Mathematics 9

A RESOURCE FOR TEACHERS

Building

Confidence

Through

Doing

BRITISH COLUMBIA
Ministry of Education

Acknowledgements

Mathematics 9: A Resource for Teachers is a response to a need expressed by many teachers faced with the challenges of implementing new curriculum and expanding their pedagogy.

The project has benefitted from the collective vision and talents of many individuals over the past two years including:

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M A T H E M A T I C S 9

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About Mathematics 9: A Resource for Teachers

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About Mathematics 9: A Resource for Teachers

What is this resource?

Mathematics 9: A Resource for Teachers complements and augments the Grade 9 Mathematics curriculum. It provides hands-on, activity-based learning experiences that support classroom instruction and appeal to a broad range of students. The opportunities for communicating, connecting, applying mathematics reasoning within an understandable context, and using technology help give students the skills, knowledge, and the attitude they need to use mathematics at home and in their future careers.

Mathematics 9: A Resource for Teachers allows for new material to be added as you experiment with different activities. The resource includes a variety of projects that you can sample and modify to meet the needs of your students. Assessment techniques appropriate for a broad range of students have also been referenced.

Finally, the material included in *Mathematics 9: A Resource for Teachers* has been piloted in Grade 9 classrooms like yours across British Columbia.

Why was this resource developed?

Mathematics 9: A Resource for Teachers was developed to support teachers in their planning for teaching and learning the Mathematics 9 curriculum. This resource helps teachers expand their repertoire of teaching strategies, so that they can better meet the wide-ranging learning outcomes found in the Grade 9 Mathematics curriculum. In particular, it helps with those learning outcomes that address the applied focus, student decision making, critical thinking and problem solving.

Mathematics 9: A Resource for Teachers has three major sections. This first section, About Mathematics 9: A Resource for Teachers, provides an orientation to this resource and its use.

The second section, Introduction to Mathematics 9, has been developed to support teachers as they make the shift to a more applied approach. This section includes:

- an overview with information about mathematics
- a brief description of the process used in teaching mathematics
- a brief description of the principles of learning mathematics

- brief descriptions of the key players involved in teaching and learning Mathematics 9 - students, teachers, parents and community.

The remaining section focuses on examples of real-world problems. Background information has been provided to ensure a context for the problems. These problems are organized as projects that include activities. By working through each phase of a project, students:

- are introduced to the problem or determine the problem for themselves by teasing it out of the situation
- propose solutions to the problem
- determine what additional skills and concepts they need in order to propose solutions and evaluate them
- acquire new skills and ways of thinking
- apply new skills, concepts and ways of thinking to solve the original problem.

The projects presented in this Resource provide opportunities for students to develop the attitudes, knowledge, skills, and levels of thinking needed to identify the mathematics within problems, and to ask mathematical questions.

Many students find it helpful for the teacher to describe the problem-based learning process in which they are engaged. That is:

- posing problems
- posing questions
- generating and identifying alternatives
- making choices
- researching and planning
- generating and constructing products
- evaluating products
- assessing choices and reflecting on them
- reflecting on the effectiveness of the products.

At the end of each project, students should realize that they have learned new mathematical concepts, skills and processes.

For students, teacher and parents, one of the most important aspects of problem-based learning is how it relates to the developmental stage of students. In Grade 9, students are examining their own lives and how they 'fit' into the universe. The Mathematics 9 curriculum encourages learning in which students use their academic understandings and abilities in a variety of in-school and out-of-school settings to solve real-world or simulated problems both alone and as part of a group. By examining real-world applications of mathematics, students learn that there is order, predictability and structure expressed through patterns, both in the mathematical world and their 'real world', reassuring them that all is not random or chaotic.

How is this resource organized?

Mathematics 9: A Resource for Teachers is organized in the following way:

- About *Mathematics 9: A Resource for Teachers* (section 1)
- Introduction to Mathematics (section 2)
- Sample Project: Counting Kokanee
- Project A: Predicting Satellite Collisions
- Project B: Determining the Relationship of Insulin to Diabetes
- Project C: Planning for a Teen Recreation Centre
- Project D: Drawing a Scale Map for a Teen Recreation Centre
- Project E: Building a Scale Model for a Teen Recreation Centre
- Appendix A: Correlation Between Projects and Mathematics 9 Learning Outcomes
- Appendix B: Blackline Masters

The first two sections of the Resource effectively set the stage for embarking on the applied mathematics journey.

Whether you are a teacher who has been on this journey for several years or one just about to begin, we suggest that you spend some time going through these first two sections. The first section provides the information you need to know about the boat on which you are sailing while the second section, provides you with the information about the journey, the destination, and the other people on the journey with you.

The third section includes six projects. Each of these projects is organized in a similar way, by phases.

Phase I sets the stage for the project by describing the problem to be solved or the project to be undertaken. It also identifies the skills and mathematical concepts needed to find solutions.

Phase II are the activities designed for students to acquire the specific knowledge, skills and mathematical concepts important to solving the problem or carrying out the project.

Phase III brings closure to the project as students develop, evaluate, and reflect upon one or more products that are solutions for the problem or project described in the first phase.

The following are synopses of the projects found in the third section of this resource.

Sample Project: Counting Kokanee

Many Grade 9 students may have wondered how the communications media know how many youth attended an outdoor concert, or how many salmon have returned to a local river to spawn. In this sample project, students learn sampling and estimation skills.

The objective of the Sample Project is to give you and your students an opportunity to become familiar with the approach used in this resource. The mathematics utilized in the Sample Project is based on the Grade 8 Mathematics curriculum.

The project contains a discussion on mathematical modelling that is designed to help you explain to your students how real-world systems can be described in terms of mathematical models. A simple heuristic is provided as well as a concrete example.

Project A: Predicting Satellite Collisions

By Grade 9, many students have their first experience buying stereo speakers or a new mountain bike. The skills they learn in this project can help them make better choices as consumers.

The objective of this project is to develop student awareness about the risk of collision between satellites, and between satellites and fragments of satellites or meteorites. The risk of collision is greater because of the dramatic increase in the number of satellites placed into specific orbits.

In this project, students review rational numbers. They learn to model situations with formulae involving powers, and to apply the Laws of Exponents.

Project B: Determining the Relationship of Insulin to Diabetes

Most students are keenly interested in their physical appearance and health. In this project, they learn to make better decisions with respect to food and exercise, and to gain a better understanding of what it means to stay healthy.

The objective of this project is to reinforce student skills in pattern recognition and in performing calculations with rational numbers and powers. Students learn more about diabetes and the role insulin plays in its prevention.

Students learn to identify and graphically represent patterns, and to apply them. They analyse two competing processes using linear models, and gain an understanding of the significance of coefficient of polynomial models.

Project C: Planning for a Teen Recreation Centre

By Grade 9, most students are increasingly independent in terms of their out-of-school activities. In this project, they investigate the need for a teen recreation centre. They learn about surveys, ways to construct questions, and skills that help them understand what survey results actually mean.

The objective of this project is for students to research the community-based recreational needs of youth.

Students learn about designing and implementing surveys, and explore the use of scatterplots and lines of best fit.

Project D: Drawing a Scale Map for a Teen Recreation Centre

Most students have had the opportunity to use a scale map or plan. They might have used these for hiking and orienteering, sewing and quilting, designing decorations for a school dance, or even rearranging their rooms.

The objective of this project is for students to learn some basic surveying techniques.

Students learn about triangulation, tangent ratio, sine and cosine, similarity, and congruence. Once they learn these concepts and skills, they prepare a scale map.

Project E: Building a Scale Model for a Teen Recreation Centre

Building a scale model is often an essential step toward building a final product. For Grade 9 students who are designing the sets for a school play, knowing how to build a scale model is an important skill. For model railway enthusiasts and doll house builders, the building of a scale model is the final product.

The objective of this project is to construct a scale model.

Students learn about solving problems using trigonometry as well as geometry in preparation for constructing their scale models.

You will note that each of Projects C, D and E focusses on a different aspect of the development of a Teen Recreation Centre. You may choose to do one, two or three of these projects. Together, they comprise a much larger investigation.

Appendices

The Appendices include additional materials to support teachers. When these materials are referred to within the text of each project, they are either recommended or required.

Appendix A contains a table correlating the five main projects with the Mathematics 9 learning outcomes. It can also be used to indicate areas where new activities may be developed.

Appendix B contains a few blackline masters that may be used in support of student investigations.

How can I use this resource to help me organize my program?

Mathematics 9: A Resource for Teachers is a tool for supporting teachers engaged in teaching mathematics. It is not a traditional mathematics text to be rigidly followed, or a prescription for creating the context for mathematics.

As the teacher, you choose the number and order of the projects to help build your mathematics program. This selection should be based on the motivation, skills, interests, needs, and abilities of your students. We have included the approximate number of class periods each project requires, but you may wish to expand or contract the scope or amount of time you spend on the projects.

Basically, there are three options for using this Resource:
1. just do it - adopt the resource as a complete program
2. pick and choose - augment an existing approach or course organization with selected activities and projects
3. create own vision - create a unique program based on large-scale problems and projects patterned after those in this resource.

Some of you have always taught mathematics in a practical and applied manner. For you, these projects may simply provide some new ideas and points of departure as you develop your own projects.

Some of you may choose to teach the projects as you became more comfortable using this mathematics approach.

Planning My Program

In order to plan your Mathematics 9 program, you will need:
- an assessment of the motivation, skills, interests, needs, and knowledge of your students
- an assessment of your own expertise, interests, and experience as a teacher of applied mathematics
- the Grade 9 Mathematics curriculum
- the *Mathematics 9: A Resource for Teachers* (this Resource)

As a starting point, refer to the Grade 9 mathematics curriculum and to the table, Correlation Between Curriculum Organizers and Projects, to familiarize yourself with the curriculum organizers and their relationship to the individual projects in this Resource. In addition, for your convenience, the learning outcomes have been reproduced and correlated with projects where the learning outcomes are addressed in a significant way. This can be found in Appendix A.

Table 1

Correlation Between Curriculum Organizers and Projects

Curriculum Organizer	Proj A	Proj B	Proj C	Proj D	Proj E
Problem Solving	•	•	•	•	•
Number (Number Concepts)	•	•			
Number (Number Operations)	•	•			
Patterns and Relations (Patterns)	•	•			•
Patterns and Relations (Variables and Equations)	•	•		•	
Shape and Space (Measurement)				•	•
Shape and Space (3-D Objects and 2-D Shapes)				•	•
Shape and Space (Transformations)	•				•
Statistics and Probability (Data Analysis)	•	•	•		
Statistics and Probability (Chance and Uncertainty)	•				

Your next step is to develop your program in conjunction with the following fundamental ideas:

- provide learning opportunities for students to develop an understanding of Grade 9 mathematics, and to integrate that knowledge into their lives so that it becomes a resource for further education and life-long learning
- actively engage students in their own learning
- provide context for all learning
- encourage students to see relevance in their learning by allowing them to focus on personal passions
- promote the use of technology that encourages the exploration of important mathematical questions
- apply context, co-operative learning, constructivist learning and communication (4 C's) as the foundation for an applied approach
- provide opportunities and support for students to take responsibility for planning, exploration, projects and exercises
- encourage students in their development of a positive attitude towards life-long learning and understanding the role of mathematics in their lives

We recommend that you:

- read through the sample project, Counting Kokanee, to get a flavour of how the other projects in this *Mathematics 9: A Resource for Teachers*, are organized
- examine the Project Planning Template, Figure 1, and review it for the sample project
- read through each project completely
- assess each project in relation to the motivation, interests, skills, knowledge and readiness of your students

- select the most appropriate project or projects
- decide whether to do all or part of each project
- determine which projects you wish to modify, if any, and how
- decide upon the order you will do the projects

Prior to implementing your program, you will need to have the following elements in place:

- a set of activities in support of the projects
- a set of learning resources for both the teacher and students; this Resource forms only part of the teacher and student materials that could be used
- a repertoire of instructional strategies that have been chosen on the basis of particular learning activities
- a range of assessment techniques and a particular assessment strategy overall
- an assessment and evaluation plan

The six projects included in this resource provide a broad range of learning experiences for your students in terms of content, process and product.

As a starting point, consider learning in the context of large-scale projects such as the building of a teen recreation centre.

Each project provides the context for the learning of:

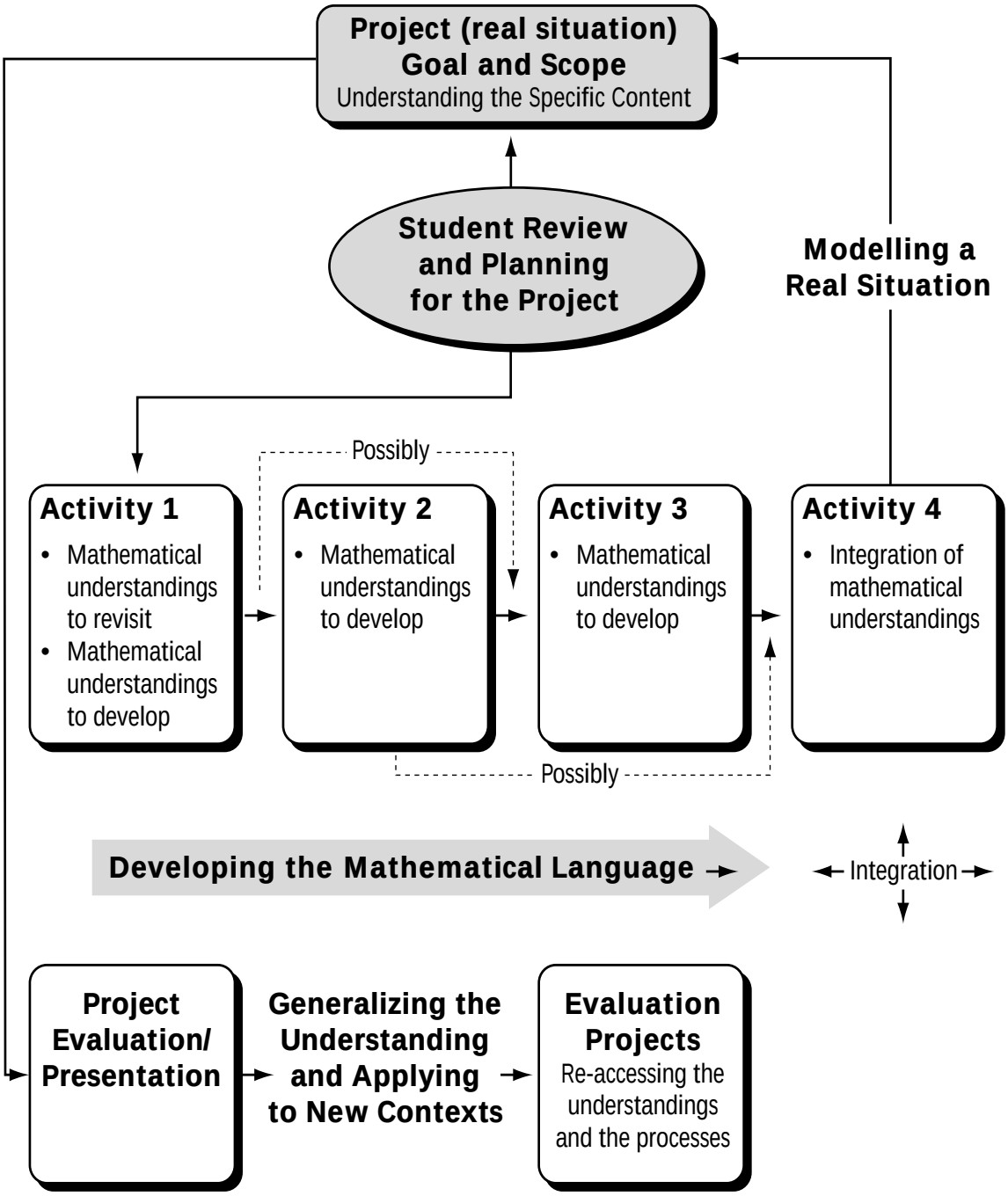
- mathematics concepts
- mathematical skills
- mathematical processes
- mathematical modelling
- attitudes towards mathematics
- ways to see and understand the world through mathematics

We have also included instructional strategies that may be used in each phase of a project. Using a variety of instructional strategies helps ensure that the learning styles of a broad range of learners are addressed.

Instructional Strategies

Teachers have found a number of instructional strategies to be particularly useful in applied mathematics programs. We hope that you find the following information helpful in your planning.

Figure 1: Project Planning Template



A teacher's role as facilitator, instructor, or mentor is both dynamic and flexible. Depending on the particular situation, the balance of responsibility for initiating and completing learning tasks shifts in favour of the teacher or the student. By focussing on an individual student's particular learning style and rate, you can select and apply appropriate strategies to increase the likelihood of success.

The selection of instructional strategies should be based upon how well they foster critical thinking and independent learning. Students will be more engaged in the learning process if you provide them with opportunities to articulate their own problems, apply appropriate strategies, and reach defensible solutions.

Student learning can be deepened and extended by combining instructional strategies. The way strategies are applied depends on several factors including:
- nature of the required activities
- nature of the class
- available resources
- available time
- experience of the teacher
- school and community support
- purpose of instruction
- student motivation
- student readiness

Several instructional strategies that may be used effectively in applied mathematics are briefly described below. It is generally highly appropriate to use a variety of strategies, allowing them to overlap or be used in combination. These instructional strategies include:
- Issues Inquiry
- Case Study
- Learning Centres
- Direct Instruction
- Field Study
- Math Labs
- Group Work or Co-operative Learning
- Simulation or Role Play
- Independent or Self-Directed Learning

Issues Inquiry

What is the Issues Inquiry Strategy?

An Issues Inquiry approach is based on identifying challenging inquiries that lead students to further questions about a problem or an issue. Students create their own questions and then ask themselves, "What do I need to know in order to answer these questions?" In mathematics, indeed in all problem-based activities, issues inquiry is a key instructional strategy.

Why use the Issues Inquiry Strategy?

Issues Inquiry requires and develops critical thinking, creative thinking, problem solving, and communication skills. These help students deal confidently and effectively with the ever-increasing quantity of information available through the news media and the Internet.

Issues Inquiry hooks students into exploring questions that are self-motivating. When exploring issues and answering their own questions, students have an opportunity to experience mathematics as a critical tool that helps them view the world in richer ways.

How can the Issues Inquiry Strategy be used?

Issues Inquiry can be initiated by either the teacher or a student with, for example, discussion of a current news story, the presentation of an engaging conundrum, or the introduction of an intriguing topic.

Issues Inquiry is facilitated when students ask questions that stimulate different aspects of thinking:

- analyzing
- assessing
- classifying
- comparing
- defining
- describing
- evaluating
- hypothesizing
- inferring
- interpreting
- observing
- predicting
- recalling
- recognizing
- reflecting
- synthesizing

The process typically involves the following without regard to a particular order:

- describing the issue or problem
- determining criteria and methods for making decisions
- deciding on constraints of the inquiry, for example, the time frame
- researching, choosing and organizing information
- analysing and synthesizing information to identify alternatives
- selecting alternatives using decision-making criteria
- presenting selected alternatives and a rationale for selection

What is the teacher's role in the Issues Inquiry Strategy?

The teacher's role involves:

- preparing students for self-directed inquiry
- ensuring that issues are of importance to the students engaged in the inquiry
- organizing and structuring appropriate groupings of students

- encouraging students to explore alternatives rather than thinking in terms of a right answer
- focussing questioning to ensure students explore appropriate pathways
- helping students work towards higher levels of questioning
- guiding students to develop assessment criteria, for example:
  - In what ways was your research appropriate and effective?
  - What are some limitations of your data?
  - What are some limitations of your analysis?
  - What are some ways you would do your analysis differently and why?
  - What are some ways you would do your research differently if you were to do it again and why?

What is the student's role in the Issues Inquiry Strategy?

The student's role involves:

- identifying and taking ownership of an interesting topic
- identifying and questioning assumptions
- identifying criteria for decision making
- gathering and assessing information
- analysing data
- identifying alternatives
- selecting and defending an alternative

Case Study

What is the Case Study Strategy?

A Case Study is a written narrative (e.g., story, drama, news article) based upon a set of events that presents a specific real-life dilemma (e.g., a resource management situation). Broad issues and several points of view are examined through a set of focussed study questions.

Why use the Case Study Strategy?

A Case Study focusses on decision-making skills, problem solving and critical thinking. Students are encouraged to explore real-world problems from their own personal perspective.

It is easier for students to understand and remember a particular example that is embedded in a meaningful context than general principles presented in isolation. A Case Study provides students with an opportunity to see the value of mathematics in understanding an issue or problem, and provides them with knowledge and opportunities to apply mathematical concepts and skills. A Case Study based in the local community, provides students with opportunities to access additional learning resources beyond those available at home or school.

What is the teacher's role in the Case Study Strategy?

The teacher's role involves:

- preparing an appropriate study that:
  - is specific and local

- is developed by students and teacher
- includes graphs, maps, photographs, text, and data
- encourages students to gather additional material
- focusses on a specific problem or situation that develops around big ideas
- develops conceptual understanding
- moves student thinking from specific to general; from abstract to concrete and back
- establishes the context for the study
- monitors depth and direction of small group discussion
- asks questions that encourage students to extend their thinking, for example, open-ended questions that cannot be answered with a simple 'yes' or 'no'
- facilitates whole-class discussion and presentations

What is the student's role in the Case Study Strategy?

The student's role involves:

- being motivated to investigate and learn from a local real-world problem
- using mathematics to think critically about issues identified in the Case Study
- developing a personal opinion based on a mathematical analysis
- discussing, communicating, and presenting personal opinion
- acknowledging other perspectives and reaching a consensus based on mathematics

Learning Centres

What is the Learning Centre Strategy?

Learning Centres are self-contained learning stations. These can be located in the classroom, around the school, or in the community. They can be focussed on the use of specific media (for example, software, website information, video, or print), on specific skill development, or on the integration of knowledge and skills. Small groups of three to five students cycle through the Learning Centres, then debrief in order to share ideas and learning.

Why use the Learning Centre Strategy?

A range of student learning styles, strengths, and interests are easily addressed through the use of Learning Centres. These can help students address specific skills through practice.

Students are encouraged to learn concepts and skills, and use them in a personal creative way through active involvement with other students. Students can contribute their own material, for example photos, samples, data, and questions, to a Learning Centre thereby enriching the experience for themselves and others.

How can the Learning Centre Strategy be used?

Learning Centres can be re-used and improved each year. The organization and structure of the centres can remain relatively stable but the content and data may change.

Teachers should select concepts and skills that can be learned and practised relatively independently. Start with three or four Learning Centres. Providing duplicate Learning Centres for large classes helps minimize preparation and class management issues.

Students can also build Centres themselves if they are given a prototype and encouraged to contribute appropriate information. Once Learning Centres have been established, groups rotate and address the questions posed by their peers. Teachers might suggest that students ask themselves, "What are three questions I want answered about the information in my display?"

What is the teacher's role in the Learning Centre Strategy?

The teacher's role involves:

- structuring concepts and skills into learning centres
- guiding and encouraging students in their use of centres
- facilitating individual groups of students by discussing the kinds of ideas and questions students are formulating
- assisting students to reflect on what they have done, clarify the objectives they want to achieve, and extend their mathematical thinking
- working with students to establish assessment criteria

What is the student's role in the Learning Centre Strategy?

The student's role involves:

- attending to specific concepts or skills
- sharing knowledge and skills with the group
- completing and recording individual work
- reflecting on and evaluating own learning

Direct Instruction

What is the Direct Instruction Strategy?

Direct Instruction is traditionally the most commonly used strategy. It can take a variety of forms including lectures, demonstrations, and video presentations. It can also be effectively integrated into other teaching strategies.

Direct Instruction may be appropriate when a list of facts, concepts, or rules needs to be presented as requisite knowledge. Discussion about the relevance or importance of the material is generally limited. Practice to consolidate learning often follows Direct Instruction.

Why use the Direct Instruction Strategy?

This is an effective means to impart information and procedures that students need in order to accomplish a task. Attention to, and retention of the material is improved when the success of an impending task depends upon specific content.

When many students are experiencing similar problems, it is often most effective to present material to the whole class using direct instruction.

Encouraging students to discuss their perspectives and present a more comprehensive view, helps them to refocus their thinking.

How can the Direct Instruction Strategy be used?

Direct instruction is particularly useful in combination with other strategies to summarize material and/or experiences. It can also be used to introduce new activities, structure choice, explain or clarify procedures, provide information in a consistent format, and summarize learning.

What is the teacher's role in the Direct Instruction Strategy?

The teacher's role involves:

- organizing and structuring the material
- relating the material to student tasks
- integrating other strategies
- providing student strategies such as taking notes, summarizing, and webbing
- assessing effectiveness of the strategy

What is the student's role in the Direct Instruction Strategy?

The student's role involves:

- listening attentively
- recording and assimilating information
- recapitulating information
- acting on information
- linking information with prior knowledge

Field Study

What is the Field Study Strategy?

Field Studies are community expeditions and activities that provide students with opportunities to do research, apply their knowledge, and observe the application of concepts to situations outside the classroom setting.

Why use the Field Study Strategy?

Field study makes learning come alive, helping students link the practical and concrete to the theoretical and abstract. First-hand experience is an excellent way to convey many mathematical concepts. Students are more likely to be successful at recalling and applying knowledge and skills in new contexts when they have had opportunities to manipulate materials and develop specific skills within a real-life context.

Field Study provides opportunities for students to develop and practise skills in settings that are different from the traditional classroom. This provides additional opportunities for a larger number of students to succeed.

How can the Field Study Strategy be used?

The immediate school surroundings and community provide a variety of field study opportunities.

Effective Field Study has specific objectives that focus the learning experience and encourage the development of problem-solving skills and higher-level thinking. Field Study should include instructions and activities at pre-determined stations.

A Field Study may be:
- expert-led; teacher-facilitated
- teacher-organized with instructions to be followed by individual students on their own time, and completed within a designated time
- a focussed investigation that includes data gathering or observation within a limited time period or over a continuous period of time
- a travel experience that includes the study of a variety of relevant physical and human manifestations of mathematical concepts

For example, learning outcomes from the Shape and Space curriculum organizer can be brought to life using a Field Study approach. Learning and applying simple surveying techniques provide an opportunity for students to realize the importance of measurement and trigonometry in many real-world situations.

What is the teacher's role in the Field Study Strategy?

The teacher's role involves:
- planning and organizing an appropriate study
- defining tasks and providing focus (including assessment criteria)
- making arrangements for students to utilize community resources
- considering transportation and safety issues
- preparing students for the learning experience
  - discussing data gathering and observation
  - considering interviewing, measuring, and sampling techniques
  - encouraging hypothesizing, analysing, and synthesizing
- debriefing, discussing, and reflecting on the experience

What is the student's role in the Field Study Strategy?

The student's role involves:
- doing the field study
- thinking critically about the objectives and tasks
- assisting in the planning of the study
- observing and collecting data
- hypothesizing and analysing
- discussing, reflecting, and assessing
- evaluating the impact of the study on their mathmatical understanding of the world

Math Labs

What is the Math Lab Strategy?

A Math Lab is a place of experimentation, exploration, and analysis. Math Labs provide students with opportunities to use a wide range of tools

including manipulatives, models, maps, photographs, data collection devices, calculators, and computer software to explore mathematical relationships in a controlled environment.

The Math Lab is a place where basic mathematical skills can be developed and where ideas, skills and tools can be applied to make sense of real-world data.

Why use the Math Lab Strategy?

A Math Lab provides hands-on interaction with equipment and physical objects. This interaction helps students construct meaning. They provide opportunities for students to work on specific skills in the context of a larger inquiry.

Math Labs help students to experience mathematics as a living and vital subject that helps them see further and understand more. Math Labs provide an opportunity for students to develop their thinking and a range of skills and processes that include:

- using technology
- visualizing
- reasoning
- problem solving
- estimating
- connecting
- communicating

How can the Math Lab Strategy be used?

Math Labs can be combined with a Field Study to focus on requisite skills, field study techniques, and data analysis. Math Labs can be conducted to model a physical situation. Students can then analyse the results of an empirical study, and discuss differences and similarities between a mathematical model and the real world.

The relationship of mathematics to other subjects, for example visual art, science and history, can be effectively explored in a laboratory setting.

What is the teacher's role in the Math Lab Strategy?

The teacher's role involves:

- selecting appropriate lab equipment
- setting up a lab and organizing access to it
- providing the context for lab activities
- assessing and discussing lab work with students

What is the student's role in the Math Lab Strategy?

The student's role involves:

- posing and testing hypotheses
- observing, measuring, classifying
- predicting, inferring, interpreting
- formulating models, designing experiments, controlling variables
- assessing data and evaluating results
- communicating and presenting lab results

Group Work or Co-operative Learning

What is the Group Work or Co-operative Learning Strategy?
Group Work or Co-operative Learning occurs when two or more students strategize and co-operatively solve problems to accomplish a shared objective.

As members of a group, students present and explain their points of view, summarize the group's position, develop a strategy to address the task, and interact synergistically to achieve an objective.

Why use the Group Work or Co-operative Learning Strategy?
Group Work or Co-operative Learning provides opportunities for students to develop interpersonal and small group communication skills, and to experience individual accountability. For many students, group work or co-operative learning provides an opportunity for positive interdependence and mutual support.

Group Work or Co-operative Learning skills become increasingly important when individuals assume greater responsibility as workers, employers, parents, and citizens, and develop a positive attitude towards the contributions of others.

How can the Group Work or Co-operative Learning Strategy be used:
Group Work or Co-operative Learning can be incorporated into many different learning activities. By assigning roles, for example, reader, checker, observer, or reporter, different skills and learning styles can be addressed.

Group Work or Co-operative Learning can be used to address a large meaningful problem that could not otherwise be considered. Students experience the power of co-operative projects and are given the opportunity to appreciate the diverse skills and perspectives of the students in their group.

What is the teacher's role in the Group Work or Co-operative Learning Strategy?
The teacher's role involves:

- emphasizing the importance of social skills such as communicating, accountability, and negotiating for agreement
- working with students to select groups appropriate for the particular task at hand
- encouraging students to discuss tasks in ways that use higher level thinking
- facilitating and supporting students as they interact in the learning process and support each other's efforts
- providing opportunities for students, both as a group and individually, to reflect on group dynamics and the value of each person's contribution to the success of the group or team
- providing strategies for students to deal with dysfunctional groups
- assessing group work appropriately based on both the process used and the product developed

What is the student's role in the Group Work or Co-operative Learning Strategy?

The student's role involves:

- selecting appropriate team members
- taking responsibility for himself and the group
- reflecting on how the group is functioning
- adjusting personal behaviour as required
- assessing personal work and the group's work

Simulation or Role Play

What is the Simulation or Role Play Strategy?

Simulation or Role Play is a scenario in which students interact within a structured context. Rules of interaction and the initial state of the scenario are presented first. Students become the actors and play out a story using available information and their mathematical understandings. This instructional strategy is a key one in applied mathematics.

The three parts to this strategy are:

1. preparation, where learning takes place about the scenario;
2. the actual simulation where the students play out their roles; and
3. a critical debriefing of the simulation.

Why use the Simulation or Role Play Strategy?

Simulation or Role Play provides students with opportunities to explore and immerse themselves in real-world or imaginary situations that they could not otherwise access. By interacting with each other and within known constraints, students are challenged to analyse their thinking in light of other ideas. Simulation or Role Play helps students see things from different perspectives, understand that there are other ways to view a situation, and that there are many ways to solve a problem.

Critical thinking, decision making, problem solving, and communication skills are emphasized in Simulation or Role Play. The consequences of a sequence of decisions, consequences not necessarily apparent at the start of the simulation, can be explored and discussed.

How can the Simulation or Role Play Strategy be used?

A Simulation or Role Play can be used to summarize or bring closure to an activity. Roles can be assigned and researched based on the interests and styles of individual students.

Students who do not participate directly in the main simulation can be assigned the role of reporter, for example, for Much Music. The student may then choose to report on the event in print, audio, or video, or through a website. The report should focus on the critical issues and arguments that emerged during the simulation activity.

What is the teacher's role in the Simulation or Role Play Strategy?

The teacher's role involves:

- setting the scene:
  - describing the circumstances of the simulation
  - identifying some of the attributes of the main characters
  - setting the objective for each character
  - identifying the rules of the simulation
- ensuring students have sufficient time and information to prepare for the activity
- guiding students as they prepare their roles
- facilitating interaction during the simulation
- helping students debrief their experience

What is the student's role in the Simulation or Role Play Strategy?

