MATHEMATICS 10 TO 12

Applications of Mathematics 10 to 12
Essentials of Mathematics 10 to 12
Principles of Mathematics 10 to 12
Calculus 12

Integrated Resource Package 2000

BRITISH COLUMBIA
The Best Place on Earth

IRP 110

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MATHE MATICS 10 TO 12 APPENDICES

This Integrated Resource Package (IRP) provides some of the basic information that teachers will require to implement Applications of Mathematics 10 to 12, Essentials of Mathematics 10 to 12, Principles of Mathematics 10 to 12, and Calculus 12. The information contained in this IRP is also available through the Internet. Contact the Ministry of Education's home page: http://www.bced.gov.bc.ca/

THE INTRODUCTION

The Introduction provides general information about the Grade 10 to 12 Mathematics curriculum, including special features and requirements. It also provides a rationale for the subject—why mathematics is taught in BC schools—and an explanation of the curriculum organizers.

THE GRADE 10 TO 12 MATHEMATICS CURRICULUM

The provincially prescribed curriculum for Grade 10 to 12 Mathematics is structured in terms of *curriculum organizers*. The main body of this IRP consists of four columns of information for each organizer. These columns describe:
- provincially prescribed learning outcome statements for Grade 10 to 12 Mathematics
- suggested instructional strategies for achieving the outcomes
- suggested assessment strategies for determining how well students are achieving the outcomes provincially
- recommended learning resources

Prescribed Learning Outcomes

Learning outcome statements are content standards for the provincial education system. Learning outcomes set out the knowledge, enduring ideas, issues, concepts, skills, and attitudes for each subject. They are statements of what students are expected to know and be able to do in each grade. Learning outcomes are clearly stated and expressed in measurable terms. All learning outcomes complete this stem: "It is expected that students will. . . . "

Outcome statements have been written to enable teachers to use their experience and professional judgment when planning and evaluating. The outcomes are benchmarks that will permit the use of criterion referenced performance standards. It is expected that actual student performance will vary. Evaluation, reporting, and student placement with respect to these outcomes depends on the professional judgment of teachers, guided by provincial policy.

Suggested Instructional Strategies

Instruction involves the use of techniques, activities, and methods that can be employed to meet diverse student needs and to deliver the prescribed curriculum. Teachers are free to adapt the suggested instructional strategies or substitute others that will enable their students to achieve the prescribed outcomes. These strategies have been developed by specialist and generalist teachers to assist their colleagues; they are suggestions only.

Suggested Assessment Strategies

The assessment strategies suggest a variety of ways to gather information about student performance. Some assessment strategies relate to specific activities; others are general. As with the instructional strategies, these strategies have been developed by specialist and generalist teachers to assist their colleagues; they are suggestions only.

Provincially Recommended Learning Resources

The Ministry of Education promotes the establishment of a resource-rich learning environment through the evaluation of educationally appropriate materials intended for use by teachers and students. The media formats include, but are not limited to, materials in print, video, and software, as well as combinations of these formats. Learning resources for Applications of Math 10 to 12 and Principles of Math 10 to 12 were reviewed and recommended as part of the Western Canadian Protocol (WCP) Mathematics Learning Resource Evaluation. The WCP recommended learning resources have been approved by the Minister and have been incorporated into the Grade Collection for each course.

The Grade Collection package contains a grade collection chart for Applications of Mathematics 10 to 12 and Principles of Mathematics 10 to 12. These charts list both the comprehensive and additional resources for each curriculum organizer for the course. Each chart is followed by an annotated bibliography. Please confirm with suppliers for complete and up-to-date information.

Provincially recommended learning resources for Calculus 12 are materials that have been reviewed and evaluated by British Columbia teachers in collaboration with the Ministry of Education.

Provincially recommended learning resources for Essentials of Mathematics 10 to 12 are being developed and will be identified at a later date. As an interim measure the ministry has provided schools with photocopy masters of learning resources developed for a similar curriculum.

It is expected that teachers will select resources from those that meet the provincial criteria and that suit their particular pedagogical needs and audiences. Teachers who wish to use non-provincially recommended resources to meet specific local needs must have these resources evaluated through a local district approval process.

THE APPENDICES

A series of appendices provides additional information about the curriculum, and further support for the teacher.

- ***Appendix A*** contains a listing of the prescribed learning outcomes for the curriculum.
- ***Appendix B*** contains a comprehensive listing of the provincially recommended learning resources for this curriculum. As new resources are evaluated, this appendix will be updated.
- ***Appendix C*** outlines the cross-curricular reviews used to ensure that concerns such as equity, access, and the inclusion of specific topics are addressed by all components of the IRP.
- ***Appendix D*** contains assistance for teachers related to provincial evaluation and reporting policy. Prescribed learning outcomes have been used as the source for examples of criterion-referenced evaluations.
- ***Appendix E*** acknowledges the many people and organizations that have been involved in the development of this IRP.
- ***Appendix F*** contains an illustrated glossary of terms used in this IRP.
- ***Appendix G*** contains illustrative examples which show the competencies an average student is expected to demonstrate for each of the prescribed learning outcomes.
- ***Appendix H*** contains more detailed information related to specific teaching strategies referred to in the IRP. This information is provided in cases where explanations of strategies and activities are too extensive to fit in the Suggested Instructional Strategies column, or where the same activity is referred to several times.

Preface: Using This Integrated Resource Package

Grade

Principles of Mathematics 10 • *Patterns and Relations (Patterns)*

Curriculum Organizer

Prescribed Learning Outcomes

The Prescribed Learning Outcomes column of this IRP lists the specific learning outcomes for each curriculum organizer.

Prescribed Learning Outcomes

It is expected that students will generate and analyse number patterns.

It is expected that students will:

- generate number patterns exhibiting arithmetic growth
- use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems
- relate arithmetic sequences to linear functions defined over the natural numbers
- generate number patterns exhibiting geometric growth

Suggested Instructional Strategies

Many natural phenomena and man-made processes grow and shrink according to easily defined mathematical rules. An understanding of arithmetic and geometric growth helps students to better understand how the world around them changes.

- Give students a collection of arithmetic or geometric sequences with blanks either between the numbers or at the end of the sequence for students to fill in. Have them state the common difference or ratio and give a word-based rule describing how one term is calculated from the previous term.
- Have students find three examples of arithmetic or geometric sequences found in nature or used by people in their day-to-day lives.
- Ask students to match a column of linear equations to a column of arithmetic sequences.
- Have students describe or model natural phenomena that incorporate geometric sequences in their structure (e.g., flowers, spiral shells, paper folding).
- Give students numerical and non-numerical patterns and ask them to define the rule(s) for each pattern. Have them use the rules to make predictions and classify the patterns.
- Have students use concepts of arithmetic and geometric growth to solve problems such as:
  - If $1000 is deposited each year into an account that earns 12% compounded annually, how much money will have accumulated after 25 years?

  Then have them:
  - generate a model of this situation
  - identify it as geometric or arithmetic
  - discuss the answer (Is it surprising? Reasonable?)

Suggested Instructional Strategies

The Suggested Instructional Strategies column of this IRP suggests a variety of instructional approaches that include group work, problem solving, and the use of technology. Teachers should consider these as examples that they might modify to suit the developmental levels of their students.

Grade

Principles of Mathematics 10 • *Patterns and Relations (Patterns)*

Curriculum Organizer

Suggested Assessment Strategies

The Suggested Assessment Strategies offer a wide range of different assessment approaches useful in evaluating the prescribed learning outcomes. Teachers should consider these as examples they might modify to suit their own needs and the instructional goals.

Suggested Assessment Strategies

Assessment strategies should demonstrate algebraic skills as well as holistic pattern recognition. Students should also be able to articulate methods to aid in pattern discovery.

Observe

- How quickly do students see the patterns in partially completed sequences:
  - with no hints (2, 5, 8…)
  - with hints (2, _, _, 16, _… - geometric)
- Can students develop a mathematical equation from a sequence?

Question

- Can students articulate what to look for in determining whether a collection of numbers is an arithmetic sequence, geometric sequence, or neither?

Collect

- Have students collect five arithmetic, five geometric, and five other sequences. Provide them with equations or descriptions of how to generate each sequence.

Peer/Self Assessment

- Have each student develop a connect-the-dots puzzle using a combination of sequences and linear equations. Ask them to have another student do the puzzle and check it for accuracy.

Recommended Learning Resources

Print Materials

- Exploring Advanced Algebra with the TI-83
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Pure Mathematics 10 (Distance Learning Package)
- A Visual Approach to Algebra

Multi-media

- The Learning Equation: Mathematics 10 Lessons 22 - 24
- Mathematics 10, Western Canadian Edition Ch. 1 (Sections 1.1, 1.2, 1.4) Ch. 5 (Section 5.5)
- MATHPOWER 10, Western Edition Ch. 2 (Sections 2.3, 2.5, 2.7)

Software

- Secondary Math Lab Toolkit
- Understanding Math Series

Games/Manipulatives

- Radical Math: Math Games Using Cards and Dice (Volume VII)

CD-ROM

- Geometry Blaster

Recommended Learning Resources

The Recommended Learning Resources component of this IRP is a compilation of provincially recommended resources that support the prescribed learning outcomes. A complete list including a short description of the resource, its media type, and distributor is included in Appendix B of this IRP.

This Integrated Resource Package (IRP) sets out the provincially prescribed curriculum for the Grade 10 to 12 Mathematics curriculum. The development of this IRP has been guided by the principles of learning:

- Learning requires the active participation of the student
- People learn in a variety of ways and at different rates
- Learning is both an individual and a group process

THE DEVELOPMENT OF THIS INTEGRATED RESOURCE PACKAGE

A variety of resources were used in the development of this IRP:

- The prescribed learning outcomes, suggested instructional strategies, suggested assessment strategies, and Illustrative Examples for Applications of Mathematics 10 to 12 and Principles of Mathematics 10 to 12 were developed with reference to *The Common Curriculum Framework for K to 12 Mathematics* (Western Canadian Protocol for Collaboration in Basic Education, 1996)
- The prescribed learning outcomes, suggested instructional strategies, suggested assessment strategies, and Illustrative Examples for Essentials of Mathematics 10 to 12 were developed with reference to *Consumer Mathematics (Senior 2, 3, & 4): A Foundation for Implementation and Student Handbooks* (Manitoba Education & Training, 1999)
- Other written resources used include
  - *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989)
  - *Guidelines for Student Reporting*
  - provincial reference sets *Evaluating Problem Solving Across Curriculum* and *Evaluating Mathematical Development Across Curriculum*
  - *Report of the BCAMT Shape/Space (Geometry) Sub-Committee* (BC Association of Mathematics Teachers, 1999)
  - *Assessment Handbook Series*
  - *Summary of Responses* to the *Draft Grade 10 to 12 Mathematics IRP*
  - *Mathematics Proficiencies for Post-Secondary Mathematics/Statistics Courses: Project Report* (Neufeld, 1999)

This IRP represents the ongoing effort of the province to provide education programs that put importance on high standards in education while providing equity and access for all learners. In addition to this print version, the Grade 10 to 12 Mathematics IRP will also be available in electronic format.

RATIONALE

Mathematics is increasingly important in our technological society. Students today require the ability to reason and communicate, to solve problems, and to understand and use mathematics. Development of these skills helps students become numerate.

> Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations. (British Columbia Association of Mathematics Teachers, 1998)

Becoming numerate involves developing the ability to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems. It also involves the development of self-confidence and the ability to use quantitative and spatial information in problem solving and decision making. As students develop their numeracy skills and concepts, they generally grow more confident and motivated in their mathematical explorations. This growth occurs as they learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life.

The provincial mathematics curriculum emphasizes the development of numeracy skills and concepts and their practical application in higher education and the workplace. The curriculum places emphasis on probability and statistics, reasoning and communication, measurement, and problem solving. To ensure that students are prepared for the demands of both further education and the workplace, the graduate years of the mathematics curriculum (Grades 11 and 12) help students develop a more sophisticated sense of numeracy. At the same time, the curriculum investigates the creative and aesthetic aspects of mathematics by exploring the connections between mathematics, art, and design.

Developing Positive Attitudes

Research, including provincial assessments, consistently indicates that there is a positive correlation between student attitudes and performance. Mathematics activities should be interesting and engaging, so that students will be more likely to take risks to develop their mathematical thinking. Classroom practice and teaching strategies should promote positive attitudes toward mathematics for all students.

Becoming Mathematical Problem Solvers

Problem solving is the cornerstone of mathematics instruction. Students must learn the skills of effective problem solving, which include the ability to:

- read and analyse a problem
- identify the significant elements of a problem
- select an appropriate strategy to solve a problem
- work alone or in groups
- verify and judge the reasonableness of an answer
- communicate solutions

Acquiring these skills can help students become reasoning individuals able to contribute to society.

As students move through the grades, the curriculum presents them with increasingly diverse and complex mathematical problems to solve. To encourage students' abilities to communicate, explore, create, adjust to changes, and actively acquire new knowledge throughout their lives, mathematical problem solving should evolve naturally out of their experiences and be an integral part of all mathematical activity. Effective problem solving consists of more than being able to solve many different types of problems. Students need to be able to solve mathematical problems that arise in any subject area and to draw upon skills developed in more than one area of mathematics. Becoming a mathematical problem solver requires a willingness to take risks and persevere when faced with problems that do not have an immediately apparent solution.

Communicating Mathematically

Mathematics is a language—a way of communicating ideas. Communication plays an important role in helping students build links between their informal, intuitive notions and the abstract language and symbolism of mathematics. Communication also plays a key role in helping students make important connections among physical, pictorial, graphic, symbolic, verbal, descriptive, and mental representations of mathematical ideas. All activities that help students explore, explain, investigate, describe, and justify their decisions promote the development of communication skills. The Kindergarten to Grade 12 mathematics curriculum emphasizes discussing, writing, and representing mathematical thinking in various ways.

Connecting and Applying Mathematical Ideas

Learning activities should help students understand that mathematics is a changing and evolving domain to which many cultural groups have contributed. Students become aware of the usefulness of mathematics when mathematical ideas are connected to everyday experiences. Learning activities should therefore help students relate mathematical concepts to real-world situations and allow them to see how one mathematical idea can help them understand others. This approach emphasizes that mathematics helps students solve problems, describe and model real-world phenomena, and communicate complex thoughts and information with conciseness and precision.

Reasoning Mathematically

Mathematics instruction should help students develop confidence in their abilities to reason and to justify their thinking. Students should understand that mathematics is not simply memorizing rules. Mathematics should make sense, be logical, and also be enjoyable. The ability to reason logically usually develops on a continuum from concrete to formal to abstract. Students use inductive reasoning when they make conjectures by generalizing from a pattern of observations; they use deductive reasoning when they test those conjectures. To develop mathematical reasoning skills, students require the freedom to explore, conjecture, validate, and convince others. It is important that their ability to reason is valued as much as their ability to find correct answers.

Using Technology

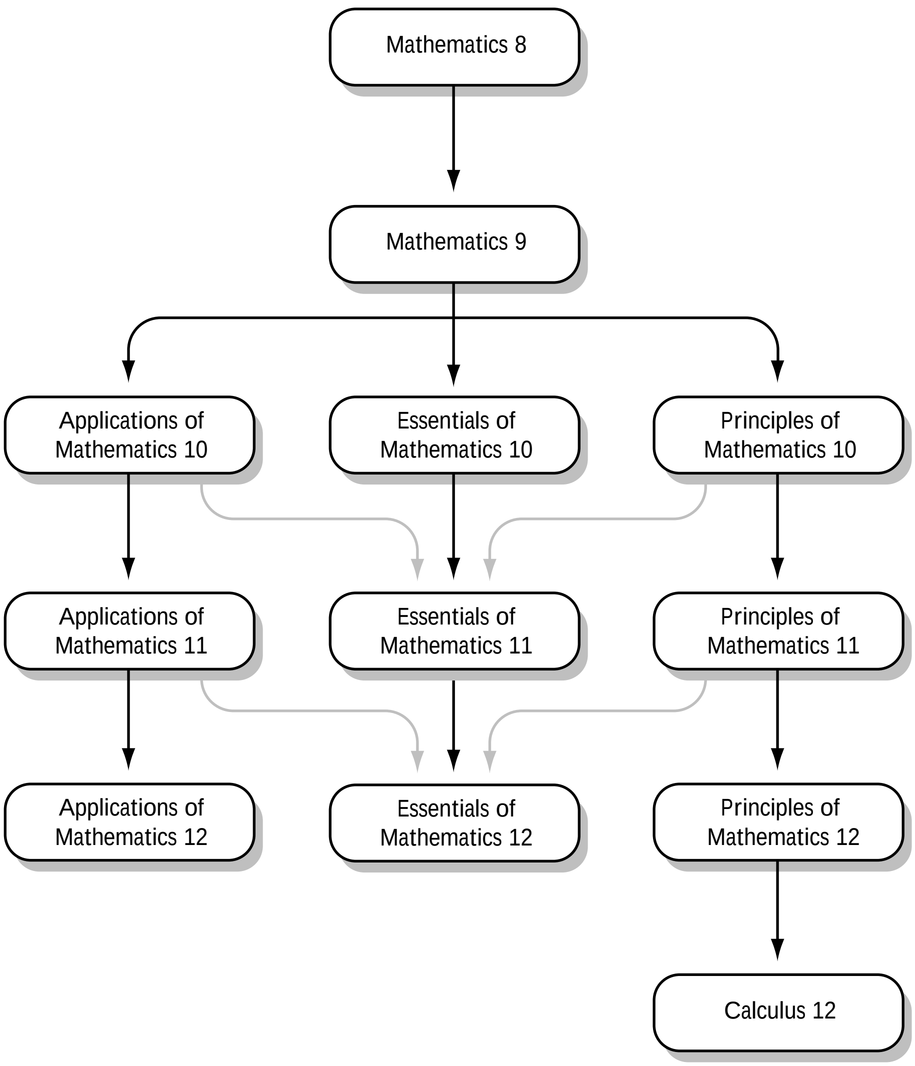
The Grade 10 to 12 Mathematics curriculum requires students to be proficient in using technology as a problem-solving tool. New technology has changed the level of sophistication of mathematical problems encountered today as well as the methods that mathematicians use to investigate them. Graphing tools such as computers and calculators are powerful aids to problem solving. The power to compute rapidly and to graph mathematical relationships instantly can help students explore many mathematical concepts and relationships in greater depth. When students have opportunities to use technology, their growing curiosity can lead to richer mathematical invention.

It is important to recognize that calculators and computers are tools that simplify, but do not accomplish, the work at hand. The availability of calculators does not eliminate the need for students to learn basic facts and algorithms. Students must have access to and be able to select and use the most appropriate tool or method for a calculation. In each of the mathematics courses, technology is used extensively to assist in the investigation and exploration of mathematical concepts.

Estimation and Mental Math

Mathematics involves more than exactness. Estimation strategies help students deal with everyday quantitative situations. Estimation skills also help them gain confidence and enable them to determine if something is mathematically reasonable. Even though they may have access to calculators from Kindergarten to Grade 12, students need to use reasoning, judgment, and decision-making strategies when estimating. Instruction should therefore emphasize the role that these strategies play.

B.C. SECONDARY MATHEMATICS COURSE STRUCTURE



Note: To simplify this diagram, not all possible student transitions between the Applications of Mathematics Pathway, the Essentials of Mathematics Pathway, and the Principles of Mathematics Pathway have been shown.

The Three Pathways

The mathematics curriculum for Grades 10 to 12 offers students a choice of routes through the different mathematics courses offered. Although each student's exact route will depend on a variety of factors, there are three main pathways:

- Applications of Mathematics
- Essentials of Mathematics
- Principles of Mathematics and Calculus 12

The Applications of Mathematics Pathway

The Applications of Mathematics pathway provides a practical, contextual focus that encourages students to develop their mathematical knowledge, skills, and attitudes in the context of their lives and possible careers. The instructional approaches used to develop the required mathematical concepts emphasize concrete activities and modeling, with less emphasis on symbol manipulation. When needed, students should have access to technology that extends their basic skills and knowledge and allows them to repeatedly investigate and model mathematical concepts and issues.

Students from the Applications of Mathematics pathway will be well prepared for many post-secondary programs that do not require calculus as part of the program of studies. The breadth of the Applications of Mathematics curriculum is intended to prepare students for entrance into many certificate programs, diploma programs, continuing education programs, trades programs, technical programs, and some degree programs.

The Essentials of Mathematics Pathway

In order to meet the challenges of society, high school graduates must be numerate. Students following this pathway will have opportunities to improve their numeracy skills and concepts. Developing a sense of numeracy will help them to understand how mathematical concepts permeate daily life, business, industry, and government. Students need to be able to use mathematics not just in their work lives, but in their personal lives as citizens and consumers. It is intended that students will learn to value mathematics and become confident in their mathematical abilities.

The Principles of Mathematics Pathway

Students following the Principles of Mathematics pathway will spend more time developing their understanding of symbol manipulation and of generalizations of more sophisticated mathematical concepts. Although there is an increased focus in this pathway on the applications of mathematics, one of the primary purposes of Principles of Mathematics will be to develop the formalism students will need to continue on with the study of calculus.

Both Applications of Mathematics 12 and Principles of Mathematics 12 have a provincial exam component. Students who successfully complete Applications of Mathematics 11, Essentials of Mathematics 11, or Principles of Mathematics 11 will meet British Columbia's graduation requirements.

Calculus 12

Calculus 12 is intended for students who have completed (or are concurrently taking) Principles of Mathematics 12 or who have completed an equivalent college preparatory course that includes algebra, geometry, and trigonometry.

Students taking Calculus 12 should be prepared to write the UBC - SFU - UVic - UNBC Challenge Examination if they choose to do so. For more information concerning the Challenge Examination contact the Mathematics Department at one of these universities.

Some schools may choose to develop articulation agreements with their local colleges. Students under these agreements may receive credit for first-term calculus (depending upon the particular agreement).

Calculus 12 Prerequisites

In *Mathematics Proficiencies for Post-Secondary Mathematics/Statistics Courses: Project Report* (Neufeld, 1999), the following concepts and skills (listed in descending order of importance) were identified as "essential" to "marginally important" for students to possess in order to attempt a course in calculus:

- the function concept
- polynomial expressions
- exponential expressions
- straight line and linear functions
- solving equations and inequalities
- circular trigonometric functions
- rational expressions
- triangle trigonometry
- the quadratic function
- logarithmic function
- radical expressions
- the geometry of lines and points
- polynomial functions
- quadratic relations
- sequences and series
- the geometry of circles

Teachers are urged to assess their students' mathematics proficiency as it relates to these topics so that any deficiencies can be addressed before new calculus concepts are taught.

Organization of the Curriculum

The prescribed learning outcomes for the courses described in this Integrated Resource Package are grouped under a number of curriculum organizers. These curriculum organizers reflect the main areas of mathematics that students are expected to address. They form the framework of the curriculum and act as connecting threads across all grade levels for each pathway. The organizers are not equivalent in terms of number of outcomes or the time that students will require in order to achieve these outcomes. Suggestions for estimated instructional times have been included in this IRP. Teachers are expected to adjust these estimated instructional times to meet their students' diverse needs.

Within each course, the prescribed learning outcomes under many of the curriculum organizers are grouped under one, two, or three suborganizers. Each set of prescribed learning outcomes is introduced by a broad statement of the associated general learning outcomes for mathematics. (Material related to the general outcomes or suborganizers is not addressed in every course.)

The ordering of organizers and outcomes in the Grade 10 to 12 mathematics curriculum does not imply an order of instruction. The order in which various outcomes and topics are addressed is left to the professional judgment of teachers.

Suggested Instructional Strategies

Instructional strategies have been included for each curriculum organizer (or suborganizer) and grade level. These strategies are suggestions only, designed to provide guidance for generalist and specialist teachers planning instruction to meet the prescribed learning outcomes. Some links to other subjects are indicated.

The strategies may be teacher directed, student directed, or both. There is not necessarily a one-to-one relationship between learning outcomes and instructional strategies, nor is this organization intended to prescribe a linear approach to course delivery; it is expected that teachers will adapt, modify, combine, and organize instructional strategies to meet the needs of students and respond to local requirements.

Context Statements

Each set of instructional strategies starts with a context statement followed by several examples of learning activities. The context statement links the prescribed learning outcomes with instruction. It also states why these outcomes are important for the student's mathematical development.

Instructional Activities

The mathematics curriculum is designed to emphasize the skills needed in the workplace, including those involving the use of probability and statistics, reasoning, communicating, measuring, and problem solving.

Additional emphasis is given to strategies and activities that:

- *foster the development of positive attitudes*

Students should be exposed to experiences that encourage them to enjoy and value mathematics, develop mathematical habits of mind, and understand and appreciate the role of mathematics in human affairs. They should be encouraged to explore, take risks, exhibit curiosity, and make and correct errors, so that they gain confidence in their abilities to solve complex problems. The assessment of attitudes is indirect, and based on inferences drawn from students' behaviour. We can see what students do and hear what they say, and from these observations make inferences and draw conclusions about their attitudes.

- *apply mathematics*

For students to view mathematics as relevant and useful, they must see how it can be applied to a wide variety of real-world applications. Mathematics helps students understand and interpret their world and solve problems that occur in their daily lives.

- *use manipulatives*

Using manipulatives is a good way to actively involve students in mathematics. Manipulatives encourage students to explore, develop, estimate, test, and apply mathematical ideas in relation to the physical world. Manipulatives range from commercially developed materials to simple collections of materials such as boxes, cans, or cards. They can be used to introduce new concepts or to provide a visual model of a mathematical concept.

- *use technology*

The use of technology in our society is increasing. Technological skills are becoming mandatory in the workplace. Instruction and assessment strategies that use a range of technologies such as calculators, computers, CD-ROMs, and videos will help students relate mathematics to their personal lives and prepare them for the future. The use of technology in developing mathematical concepts and as an aid in solving complex problems is encouraged to a greater extent as the student moves from grade to grade.

- *require problem solving*

For students to develop decision-making and problem-solving skills, they need learning experiences that challenge them to recognize problems and actively try to solve them, to develop and use various strategies, and to learn to represent solutions in ways appropriate to their purposes. Problems that occur within the students' environment can be used as the vehicle or context for students to achieve the learning outcomes in any of the curriculum organizers.

Instructional Focus

The Grade 10 to 12 mathematics courses are arranged into a number of organizers, including the Problem Solving organizer. Decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations allows more time for concept development.

In addition to problem solving, other critical thinking processes—reasoning and making connections—are vital to increasing students' mathematical power and must be integrated throughout the program. A minimum of half the available time within all organizers should be dedicated to activities related to these processes.

Instruction should provide a balance between estimation and mental mathematics, paper and pencil exercises, and the appropriate use of technology, including calculators and computers. (It is assumed that all students have regular access to appropriate technology such as graphing calculators, or computers with graphing software and standard spreadsheet programs.) Concepts should be introduced using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

INTEGRATION OF CROSS-CURRICULAR INTERESTS

Integration of Cross-Curricular Interests

Throughout the curriculum development and revision process, the advice of experts has been invited to ensure that relevance, equity, and accessibility issues are addressed in all Integrated Resource Packages.

The recommendations of these cross-curricular reviews have been integrated into the prescribed learning outcomes, suggested instructional strategies, and assessment strategies components of all curricula with respect to the following:

- Applied Focus
- Career Development
- Multiculturalism and Anti-Racism
- English as a Second Language (ESL)
- Special Needs
- Aboriginal Studies
- Gender Equity
- Information Technology
- Media Education
- Science-Technology-Society
- Environment and Sustainability

See Appendix C: Cross-Curricular Interests for more information.

Gender Issues in Mathematics

The education system is committed to helping both male and female students succeed equally well. In British Columbia, significant progress has been made in improving the participation and success rate of female students in secondary math courses. They now take about the same number of secondary math courses as males. There continues, however, to be a relatively low rate of female participation in math-related careers and education. Positive attitudes toward the practice of mathematics, as well as skill in mathematics, are essential to the workplace and to everyone's ability to participate fully in society. Teaching, assessment materials, learning activities, and classroom environments should place value on the mathematical experiences and contributions of both men and women and people of diverse cultures.

Research regarding gender and mathematics has raised a number of important issues that teachers should consider when teaching mathematics. These include the diversity of learning styles, gender bias in learning resources, and unintentional gender bias in teaching. The following instructional strategies are suggested to help the teacher deliver a gender-sensitive mathematics curriculum.

- Feature both females and males who are mathematicians, or who make extensive use of mathematics in their careers, as guest speakers or subjects of study in the classroom.
- Design instruction to acknowledge differences in experiences and interests between young women and young men.
- Demonstrate the relevance of mathematics to a variety of careers and to everyday life in ways that are apt to appeal to particular students in the class or school. Successful links include biology, environmental issues, and current topics in mass media.
- Explore not only the practical applications of mathematics, but also the human elements, such as ways in which ideas have changed throughout history and the social and moral implications of mathematics.
- Explore ways of approaching mathematics that will appeal to a wide variety of students. Use co-operative rather than competitive instructional strategies. Focus on concept development, encouraging students to question until they can say "I've got it." Include a wide variety of applications that demonstrate the role of math in the social fabric of our world. Varying approaches appeal to a wider variety of students.
- Emphasize that mathematics is used by ordinary people with a variety of interests and responsibilities.
- Allowing for informal social interaction with successful "math using" members of the community will help change the negative stereotypes of mathematicians and their social style.
- Provide opportunities for visual and hands-on activities, which most students enjoy. Experiments, demonstrations, field trips, and exercises that provide opportunities to explore the relevance of mathematics are particularly important.

Adapting Instruction for Diverse Student Needs

When students with special needs are expected to achieve or surpass the learning outcomes set out in the mathematics curriculum, regular grading practices and reporting procedures are followed. However, when students are not expected to achieve the learning outcomes, adaptations and modifications must be noted in their Individual Education Plans (IEPs). Instructional and assessment methods should be adapted to meet the needs of all students.

The following strategies may help students with special needs succeed in mathematics:

- Adapt the Environment
  - Change the student's seat in the classroom.
  - Make use of co-operative grouping.
- Adapt Presentations
  - Provide students with advance organizers of the key mathematical concepts.
  - Demonstrate or model new concepts.
  - Adapt the pace of activities as required.
- Adapt Materials
  - Use techniques, such as colour coding the steps to solving a problem, to make the organization of activities more explicit.
  - Use manipulatives such as large-size dice, cards, and dominoes.
  - Use large-print charts such as a 100s chart or a times-table chart.
  - Provide students with a talking calculator or a calculator with a large keypad.
  - Use large print on activity sheets.
  - Use opaque overlays on text pages to reduce the quantity of print that is visible.
  - Highlight key points on activity sheets.
- Adapt Methods of Assistance
  - Have peers or volunteers assist students with special needs.
  - Have students with special needs help younger students learn mathematics.
  - Have teacher assistants work with individuals and small groups of students with special needs.
  - Work with consultants and support teachers to develop problem-solving activities and strategies for
  - mathematics instruction for students with special needs.
- Adapt Methods of Assessment
  - Allow students to demonstrate their understanding of mathematical concepts in a variety of ways, such as murals, displays, models, puzzles, game boards, mobiles, and tape recordings.
  - Modify assessment tools to match student needs. For example, oral tests, open-book tests, and tests with no time limit may allow students to better demonstrate their learning than a traditional timed paper-and-pencil test.
  - Set achievable goals.
  - Use computer programs that provide opportunities for students to practise mathematics as well as record and track their results.
- Provide Opportunities for Extension and Practice
  - Require the completion of only a small amount of work at a given time.
  - Simplify the way questions are worded to match the student's level of understanding.
  - Provide functional, everyday contexts (e.g., cooking) in which students can practise measurement skills.

SUGGESTED ASSESSMENT STRATEGIES

Teachers determine the best assessment methods for their students. The assessment strategies in this document describe a variety of ideas and methods for gathering evidence of student performance. The assessment strategies column for a particular organizer always includes specific examples of assessment strategies. Some strategies relate to particular activities, while others are general and could apply to any activity. These specific strategies may be introduced by a context statement that explains how students at this age can demonstrate their learning, what teachers can look for, and how this information can be used to adapt further instruction.

About Assessment in General

Assessment is the systematic process of gathering information about students' learning in order to describe what they know, are able to do, and are working toward. From the evidence and information collected in assessments, teachers describe each student's learning and performance. They use this information to provide students with ongoing feedback, plan further instructional and learning activities, set subsequent learning goals, and determine areas requiring diagnostic teaching and intervention. Teachers base their evaluation of a student's performance on the information collected through assessment.

Teachers determine the purpose, aspects, or attributes of learning on which to focus the assessment; when to collect the evidence; and the assessment methods, tools, or techniques most appropriate to use. Assessment focuses on the critical or significant aspects of the learning to be demonstrated by the student.

The assessment of student performance is based on a wide variety of methods and tools, ranging from portfolio assessment to pencil-and-paper tests. Appendix D includes a more detailed discussion of assessment and evaluation.

Applications of Mathematics Pathway

Problem Solving	Applications of Math 10	Applications of Math 11	Applications of Math 12
	• use a variety of methods to solve real-life, practical, technical, and theoretical problems	• use a variety of methods to solve real-life, practical, technical, and theoretical problems	• use a variety of methods to solve real-life, practical, technical, and theoretical problems
Number			
Number Concepts	• analyse the numerical data in a table for trends, patterns and interrelationships.		
Number Operations	• use basic arithmetic operation on real numbers to solve problems. • describe and apply arithmetic operations on tables to solve problems, using technology as required.	• solve consumer problems, using arithmetic operations.	• describe and apply operations on matrices to solve problems, using technology as required. • design or use a spreadsheet to make and justify financial decisions.
Patterns and Relations			
Patterns			• generate and analyse cyclic, recursive and fractal patterns.
Variables and Equations		• represent and analyse situations that involve expressions, equations and inequalities. • use linear programming to solve optimization problems.	
Relations and Functions	• examine the nature of relations with an emphasis on functions. • represent data, using function models.	• represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.	
Shape and Space			
Measurement	• demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. • solve problems involving triangles, including those found in 3-D and 2-D applications.	• demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. • use measuring devices to make estimates and to perform calculations in solving problems.	• analyse objects, shapes and processes to solve cost and design problems.
3-D Objects and 2-D Shapes	• solve coordinate geometry problems involving lines and line segments.	• develop and apply the geometric properties of circles and polygons to solve problems.	• solve problems involving polygons and vectors, including both 3-D and 2-D applications.
Statistics and Probability			
Data Analysis	• implement and analyse sampling procedures, and draw appropriate inferences from the data collected.	• analyse graphs or charts of given situations to derive specific information.	
Chance and Uncertainty			• use normal and binomial probability distributions to solve problems involving uncertainty. • solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations. • model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

Essentials of Mathematics Pathway

Essentials of Math 10	Essentials of Math 11	Essentials of Math 12
Problem Solving • use a variety of methods to solve real-life, practical, technical, and theoretical problems	**Problem Solving** • use a variety of methods to solve real-life, practical, technical, and theoretical problems	**Problem Solving** • use a variety of methods to solve real-life, practical, technical, and theoretical problems
Personal Banking • prepare bank forms including cheques, deposit slips, cheque-book activity record and reconcilation statements	**Income and Debt** • demonstrate an awareness of selected forms of personal income and debt	**Personal Finance** • solve consumer problems involving insurance, mortgages and loans
Wages, Salaries and Expenses • solve problems involving wage, salaries and expenses	**Personal Income Tax** • prepare a simple income tax form	**Taxation** • demonstrate an ability to fill out an income tax form
		Government Finances • demonstrate an awareness of the income and expenditures of federal, provincial and municipal governments
Spreadsheets • design and use a spreadsheet to make and justify decisions	**Owning and Operating a Vehicle** • analyse the cost of acquiring and operating a vehicle	**Investments** • demonstrate and recognize the differences concerning different types of financial investments
Rate, Ratio and Proportion • apply the concepts of rate, ratio and proportion to solve problems	**Relations and Formulas** • represent and interpret relations in a variety of contexts	**Variation and Formulas** • use algebraic and graphical models to generate patterns, make prediction and solve patterns
Trigonometry • demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles		
Geometry Project • complete a project that includes a 2-D plan and a 3-D model of some physical structure	**Measurement Technology** • determine measurements in Systeme International (SI) and Imperial systems using different measuring devices	**Design and Measurements** • analyse objects, shapes and processes to solve cost and design problems
Probability and Sampling • develop and implement a plan for the collection, display and analysis of data using technology as required	**Applications of Probability** • demonstrate an awareness of the applications of probability to real world situations	
	Data Analysis and Interpretation • analyse data with a focus on the validity of its presentation and the inferences made	
	Business Plan • prepare a plan and operate a successful business	**Life/Career Project** • research career choices and perform a comparative study

Principles of Mathematics Pathway

Problem Solving	**Principles of Math 10**	**Principles of Math 11**	**Principles of Math 12**
	• use a variety of methods to solve real-life, practical, technical, and theoretical problems	• use a variety of methods to solve real-life, practical, technical, and theoretical problems	• use a variety of methods to solve real-life, practical, technical, and theoretical problems
Number			
Number Concepts	• analyse the numerical data in a table for trends, patterns and interrelationships. • explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.		
Number Operations	• describe and apply arithmetic operations on tables to solve problems, using technology as required. • use basic arithmetic operations on real numbers to solve problems.	• solve consumer problems, using arithmetic operations.	
Patterns and Relations			
Patterns	• generate and analyse number patterns.	• apply the principles of mathematical reasoning to solve problems and to justify solutions.	• generate and analyse exponential patterns.
Variables and Equations	• generalize operations on polynomials to include rational expressions.	• represent and analyse situations that involve expressions, equations and inequalities.	• solve exponential, logarithmic and trigonometric equations and identities.
Relations and Functions	• examine the nature of relations with an emphasis on functions. • represent data, using linear function models.	• examine the nature of relations with an emphasis on functions. • represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.	• represent and analyse exponential and logarithmic functions, using technology as appropriate.
Shape and Space			
Measurement	• demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. • solve problems involving triangles, including those found in 3-D and 2-D applications.	• solve problems involving triangles, including those found in 3-D and 2-D applications. • solve coordinate geometry problems involving lines and line segments, and justify the solutions.	
3-D Objects and 2-D Shapes	• solve coordinate geometry problems involving lines and line segments.	• develop and apply the geometric properties of circles and polygons to solve problems.	• classify conic sections, using their shapes and equations.
Transformations			• perform, analyse and create transformations of functions and relations that are described by equations or graphs
Statistics and Probability			
Data Analysis	• implement and analyse sampling procedures, and draw appropriate inferences from the data collected.		
Chance and Uncertainty			• use normal and binomial probability distributions to solve problems involving uncertainty. • solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations. • model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

Curriculum

Applications of Mathematics 10

ESTIMATED INSTRUCTIONAL TIME

Applications of Mathematics 10 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

APPLICATIONS OF MATHEMATICS 10

Organizer (Suborganizer)	% of Time
Problem Solving	Integrated Throughout
Number (Number Concepts)	5 - 15
Number (Number Operations)	15 - 25
Patterns and Relations (Relations and Functions)	20 - 30
Shape and Space (Measurement)	10 - 20
Shape and Space (3-D Objects and 2-D Shapes)	10 - 20
Statistics and Probability (Data Analysis)	10 - 20

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

It is expected that students will:

- solve problems that involve a specific content area such as, geometry, algebra, trigonometry, statistics, probability, etc.
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems involving a range of mathematical disciplines can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Applications of Mathematics 10.

- In a class discussion, define *problem solving* with students, pointing out that in mathematics problem solving involves many disciplines, including algebra, geometry, trigonometry, statistics, and probability.
- Introduce new types of problems directly to students (i.e., without demonstration), and facilitate as they attempt to solve them.
- Have students work in small co-operative groups of three to five when introducing new types of problems.
- Model and reinforce student use of a variety of approaches to problem solving (e.g., algebraic and geometric).
- Emphasize that many problems may not be solved in one try and that it is sometimes necessary to revisit the problem, start over, and try again.
- Have students or groups discuss their thought processes as they attempt to solve a problem. Point out the strategies inherent in their thinking (e.g., guessing and checking, looking for a pattern, making and using a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- When students arrive at solutions to particular problems, encourage them to generalize from or extend the problem situations.

Note: See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These problems are indicated with an asterisk (*)

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe

- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question

- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work

Collect

- On selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not for particular problems.
- Have students use mind maps and flowcharts to describe and display problem solutions.

Self-Assessment

- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set, *Evaluating Problem Solving Across Curriculum*, may be helpful in identifying such criteria

.

RECOMMENDED LEARNING RESOURCES

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises.

These resources, while not part of the Grade Collections, have Provincially Recommended status.

Appendix B includes an annotated bibliography of these resources for ordering convenience.

Prescribed Learning Outcomes

It is expected that students will analyse the numerical data in a table for trends, patterns and interrelationships.

It is expected that students will:

- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data)
- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data)

Suggested Instructional Strategies

Using tables enhances students' ability to see data in a compact and easily readable format. Students can extract specific information from tables, and use them to do general comparisons and find relationships.

- Distribute single newspapers to groups of three or four students and have them browse through the paper to find the sections that contain the most tables. Ask the groups to give two reasons why those particular sections use tables extensively.
- Have each student find three specific pieces of information in a table from a newspaper, magazine, or the Internet and share with a partner. Have pairs share their findings with the rest of the class.
- Place a multi-column, non-recursive table (e.g., hockey standings, stock market data) on an overhead. In a class discussion, encourage students to compare and contrast the data in the table. As a class, produce a word list of connections between the rows and columns of data.
- Provide students with a recursive table (e.g., mortgage, credit card account, savings account). Point out that the closing balance for any month is the opening balance of the next month. Ask students to analyse the repetitive nature of the calculations. Have them calculate the balance after three payments, given an opening balance, a monthly rate of interest, and the payment amount.

SUGGESTED ASSESSMENT STRATEGIES

Tables are commonly used to represent real life data. Assess students' ability to use tables in a practical way.

Observe

- While students are drawing meaning from tabular representations of data, note their ability to:
  - use words to describe data
  - identify specific facts from tables
  - determine trends where they exist
  - develop recursive tables in the form of spreadsheets

Collect

- Give students multi-column tables such as sports standings, and note the extent to which students are able to glean information. Have students write a newspaper article based on the information and note the extent to which they are able to interpret data.

Question

- Ask students to suppose that they are responsible for adding 10 new items to the inventory of a clothing store. Have them create a spreadsheet that shows price, GST, PST, and cost. Ask them to explain how the value for each cell in their spreadsheet is calculated.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 10, Western Canadian Edition
  Ch. 2 (Sections 2.1 - 2.5)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- use basic arithmetic operations on real numbers to solve problems.
- describe and apply arithmetic operations on tables to solve problems, using technology as required.

It is expected that students will:

- communicate a set of instructions used to solve an arithmetic problem
- perform arithmetic operations on irrational numbers, using appropriate decimal approximations.
- create and modify tables from both recursive and nonrecursive situations
- use and modify a spreadsheet template to model recursive situations
- solve problems involving combinations of tables, using:
  - addition or subtraction of two tables
  - multiplication of a table by a real number
  - spreadsheet functions and templates

.

SUGGESTED INSTRUCTIONAL STRATEGIES

Many calculations in science, industry, and finance involve irrational numbers. Students need to be able to perform simple calculations with irrational numbers and to properly interpret the results. Although many questions can be solved adequately using approximate values, students should be able to work with exact representations of irrational numbers where possible.

Tables enable organized data to be recognized and facilitate calculations.

- Provide students with a descriptive paragraph or set of written instructions for guessing a secret number, and have them work out the series of mathematical steps required. Ask them to confirm their process with a partner to test its accuracy.
- Provide students with a descriptive paragraph containing numeric data (e.g., mutual funds over a 10-year period) and ask them to organize the data into a table.
- Give students a collection of sale papers and have them comparison shop for specific items. Have them place the results of the comparison in a table and underline the best price.
- Provide students with a basic spreadsheet template for calculating the cost of building a house. Have them change the dimensions of the house and number of bathrooms and bedrooms to see how the construction, electrical, plumbing, and total costs change.
- Have students work in pairs to design a template to show balance owing on a sample credit card account at the end of one month after a payment has been made.

Suggested Assessment Strategies

Number operations are the tools students use to solve problems. Students need to demonstrate a clear understanding of the processes involved in performing various operations involving irrational numbers and how the operations are used.

Observe

- Ask students to draw several regular polygons and have them count the number of sides, diagonals, and intersection points and prepare a table showing this data. Observe students' abilities to use the tables to describe how the values of each polygon can be predicted without counting.

Self-/Peer Assessment

- Have students create a three- or four-step expression that requires order of operation steps to solve, then use it to develop a written flow chart that describes the solution process.
- Given a spreadsheet template of a recursive situation, have students modify it. Have students describe their ability to modify spreadsheets and understand the implications of the changes.

Collect

- Ask students to create a spreadsheet that would show bills for a credit card account that would occur in a situation in which no payments are made. Assume the amount borrowed was $1000 and the annual interest rate is 18% compounded monthly. Have them extend the table to show 12 months of account balances. Ask students to write a letter from the bank explaining the table to the credit card holder.
- Give students a table that is missing values and have them use the values presently there to complete the table.

Presentation

- Have students collect data from a school sports team (e.g., free-throw percentage, penalties or fouls vs. minutes played) and create a table that shows an analysis of calculated values. Have students present their results to the class.

Recommended Learning Resources

Print Materials

- Applied Mathematics 10, Western Canadian Edition
  Ch. 1 (Section 1.7)
  Ch. 2 (Sections 2.1, 2.3 - 2.5)
  Projects: Sports Statistics, Health Care Planning, Snow Board Design, Hockey Pool, The Sawmill, Cross Canada Tours

PRESCRIBED LEARNING OUTCOMES

It is expected that students will :

- examine the nature of relations with an emphasis on functions.
- represent data, using function models.

It is expected that students will:

- plot linear and nonlinear data, using appropriate scales
- represent data, using function models
- use a graphing tool to draw the graph of a function from its equation
- describe a function in terms of:
  - ordered pairs
  - a rule, in word or equation form
  - a graph
- use function notation to evaluate and represent functions
- determine the domain and range of a relation from its graph
- determine the following characteristics of the graph of a linear function, given its equation:
  - intercepts
  - slope
  - domain
  - range
- use partial variation and arithmetic sequences as applications of linear functions

SUGGESTED INSTRUCTIONAL STRATEGIES

Understanding and describing functional relationships is essential to interpreting and predicting the behavior of the world around us. Students will explore how to translate between algebraic and graphical representations and use these skills to make inferences and solve problems.

- Ask students to explore non-linear functions with graphing technology (calculators or software) and discuss the similarities and differences in the graphs and equations.
- Have students use graphing calculators or computers to graph $y = x$ and investigate the graphic changes as a number is added (e.g., $y = x + 3$) and when x is multiplied by a constant (e.g., $y = -2x$) to develop the concept $y = mx + b$. Have them record their findings in their journals.
- Ask students to work in groups to determine techniques that will generate linear equations, given information other than slope and intercept.
- To explore the relationship of linear equations in computer programming, have students work in co-operative groups to generate short programs that produce linear drawings, then share these with the class.
- Organize a carousel activity in which students move in groups around various stations set up with activities to generate data. Students decide as a group how best to graph or display the data. Discuss as a class the similarities and differences in the solutions.
- Use a bakery model (ingredients in $\rightarrow$ pastry out) or a sawmill/pulp model (logs in $\rightarrow$ lumber pulp/paper out) to illustrate the concept of domain and range.
- Have students use a function machine to calculate the range of a function from its domain.
- Have students generate two data points in a linear function (e.g., kilometres driven and car rental cost) and graph them. Ask them to generate model functions to answer questions about the intercepts (fixed costs) and slope (variable costs).
- Ask students to use maps and scale diagrams to calculate distances and sizes.

SUGGESTED ASSESSMENT STRATEGIES

Plotting data, and the subsequent analysis of relations and functions, are key to students' understanding of patterns. Assessment in this area should focus on students' ability to perform individual outcomes (such as graphing and analyzing functions derived from the physical world), and reflect students' understanding of the uses of graphs.

Collect

- Have students collect data on relations between pizza costs and diameters. This data could be represented as a list of ordered pairs, expressed as a rule, described in equation form, as a hand sketched graph, and finally drawn using a graphing calculator. Check particularly for students' ability to select scale, find slope in appropriate units, and select window parameters.

Question

- As students work on using function notation, evaluating functions and determining domain, range, intercepts and slope, check for their ability to work competently using both sketches and graphing tools. Note students' abilities to translate their graphing skills to understand the real-world implications of the data.

Presentation

- Give students ordered pairs such as (0,32), (100,212), and (-40,-40) that show converted temperatures in both Fahrenheit and Celsius scales. Ask students to graph the data to see if it is linear, then determine the slope and y-intercept. Have students then reverse the axes and re-graph the data. Circulate through the classroom to ensure that students' graphs are correct. Ask students to determine where the two temperature scales meet. Have them generate a conversion formula for Fahrenheit and Celsius.

Peer Assessment

- Ask students to work in pairs using graphing calculators. Students can display graphs on their calculators, then challenge their partners to reproduce the graphs on their own calculators.
- Have one student generate an equation in slope-intercept form and a partner change it to general form and vice versa. Students can check each other's work for accuracy.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 10, Western Canadian Edition
  Ch. 3 (Sections 3.1 - 3.6)
  Ch. 6 (Sections 6.1 - 6.6)
  Projects: Safety Technology, Scuba Diving Tours, Hair Products, Anthropology, Space Technology

- Exploring Functions with the TI-82 Graphics Calculator
- Graphing Calculator Activities for Enriching Middle School Mathematics
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Using the TI-81 Graphics Calculator to Explore Functions
- A Visual Approach to Algebra
- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator
- What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Software

- GrafEq (Macintosh & Windows Version 2.09)
- Green Globs & Graphing Equations

Games/Manipulatives

- Radical Math: Math Games Using Cards and Dice (Volume VII)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects
- solve problems involving triangles, including those found in 3-D and 2-D applications

It is expected that students will:

- calculate the volume and surface area of a sphere, using formulas that are provided
- determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects
- solve problems involving two right triangles
- extend the concepts of sine and cosine for angles through to 180˚
- apply the sine and cosine laws, excluding the ambiguous case, to solve problems
- select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes
- analyse the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy
- solve problems involving length, area, volume, time, mass and rates derived from these
- interpret drawings, and use the information to solve problems

SUGGESTED INSTRUCTIONAL STRATEGIES

Many worthwhile and practical problems in mathematics involve calculations of shape and space and are most easily solved by making and labeling a drawing. Some drawings work best when reduced to scale, while in other cases an annotated sketch is sufficient. Visualizing shape and space is a critical component of learning mathematics.

- Bring a collection of spherical balls related to various sports into the classroom. Have students find the circumference of each ball using a fabric tape, and, from the circumference, calculate radius-diameter-area of the greatest circle. Ask them to place the results in a table and find the relationship between each column and classify it as linear or non-linear.
- Discuss why smaller animals require faster metabolisms to maintain a constant body temperature, and why they require more food as volume decreases faster than surface area decreases.
- Have students use equipment such as a parallax viewer (for range) and clinometer (for height) to analyse the motion of a football thrown on a field, resolving the motion into horizontal and vertical components. Point out that the horizontal motion vs. time is linear while the vertical motion vs. time is described using a quadratic function (i.e., non-linear).
- Provide students with pictures of non-right triangles that show either two sides and one angle or two angles and one side. Have students calculate the missing side or angle. A practical example might involve using an orienteering map.
- Give instructions to students (in terms of angles and sides) to move to a certain point in two steps (two sides of a triangle), using Law of Cosines or Law of Sine to calculate how to return to the starting point.
- Divide the class into small groups and supply each group with a measuring instrument (e.g., ruler, meter stick, measuring tape, 10 cm piece of string, 5 cm x 5 cm square of paper). Give students a list of objects of various sizes and shapes in the classroom and have them measure specific aspects of each one. In a class discussion, ask students to determine which measuring instruments were best for measuring which object, which measurements were most precise, and which measurements were most accurate.
- Develop a five-station carousel where each station has one type of measurement problem (e.g., length, area, volume, mass, time). Include time dependent questions involving rate. Design a treasure hunt or rally to send students from station to station to solve problems.

SUGGESTED ASSESSMENT STRATEGIES

Being able to measure using instruments and understanding measurement limitation is fundamental to applying mathematics. Assessing students' abilities to solve formula type, or drawing interpretation type problems should be done in a variety of ways, reflecting real world methodologies.

Observe

- As students approach measurement problems check on their ability to use instruments and obtain results that are both precise and accurate. Particularly where group work is involved, check for individual students' abilities to measure and report.
- Note students ability to translate written problems into sketches, including those whose angles range from 0° to 180°. Observe students' ability to use calculators and manipulate angle representations to solve problems.

Collect

- Check students' solutions to problems for clarity and presentation. Students should be able to explain the strategy they used to solve selected problems.
- Check that students develop criteria that can be used to review their plans for a parking design. Criteria might include:
  - Will a large car fit in the designed spaces?
  - Can cars easily get in and out of the spaces?
  - Are the identified dimensions reasonable?
  - Is there a sufficient number of spaces?

Self-/Peer Assessment

- Ask students to comment or reflect on their ability to visualize relative size and predict the scaling of 3-D objects. For example, ask them if they feel able to predict surface areas given dimensions of objects such as spheres.
- Have students create two or three problems involving sine and cosine laws, including solutions. Have them exchange problems with a partner. Partners complete, and then comment on, the construction of the problem and its solution.
- Encourage students to check each other's measurements and compare results. Team and group analysis of others performance is an excellent way of providing alternate assessments.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 10, Western Canadian Edition
  Ch. 1 (Sections 1.1 - 1.8)
  Ch. 7 (Sections 7.1 - 7.7)
  Projects: Jewelry Design, Candy Boxes, Jet Propulsion, Reach for the Top, Remote Sensing

Software

- The Geometer's Sketchpad

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve coordinate geometry problems involving lines and line segments.

It is expected that students will:

- solve problems involving distances between points in the coordinate plane
- solve problems involving midpoints of line segments
- solve problems involving rise, run and slope of line segments
- determine the equation of a line, given information that uniquely determines the line
- solve problems using slopes of:
  - parallel lines
  - perpendicular lines

SUGGESTED INSTRUCTIONAL STRATEGIES

Describing linear relationships is essential to interpreting and predicting the behaviour of the world around us.

- Using a geoboard, have students stretch an elastic between two posts to form a line segment, then stretch one side of the elastic around the appropriate post to form a right triangle. Have them measure the orthogonal sides by counting the posts and use the Pythagorean Theorem to find the length of the original line segment (hypotenuse of triangle).
- Have students use a clinometer to measure the angle to the top of a building and connect the tangent of the angle to the slope of the line to the top of the building.
- Develop a five-station carousel where each station has a different type of word problem (e.g., distance, midpoint, slope, equations of lines, parallel and perpendicular lines). Design a treasure hunt or rally to send students from station to station to solve problems. Use a point system to award prizes for the most accurate solutions.
- Given an equation for a line, have students list five unique pieces or types of information that they could use to find that equation. For example, given $2x - 3y = 6$ students could list the following:
  - different points on the line (6,2),(9,4),...
  - slope of line is m=2/3
  - y-intercept is b = -2 or the point (0,-2)
  - x-intercept = 3 or the point (3,0)
  - standard form of equation, $2x - 3y = 6$
  - slope intercept form is: $y = \frac{2}{3}x - 2$
- Have students use a graphing calculator or graphing software to explore the similar attributes of parallel or perpendicular lines. Give students six lines to graph and have them note which lines are parallel or perpendicular, analysing the similarities in their equations.
- Have students use technology with dynamic geometry software to draw two lines, then rotate one line. Have them note how its slope changes as it first becomes parallel, then perpendicular to the other line.

SUGGESTED ASSESSMENT STRATEGIES

Solving problems in the coordinate plane is a natural extension of graphing, geometry, and algebraic skills. Assessment should focus students' conceptualization of the coordinate plane and its uses in understanding the world around them.

Observe
- Take note of students' approaches to word problems that require line and segment solutions. Students should be able to explain the steps they took to translate word problems into a drawing on the coordinate system. Observe students' abilities to draw and label clearly.

Question
- For select problems, question students as to the method they used. For example, ask if the student can check a pen-and-paper solution using a graphing tool or vice versa.

Presentation
- Ask students to select a graphic representation of a solution to a problem they have created. Have them present the problem to members of a small group and then teach the problem solution. Use student created problems on teacher tests where appropriate.
- Ask students to create a poster that shows the relationship between a linear equation and its components.

Collect
- Place information about various linear equations on file cards on a table. Give students one file card and have them list or collect cards with data related to the equation on their card.

Peer Assessment
- Ask students to work in pairs using graphing calculators. Students can display graphs on their calculators, then challenge their partners to reproduce the graphs on their own calculators.
- Have one student generate an equation in slope-intercept form and a partner change it to general form and vice versa. Students can check each other's work for accuracy.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 10, Western Canadian Edition
  Ch. 5 (Sections 5.1 - 5.5)
  Ch. 6 (Section 6.5)
  Project: Skiing in San Diego

Software

- The Geometer's Sketchpad
- Green Globs & Graphing Equations

Games/Manipulatives

- Radical Math: Math Games Using Cards and Dice (Volume VII)

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will implement and analyse sampling procedures, and draw appropriate inferences from the data collected.

It is expected that students will:

- choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population
- defend or oppose inferences and generalizations about populations, based on data from samples
- determine the equation of a line of best fit, using:
  - estimate of slope and one point
  - least squares method with technology
- use technological devices to determine the correlation coefficient r
- interpret the correlation coefficient r and its limitations for varying problem situations, using relevant scatterplots

SUGGESTED INSTRUCTIONAL STRATEGIES

Most data collection involves the use of samples rather than populations. Understanding the uses, advantages, and disadvantages of sampling is critical to interpreting statistical information.

- Ask students to identify ways of using sampling to deliberately skew survey results (e.g., survey of probable Stanley Cup winners by polling only fans of one team, survey of favourite musical groups by polling people from only one age group). Have students identify examples of biased or invalid samples on data reported in the media. Ask them to work in groups to analyse the data and determine what the sample population might have been. Discuss as a class.
- Have students suggest real-world applications for using distributions (e.g., knowing the size distribution, how many of each shoe size a shoe store should stock).
- Give students a series of case studies involving survey information and have them critique the studies and support or reject conclusions in the case studies. Alternatively, limit the case study to the survey setup and data and then have students develop their own conclusions.
- Have students plot anonymous class scores from a math test to develop a median-median best fit line, then find its equation. Have them use a graphing calculator or graphing software to find the least squares best fit line then calculate its equation. Ask students to compare and contrast the two and comment on which one is the best representation. Ask them to analyse the point-slope method for accuracy and ease of use.
- Produce six scatterplots with correlation coefficients approximately equal to -1, -0.5, 0, 0.25, 0.75, and 1. Have students calculate the correlations for each graph (perhaps as a small group exercise) using graphing calculators and have them define the difference between strong, weak, and zero coefficients as well as positive and negative coefficients.
- Present students with a series of business- or school-based case studies involving either raw data, scatterplots or pre-calculated correlation coefficients. Have students use the data to make decisions within the context of the case study.

SUGGESTED ASSESSMENT STRATEGIES

While many students are able to collect experimental data, their ability to make meaning of it mathematically will help their understanding of science, social sciences, and technology.

