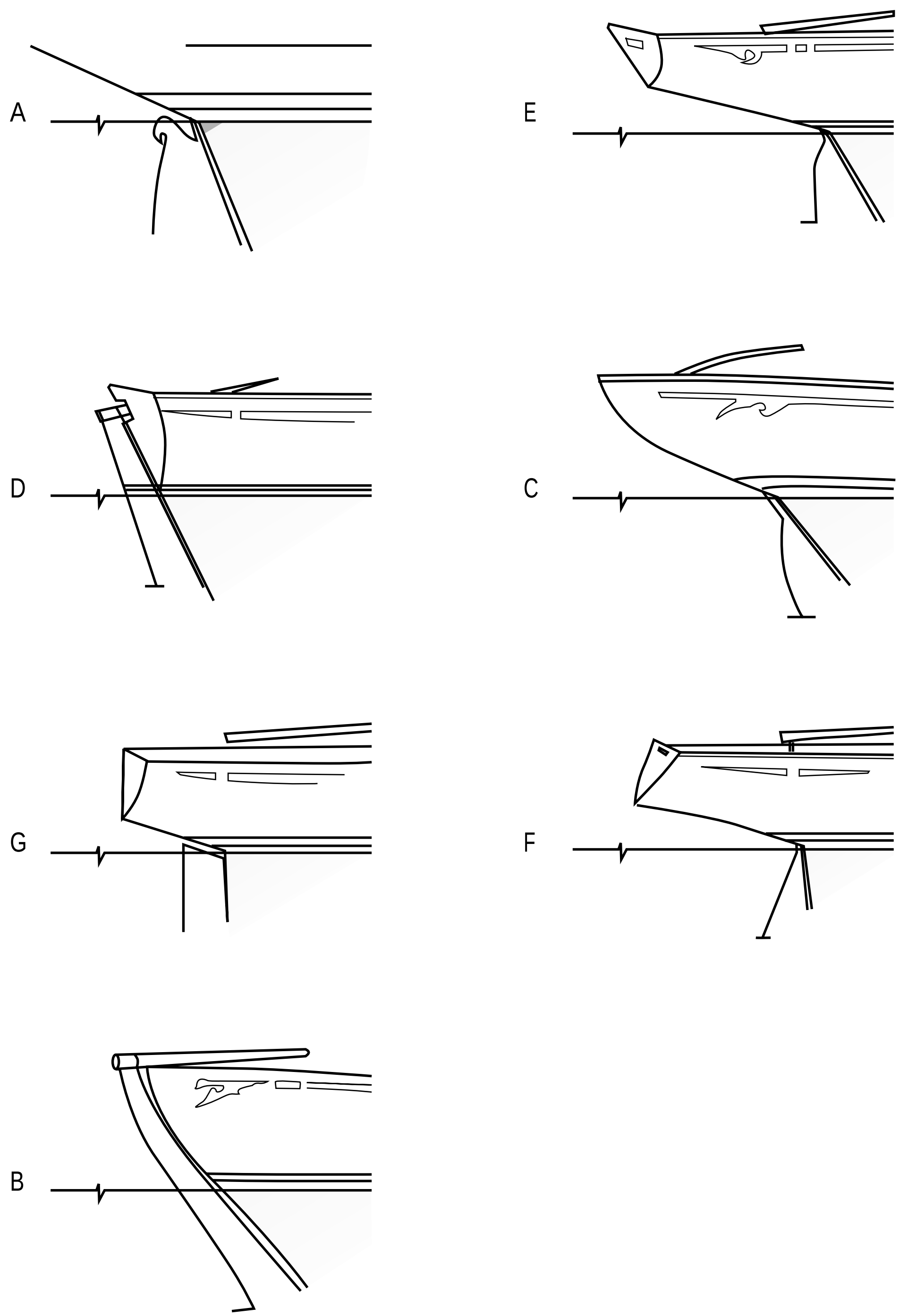
Student Activity: Bow and Stern Design

Figure 50: Typical Bow Designs



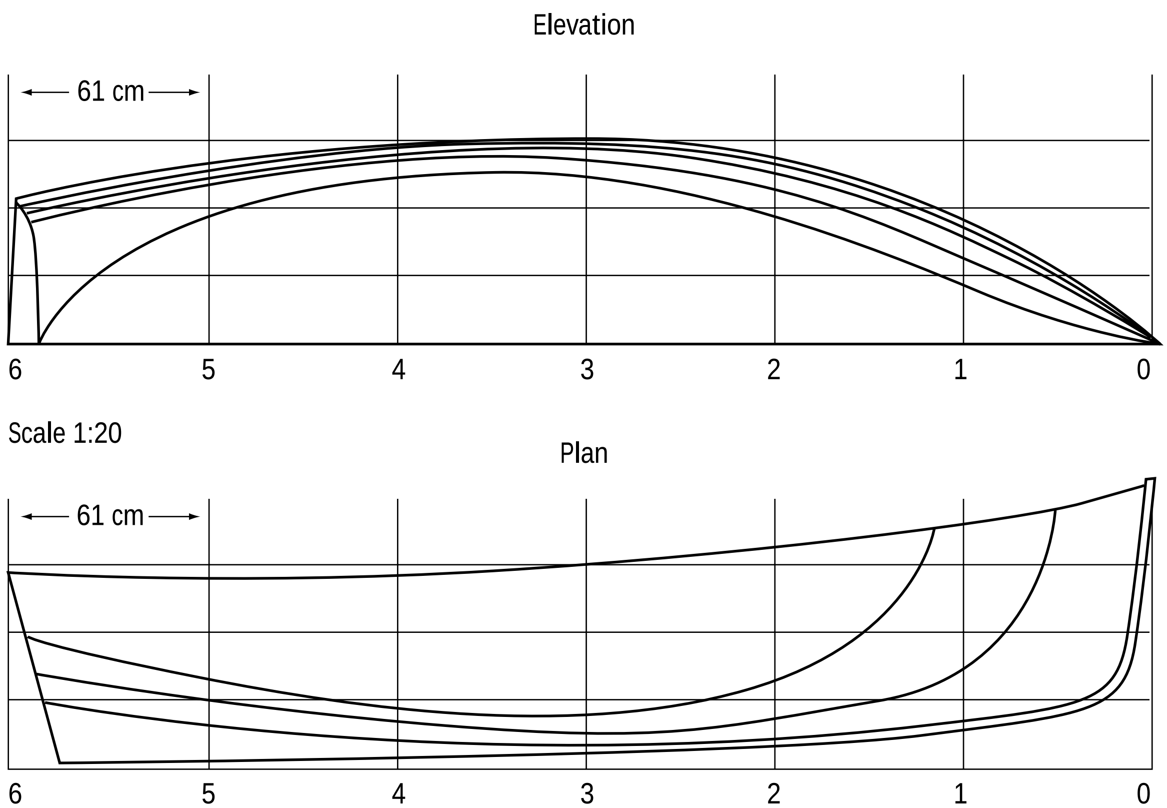
Student Activity: Bow and Stern Design

Figure 51: Typical Stern Designs



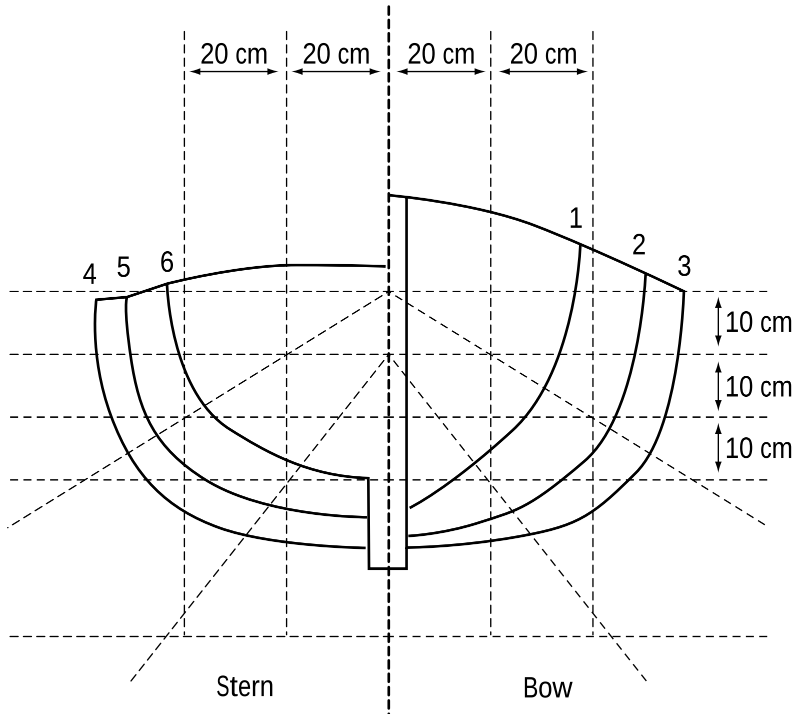
Student Activity: Bow and Stern Design

Figure 52: Hull Elevation and Plan



Student Activity: Bow and Stern Design

Figure 53: Hull Profile



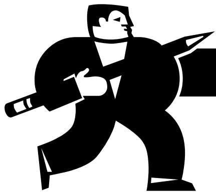
Boat designers draw hull plans in an unusual way. Because boat hulls are symmetrical, they often show only half of the bow and half of the stern. The boat builder simply folds the hull plan in half and draws its mirror image to get the full bow or stern plan.

The numbers 1, 2, 3, 4, 5, and 6 represent the templates for the bow and the stern.

The vertical dotted lines represent 20 cm distances, while the horizontal dotted lines represent 10 cm distances.

Student Activity: Bow and Stern Design

Draw your hull design:



Student Activity: Self-evaluation

Student Name: ____________________________ Date: ___________________

1. Some things I learned about the properties of triangles include:

2. Some things I learned about real-world construction techniques include:

3. Some things I want to learn about planning a construction project include:

4. Some advice I would give to a friend who is planning a construction project is: