Project D: Drawing a Scale Map for a Teen Recreation Centre

Project D: Drawing a Scale Map for a Teen Recreation Centre

Orientation

Time:

12 to 13 hours

Instructional Strategies:

- Direct Instruction
- Field Study
- Group Work

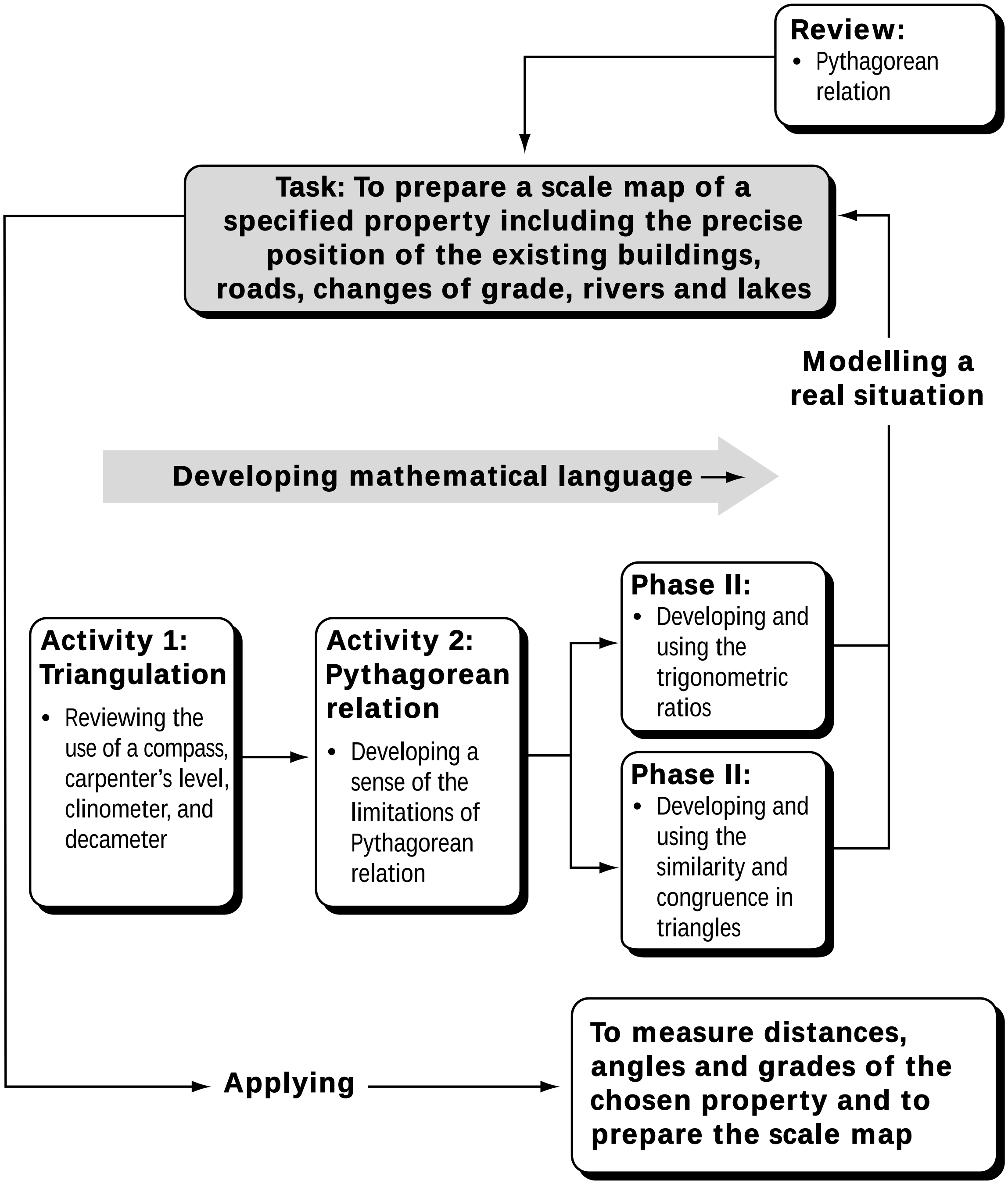
Real-World Applications:

Some real-world applications of the skills and concepts learned in this project include:
- describing activities that are done by surveyors, engineers, architects, timber cruisers, dress designers and interior designers
- using a scale map for outdoor recreation activities and travel
- adapting patterns for sewing and quilting
- designing the layout for chairs and tables in the school gym for a dance
- designing a bird house or stereo cabinet

Project Overview:

Figure 1 on the following page is a schematic to help you conceptualize Project D: Drawing a Scale Map for a Teen Recreation Centre.

Figure 1: Unit D: Drawing a Scale Map for a Teen Recreation Centre



Project Synopsis

The class is expected to prepare a scale map of the specified property. The scale map must include major features, for example, existing buildings, roads, changes of grade, rivers, lakes.

The objective of the project is to provide students with the trigonometry and geometry tools necessary to draw a scale map.

In Phase I students learn more about the problems associated with drawing a scale map through activities that apply triangulation and the Pythagorean relation.

In Phase II, students learn about trigonometric and geometric solutions throughout the course of the activities.

Finally, in Phase III students draw both a sketch map and their final scale map.

Phase I: Triangulation

Objective:

The objective of this phase is for students to learn more about the challenges associated with drawing maps. In particular, they learn about triangulation and the Pythagorean relation.

Materials:

- pair of compasses
- 2 m graduated pole
- decameter and/or rope
- stand for level
- two carpenter's levels
- protractor
- clinometer (for an example, refer to Figure 4: Clinometer)
- drawing instruments (that is, pens, pencils, ruler)

Procedure:

Ideally, the property to be surveyed for the project should have an irregular shape and surface. Obstacles like bushes, a river, a lake, or existing buildings add interesting complexity to the activity ensuring that students develop indirect measurement strategies. Students should include measurements of distances, angles, and levels.

Ask students how they might measure the distance and the direction of a line between two points when one point is not visible from the other, or where one point is not accessible.

Relate their suggestions to the project by giving examples of the need to find indirect measurement methods in situations where one point is not accessible for direct measurement.

Suggestions for introducing the concepts
- Borrow a video on surveying or invite a surveyor to do a class presentation.
- Briefly describe the historical development of indirect methods such as trigonometry. The term 'trigonometry' is derived from the ancient Greek terms for 'trigone' for triangle and 'metric' for measure. The measuring techniques involving triangles were developed by the ancient Greeks, mainly to calculate distances and angles in astronomy. Find illustrations of this to show your students.

Give examples where triangulation is necessary and stress the similarity of the problems (indirect measurement) among the examples.

- Loggers often need to estimate the height of a tree before cutting it down to determine where the tip of the tree will fall, or whether the tree will hit power lines or other trees. Loggers use an instrument called a 'clinometer' to estimate the height of trees. Show the students a clinometer or explain how to build one.
- In surveying, one has to measure the distance and the direction between two points that are either not accessible, or not visible at the same time. Surveyors use trigonometric techniques all the time. (If available, show a theodolite and explain its use by surveyors.) This was the case during the construction of the Railways in the 19th century. Recently, during the construction of the Chunnel, (the tunnel under the English Channel that connects England and France), surveyors and engineers needed to use indirect measurement techniques.
- Traffic accident investigators need to apply indirect measurement techniques to determine the speed of vehicles involved in accidents or to prepare scale diagrams as evidence in court. Show illustrations or pictures of an accident and the scale diagram that has been presented in court.
- Astronomers cannot walk the distance between two planets so they rely on indirect measurement methods. Show the illustration of how Eratosthenes measured the radius of the Earth in 195 BC.
- Brain surgeons cannot directly measure the distances and the angles in the brain of the patient before operating with a laser. Show the illustration of the measurements done on a brain before surgery.
- Dentists need to rely on indirect measurements when analysing the X-rays of a patient's jaw in order to design braces. Show an X-ray obtained from a dentist and the construction of triangles that helps the dentist decide upon the shape of the braces.
- In navigation, determining the exact location of a boat requires a very good understanding of trigonometry. Show an example. Demonstrate the use of a sextant as well if there is one available to you.

Ask students to find other examples where triangulation techniques are necessary (see Student Activity: Research).

Ask students to answer the following questions.

- What is triangulation?
- Why is it easier and more precise to measure angles rather than distances?
- How are the trigonometric ratios used in surveying?
- Why is similarity and congruence of triangles so important in surveying?

Activity 1: Triangulation

Objective:

The objective of this activity is for students to use triangulation as a means of measuring something they cannot see.

Materials:

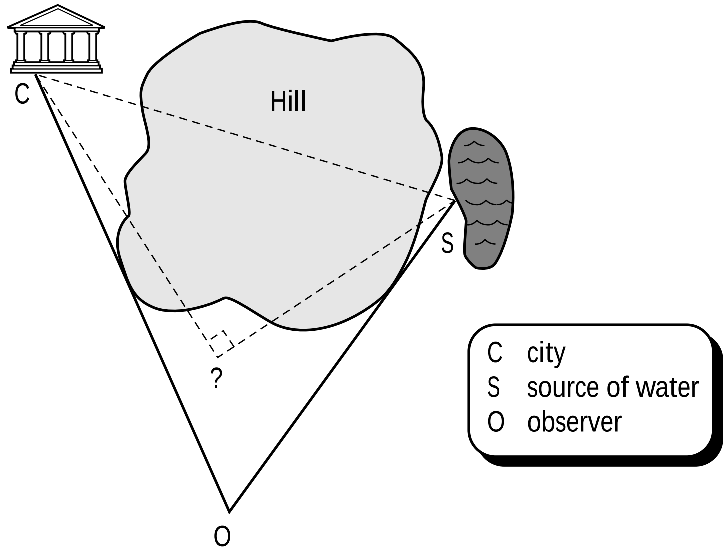
- pair of compasses
- 2 m graduated pole
- decameter and/or a rope
- stand for level
- two carpenter's levels
- protractor
- clinometer (for an example, refer to Figure 4: Clinometer)
- drawing instruments (that is, pens, pencils, ruler)

Procedure:

Because of an increasing population, a certain city of ancient Greece found its water supply insufficient. Residents decided that water had to be channelled from a source in the nearby mountains. Unfortunately, a large hill intervened, and there was no alternative to tunnelling. Working both sides of the hill, the tunnellers met in the middle as planned (see Figure 2: Water Supply in Ancient Greece). How did they do this?

How did planners determine the correct direction to ensure that the two crews would meet? How would you have planned the job? Remember that the Greeks could not use radio signals or telescopes, for they had neither. Nevertheless, they devised a method and actually succeeded in making their tunnels from both sides meet somewhere inside the hill.

Figure 2: Water Supply in Ancient Greece



Essentially, the problem is this: How do the tunnellers determine the direction and the length of the would-be line of sight between the two points *C* and *S* when a hill intervenes?

Ask students to list and describe concepts and techniques they already have in order to solve such a problem. Suggest that they use the Pythagorean relationship for it permits the calculation of the third side of a right triangle. This implies that a right triangle whose hypotenuse is the line of sight *CS* is to be constructed. How do they draw such a right triangle? What do they know? How many different right triangles can they construct? How would they actually do this in a field?

This takes care of the length but the directions are not yet determined. What is missing? Students realize that they need to measure an angle. Present different right triangles having the same hypotenuse demonstrating to students that the angles differ. Introduce the necessity of determining the relationships between sizes of angles and lengths of the sides of triangles.

Now present a different situation (see Figure 2: Water Supply in Ancient Greece) where it is not possible to construct a right triangle such that the observer, *O*, can see both points *C* and *S* (if it were possible the line *OC* or *OS* would pass through the hill!). The triangle *OCS* is not a right triangle.

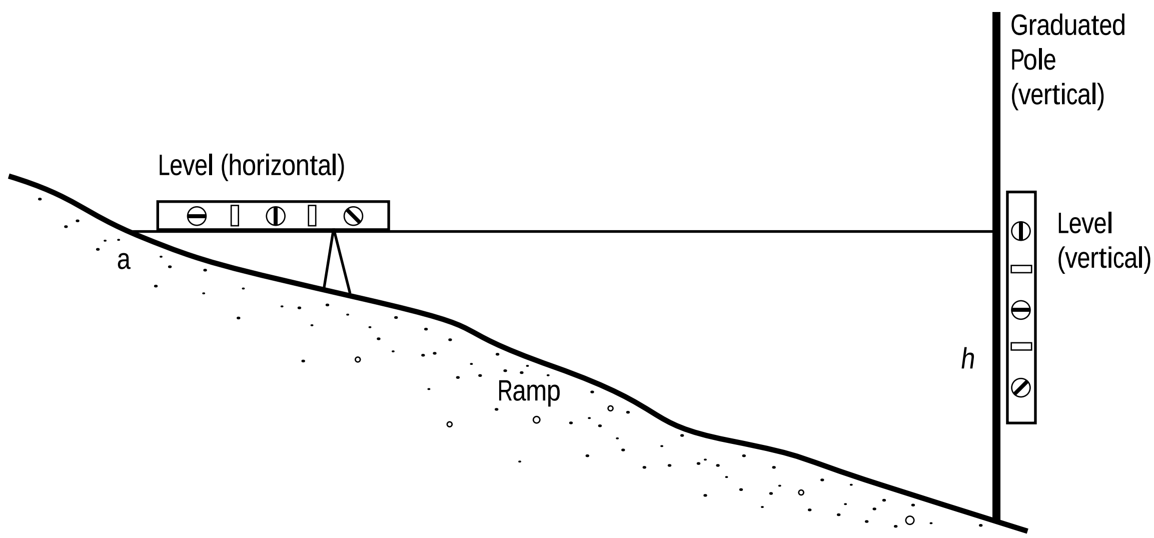
Ask students how they would approach this new situation. Introduce the need for the concept of similarity of triangles and show students it is possible to calculate the ratio of the corresponding sides. Through discussion help students understand that the corresponding angles in similar triangles are congruent.

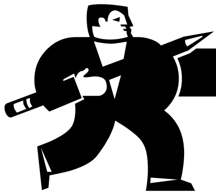
Outdoor Demonstrations:

Demonstrate, in the school yard, how to use a compass, a carpenter's level, a clinometer, and a decameter. For example, have the class identify two points, *C* and *S*, such that *C* cannot be seen from *S* (using a building or a hill). Walk away from the obstacle with a compass until you can see *C* and *S* with a right angle. Ask students to measure *OC* and *OS*. From *O*, walk along a circular arc (traced with the rope) making sure that the angle remains 90° and ask students to measure the distances.

Demonstrate, in the school yard, how to measure a slope with the two levels (see Figure 3: Levels) and how to use the clinometer to measure an elevation.

Figure 3: Levels





Student Activity: Triangulation

Student Name: ____________________________ Date: ___________________

What is triangulation?

In this activity, you will learn how to measure distances and angles for points that do not allow direct measurements. This is the case when the Pythagorean relation cannot be used.

You will derive useful formulae to determine the length of a side of a right triangle. These formulae are the trigonometric ratios of a right triangle.

The methods you are going to learn are necessary for measuring distances and angles in open fields. These methods are used regularly by surveyors, traffic accident analysts, navigators, and even brain surgeons and dentists.

Why do triangulation?

In the logging industry, the height of a tree is often estimated before it is cut down. Questions such as, "Where will the tip of the tree fall?" or " Is the tree going to hit power lines or damage another tree?" need to be answered.
In surveying, the angle and the direction between two points that are either not accessible or not visible at the same time need to be measured.

- Traffic accident investigators need to apply indirect measurement techniques to determine the speed of vehicles involved in accidents or to prepare scale diagrams as evidence in court.
- Astronomers cannot walk the distance between two planets so they depend on indirect measurement methods.
- Brain surgeons cannot directly measure the distances and the angles in a patient's brain before operating with a laser.
- Dentists need to rely on indirect measurements to analyse the X-rays of a patient's jaw in order to design braces.

Materials:

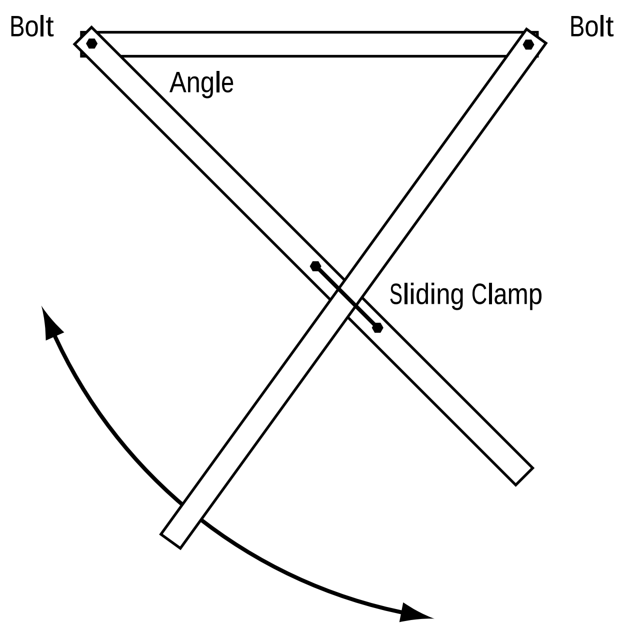
You will need the following materials and equipment. Make sure that they are available and that you know how to use them properly. Ask your teacher if necessary.

- pair of compasses
- 2 m graduated pole
- decameter and/or a rope
- stand for level
- two carpenter's levels
- protractor
- clinometer (see Figure 4: Clinometer)

Student Activity: Triangulation

- drawing instruments (that is, pens, pencils, ruler)
- graph paper

Figure 4: Clinometer

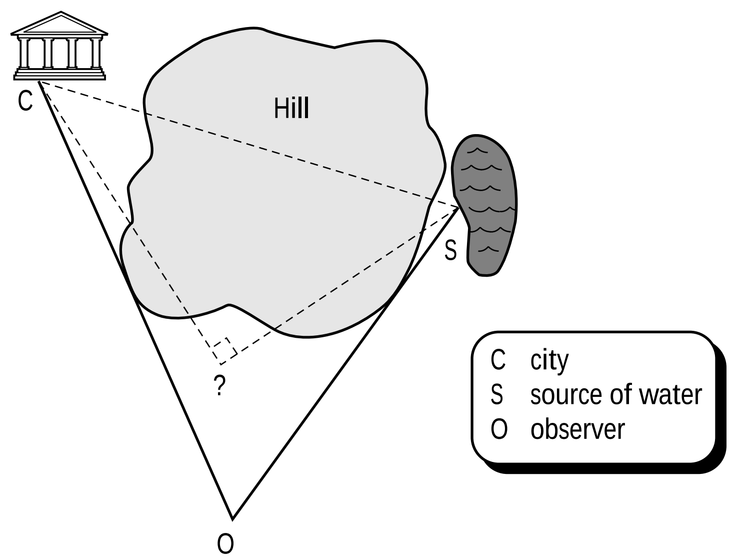


Propose a Strategy to Solve the Following Problem:

Due to an increasing population, a certain city of ancient Greece found its water supply insufficient. It was decided that water had to be channelled in from a source in the nearby mountains. Unfortunately, a large hill intervened and there was no alternative except to tunnel through the hill. Working from both sides of the hill, the tunnellers met in the middle as planned. (See Figure 5: Water Supply in Ancient Greece, below.) How did they do this?

Student Activity: Triangulation

Figure 5: Water Supply in Ancient Greece



Student Activity: Research

Student Name: ______________________ Date: ________________

Using the Internet, a magazine or a newspaper, find one picture of a situation where distance or length is important but cannot be measured directly. Your picture can be a drawing or a photograph. Paste the picture into the space below, describe the measurement, and draw a sketch of how the measurement can be made using triangles.

Picture:

This is a picture of:

The source of this picture is:

Description of the measurement:

Sketch of how the measurement can be made by using triangles:

Activity 2: Pythagorean Relation

Objective:

The objective of this activity is to practise using the Pythagorean relation.

Materials:

- student activity sheets

Procedure:

Review the Pythagorean relation if necessary. Confirm that students:
- understand the Pythagorean relation [Shape and Space - Measurement I (Mathematics 8)]; and
- can use their calculators to compute square roots with decimals.

Suggest the following situation:

If the lengths of two sides of a right triangle are known, the third side may be calculated using the Pythagorean relation. The Pythagorean relation follows:

In a right triangle, the square of (on) the side opposite the right angle is equal to the sum of the squares of (on) the other two sides

$$c^2 = a^2 + b^2$$

or $c = \sqrt{a^2 + b^2}$

In Figure 6: Accident Investigation #1, the height of the cliff *a* has been measured as 18.75 m. The car landed 9.14 m from the bottom of the cliff, a distance *b*. The length of the hypotenuse *c* would be:

$$c = \sqrt{351.56 + 83.54}$$

$$c = \sqrt{351.56 + 83.54}$$

$$c = \sqrt{435.10}$$

$$c = 20.86 \text{ m}$$

Verify that students can identify the hypotenuse and the right angle sides (legs) of the triangle. Also ensure that they can enter the proper data and use the calculator's square root function.

Remind students that the length of the hypotenuse is longer than either of the other sides. Ask them why this is so. Then have them estimate the result from the figure.

Make sure that they use their calculators properly and that they are able to round the result (in this case, two decimals). Can they calculate the result with a minimum of operations?

Reinforce with the following problem:

When the length of one side of a right triangle is known and the length of the hypotenuse is also known, the length of the remaining side can be calculated. A traffic accident investigator has measured, by using a tape, the embankment (hypotenuse c of the right triangle) and the height of the embankment (side a of the triangle).

$c = 20.86$ m
$a = 18.75$ m

Using the Pythagorean relation, determine that $b = 9.14$ m

Make sure that the students are able to manipulate the formula properly. Ask them to estimate the result (from Figure 6: Accident Investigation #1) and from the fact that the answer needs to be smaller than the hypotenuse and the other side of the right angle). You could mention that if they do the wrong subtraction, they will end up with a negative number and the calculator will indicate that with an 'ERROR' message.

When students suggest situations where the Pythagorean relation can be used, ensurethat they realize these situations have one aspect in common—two sides of a right triangle are known.

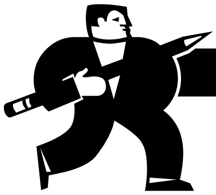
You may use an evaluation form to assess the prerequisite skills and knowledge of your students. An example, Table 1: Assessing the Prerequisites, is on the following page.

Table 1

Assessing the Prerequisites

Code	Criteria	Rating	Comments

The Code column refers to the requisite learning outcomes in the matrix of the prerequisites in Appendix A. Additional criteria may be added as required.



Student Activity: Pythagorean Relation

Student Name: ____________________________ Date: ___________________

If the lengths of two sides of a right triangle are known, the third side can be calculated by using the Pythagorean relation.

In a right triangle, the square of the side opposite the right angle is equal to the sum of the squares of the other two sides

$$c^2 = a^2 + b^2$$

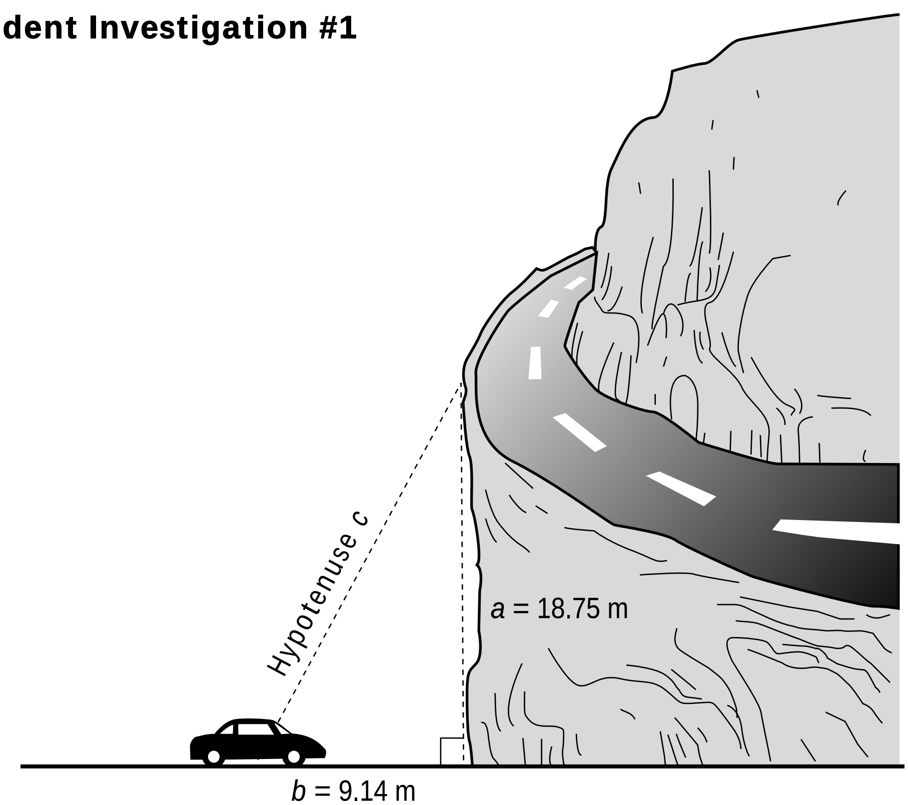
or $c = \sqrt{a^2 + b^2}$

Accident Investigation Example #1

In Figure 6: Accident Investigation #1, below, the height of the cliff is 18.75 m and is represented by a.
The car landed 9.14 m from the bottom of the cliff. This distance is b.
The length of the hypotenuse c would be:

c = ____________________________

Figure 6: Accident Investigation #1



Student Activity: Pythagorean Relation

Accident Investigation Example #2

When the length of one side of a right triangle is known and the length of the hypotenuse is also known, the length of the remaining side may be calculated using the Pythagorean relation.

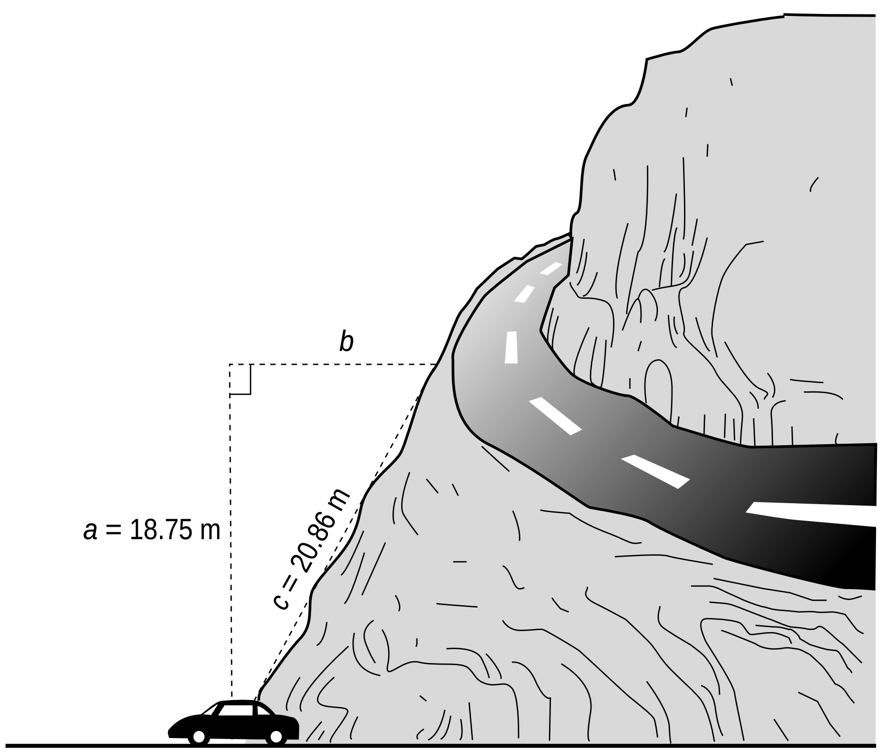
As shown below in Figure 7: Accident Investigation #2, a traffic accident investigator used a tape measure to determine the length of the embankment (hypotenuse c of the right triangle). The height of the embankment (side a of the triangle) was obtained from records kept during the time of road construction.

$c = 20.86$ m
$a = 18.75$ m

Use the Pythagorean relation to determine b.

b = ____________________

Figure 7: Accident Investigation #2



Student Activity: Pythagorean Relation

Describe another situation where the Pythagorean relation can be used. Draw a diagram.

Describe a situation where the Pythagorean relation cannot be used.

What are some of the ways in which these two situations are different?

Phase II
Part 1: Solving the Problem Using Trigonometry

Objective:

The objectives of Phase II are twofold. The first objective is to guide students in the development of the tangent ratio formula from a real-world situation. Students record measurements, graph the relations, and then develop the formula. There is a motivation to develop the formula because students discover that it is impossible to solve some problems using the Pythagorean relation.

The second objective of this activity is to have students realize the limitations of the tangent ratio in solving concrete problems and to introduce other trigonometric ratios. Once an angle and any two sides of a right triangle are known, the tangent ratio method is not the best way to solve the right triangle. It is possible to solve the problem indirectly by using the Pythagorean relation.

This activity can be used to reinforce or evaluate the learning outcomes relating to Patterns and Relations. See Appendix A: Correlation Between Prescribed Learning Outcomes and Projects.

Materials:

- clinometer
- decameter
- protractor
- pair of compasses
- 2 carpenter's levels
- stand
- 2 m pole
- scientific calculator

Procedure for Tangent Ratio:

Students have identified situations where the Pythagorean relation cannot be used. Such a situation might be to measure the height of a pole or a tree. Encourage students to find reasons that make such a calculation necessary. An example might be that a diseased tree must be cut down and the faller wants to make sure that, in falling, it will not crash into another tree, or fall on another object or person. Encourage students to estimate the height of the tree. One of the ways they might do this is by comparing the height of the tree to the known height of a nearby structure.

Verify that students can use their clinometer properly. Verify that they have used Table 2: Distances and Angles (Student Activity: Measuring a Tree) to record the distances measured with the decameter and the angles measured with the protractor and clinometer.

Can students deduce that the angle at a long distance will eventually be 0° and that the angle at the tree base is 90°?

Note the details students use to express the relation between the distance and the angle. Do they realize that the relation is not linear, that is, if the distance doubles, the angle does not double? Moreover, do they realize that the way the angle changes depends on the distance they measure? Far away from the tree, the angle varies a small amount with distance but when they are close to the base of the tree it varies a large amount.

Encourage students to use the information above to help them while they plot angles versus distances. They should visualize that the distance is growing longer and longer as they approach 0°. Ask them, "What happens at 0°?"

This representation of a relation is more abstract than the numerical representation and some students may have difficulties. Help them compare this relation with a linear relation on the same graph and show them how they differ.

For some students, you will have to use numbers to represent the relation in words.

Demonstrate how to use the [TAN] mode on the calculator. Note that calculators differ. On some calculators, the angles can be expressed in degrees, grad, or rad. Make sure that students use the [DEG] mode. They should round the result to 3 decimals places.

While completing Table 3: Tan, encourage students to first complete the first and third columns (angles and distances, respectively) and then use a minimum of operations on the calculator:

Angle [TAN] x Distance [=]

What is the meaningful number of decimals (or number of significant figures) in the product? The tangent has 3 decimals and the distance had 2 decimals. In this case the answer should be rounded to 2 decimal places.

The results in the last column should be constant. This is the height of the tree. Observe how the students deduce this result. The angle and the distance vary. The only fixed measure is the height of the tree.

Students should be able to express the relationship in their own words. For example:

"If I multiply the distance from the tree by the tangent of the angle, I obtain the height of the tree no matter what the distance is."

Students should also be able to express the relation in the form of a formula. If d is the distance, *A* is the angle, and *H* is the height, then

$$d \times tan(A) = H$$

Give students several angles and ask them to calculate the tangent ratio on their calculators. Ask them to explain why tan 90° gives an error on the calculator (refer to the graph).

Encourage students to manipulate the tangent ratio formula in order to obtain an expression for *d* and an expression for tan(A):

$$d = \frac{H}{tan\ (A)} \quad \text{and} \quad tan(A) = \frac{H}{d}$$

Some students may prefer to write the formula in terms of adjacent and opposite sides of the right angle. Ask them to record this on their Student Activity sheet:

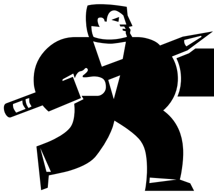
$$tan(A) = \frac{\text{opposite side}}{\text{adjacent side}}$$

Ask students to solve the three problems posed in Diagrams 1, 2, and 3 on the Student Activity: Measuring a Tree, and to imagine real situations that can be represented by the diagrams. They should solve the problem from Activity 2 of the Student Activity: Measuring a Tree for homework.

Assessment:

While students are engaged in the activity, informally evaluate how well they are able to:

- assess the relevance of this activity to the overall project;
- identify situations where the Pythagorean relation cannot be used to solve a particular concrete problem;
- identify situations where the tangent ratio can be used to solve a concrete problem;
- express the tangent ratio as the ratio of the opposite side to the adjacent side of right triangle;
- use their calculators to correctly find the tangent of an angle;
- round the result to three decimal places; and
- manipulate the tangent ratio formula to find one quantity in terms of the other two.



Student Activity: Measuring a Tree

Student Name: ___________________________ Date: __________________

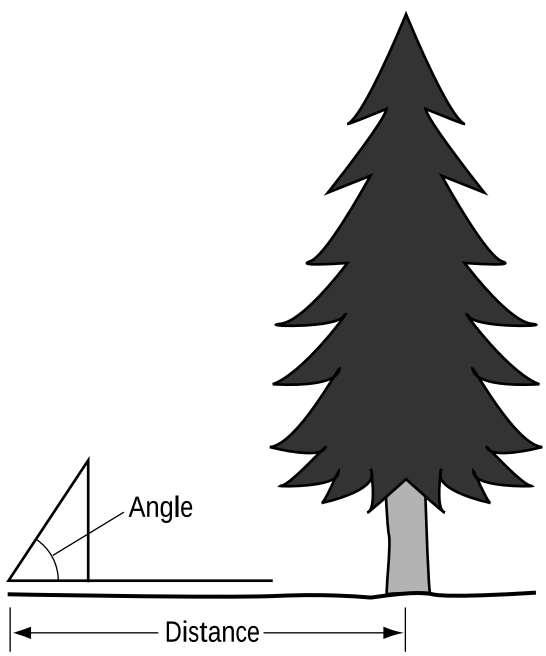
Activity 1: Measuring a Tree

Measuring the height of a tree or a pole is a situation where the Pythagorean relation cannot be used.

What are some reasons you might need to know these heights?

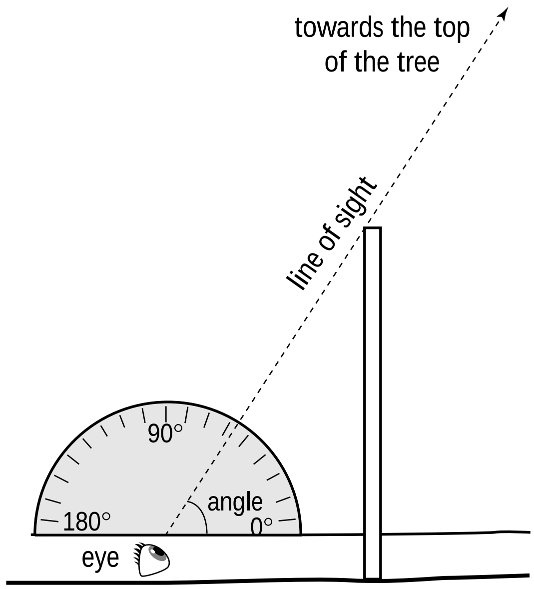
With your partner, choose a tree, pole, or building on the school property. Estimate the height of the object you chose:_______________metres

Figure 8: Viewing the Tree



Student Activity: Measuring a Tree

Figure 9: Using a Clinometer



Walking away from the tree, notice how the angle at which you see the top of the tree changes (Figure 8: Viewing the Tree). Use your clinometer and the protractor (as indicated in Figure 9: Using a Clinometer) to determine the angle at the base of the object, at 10m, and at 25m.

at the base of the object ____________________

10 m away ____________________

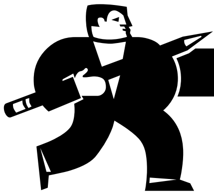
25 m away ____________________

Walk away from the tree so you see the top of the object under an angle of 60°, 45°, 30° and measure the distance you walked.

60° ____________ m

45° ____________ m

30° ____________ m



Student Activity: Measuring a Tree

Table 2

Distances and Angles

Distance (metres)	Angle (degrees)
10 meters	
	45°

What are some things you notice about the distance/angle relationship in Table 2: Distances and Angles?

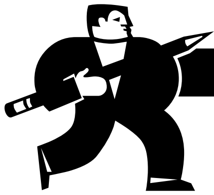
Questions:

What is the angle at a horizontal distance of 500 m?

What is the horizontal distance if the angle is 90°?

What is the horizontal distance if the angle is 0°?

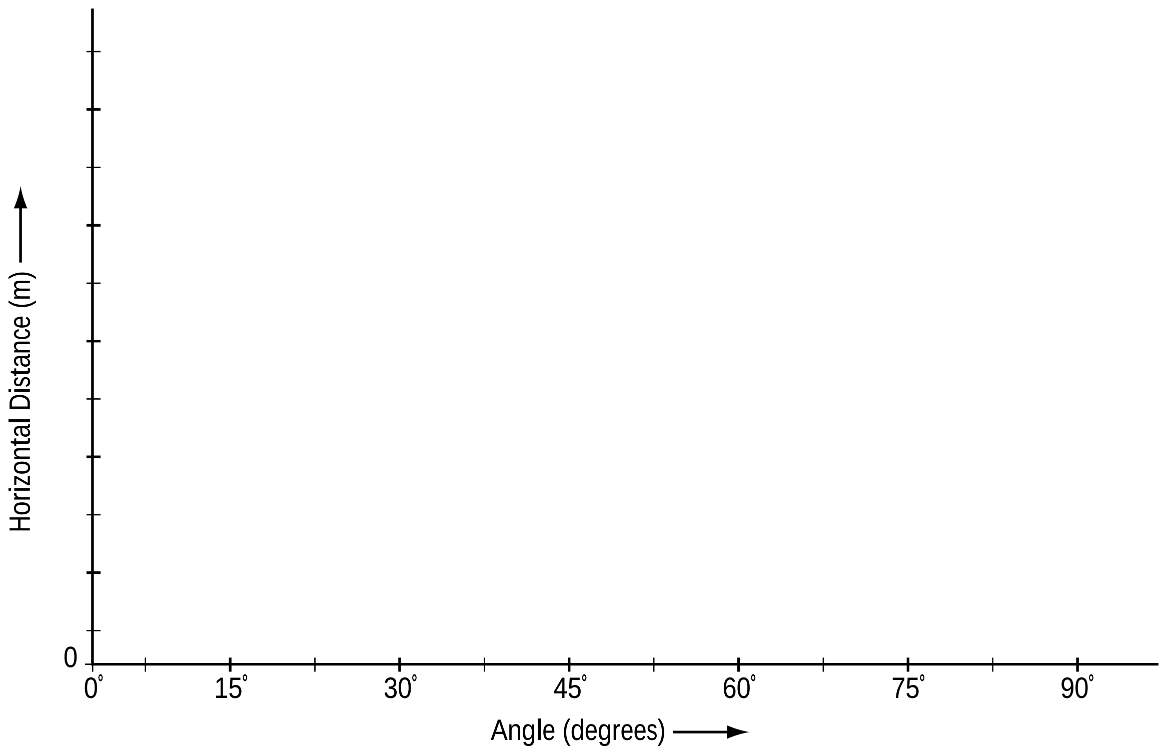
Use your own words to describe the change of the angle with respect to the change of the distance? (e.g., if the distance doubles, what happens to the angle, and is this a linear relation?)



Student Activity: Measuring a Tree

On the grid below, plot the angles against the distances as recorded in Table 2: Distances and Angles.

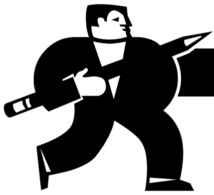
Figure 10: Relation Between Distances and Angles



What are some things you notice about the relationship between the horizontal distances and the angles?

Describe how the angle varies with the distance.

Use the TAN mode on your calculator for the angles in Table 2: Distances and Angles and record your results in Table 3: Tan, below.



Student Activity: Measuring a Tree

Table 3

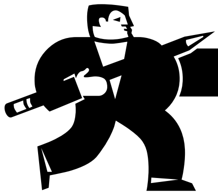
Tan

Angle (degrees)	TAN of Angle (3 decimal places)	Distance (metres)	Product (m) (TAN of Angle x Distance)

What is the meaning of the product calculated in the last column?

Use your own words to describe how angle *A*, the horizontal distance from the bottom of the tree, *d*, and the height of the tree, *H*, are related.

Write an equation to describe the relationship.

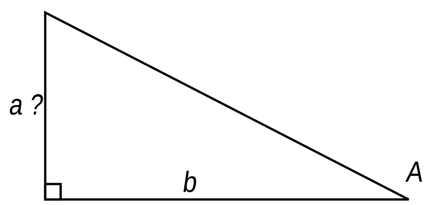


Student Activity: Measuring a Tree

Develop formulae to calculate the distances and angle *A* in the diagrams below.

In Diagram 1, you know the adjacent side, *b*, and the angle *A*, and you want to determine the opposite side, *a*. Use the space provided to show the steps you used to determine the formula expressed in terms of *a*.

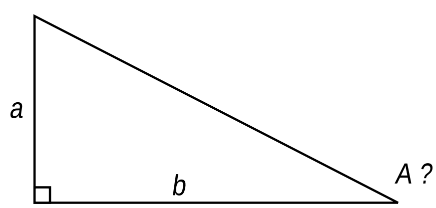
Diagram 1



Formula: $a =$ ___________

In Diagram 2, you know the adjacent side, *b*, the opposite side, *a*, and you want to determine the angle *A*. Use the space provided to show steps.

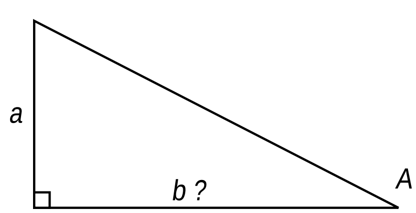
Diagram 2



Formula: $\angle A =$ ___________

In Diagram 3, you know the opposite side, *a*, and the angle *A*, and you want to determine the adjacent side *b*. Use the space provided to show steps.

Diagram 3



Formula: $b =$ ___________

Tangent ratio in a right triangle:

Describe a real-world situation to illustrate Diagrams 1, 2, and 3 above.

Diagram 1:

Diagram 2:

Diagram 3:

Student Activity: Revisiting the Accident

Student Name: ______________________________ Date: ____________________

Activity 2: Revisiting the Accident

(using an angle and the Tangent Ratio to measure a distance)

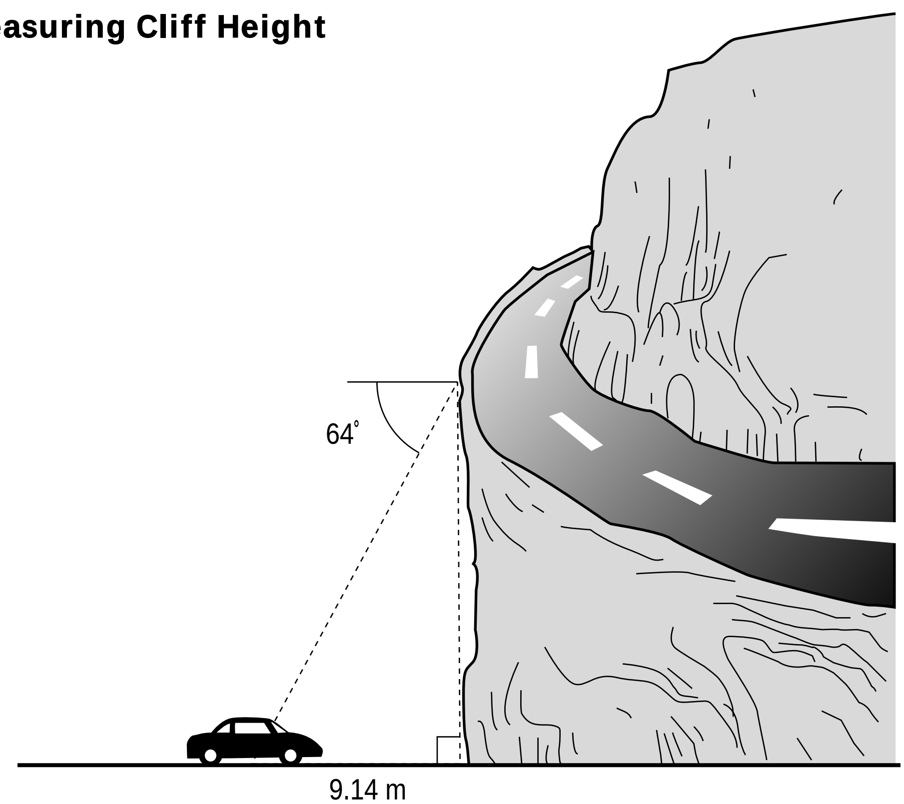
The following measurements were taken after an accident (see Figure 11: Measuring Cliff Height, below).

The vehicle left the highway and landed 9.14 m from the base of the cliff.

The angle from the horizontal to the point of landing, *A*, was 64°. It was measured with a clinometer.

In order to draw a scale diagram, the investigator needs to determine the height of the cliff. What is the vertical height of the cliff? What is the formula you used?

Figure 11: Measuring Cliff Height



Procedure for Sine and Cosine:

A major concern at this point, is to ensure that students are able to decide which ratio of sides is the most appropriate to use in a given situation. It is also important to mention that any ratio can be used but extra work may be required.

Introduce students to inverse trigonometric functions and have them use their calculators to determine an angle given the sine or the cosine ratio when they have to solve a concrete problem such as determining an angle of elevation or the angle of depression of an access ramp. (See Figure 12: Determining Angle of Depression).

Ask students to brainstorm some reasons for needing to know the angle of a ramp or a road, for example, a provincial construction regulation. Have students find other situations that can be modelled by a right triangle where the hypotenuse and one side are known.

Suggest the following situation: "What direct information can be obtained using radar (not height-finding) and how is the missing information determined?" The radar screen indicates the direction and the distance of an airplane but not the elevation (Figure 12: Determining Angle of Depression).

Have students determine the connection between these two situations and explain why none of the previous methods is directly applicable. Ask students to solve the problem indirectly by combining the tangent ratio and the Pythagorean relation. In Figure 12: Determining Angle of Depression,
h is the opposite side of angle A (measured quantity);
a is the adjacent side of angle A (not a measurable quantity); and
d is the hypotenuse (measured quantity).
From the tangent ratio, $h = a \times \tan(A)$ but a is unknown.

From the Pythagorean relation, $d^2 = a^2 + h^2$, so that $a = \sqrt{d^2 - h^2}$.
Now you are back to the tangent ratio method.

NOTE: there are two values for a (positive and negative). Students could test this with their calculator, by taking a number, taking the square root, changing the sign, and squaring the result.

If there is time, students can also derive the sine or cosine ratio in the same way they derived the tangent ratio.

Ask students to find a ramp or road and to determine the angle of elevation. A ladder could be used instead of a ramp or road. What are the direct measurements they can take? How is this related to the project?

Students should model the situation with a right triangle and correctly name the different sides of the triangle: a and h are the sides of the right angle, d is the hypotenuse.

Have students calculate the ratio, $\frac{\text{Height}}{\text{Distance}}$, at different distances and complete Table 4: Ramping Up. This is the ratio of the opposite side h (numerator) and the hypoteneuse, d, and is called the 'SINE' ratio for angle A. It is abbreviated 'sin A'. Introduce the cosine ratio in the same way.

Show students how to use the [2nd] mode [OR: depending on the calculator, [SHIFT] or [INV]] followed by the [SIN] function on their calculator. Explain that they are doing the reverse, that is, if they know the value of the ratio, they can determine the corresponding angle.

Ask students to use both their own words and then a formula, to describe the relation between the height measured on the pole (opposite side to angle *A*), the distance on the ramp (hypotenuse), and the angle of the ramp (*A*).

Insist that students determine the appropriate ratio to be used in given situations. Have students work on the four diagrams found in the Student Activity: Ramping Up, and ask them to find alternate ways by combining two methods.

By using their calculators, have students realize that if the sine ratio of an angle = cosine ratio of an angle, then the angles are complementary. Have them practise with different values.

Enrichment: Accuracy and Rounding

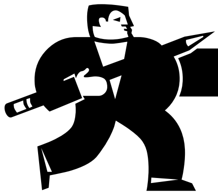
Have students use the calculator to calculate the sine and cosine ratios of different angles (accurate to one decimal place) near 0°, 45°, and 90°. Then have students discuss how the two ratios vary. They should notice that near 90° the sine varies slowly while the cosine varies rapidly and that the opposite occurs near 0°. What can they deduce about the ratios that would be useful in different situations? What can they say about the rounding of the ratios?

Assessment:

Have students propose a strategy for solving cliff heights (Student Activity: Pythagorean Relation) before taking measurements in the school yard and preparing the scale diagram. This activity could be used as an assessment for both the tangent ratio and the other trigonometric ratios, and for reviewing the concept of scaling. Refer to the Grade 8 Mathematics curriculum - Shape and Space; Measurement and Transformations. You may need to have students recall the concepts of enlargement and reduction, and how to use scaling factors.

Evaluate how well students:
- correctly identify the measurable quantities and the unknown ones;
- select another appropriate method to solve the problem;
- model the problem with the correct formula;
- use the correct functions on their calculators;
- make the appropriate rounding and approximations;
- transfer and apply their knowledge to other situations;
- relate their knowledge to the overall project;
- choose an acceptable scaling factor;
- accurately use the drawing instruments; and
- accurately transpose the scale map back to reality.

Note: For Student Activity: Mapping, Activity 6: Mapping, provide students with copies of Blackline Master 5 as needed.



Student Activity: Ramping Up

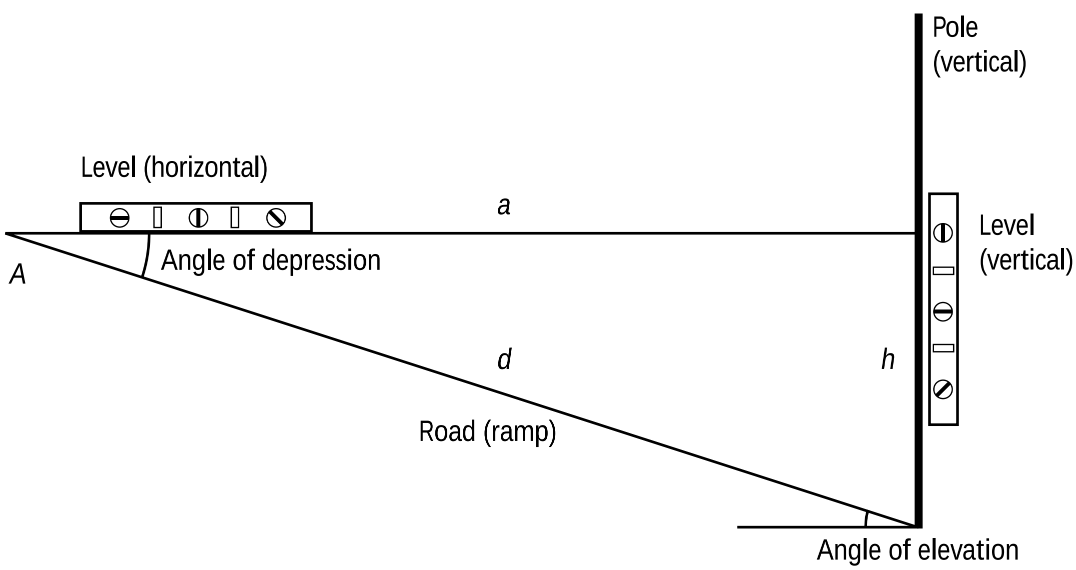
Student Name: __________________________ Date: __________________

In some situations, neither the Pythagorean relation nor the tangent ratio can be used because some distances or angles cannot be measured.

Activity 3: Ramping Up

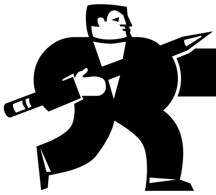
The following problem is an example of such a situation (see Figure 12: Determining Angle of Depression).
Determine the angle of depression of an access ramp by using a pole and a carpenter's level.

Figure 12: Determining Angle of Depression



What are some situations where you may need to determine the angle of depression of a ramp or a road?

Why can't you solve this problem by using the Pythagorean relation or the tangent ratio?



Student Activity: Ramping Up

With your partner, find a ramp or a road. You will need the 2 m graduated pole, the decameter, and the carpenter's level. Describe how you could measure the angle of depression.

What distances can you measure directly? Record these. Remember to use the same units (centimetres or metres) when you record the distances.

Model the situation with a right triangle and name the different sides of the triangle (where *a* and *h* are the sides of the right angle, *d* is the hypotenuse).

Repeat the same operation two more times at different places along the slope. Record your measurements below in Table 4: Ramping Up.

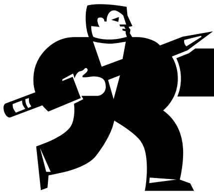
Calculate the ratio, $\frac{\text{Height}}{\text{Distance}}$. This is the ratio of the opposite side *h* to the hypoteneuse. This ratio is called the 'SINE' ratio corresponding to the angle *A*. It is abbreviated *sin A*.

Press 2nd [OR: depending on the calculator, SHIFT or INV] followed by SIN on your calculator and enter the results below in Table 4: Ramping Up.

Table 4

Ramping Up

Ramp Distance (*d*)	Pole Height (*h*)	Ratio *h*/*d*	



Student Activity: Ramping Up

Describe the relationship among the height measured on the pole (opposite side), the distance on the ramp (or road) (hypotenuse), and the angle of depression of the ramp (or road) (*A*).

Write the formula describing the relation between *h*, *d*, and angle *A*.

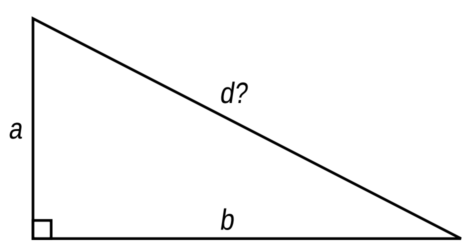
The sine ratio can be used to solve problems where two quantities can be measured. List all the combinations of two quantities that can be measured directly so that you can then determine any of the other quantities that cannot be measured:

Measured quantities	Calculated quantity
and	
and	
and	

In each of the following diagrams:

- describe a situation in the real world that is like the one shown in the diagram;
- determine the quantities that are measurable;
- decide what method (Pythagorean relation, tangent ratio, sine ratio) to use to calculate the 'non-measurable' quantity; and
- calculate the 'non-measurable' quantity.

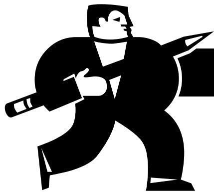
Diagram 1



Real Situation:

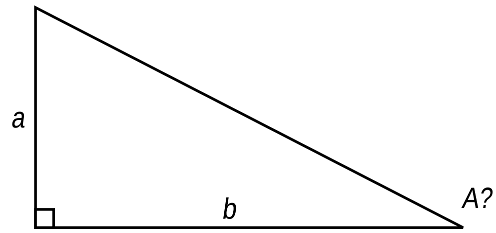
Method:

Solution:



Student Activity: Ramping Up

Diagram 2

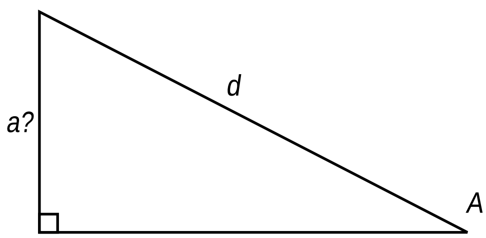


Real Situation:

Method:

Solution:

Diagram 3

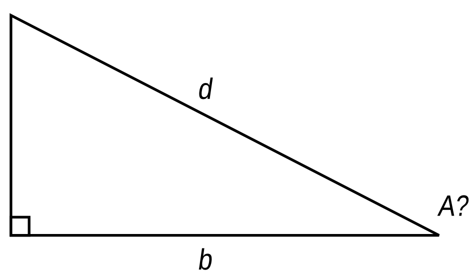


Real Situation:

Method:

Solution:

Diagram 4



Real Situation:

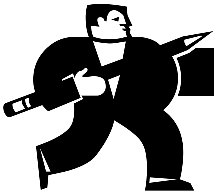
Method:

Solution:

In Diagram 4 above, none of the previous methods can be used directly. You may have found a combination of two methods. If not, try to combine any two of the other methods. This is approach is both complicated and time consuming.

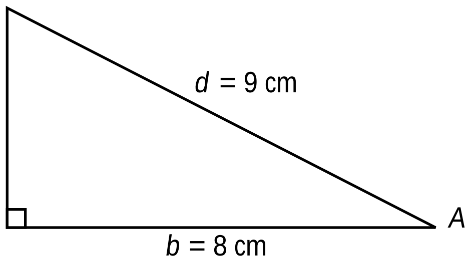
Describe some of the ways that this situation differs from the previous one?

In problems where the hypotenuse *d* and the adjacent side *b* are known the ratio $\frac{b}{d}$ is called the cosine ratio, abbreviated *cos*. (Figure 13: Cosine Ratio)



Student Activity: Ramping Up

Figure 13: Cosine Ratio



What are some real-life situations where it is advantageous to use the cosine ratio?

For Figure 13: Cosine Ratio, use your calculator to determine the angle whose cosine ratio is $\frac{b}{d}$. This time, press SHIFT [OR 2nd, or INV] followed by COS.

(Answer: 27.3°)

Use your calculator (or a trigonometric table) to determine the following ratios.

sine ratio = 0.500: angle = __________ °
cosine ratio = 0.500: angle = __________ °

sine ratio = 0.501: angle = __________ °
sine ratio = 0.505: angle = __________ °
sine ratio = 0.510: angle = __________ °

cosine ratio = 0.101: angle = __________ °
cosine ratio = 0.105: angle = __________ °
cosine ratio = 0.110: angle = __________ °

How do you describe the accuracy of the measurements?

Student Activity: Ramping Up

How does an error in the calculation of the ratio affect the size of the angle?

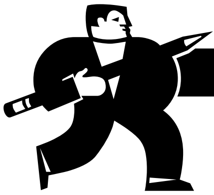
Use your calculator to evaluate the sine and the cosine ratios of the following angles. Round your answers to three decimals and record in Table 5: Sine and Cosine Ratios.

Table 5

Sine and Cosine Ratios

Angle	Sine	Cosine
0		
1		
5		
44		
45		
50		
85		
86		
90		

Is the accuracy of the measurements more or less important near 90° or near 0°? Explain.



Student Activity: Brain Surgery

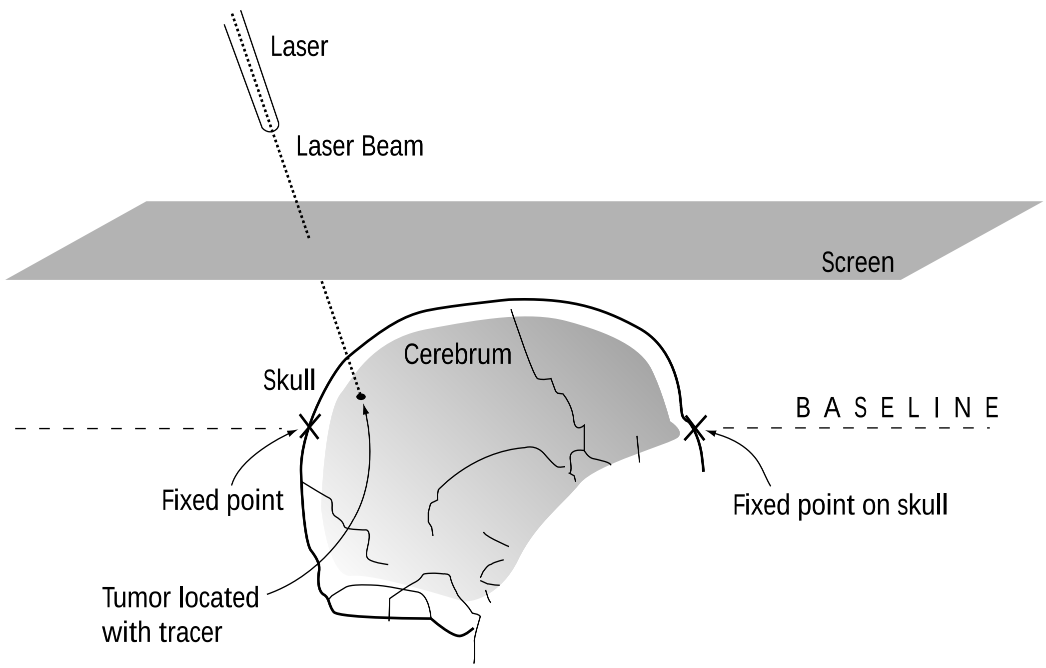
Student Name: ______________________ Date: ______________

Activity 4: Brain Surgery

Brain surgeons need to direct a laser beam with very high precision in order to remove a tumour without damaging the healthy parts of the brain. The tumour is located with a ‘tracer’ and the distance from the tracer to a screen is measured. Refer below to Figure 14: Laser Brain Surgery. A baseline is drawn by using two fixed points on the skull.

Model the situation with a right triangle and identify, on the diagram, the direct measurements you can take and the measurements you must deduce in order to direct the laser beam.

Figure 14: Laser Brain Surgery



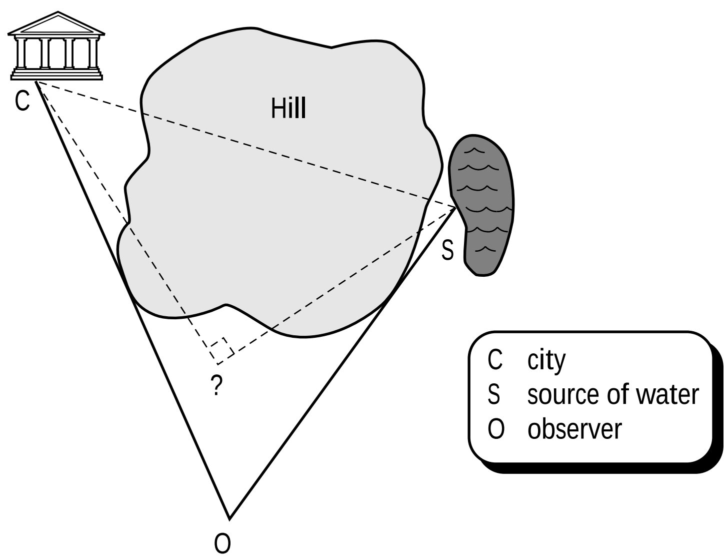
Student Activity: Ancient Greece

Student Name: ___________________________ Date: __________________

Activity 5: Ancient Greece

Due to an increasing population, a certain city of ancient Greece found its water supply insufficient. Water had to be channelled from a source in the nearby mountains. Unfortunately, a large hill intervened and there was no alternative to tunnelling. Working both sides of the hill, the tunnellers met in the middle as planned. (See Figure 15: Water Supply in Ancient Greece.)

Figure 15: Water Supply in Ancient Greece



How did the planners determine the correct direction to ensure that the two crews would meet? How would you have planned the job? Remember that the ancient Greeks could not use radio signals or telescopes. Nevertheless, they devised a method and actually succeeded in making their tunnels from both sides meet somewhere inside the hill.

Use your knowledge of trigonometric ratios to solve this problem.

Student Activity: Mapping

Student Name: ______________________ Date: ______________

Activity 6: Mapping

Identify two points, *C* and *S*, where *C* cannot be seen from *S* such as points along the sides of a building or a hill.

Walk away from the obstacle with a compass until you can see both *C* and *S* with a right angle. (See your teacher for the proper use of a compass.)

Measure *OC* and *OS* with the decameter.

From *O*, walk along a circular arc making sure that the angle remains 90° and measure the distances.

Sketch the locations and record below all the information for your scale diagram. You need to decide on references (a point and a direction) so you can place them on the scale diagram.

Scale Diagram
Use the scale map pages provided by your teacher.

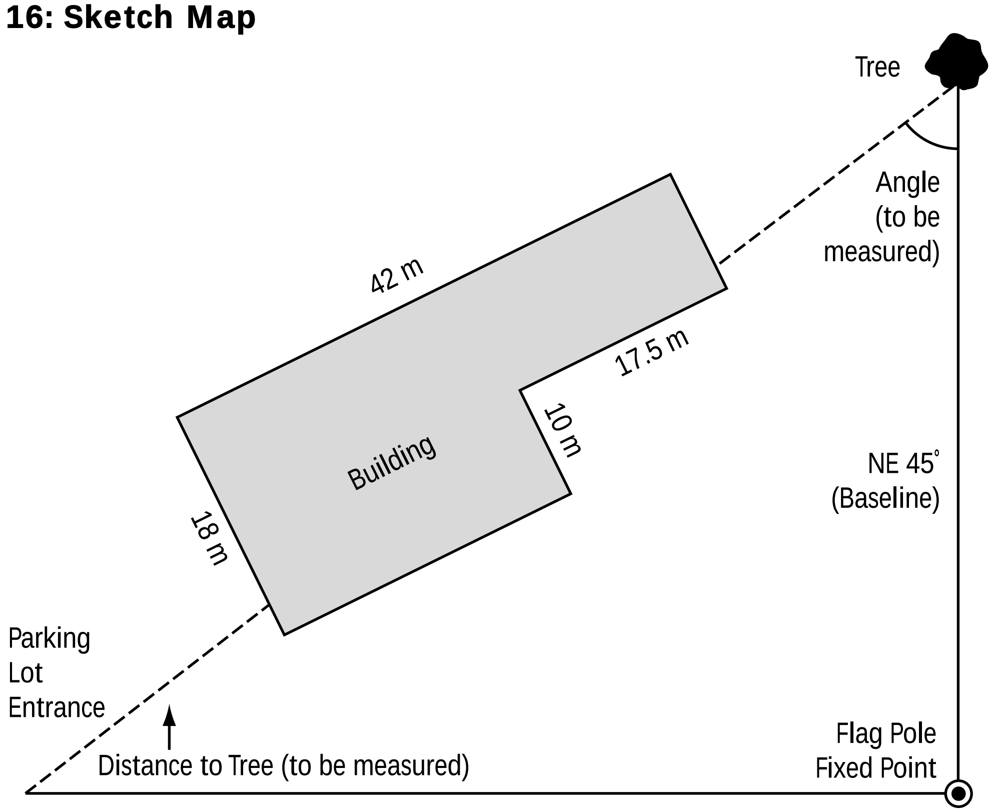
Select the scale you want to use. Your selection depends on how large the distances are on the field. For example, if the distances are quite large, let 1 mm on the diagram = 1 m in actual distance; for smaller distances, let 1 cm on the diagram = 1 m in actual distance.

On your paper, indicate the 'fixed point' and the 'baseline' first. From your sketch, you are able to decide where to place the fixed point and the baseline in order that the entire situation can be drawn on the paper. (See Figure 16: Sketch Map.)

Indicate the direction of the baseline (in degrees) from your measurements and then indicate all the measurements by using the scale.

Student Activity: Mapping

Figure 16: Sketch Map



Phase II
Part 2: Solving the Problem Using Geometry

Objective:

The objective of Phase II: Part 2, is to develop solutions to the problem by using geometry, particularly similarity and congruence.

Materials:

- rope
- decameter
- levels
- stand
- pole
- posts
- flags (or red cones)

Procedure: Similarity of Triangles

The intent of this activity is to introduce students to the concept of similarity of triangles. Students are asked either to determine the unknown height of a tree (by constructing similar triangles) or to determine the grade of a road and then, to solve the problem of the ancient Greek City in a different way. They discover the conditions under which two triangles are similar through direct measurement.

In the first situation, the triangles are right triangles. They then extend the similarity conditions to any triangles in the last situation.

These concepts become helpful in their project because they provide alternate methods for determining indirect measures in a field when right triangles cannot be constructed.

In the Student Activity: Measuring Grade, students experimentally determine the ratios of corresponding sides of two similar triangles. Students note the congruence of the corresponding angles and deduce that the ratios of corresponding sides are equal numerically. Students then express the equality of the ratios formally and use this property to solve the problem.

If there is not enough time to do this activity outside, students can use the results of the measurements from Phase II Activity 1: Measuring a Tree.

Measuring the Grade of a Roadway

Students revisit the concept of similarity in measuring the grade of a ramp (see Phase II Activity 3: Ramping Up). They can use the results found in that activity, if time is a problem.

Similar triangles can be used to measure the grade or elevation of a roadway using a carpenter's level (see Figure 3: Levels).

The percentage grade is the vertical rise or the fall of the road over a horizontal distance of 100 m.

In Figure 21: Measuring Grade, the triangles *BTR* and *BDF* are similar. The board *BT* is 3 m (or 300 cm) long, the fall is 30 cm. Over a distance of 100 m, the fall will be 10 m. This is called the 'percentage grade' of the road. The grade is 10%.

By using the tangent ratio, tan(*B*) = 0.1. Using the inverse tangent function on the calculator gives 5.71°.

Measurement in the School Yard (Part 2) (Partial evaluation)

This activity illustrates the inverse process. That is, it goes from a representation to reality. It also provides students with a new tool for making measurements in their project.

Referencing Figure 23: Mapping Using Similar Triangles #2 the two points, *C* and *S*, are such that *C* cannot be seen from *S* and, also, a right triangle cannot be constructed. Here, it is not possible to construct a right triangle such that the observer, *O*, can see both points *C* and *S*.

In this new situation, the triangle *SOC* is not a right triangle. Can students apply the method of similarity? How are students approaching this new situation?

Have students draw a scale diagram as precisely as possible using the field measurements from their portfolio. To help them understand how crucial accuracy is in a scale diagram, have students estimate:
- how an error of 2 mm on the diagram would be transposed in the field if their scale is 1 cm : 1 m (answer: 20 cm)
- how an error of 1° (with the protractor) would be reflected in the field over a distance of 35 m.

Students construct similar triangles and use the property of similarity to determine where *E* would be on line joining *O* to *C* such that the line *ED* is parallel to the line *CS*. They should notice the congruence of the angles and apply the equality of the ratios of sides to determine the precise direction and length of line *CS*.

Review the conditions of similarity of triangles by recapping the congruence of the angles and the proportionality of the corresponding sides. Indicate to students that there will be three cases.

Mention to students that in their project they will likely have to use this method because it is not always possible to construct right triangles in the real world.

Procedure: Congruence of Triangles: How to Measure an Object's Distance from the Shoreline

If necessary, review transformations, that is, translation, rotation, and reflection. During transformations, the angles and distances are conserved (by translating a triangle, one obtains a 'congruent triangle').

Congruent triangles are a special case of similar triangles. Congruent triangles have ratios 1 : 1, that is, all corresponding sides are equal and the scaling factor is 1. Congruent triangles are triangles that are equal in all their corresponding sides and angles.

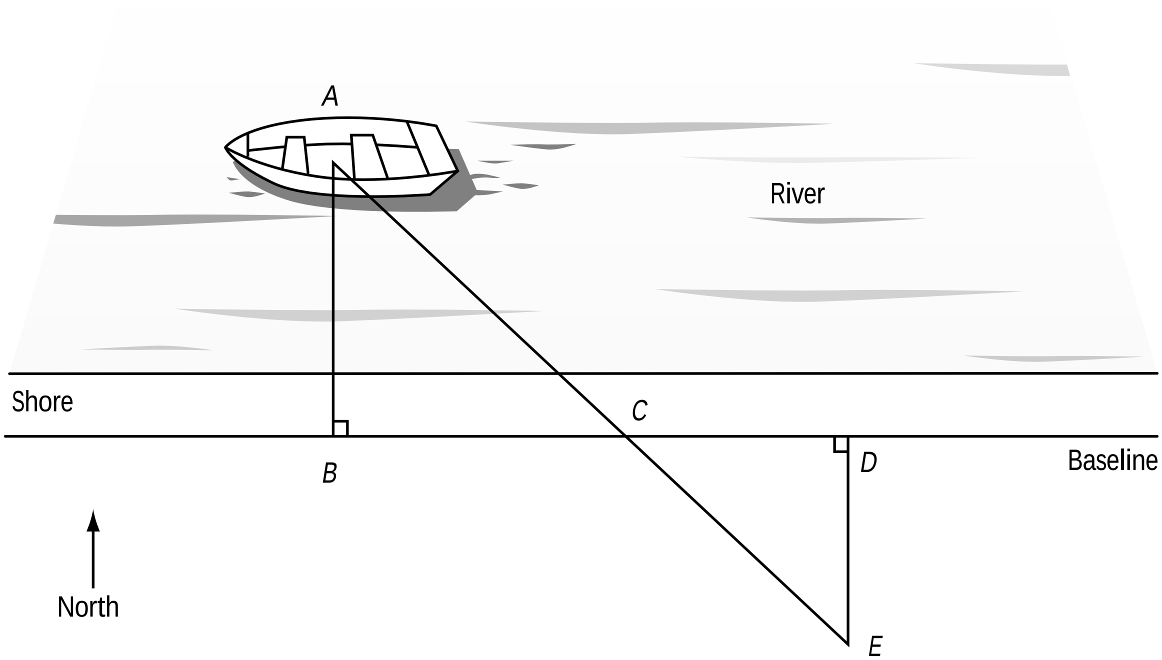
The task is to estimate the position of a boat in a lake, a river, or other area that is difficult to reach. A real-world example is that of a coast guard need to estimate the distance from shore to a boat in trouble.

The distance may be measured indirectly by using a baseline along the shoreline, and sighting and preparing two congruent triangles. You may use Figure 17: Measuring Distance From the Shore, below as a rough guide to the following construction:

- Lay a baseline *BD* of any convenient length. At point *B*, the baseline is at a right angle (90°) with the line of sight to the boat at *A*.
- Find the baseline's midpoint and mark it *C*. The lines from *B* to *C* and *C* to *D* are equal in length.
- From *D*, proceed at an angle of 90° (south in this example) to a point where *C* may be sighted in direct line with point *A*. Mark this point *E*.
- Measure the distance from point *D* to point *E*. This represents the distance the boat is out in the river (*BA*) because in congruent triangles, all corresponding sides and angles are equal.

As a reinforcement activity, ask students to determine a way to solve the problem if there are some restrictions on the length of the baseline. For example, some obstacles restrict the construction of a baseline that is long enough to construct congruent triangles. Again reference Figure 17: Measuring Distance From the Shore, below.

Figure 17: Measuring Distance From the Shore



This is a good opportunity to place the students in a problem situation where they have to apply their knowledge of similar triangles.

Students should recognize that the two triangles are similar and explain that corresponding angles are congruent.

Students find the corresponding ratios and evaluate the unknown distance from the proportionality:

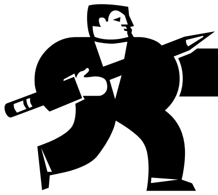
$\frac{ED}{AB} = \frac{DC}{BC}$ implies that ED = AB $\left(\frac{DC}{BC}\right)$.

For example, if DC = 3 times BC, then DE = 3 times AB.
Make sure the students have A, E, and C on the same line.

Evaluation Criteria

To what extent can students in a given situation:

- demonstrate that there is often more than one method that may be used
- choose the most appropriate and precise methods from a range of methods
- apply their knowledge of indirect measurement methods
- use measurement instruments with accuracy and drawing equipment with precision
- manipulate similarity ratios to determine an unknown measure
- perform calculations with precision and round results properly
- determine the reasonableness of results
- explain the impact of imprecision of field measurements on scale diagrams



Student Activity: Revisiting the Tree

Student Name: ___________________________ Date: __________________

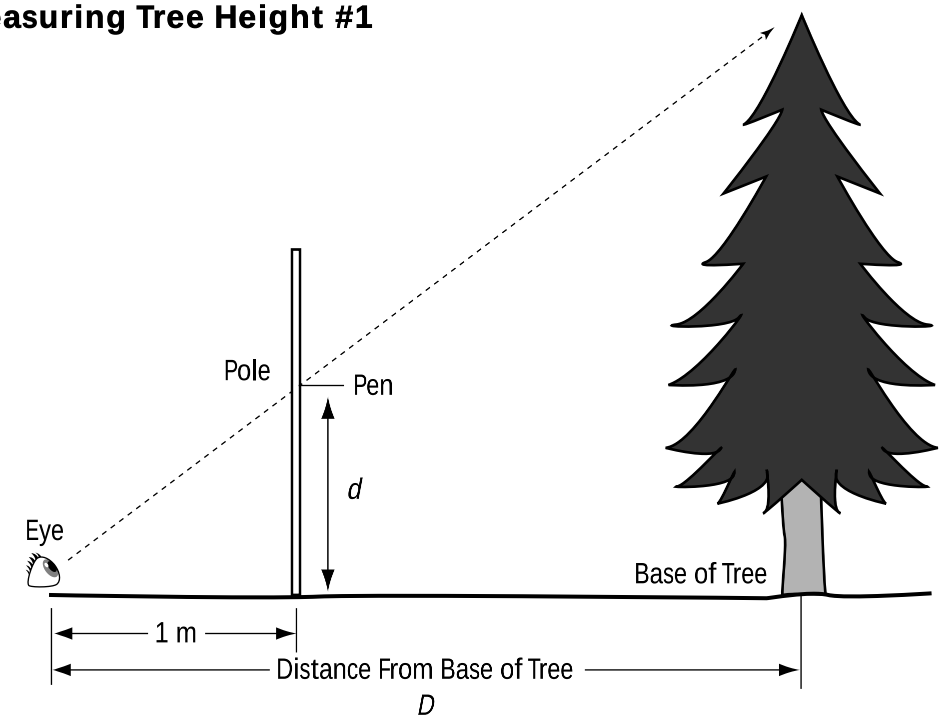
Activity 7: Revisiting the Tree

Sometimes it is more convenient to use techniques that do not involve trigonometric ratios.

Here is a situation where you can determine the height of a tree without using the tangent ratio as you did in the first activity.

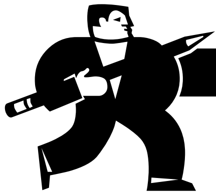
At some distance away from the tree, lie down on the ground and ask your partner to hold the 2 m pole vertically at a distance of 1 m from your eyes (see Figure 18: Measuring Tree Height #1).

Figure 18: Measuring Tree Height #1



Look up at the top of the tree and ask your partner to move a pen along the pole until you see the pen and the top of the tree on the same line. Record the distance of the pen on the pole and measure how far you are from the base of the tree.

Repeat this at different distances from the tree, always keeping the pole 1 metre away from you, and record your results in Table 6: Measuring Tree Height.



Student Activity: Revisiting the Tree

Table 6

Measuring Tree Height

Distance of the Pen on the Pole (*d*)	Distance From the Tree (*D*)	Product (*D* x *d*)

What are your conclusions?

Repeat the same procedure placing the pole at 2 m and 3 m from your eyes.

Calculate, in each case, the product *d* x *D* when the pole was positioned:
at 1 m, *d* x *D* =

at 2 m, *d* x *D* =

at 3 m, *d* x *D* =

What is the product of *d* x *D* when the distance of the pole from eyes is:

0.2 m? *d* x *D* =

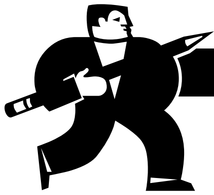
0.5 m? *d* x *D* =

5 m? *d* x *D* =

Let *x* represent the distance between your eyes and the pole (Figure 19: Measuring Tree Height #2 below).

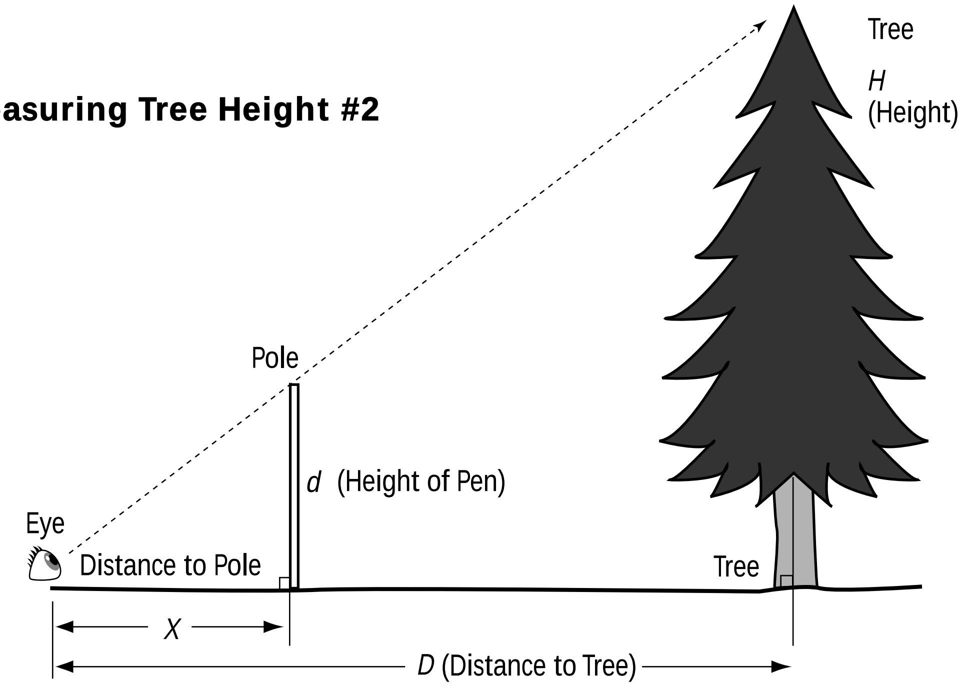
In your own words, relate *x*, *d*, *D*, and the height of the tree, *H*.

Express this relation in the form of an algebraic function (formula)?



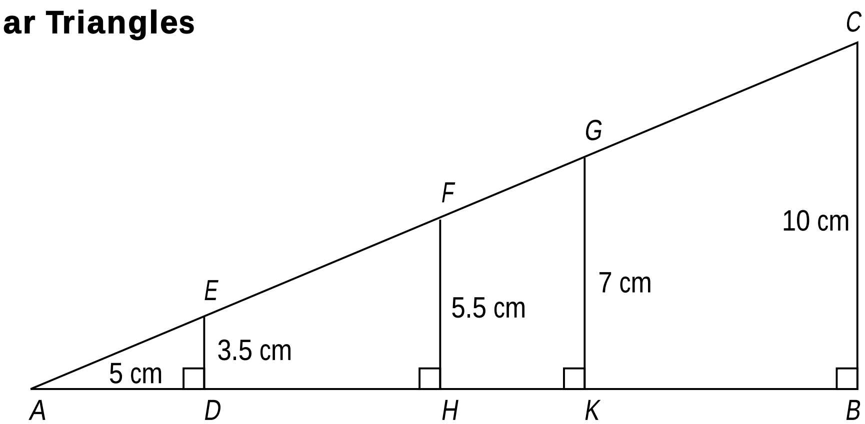
Student Activity: Revisiting the Tree

Figure 19: Measuring Tree Height #2



For the Figure 22: Similar Triangles, what can you say about angles *E*, *F*, *G*, and *C*?

Figure 20: Similar Triangles

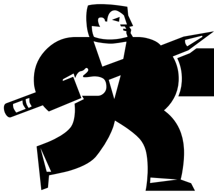


Triangles *ADE*, *AHF*, *AKG*, and *ABC* each have equal corresponding angles. Triangles with each pair of corresponding angles equal are called 'similar triangles'.

The corresponding sides of similar triangles are in the same ratio. To convince you, measure the lengths with a ruler and evaluate the ratios. For example, measure *AD* and *AH*, *ED* and *FH*, and *AE* and *AF*. Calculate the ratios

$\frac{AD}{AH}$, $\frac{ED}{FH}$, and $\frac{EE}{AF}$.

$\frac{AD}{AH} =$ $\qquad$ $\frac{ED}{FH} =$ $\qquad$ $\frac{AE}{AF} =$



Student Activity: Measuring Grade

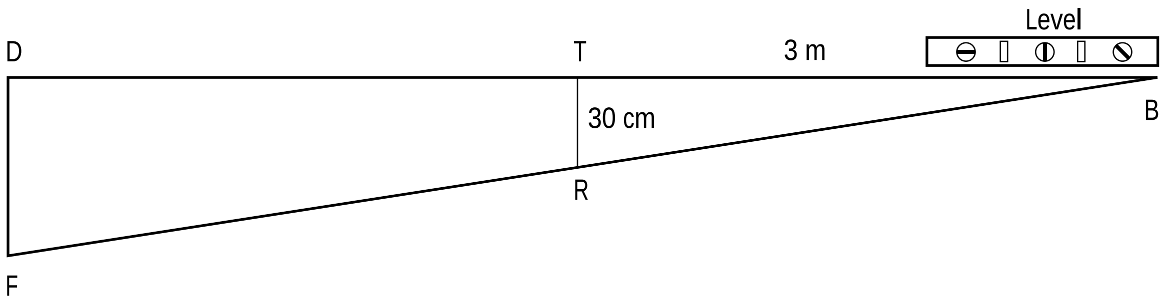
Student Name: ___________________________ Date: __________________

Activity 8: Measuring

Similar triangles can be used to measure the grade or elevation of a roadway with a carpenter's level (see Figure 21: Measuring Grade).

The percentage grade is the vertical rise or the fall of the road over a horizontal distance of 100 m.

Figure 21: Measuring Grade



In the figure, triangles *BTR* and *BDF* are similar triangles having the same shape but not the same size . The board (*BT*) is 3 m (or 300 cm) long, and the vertical fall (*TR*) is 30 cm. What is the vertical fall (*DF*) for a distance of 100 m?

This ratio expressed as a percentage (%) is called the 'percentage grade' of the road.

Determine the depression angle of this road. What is the trigonometric ratio you used to determine this?

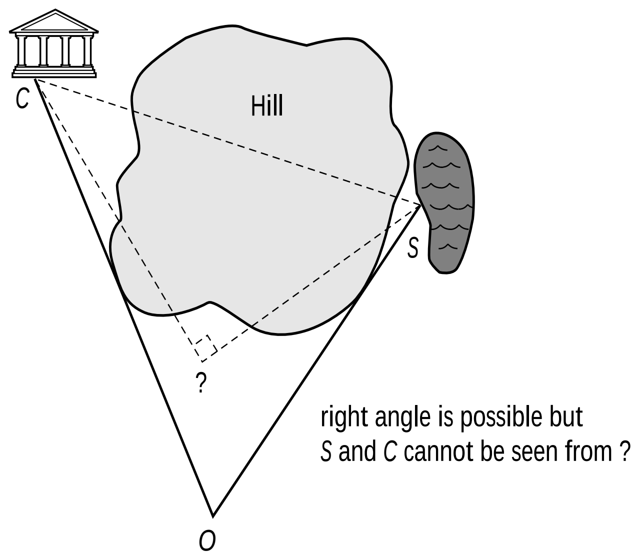
Student Activity: Mapping Using Similarity

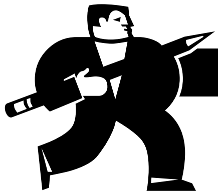
Student Name: ___________________________ Date: __________________

Activity 9: Mapping Using Similarity

Assume that the two points *C* and *S* are such that *C* cannot be seen from *S* and where you cannot construct a right triangle *SOC* when observing both *C* and *S* from *O* (Figure 22: Mapping Using Similar Triangles #1).

Figure 22: Mapping Using Similar Triangles #1





Student Activity: Mapping Using Similarity

It is not possible in this situation to construct a right triangle such that the observer, *O*, can see both points *C* and *S* (such a triangle would have the line *OC* or *OS* passing through the hill).

In this situation, the triangle *SOC* is not a right triangle. Can the method of similarity be applied?

How can you approach this new situation?

In the school yard, walk away from a building until you are able to see both points *C* and *S* (see Figure 23: Mapping Using Similar Triangles #2 below).

Measure *OC* and *OS* with the decameter.

From *O*, walk a certain distance (somewhere between the building and *O*) in the exact direction of *S*, call this point *D*. Measure and record this distance *OD*.

Draw a scale diagram of the situation.

On the scale diagram, use the property of similarity to determine where point *E* would be on the line joining *O* to *C* so that line *ED* is parallel to line *CS*.

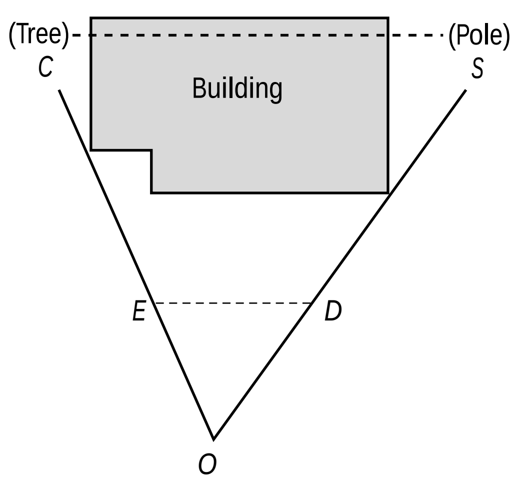
Do you see a way of using the idea that *ED* and *CS* are parallel? Discuss this with students in your group.

Similarity is not a property for right triangles only. The same rules apply for all triangles that do not have right angles, that is, for scalene and isosceles triangles.

Similarity gives you a way to calculate an unknown side of a scalene triangle when another side is known and two corresponding sides of a similar triangle are known.

You will have an opportunity to apply this feature in another activity.

Figure 23 : Mapping Using Similar Triangles #2



Student Activity: Mapping Using Congruence

Student Name: ______________________________ Date: ____________________

Activity 10: Mapping Using Congruence

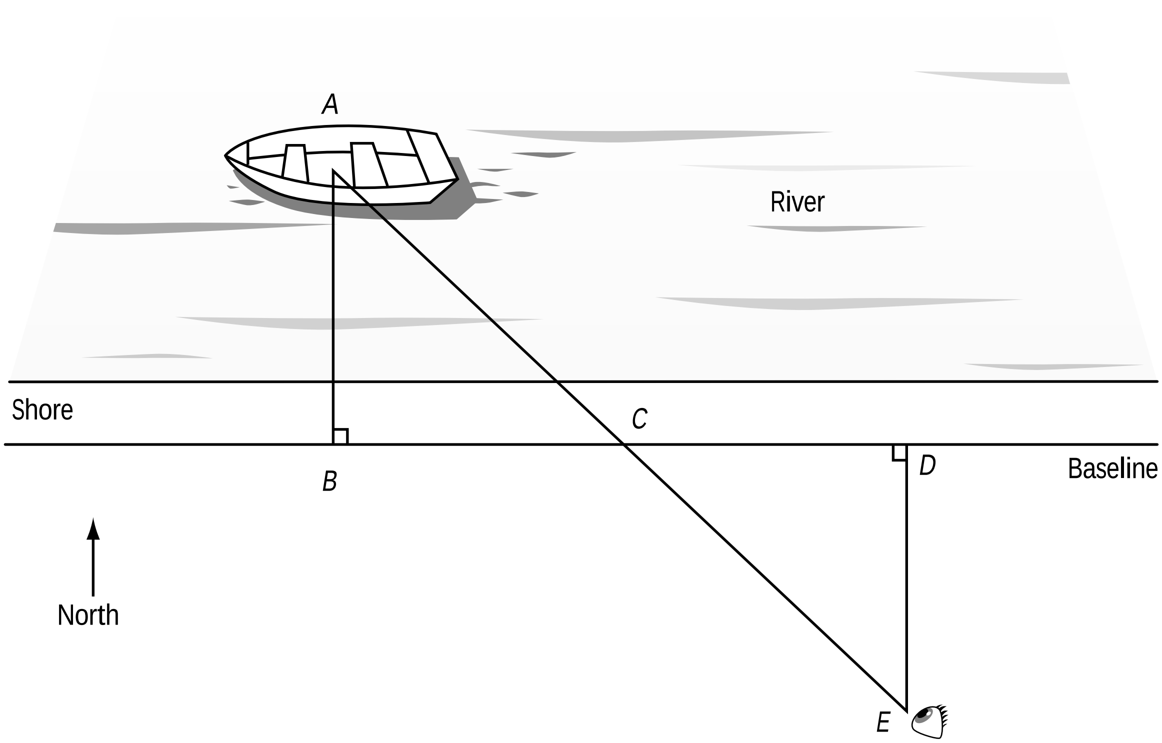
Congruent triangles are those equal in all their corresponding sides and angles.