Collect

- Have students design and conduct research projects requiring them to choose and apply sampling techniques. As students work on their projects, ask questions such as:
  - How did you choose your sampling technique?
  - Can you justify your choice?
  - Can you identify possible sources of bias or error in your sample?
  - When might someone want to bias a sample and how might they do it?
  - What makes the graphical representations you are using appropriate for your data?
  - How are your conclusions affected by your sample?
- Work with students to develop criteria they can use to assess their projects. Students use the criteria to assess their own work, then describe how their projects could be changed to correct identified problems. Criteria might include:
  - a clear description of sampling procedures
  - justification for the sampling technique
  - accurate identification of possible reasons for bias or error
  - effective graphical representations of data summaries
  - appropriate references, clearly linked to the data, about the population from which the sample was taken

Question

- Where student collected data contains obvious anomalies (or outliers) ask students if these should be included or excluded and have them explain why.

Self-/Peer Assessment

- Provide students with materials necessary to develop study cards describing terms and concepts related to data analysis. Have students quiz one another using the cards. Small groups of students can use the cards to compose tests for other groups to take. Give students basic test requirements such as length and type of items.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 10, Western Canadian Edition
  Ch. 4 (Sections 4.1 - 4.5)
  Ch. 6 (Sections 6.6 - 6.9)
  Projects: Student Choice Awards, Ecosystem Study, The News, Hair Products, Anthropology
- Exploring Statistics with the TI-82 Graphics Calculator
- Graphing Calculator Activities for Enriching Middle School Mathematics
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Using the TI-81 Graphics Calculator to Explore Statistics
- What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Curriculum

Applications of Mathematics 11

ESTIMATED INSTRUCTIONAL TIME

Applications of Mathematics 11 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

APPLICATIONS OF MATHEMATICS 11

Organizer (Suborganizer)	% of Time
Problem Solving	Integrated Throughout
Number (Number Operations)	15 - 25
Patterns and Relations (Variables and Equations)	15 - 20
Patterns and Relations (Relations and Functions)	15 - 20
Shape and Space (Measurement)	10 - 20
Shape and Space (3-D Objects and 2-D Shapes)	10 - 20
Statistics and Probability (Chance and Uncertainty)	10 - 20

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, or probability
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems involving a range of mathematical disciplines can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Applications of Mathematics 11.

- In a class discussion, define *problem solving* with students, pointing out that in mathematics problem solving involves many disciplines, including algebra, geometry, trigonometry, statistics, and probability.
- Introduce new types of problems directly to students (i.e., without demonstration), and facilitate as they attempt to solve them.
- Have students work in small co-operative groups of three to five when introducing new types of problems.
- Model and reinforce student use of a variety of approaches to problem solving (e.g., algebraic and geometric).
- Emphasize that many problems may not be solved in one try and that it is sometimes necessary to revisit the problem, start over, and try again.
- Have students or groups discuss their thought processes as they attempt to solve a problem. Point out the strategies inherent in their thinking (e.g., guessing and checking, looking for a pattern, making and using a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- When students arrive at solutions to particular problems, encourage them to generalize from or extend the problem situations.

Note: See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These problems are indicated with an asterisk (*)

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe

- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question

- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used to new situations
  - link mathematics to other subjects and to the world of work.

Collect

- For selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not work for particular problems.

Self-Assessment

- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set, *Evaluating Problem Solving Across Curriculum*, may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises.

These resources, while not part of the Grade Collections, have Provincially Recommended status.

Appendix B includes an annotated bibliography of these resources for ordering convenience.

Prescribed Learning Outcomes

It is expected that students will solve consumer problems, using arithmetic operations.

It is expected that students will:

- solve consumer problems, including:
  - wages earned in various situations
  - property taxation
  - exchange rates
  - unit prices
- reconcile financial statements including:
  - cheque books with bank statements
  - cash register tallies with daily receipts
- solve budget problems, using graphs and tables to communicate solutions
- solve investment and credit problems involving simple and compound interest

Suggested Instructional Strategies

A working knowledge of the skills involved in money management is of advantage to all students throughout their lives. An understanding of procedures and applications of mathematics can help students make rational economic decisions.

- Explain various deductions from earnings, then have students determine the effects of these deductions on pay. Students who have jobs could use their pay slips as examples.
- Have students use tables, calculators, and computer spreadsheets to compare the effects of various rates of compounding interest on personal investments and loans.
- Invite a guest speaker (e.g., accountant, banker, financial planner) to talk about credit and borrowing. Have students prepare questions that focus on the underlying mathematics involved.
- Ask students to use information collected from advertising (e.g., print, television, on-line) to compare the cost of paying cash for an item to the cost of buying on a variety of credit plans.
- Have students use spreadsheet software to:
  - record and track expenditures
  - build mortgage and other repayment schedules
- Have each student use spreadsheet software to prepare a budget, given:
  - an income (e.g., salary, wages, commissions, investments, pension)
  - fixed expenses (e.g., utilities, rent, insurance, deductions, savings, investment)
  - variable expenses (e.g., clothes, entertainment)
- Ask students questions that require interpretation of a budget (e.g., How long would it take to save enough money to make a major purchase?).

SUGGESTED ASSESSMENT STRATEGIES

Consumer problems play an important role in the daily lives of students. To assess students' thinking and the strategies they are using, note their abilities to estimate, predict, calculate (using appropriate formulae and technology), make financial decisions, and verify the reasonableness of their conclusions. As they check their solutions and justify their mathematical thinking, look for evidence of developing abilities to communicate mathematically.

Observe

- Observe how well students perform basic calculations manually and how effectively they use available technology to solve problems involving simple and compound interest. Use these observations to determine what practices would benefit the student.
- While students solve consumer problems, note the extent to which they:
  - estimate and verify their results using appropriate technology
  - apply a variety of strategies
  - generalize from their solutions
  - observe soundness of reasoning for financial decisions

Question

- Have each student interview someone who works in a financial field, write a report summarizing the research, and develop an educational plan for the occupation. Review each report, looking for evidence that the student is able to:
  - identify all facets of the job
  - give information about the use of technology in the industry
  - identify personal attributes and required skills for workers in the field
  - generate a list of educational requirements

Collect

- Have students use spreadsheets to develop examples that illustrate the relative advantage of using compound versus simple interest over a specified period of time.
- Have students create budgets of income and expenses for their personal situations using spreadsheet and graphics programs.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 11, Western Canadian Edition

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- represent and analyse situations that involve expressions, equations and inequalities.
- use linear programming to solve optimization problems.

It is expected that students will:

- graph linear inequalities, in two variables
- solve systems of linear equations, in two variables:
  - algebraically (elimination and substitution)
  - graphically
- solve nonlinear equations, using a graphing tool.
- solve systems of linear inequalities in two variables using graphing technology
- design and solve linear and nonlinear systems, in two variables, to model problem situations
- apply linear programming to find optimal solutions to decision-making problems

SUGGESTED INSTRUCTIONAL STRATEGIES

When students create, identify, and interpret linear and non-linear function graphs that represent real-world situations or events, they connect their knowledge of linear systems and graphs with their technology and problem-solving skills.

- Consult with science teachers to obtain laboratory data that represents real-world events and that can be modeled with linear or non-linear relations. Have students use a graphing utility to generate a line of best-fit and to analyse the function in context. Technology such as a Calculator Based Laboratory (CBL) could also be used to generate data.
- Model the process of creating and analysing graphs on a variety of topics (e.g., population growth, car value depreciation, radioactive decay), paying special attention to the patterns and characteristics of each graph.
- Emphasize the relationship between two variables by encouraging students to use relational terminology (e.g., the distance an object falls depends on or *is a function of*, the time that it falls).
- Ask students to compare various company rates (e.g., vehicle towing, cellular phones, automobile rentals) to conduct a break-even analysis.
- Emphasize that relations between two variables give a line on the Cartesian plane, and that problems of multiple relations can be solved graphically and algebraically.
- Use graphing calculators or computer software to introduce the concepts of linear inequalities and systems of linear inequalities. Have students:
  - shade the solution planes and mark the boundaries of graphs of linear inequalities
  - shade and find areas of overlap for systems of linear inequalities
  - determine vertices and possible restrictions to the solution of systems of linear inequalities
  - determine maximum and minimum values
- Present students with models of problem situations and have them determine the linear equations, inequalities, and/or systems that definine the problem. Have students discuss the problems and ways of solving them, analyse constraints on the problems, and discuss feasibility of the constraints.

SUGGESTED ASSESSMENT STRATEGIES

Linear and non-linear functions occur in real-life situations (e.g., maximizing profit and productivity, exponential growth of bacteria, earthquakes, radiocarbon dating). It is important to observe students' abilities to recognize patterns and functional relationships as well as their abilities to select and use techniques to solve various types of problems. Assessment of students' linear programming and graphing skills should focus on the extent to which students apply the concepts to model and solve optimization problems.

Observe

- While students are designing and solving linear systems, note the extent to which they:
  - use linear programming to accurately model various problems
  - use correct terminology such as optimization, objective functions, constraints
  - draw graphs accurately
  - correctly use graphing calculators with shade and trace capabilities
  - draw appropriate conclusions and interpretations
  - recognize discrete or continuous data

Collect

- Assign a series of problems that require students to:
  - use graphing calculators
  - identify the appropriate graph with its associated function

Check students' work for evidence that they:
  - clearly understand the requirements of the problem
  - use efficient strategies and procedures to solve the problem
  - can verify the accuracy and reasonableness of their answers

Presentation

- Have students work in small groups to complete a project to design and analyse a profit and productivity problem using linear programming. Look for evidence that they:
  - interpreted and modeled the problem correctly
  - arrived at logical conclusions or solutions
  - logically and clearly presented the work

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 11, Western Canada Edition
- Explore Quadratic Functions with the TI-83 or TI-82
- Exploring Functions with the TI-82 Graphics Calculator
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR

PRESCRIBED LEARNING OUTCOMES

It is expected that students will represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.

It is expected that students will:

- determine the following characteristics of the graph of a quadratic function:
  - vertex
  - domain and range
  - axis of symmetry
  - intercepts

SUGGESTED INSTRUCTIONAL STRATEGIES

In order to fully describe a function or relation, students need to be able to extract key data from either the graph of the function or the equation. Students should be able to describe, where appropriate the vertices, domain and range, symmetry, or intercepts of a relation.

- Provide real-life situations modeled by quadratic equations (e.g., parabolic sound reflectors in auditoriums, curves created by origami art, reflecting surfaces of search lights) and have students use graphing calculators to determine:
  - maximum (or minimum) value and its meaning in context
  - x-intercept(s) and its meaning in context
  - y-intercept(s) and its meaning in context

  Ask students to explain how these values relate to the real-life situations.
- Using visual aids such as flip cards, overhead projectors, or graphing calculators with overhead displays, present students with examples of several quadratic equations of varying levels of complexity and their corresponding graphs. For example:
  $y = x^2$, $y = -3x^2$, $y = -\frac{1}{2}x^2$, $y = 2x^2 + 3$.
  Ask students to make generalizations about the relationship between the form of the equation and the shape of its corresponding graph. Repeat this process with the other types of functions.
- Work with science teachers to set up experiments to obtain parabolic data (e.g., use a Calculator Based Laboratory to obtain the heights of falling objects or projectiles and generate a model function to solve for roots, vertices, and time at a given height).
- Make BINGO Cards with the following setup:

B	I	N	G	O
max/min	x-int	y-int	axis of symmetry	domain & range

Provide a page with a number of options that students can fill in under each category. Using an overhead projector, display the graph or equation of a function. Students may mark under any category a correct feature of the graph or equation which appears on their card.

SUGGESTED ASSESSMENT STRATEGIES

Assessment strategies must emphasize the students' ability to extract key descriptive information from parabolic graphs or equations.

Observe

- Parabola Aerobics: Students begin with their arms up in a parabola shape. Ask students to describe where the vertex is on their body; where the axis of symmetry is. Ask them to show you what the graph would look like if the *x*-intercepts (in this case the vertex) went from (0,0) to (-3,0) and (3,0). Observe that students make their parabola move down. Ask them to show you what would happen if the new axis of symmetry was x=2. All students should move sideways, etc.
- As students use the BINGO Cards, keep track of possible correct solutions. As they play the game, observe the extent to which they can glean the graph or equation features from the given graph or equations.