The student's role involves:

- preparing for her individual role by gathering information and developing mathematical skills to make sense of information
- engaging in the simulation by interacting and exploring possibilities
- reflecting on the simulation to arrive at meaningful conclusions about the importance of mathematics in real-life scenarios

Independent or Self-Directed Learning

What is the Independent or Self-Directed Learning Strategy?

Independent or Self-Directed Learning consists of a time-limited research or learning project chosen by the student in consultation with the teacher. Self-directed learning should always be an option for students.

Why use the Independent or Self-Directed Learning Strategy?

This strategy provides students with the opportunity to take responsibility for:

- identifying learning objectives
- working independently towards achieving those objectives
- actively assessing their accomplishments in relation to predetermined criteria

This strategy has the added benefits of:

- appealing to many students by helping them focus on a particular passion
- appealing to many students who do not work well in groups
- helping students develop research skills that will benefit them in later years

How can the Independent or Self-Directed Learning Strategy be used?

Independent or Self-Directed Learning requires a contract with which the student feels comfortable before starting the project. Students must develop a vision of their project, identify an objective, and select a specific challenge within that objective.

Students write a contract either developing their own proposal or working from an initial list of challenges provided by the teacher. The contract should state that, "I've chosen this topic/challenge; I'm going to do these particular

things and this is how I'm going to do it; I'll be finished by this date and will deliver a product (e.g., report, presentation, video, model) that will demonstrate my learning."

An additional component of the contract with the student could include an assessment of the experience doing the topic. For example: "My experience doing this project has helped me to understand the world of mathematics in the following ways..." If assessment has been incremental, it will be nearly complete when the project is completed.

What is the teacher's role in the Independent or Self-Directed Learning Strategy?

The teacher's role involves:

- guiding the student in developing a plan and a timeline that address the student's learning style, include appropriate milestones, and anticipate obstacles
- facilitating implementation of the plan
- monitoring the learning
- assessing learning incrementally according to agreed-upon criteria

What is the student's role in the Independent or Self-Directed Learning Strategy?

The student's role involves:

- working with the teacher to determine a preferred learning style
- developing a learning plan around a topic of personal interest and including appropriate assessment criteria
- entering into a project contract
- working through the project and providing deliverables on time

A note about using technology...

Technology supports and enhances the instructional strategies described above.

Technology not only engages students but it is also intricately linked to past and present mathematical exploration. Similarly, mathematics is embedded and entwined with all technological development. Using technology can help students understand the fundamental role mathematics plays in our lives. The use of technology helps prepare students for the workplace. Various learning styles can also be addressed by using technology.

One of the expectations of the Mathematics curriculum is that technology will be used as a tool to help students develop mathematical concepts and solve complex problems.

Technology can be used for:

- facilitating interaction (e.g., e-mail, Internet, fax)
- conducting simulations
- practising skills

- manipulating information (e.g., spreadsheets, databases, calculators),
- collecting data (measuring devices)
- modelling and analysing real-life phenomena
- stimulating critical thinking, problem solving, and decision making

The teacher's role involves:
- providing students with access to a wide variety of technology
- identifying available technology
- determining appropriate use of technology
- organizing access to and use of the technology
- providing context for using technology to support specific mathematical activities
- using technology to facilitate the applications of mathematics
- providing alternatives to using a specific technology

The student's role involves:
- attending to the mathematics rather than dwelling on the technology
- selecting and using appropriate technology
- using technology effectively

Assessment

Assessment is the systematic process of gathering information to provide ongoing feedback about critical or significant aspects of a student's learning. Students benefit from assessment when they clearly understand the learning objectives and expectations.

Student performance assessment is based on a wide variety of methods and tools. These range from portfolio assessment to pencil-and-paper tests. Appendix D of the IRP includes a detailed description of assessment and evaluation.

Provincial reference sets can also help you assess the skills that students acquire across curricular areas, in particular:
- Evaluating Problem Solving Across Curriculum (RB 0053)
- Evaluating Group Communication Skills Across Curriculum (RB 0051)
- Evaluating Mathematical Development Across Curriculum (RB 0052)

A series of assessment handbooks that provide guidance for teachers as they explore and expand their assessment repertoires includes:
- Performance Assessment (XX0246)
- Portfolio Assessment (XX0247)
- Student-Centred Conferencing (XX0248)
- Student Self-Assessment (XX0249)

The assessment strategies listed in Column 3 of the IRP suggest several approaches to gathering information about student performance. These assessment strategies have been developed by specialist and generalist teachers to assist their colleagues. Ideas for developing a repertoire of assessment

strategies may come from reviewing the earlier section on Instructional Strategies, and reviewing the suggested instructional strategies in each Column 2 of the Grade 9 Mathematics curriculum.

Feedback, Please

We welcome any comments or questions you might have about this resource and encourage you to share the experiences you and your students have had using it, for example, through a listserv or website. Please contact:

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