The position of a boat in a lake, river, or other area that is difficult to reach may be measured indirectly by using a baseline along the shoreline, sighting, and drawing two congruent triangles.

Example (Figure 24: Mapping Using Congruent Triangles, below):

- Lay a baseline, *BD*, of any convenient length. The end of the baseline, *B*, must be such that *BD* is at right angles (90°) with the line of sight from *BD* to the boat at *A*. (Assume A lies north of *BD*.)
- Find the midpoint of the baseline and mark it *C*. *BC* and *CD* are equal in length.
- From *D*, proceed at an angle of 90° (south, in this example) to a point where *C* is sighted in direct line with point *A*. Mark this point *E*.
- Measure the distance from point *D* to point *E*. This represents the distance from shore the vehicle is out in the river. This is true because for congruent triangles, all corresponding sides and angles are equal.

Figure 24: Mapping Using Congruent Triangles



Phase III: Preparing the Scale Map

Objective:

The objective of this phase of the project is the preparation of the scale map. This phase provides students with the opportunity to apply the trigonometric and geometric concepts they have developed over the course of this project. These include how they use their knowledge both to approach new situations in the field and to apply it in different contexts.

Materials:

- graph paper
- ruler
- pair of compasses
- sharp-pointed pencil

Procedure:

Keep in mind the learning outcomes covered during this phase of the project, in particular those that address: Problem Solving, Mathematical Reasoning, Group Skills, Communication Skills and Attitude. The use of the checklist of Appendix A may be helpful.

Groups of students walk the property taking note of points of interest, structures that can serve as reference points, and compass directions.

Remind students to use a compass to record North and the baseline direction.

Students mark the property limits by placing posts at every corner and numbering them. The first step in the sketch will be to draw the property boundaries.

Students sketch the relative positions of objects such as buildings, fences, roads, and bushes and anticipate the points they will have to locate. For each point, they should note what is visible and what is not visible. For example, in Figure 25: Diagram of a Sketch Map, post *Z* is visible from posts *U* and *R* but not from post *V*.

Back in the classroom, students can now divide the tasks. For instance, one group could be responsible for the boundaries of the parking lot, another responsible for locating a building, and another for the levels, and so on.

All students should participate in determining the location of the boundaries of the property.

Each group develops a strategy. For an example of a strategy, refer to Table 7: Measurements.

Observe the students and advise them about the best strategy. Note how they approach the problem and how they apply the concepts of trigonometry to find and justify ways to perform indirect measurements. (See Figure 25: Diagram of a Sketch Map.)

Tell students that more than one strategy is possible. Encourage them to find the most appropriate and most reliable strategies.

Evaluate their strategies and make comments and corrections before sending them out in the field.

In the field, check if students are using their equipment properly and following the strategies they developed.

Encourage student groups to exchange information about previously located points.

Ensure that students are aware that measuring a distance on an uneven surface leads to inaccuracy (because the surface is not flat).

Help students construct their scale diagrams using construction of similar triangles.

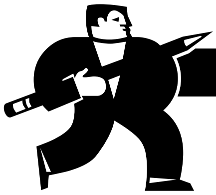
Back in the classroom, students use graph paper, a ruler, a pair of compasses, and a sharp-pointed pencil to draw the scale map. Using a pencil with a broad or dull point may cause inaccuracy.

Remind students to indicate the scale and the legend on their scale maps. Encourage them to use a straight edge and a pair of compasses (instead of using a ruler) to record distances on the map.

Students should proceed first with the most obvious points and continue by using the results obtained in their Table 7: Measurements. Regularly check the sketches produced by other groups to avoid mistakes.

Some points have been used in the measurements more often than necessary. In some cases, you will observe some discrepancies. Rely on the most precise measurements.

For indirect measurements, have students prepare separate maps including all the constructions and calculations.



Student Activity: Sketch Mapping

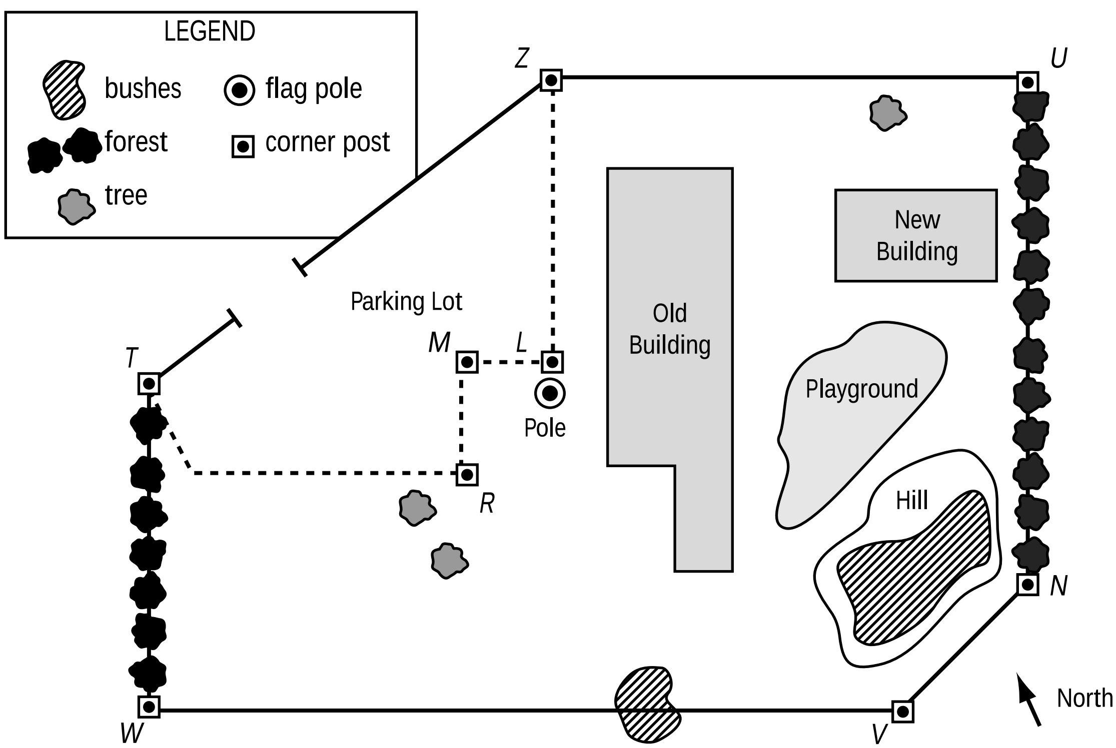
Student Name: ____________________________ Date: ____________________

Activity 1: Sketch Mapping

Go to the site and walk the property, taking note of every interesting point such as a building or hill.

On a piece of paper, sketch the boundaries of the property. The boundaries might include roads, fences, trees and rivers. Roughly indicate the interesting points you have noticed. This sketch does not need to be precise. Use symbols to identify the particular points. Make sure you identify North precisely. See Figure 25: Diagram of Sketch Map below.

Figure 25: Diagram of Sketch Map

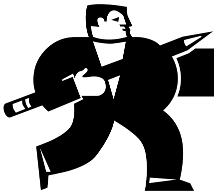


Procedure:

The tasks are divided among the groups (one group may locate the old building, another group is responsible for locating the new building, and so on).

All groups are responsible for the boundaries of the property.

Take a few minutes to develop and elaborate upon a strategy within your group.



Student Activity: Sketch Mapping

A good strategy would include the following operations:

Sample:

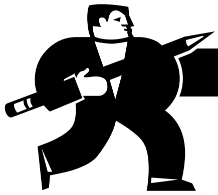
- Determine all the points, for example buildings and hills that you need to locate on the map. Mention if they can be seen from each of the other points. Points could include: property corner posts, old buildings, new buildings, playground, shrubs, and hills.
- Find a fixed point, for example a flag pole.
- Find a fixed direction (this will be your baseline). Use the fixed point and another reference point, note the direction with a compass as precisely as possible. For example, the direction from the flagpole to the corner pole, L, is 40° East (N 40° E).
- Then you need to locate the directions and the distances of the property boundaries. This must be done accurately because you may use some of these points later as secondary reference points (points for verification).
- You should use a table to record your measurements (See Table 7: Measurements).

From the flag pole, you can see the corner poles at *W*, *T*, *R*, *M*, *L* and *Z*. Determine direct measurements with the decameter and the compass. For other corners, you need to use indirect measurements.

Table 7

Measurements

Point	Distance to Fixed Point (*m*)	Direction from North	Method
Corner *L*	5	40 E	
Corner *M*	9	330 E	
Corner *R*	4	260E	
Corner *Z*	35	40 E	
Corner *U*			
Corner *N*			



Student Activity: Sketch Mapping

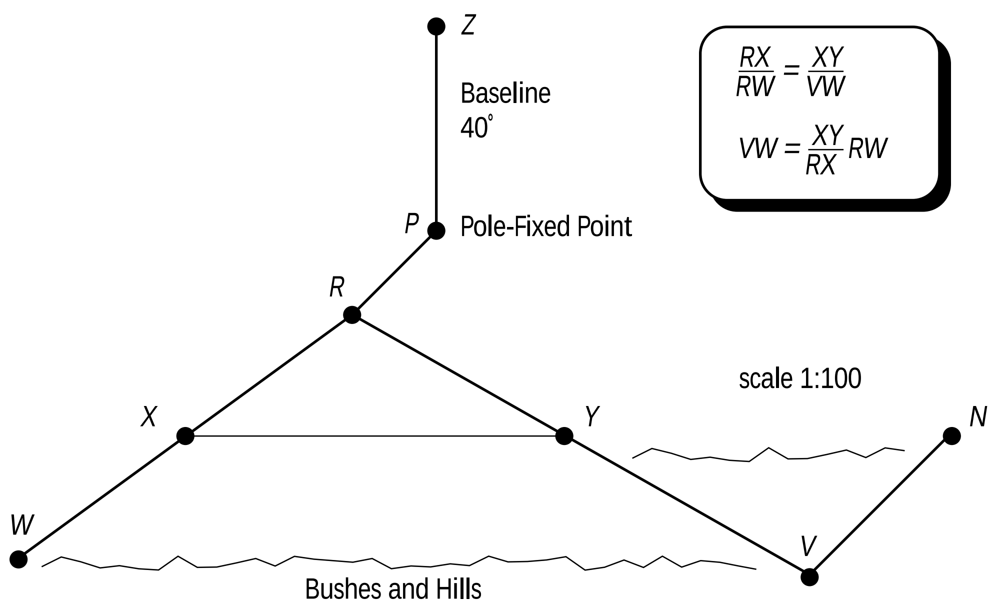
Detailed procedure to locate corner *U*, given location of pole, *P*, and corner *Z*:

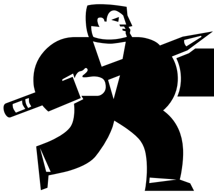
- right triangle is visualized with pole, *Z* and corner *U*
- distance pole to corner *Z* is known: 35 m with decameter
- distance corner *Z* to corner *U* is measured: 42 m with decameter
- distance pole to corner *U*: calculated using Pythagorean relation 54.67 m
- direction: angle *ZPU*: tangent ratio *ZU/ZP* - calculator - 50.2°

Procedure to locate corner *N*: (A suggested strategy follows; others are possible. See Figure 26: Locating Corner 3 Construction)

- corner *N* cannot be seen from any other located points but can be seen from corner *V*, but corner *V* cannot be seen from corner *W* for bushes and hills
- distance *VW* cannot be measured directly
- corner *R* can be seen from corner *V* and distance *VW* can be measured
- *WR* can be measured and *W* can be seen from *R*
- construct triangle *RWV* and use similarity of triangles technique to locate corner *V*
- construct triangle *VWN* and use similarity of triangles to locate corner *N*

Figure 26: Locating Corner 3 Construction





Student Activity: Sketch Mapping

Field Measures

For each point you need to locate, devise a similar action plan with your partners.

During this operation, take turns within your group doing the different tasks. These tasks include: measuring distances, angles and levels; noting the results; and double-checking the measures. Make sure that your group follows the strategy.

Record the measurements in the field book and include as much detailed information as you can. This will be helpful during the drawing of the scale map.

Exchange information with the other groups and double-check your results against theirs. You may save time by using points located by another group.

Measure the distances accurately. The decameter must be held as accurately as possible in a vertical position. You may tie the rope first between the two points and follow the rope with the decameter. Measurements that use only the rope can be inaccurate because the rope can stretch.

When measuring distances, make sure that you measure only flat surfaces. Measuring distances between two points on an uneven surface leads to major error.

Student Activity: The Scale Map

Student Name: ______________________________ Date: ____________________

Activity 2: The Scale Map

Materials:

- Scale Map blank sheet
- graph paper
- ruler
- pair of compasses
- sharp-pointed pencil

Procedure:

From the sketch, place the fixed point and the baseline in such a way that the whole map can be drawn on the paper. Indicate North and the direction of the baseline. Prepare a legend that everybody is able to understand.

Show the scale at the bottom of the plan. It is a good idea to draw the baseline long enough for accuracy and properly divided. Then you can use your pair of compasses to measure distances from the graduated line.

Now add the most obvious points and continue by using the results obtained in your Table 7: Measurements. Compare your sketch with those of the other groups on a regular basis to avoid mistakes.

Some points have been measured more than necessary. You can use this extra information to double-check your measurements. In some cases, there will be some discrepancies. You should rely on the measurements you think are most precise.

For indirect measurements, prepare separate maps that include all the constructions and calculations.