Question

- Circulate and ask students:
  - how they select the window for their graphs
  - what happens when the TRACE feature is used
  - what the *x* and *y* values mean
  - why the *y* value changes when the *x* value changes
  - why a regression equation is used - what are its benefits

Collect

- Writing assignment: Given a particular problem or application, ask students to explain the importance and use of:
  - maximum or minimum points
  - *x*-intercepts
  - *y*-intercepts
  - the axis of symmetry
  - the domain
  - the range

  Assess student responses according to a previously developed set of criteria developed with the class.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 11, Western Canadian Edition
- Explore Quadratic Functions with the TI-83 or TI-82
- Exploring Functions with the TI-82 Graphics Calculator
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Using the TI-81 Graphics Calculator to Explore Functions
- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator
- What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Software

- GrafEq (Macintosh & Windows Version 2.09)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.
- use measuring devices to make estimates and to perform calculations in solving problems.

It is expected that students will:

- enlarge or reduce a dimensioned object, according to a specified scale
- calculate maximum and minimum values, using tolerances, for lengths, areas and volumes
- solve problems involving percentage error when input variables are expressed with percentage errors
- design an appropriate measuring process or device to solve a problem

SUGGESTED INSTRUCTIONAL STRATEGIES

Understanding the concept of scale, including the implications of tolerances and errors involved, is important in many jobs and projects that require accuracy and precision in measurement. Knowledge of measurement processes transcends many fields.

- Demonstrate the concept of tolerance, using a high-quality machined mechanical component and a low-quality mechanical component.
- Discuss the effect of absolute error and percentage error when comparing the quality of machines.
- Have students compare the errors generated when a number of small values are added to those generated when fewer but larger amounts are added.
- To bring relevant labs and experiences into the classroom, consult with science teachers for current applications of error in measurements and with shop teachers regarding precision of tools.
- Have students investigate applications of scale, including:
  - enlargement techniques for artists
  - metal tooling machines making small parts from larger initial diagrams
  - blueprints produced with Computer Aided Design programs
  - links to trigonometry and similar triangles
- Have students analyse the difference in accuracy when measuring using:
  - an Erlenmeyer flask
  - a volumetric flask
  - a graduated cylinder

  Discuss effects the shape of each has on the measuring error.
- Provide students with opportunities to use various methods of showing scale (e.g., the linear scale, the ratio method, the statement method, the scale factor method). Encourage students to become efficient and confident at making conversions and determining scale factors and ratios.
- Have each student select a scale and use it to make a scale diagram of the classroom.
- Provide a number of small items such as thumbtacks, paper clips, or stickers, and have each student make an enlargement scale diagram of one item. As a class, discuss differences in how selected scales are stated.

SUGGESTED ASSESSMENT STRATEGIES

Assessment should centre on students' ability to calculate errors as well as the development of an intuitive understanding of how much error is allowed and how much error will be introduced by various measurement procedures.

Observe
- Assign projects that involve 3-D scales and observe the extent to which students:
  - use appropriate ratios and scale
  - can use connecting blocks to build an object from an orthographic diagram

Question
- Have students critique different methods used in making scale diagrams and suggest changes in procedures.
- Analyse situations when percent accuracy is more favorable than absolute accuracy.
- Have students suggest methods of minimizing various types of errors in measurement.

Collect/Observe
- Have students make a list of various devices used for measuring a particular attribute (e.g., length) and rank them in terms of their precision.
- Have students sort a list of questions according to which would require absolute error and which would require percent error to solve
- Have students list various measuring devices used in other shop or science courses at your school. In small groups have students discuss the precision of each tool. Ask students to:
  - give examples of projects which might require two or more measuring tools
  - explain how the precision of their measurements are affected when two or more tools are used

Peer/Assessment
- Have students perform a simple physics lab (such as rolling a marble down a ramp) and do the calculations to find the marble's average velocity. Have students calculate all experimental error and share their findings with peers. Groups can switch calculations and check each other's work

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 11, Western Canadian Edition

Software

- The Geometer's Sketchpad

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

It is expected that students will:

- use technology with dynamic geometry software to confirm and apply the following properties:
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle, from any external point, are congruent
  - the sum of the interior angles of an n-sided polygon is equal to $(2n - 4)$ right angles
- use properties of circles and polygons to solve design and layout problems

SUGGESTED INSTRUCTIONAL STRATEGIES

Applications of geometry, which connects the physical world with students' mathematical knowledge, will help students understand the practical importance of geometry and reinforce the concept of rules and patterns in mathematics. Exploring geometric ideas encourages the development of critical thinking and analysis skills, and helps students recognize geometry's wide applicability in solving problems.

- Have students apply the properties of a circle to:
  - create a graphic design using computer graphics software
  - determine the centre of curvature of an arc in order to replicate it
- Encourage divergent thinking by providing students with geometry applications that include problems with more than one possible solution or that require multiple methods for solving the problem.
- Have students research and report on an area of geometry of personal interest or build a model that illustrates a geometric concept. Examples could include:
  - fractal geometry
  - transformational geometry
  - models of Platonic solids
  - exploring optical illusions
  - researching careers that use geometry
  - topology
  - tessellations

  Have them explain the model to the class.
- Have students develop experiments in which they measure chord and tangent circle properties and use inductive reasoning to draw conclusions from results.
- Show students how to construct hexagons and octagons using a straightedge and compass. Ask students to find ways to construct other regular polygons.
- Have students form cooperative groups and provide each group with a length of string that represents either the radius or the length of one side of a regular polygon. Have each group find a way to lay out a foundation for a small structure (e.g., a regular polygon-shaped patio, a gazebo foundation).
- Arrange a class field trip to a local community college or engineering office to observe a demonstration of computer-aided design.
- Using examples of applications of geometry (e.g., construction projects, landscape design, computer graphics, quilting or tiling) have students complete a layout project.

SUGGESTED ASSESSMENT STRATEGIES

Students demonstrate their understanding of geometric principles when solving problems that involve making connections between the physical world and the geometric models that represent it.

Observe

- Provide students with examples of art from different cultures, such as Aboriginal beadwork, mandalas from India and Mexico, Celtic knot designs, Icelandic sweater designs, Japanese and Chinese lattice designs, and patterns in Islamic art. Have them analyse the art for the extent to which the artist used geometric principles (e.g., symmetry, circle properties, transformations).

Collect

- Have students keep a scrapbook or math journal of practical relationships or applications in which geometry and corresponding terms are used. Require them to provide a brief written explanation of how each entry is an example of some aspect of geometry they have studied. Assess their collections based on criteria developed with students. Criteria might include:
  - knowledge of properties
  - representation of geometric ideas with models or diagrams
  - recognition of mathematical interrelationships
  - application of geometry to solve problems

Self-/Peer Assessment

- Have students work in pairs to create geometry problems involving properties of circles. Each pair exchanges problems with another pair of students, solves the problems, then compares the different solutions and processes used.
- Ask students to exchange the written communication of a design with a partner. Have the partner reproduce the design from this communication. Was the communication clear enough for reproduction of the design? What would improve the communication?

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 11, Western Canadian Edition

Software

- The Geometer's Sketchpad

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will analyse graphs or charts of given situations to derive specific information.

It is expected that students will:

- extract information from given graphs of discrete or continuous data, using:
  - time series
  - continuous data
  - contour lines
- draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data
- design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation
- collect experimental data and use best-fit exponential and quadratic functions, to make predictions and solve problems

SUGGESTED INSTRUCTIONAL STRATEGIES

Creating and interpreting graphs and charts are important real-life skills. Students will encounter many day-to-day situations in which they will be required to interpret and analyse graphs and make reasonable inferences about the data.

- Have students use previously learned sampling skills to collect bivariate data such as:
  - the number of absences from class and corresponding course grade
  - age of the school bus and cost of upkeep of the bus the previous month
  - points scored by a professional sport team at home compared to the attendance at the game and attendance at the next game
- Use the calculator based laboratory (CBL) to :
  - generate data and scatter plots (e.g., motion data, data from science labs
  - plot data for populations from various age groups to confirm or disprove the occurrence of a baby boom
- Present data from spreadsheets in different forms (e.g., pie chart, histogram, broken line) and determine the appropriate use of differing forms of data presentation. Note: Translating from data → functions is a good way to relate tangible problems to theoretical mathematical manipulations.
- With collected data, have students create a scatter plot and then determine the line of best fit. They compare the resulting plots and determine the linear correlation coefficient of the data. Finally, they draw conclusions based on the lines of best fit, keeping in mind the limitations associated with the correlation coefficient.
- Have students research examples that exhibit misuse of statistics such as:
  - assumed cause and effect based on calculated correlation
  - linear projections made beyond the relevant range of the data
  - use of inappropriate data (e.g., outliers) to calculate the correlation coefficient
- Provide samples that may be misleading from sources such as political pamphlets or advertisements, and encourage students to question conclusions drawn from the data.

SUGGESTED ASSESSMENT STRATEGIES

Statistics is the science of collecting, organizing, and interpreting data. In assessing student progress in data analysis, look for evidence that students can determine the curve of best fit for the data.

Observe

- In assessing student performance in analysing data, look for evidence that they can:
  - differentiate between discrete and continuous data
  - determine the equation of the curve of best fit
  - use functions to extrapolate and interpolate trends
  - use reasoning to choose the appropriate regression curves based on trend of data and the context from which the problem is based
- As students work with graphing calculators and tables of data obtained from experiments, circulate, asking questions and observing how effectively they are able to:
  - enter the data into their calculators
  - sketch, on graph paper, an accurate representation of the calculator's resulting graph
  - determine the regression equation that best models the date
  - use the regression equation to solve applied problems

Collect

- Have students investigate real-life situations to determine possible relationships among the data (e.g., education and income, deaths and car speed). For each data set, ask them to construct the scatter plot, determine the curve of best fit, and calculate the correlation coefficient. Assess their work for the extent to which they accurately determine the line of best fit.

Self Assessment

- Students can discuss and debate the validity of other student's analyses of data. Look for reasons why they choose particular regression lines, displayed data in particular way, and how they reached a conclusion based on data.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 11, Western Canadian Edition
- Exploring Statistics with the TI-82 Graphics Calculator
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Using the TI-81 Graphics Calculator to Explore Statistics
- What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Curriculum

Applications of Mathematics 12

ESTIMATED INSTRUCTIONAL TIME

Applications of Mathematics 12 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

APPLICATIONS OF MATHEMATICS 12

Organizer (Suborganizer)	% of Time
Problem Solving	Integrated Throughout
Number (Number Operations I)	10 - 20
Number (Number Operations II)	10 - 20
Patterns and Relations (Patterns)	10 - 20
Shape and Space (Measurement)	10 - 20
Shape and Space (3-D Objects and 2-D Shapes)	15 - 25
Statistics and Probability (Chance and Uncertainty)	15 - 25

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, probability, etc.
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems involving a range of mathematical disciplines can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Applications of Mathematics 12.

- In a class discussion, define *problem solving* with students, pointing out that in mathematics problem solving involves many disciplines, including algebra, geometry, trigonometry, statistics, and probability.
- Introduce new types of problems directly to students (i.e., without demonstration), and facilitate as they attempt to solve them.
- Have students work in small co-operative groups of three to five when introducing new types of problems.
- Model and reinforce student use of a variety of approaches to problem solving (e.g., algebraic and geometric).
- Emphasize that many problems may not be solved in one try and that it is sometimes necessary to revisit the problem, start over, and try again.
- Have students or groups discuss their thought processes as they attempt to solve a problem. Point out the strategies inherent in their thinking (e.g., guessing and checking, looking for a pattern, making and using a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- When students arrive at solutions to particular problems, encourage them to generalize from or extend the problem situations.

  Note: See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These problems are indicated with an asterisk (*)

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe
- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question
- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in theirown words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work.

Collect
- For selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not work for particular problems.

Self-Assessment
- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set, *Evaluating Problem Solving Across Curriculum,* may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises.

These resources, while not part of the Grade Collections, have Provincially Recommended status.

Appendix B includes an annotated bibliography of these resources for ordering convenience.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will describe and apply operations on matrices to solve problems, using technology as required.

It is expected that students will:

- model and solve problems, including those solved previously, using technology to perform matrix operations of addition, subtraction, and scalar multiplication as required
- model and solve consumer and network problems using technology to perform matrix multiplication as required

SUGGESTED INSTRUCTIONAL STRATEGIES

Matrices offer an excellent way for students to explore problem solving, modeling and its limitations, and interpretations of results. Matrix operations give students the tools to analyse and solve complex problems that they may have been previously unable to solve.

- As a class, brainstorm situations that require the use of matrices (e.g., scheduling a tournament, finding the most direct and economical route to travel to a desired location, tracing consumer preferences).
- Have students discuss various ways of organizing raw data so that it can be displayed and analysed in a data table, diagram, or graph. Ask students to determine when using a matrix is the most appropriate way to display certain types of data.
- Provide students with various scenarios that allow them to explore the specific matrix operations needed to generate new information for (or solutions to) a problem. Have students analyse the resulting matrix to verify hypotheses or generate new questions.
- Have students work in groups on projects that require the use of matrices to make informed decisions. For example, they might determine:
  - the type and quantity of coffee to purchase for a coffee shop, based on consumer preferences
  - the number of T-shirts and baseball caps that should be purchased for the year, given a set school store budget and a known school population
- Ask each student to develop a project related to matrices using resources such as the NCTM Addenda Series, the Internet, or consumer preference surveys of their school or neighborhood.

SUGGESTED ASSESSMENT STRATEGIES

Matrices provide an effective way to organize, manipulate, and interpret data in tabular form; assessment should focus on students' abilities to solve complex problems using matrix operations.

Observe

- In their oral or written work, look for evidence that students are able to :
  - use matrix terminology such as elements, rows, columns, dimensions, scalar, scalar product, product, and identity matrix
  - use matrices to represent and organize information
  - generate new information
  - use appropriate technology to perform matrix calculations
  - interpret the results

Presentation

- Have students present data in matrix form generated from business or industry, and have them explain what problem is being analysed or solved using this model. Look for evidence that students have:
  - accurately represented the information in matrix form
  - chosen and used appropriate operations
  - interpreted results correctly
  - understood the limitations of the data

Peer Assessment

- Ask small groups of students to design a test for their classmates, given parameters for the test such as length, topics, and type of questions. A condition of administering their tests to other groups is that all students in the author group must be able to answer each question correctly.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 12 available June 2001

Software

- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will design or use a spreadsheet to make and justify financial decisions.

It is expected that students will:

- design a financial spreadsheet template to allow users to input their own variables
- analyse the costs and benefits of renting or buying an increasing asset, such as land or property, under various circumstances
- analyse the costs and benefits of leasing or buying a decreasing asset, such as a vehicle or computer, under various circumstances
- analyse an investment portfolio applying such concepts as interest rate, rate of return and total return

SUGGESTED INSTRUCTIONAL STRATEGIES

Financial decisions are justified by analysing how variables affect the time value of money. This analysis is readily achieved when using financial spreadsheet templates.

- Give students an existing spreadsheet template (e.g., compound interest, annuity) and have them input values and analyse how the output changes.
- Have students modify an existing template so that it will generate additional information. For example:
  - modify amortization tables to find the total interest over a specific period or change the interest rate part way through the term (refinancing plan)
  - personalize budget templates to reflect students' individual financial situations
- Have students create a template (financial spreadsheet) that will give monthly details of borrowing $80,000 for 20 years at 8.5% interest. The information wanted is monthly payment, monthly debt remaining, interest paid monthly, and amount of principle paid monthly.

SUGGESTED ASSESSMENT STRATEGIES

To make wise financial decisions, consumers must understand the mathematics involved in borrowing and investing money. Assessment should involve the use of spreadsheet templates and provide evidence of students' abilities to analyse situations so they can make informed decisions.

Observe
- While students are working on the templates, look for evidence they can:
  - manipulate the variables within the template to solve the given problem
  - modify the template to include additional information to address a different problem situation
  - modify the template to address different problems
  - use technology with relative ease and confidence

Collect
- Provide each student with a unique financial profile, including income and debt load. Have students prepare five versions of an amortization template to reflect various input variables (e.g., periods, interest rates, down payments). Have them analyse the results of the manipulations and determine which combination of variables is most appropriate.

Self-Assessment
- Have students select and use a template or spreadsheet of personal finances, money management in general, or a home mortgage. Ask them to answer the following questions:
  - What would happen if ____________________?
  - What would be true if ____________________?
  - What would be the result if ________________?
  - What if _____________ had happened instead?
  - What does this information suggest?
  - What rule applies in this case?

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 12 available June 2001

Software

- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will generate and analyze cyclic, recursive, and fractal paterns

It is expected that students will:

- describe periodic events, including those represented by sinusoidal curves, using the terms amplitude, period, maximum and minimum values, vertical and horizontal shift
- collect sinusoidal data; graph the graph using technology, and, represent the data with a best fit equation of the form:
  - $y = a \sin (bx + c) + d$
- use best fit sinusoidal equations, and their associated graphs, to make predictions (interpolation, extraction)
- use technology to generate and graph sequences that model real-life phenomena
- use technology to contruct a fractal pattern by repeatedly applying a procedure to a geometric figure
- use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns

SUGGESTED INSTRUCTIONAL STRATEGIES

The study of cyclic, recursive, or fractal patterns requires the use of spatial reasoning to model and analyse complex natural patterns such as biological structures, coastlines, or snowflakes. Periodic events and fractal geometry also offers students a chance to explore the aesthetic aspects of mathematics.

- Have students analyse patterns found in computer simulations that model transformations of geometric figures. Ask them to identify examples of divergent, convergent, oscillating, or static patterns.
- Encourage students to use their geometric and analytical skills to construct pencil-and-paper drawings (e.g., Koch snowflake, Sierpinski triangle), paper-and-scissors constructions (e.g., pop-up fractals), and computer software (e.g., fractal generators and drawing software). Ask students to record the changes observed in the perimeters, areas, and volumes of the fractals they construct. Ask them to use their observations to make generalizations such as the volume-to-area ratio found in naturally occurring fractal-like objects (e.g., sponges, lungs).
- Invite students to conduct a group or individual research project on fractal geometry examining:
  - its historical development
  - specific career applications (e.g., botany, animation, cartography, climatology, medicine, art)
- Have students use periodic functions to model and graph common periodic patterns (e.g., tidal flow, the beating of a heart, and seasonal changes), and to analyse and make predictions about them.
- Have students identify and collect data about a number of events that are cyclic over time. They then sketch graphs of these periodic events with time represented on the x-axis. Examples that may help stimulate ideas include:
  - the depth of water at a beach over a 48-hour period
  - the temperature in a house where the heat is controlled by a thermostat
  - the height of a pendulum in a grandfather clock as the pendulum swings
  - the firing sequence of cylinders in a four-cylinder car
  - the current in a household circuit

SUGGESTED ASSESSMENT STRATEGIES

Many complex phenomena can only be modeled using periodic or fractal patterns. Fractal geometry provides a way to solve problems involving irregularity and similarity under changes of scale. Assessment of students' representations of cyclic data should focus on the accuracy of their graphs of the data, their abilities to interpret and make predictions based on their graphs, and the extent to which they accurately represent sinusoidal graphs with equations.

Observe

- While students are working with computer software that generates sequences or drawings, look for evidence of students' abilities to:
  - use computers with relative ease and confidence
  - analyse patterns in the simulation runs
  - use the software application to calculate lengths, areas, and volumes of resulting patterns
- While students are constructing paper-and-pencil drawings, computer-generated images, or 3-D constructions, look for evidence that they use the concept of self-similarity.
- While students are graphing sinusoidal data, look for evidence that they:
  - use correct terminology
  - accurately construct the graphs from different sets of cyclic data
  - use the graphs to make predictions
  - represent the graphs with the correct equations
- Have small groups of students each research a problem that requires the collection of data about naturally occurring events. Have them graph the data and use the graphs to make predictions to solve the problems. As groups present their work, look for evidence of the extent to which they:
  - understand periodic functions
  - accurately graph the data
  - use the graphs to solve problems (e.g., make predictions)
  - present their solutions to the problems in a clear and logical manner

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 12 available June 2001
- Exploring Advanced Algebra with the TI-83
- An Introduction to the TI-82 Graphing Calculator

Software

- Secondary Math Lab Toolkit
- GrafEq (Macintosh & Windows Version 2.09)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will analyse objects, shapes and processes to solve cost and design problems.

It is expected that students will:

- use dimensions and unit prices to solve problems involving perimeter, area and volume
- solve problems involving estimation and costing for objects, shapes or processes when a design is given
- design an object, shape, layout or process within a specified budget
- use simplified models to estimate the solutions to complex measurement problems

SUGGESTED INSTRUCTIONAL STRATEGIES

Real-world activities involving measurement formulae and trigonometry enable students to explore the physical world.

- Have students investigate building costs, technical drawings, and related documents, then invite a builder, architect, interior designer, or other appropriate professional to explain her or his measurement techniques, procedures, and "tricks of the trade."
- Have students work in pairs using two methods to determine the trigonometric ratios of right triangles. One student measures the sides and uses these measurements to calculate the ratios; the other student measures the angles and uses a calculator or trigonometric table to determine the ratios. They then compare their results and identify and discuss discrepancies between calculated and measured results.
- Where possible, have students estimate inaccessible distances or dimensions (e.g., the height of a building, width of a river, gradient of a hill). Have them confirm the estimates by using calculations that include:
  - ratio and proportion
  - the Pythagorean theorem
  - trigonometry

SUGGESTED ASSESSMENT STRATEGIES

Measurement experiences are a powerful application of mathematical theory to everyday phenomena. Students demonstrate their knowledge of measurement concepts by generalizing about patterns and procedures and by solving problems involving area, perimeter, surface area, and volume.

Observe

- While students are working on measurement activities, circulate and provide feedback on:
  - their abilities to match the correct measurement formula to a given figure
  - their abilities to recognize and apply the sine and cosine laws
  - the extent to which they consider the reasonableness of their answers
  - their overall understanding of measurement concepts in solving problems
  - their abilities to select the appropriate trigonometric ratio to solve problems involving right triangles

Collect

- Review students' written work in solving measurement problems for evidence that they can:
  - explain the method used in finding the area or volume of a composite figure
  - use various methods for finding the answers
  - solve various types of problems based on the interpretation of measurements (e.g., from a set of blueprints)
  - show the connection between a model, its related equation, and the physical situation (e.g., string art, building application, computer animation, business applications, graphic design)

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 12 available June 2001

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve problems involving polygons and vectors, including both 3-D and 2-D applications.

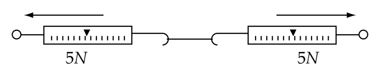
It is expected that students will:

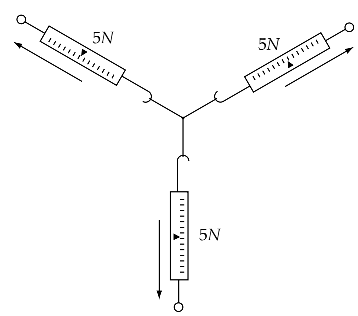
- use appropriate terminology to describe:
  - vectors (i.e., direction, magnitude)
  - scalar quantities (i.e., magnitude)
- assign meaning to the multiplication of a vector by a scalar
- determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods
- model and solve problems in 2-D and simple 3-D, using vector diagrams and technology

SUGGESTED INSTRUCTIONAL STRATEGIES

The study of vectors helps students connect mathematics to the real world. Situations involving forces, velocities, electricity, and navigation are often best described and resolved using vectors.

- Help students recognize a wide variety of situations that can be described using vectors. Provide students with lists of quantities to classify as either vector or scalar.
- Design experiments where students use spring balances. For example, have students:
  - use two balances to examine collinear force vectors
  - use three balances to examine an equilibrium situation in two dimensions





  - verify the experimental findings using trigonometry
- Provide opportunities for students to identify practical situations that can be analysed using vectors (e.g., ferries crossing a river, guy wires supporting poles, hanging a clothesline).
- Engage students in a three-way or four-way tug-of-war. Ask students to repeat the exercise using different angles, and then draw conclusions.
- Ask each student to research a favourite sport to determine where vectors may be used to understand the physics of the sport. Have students report their findings to the class. For example, vectors have significant applications in windsurfing, snowboarding, and skiing. The force of the wind, the strength and direction of the currents in the water, and the resistance of the snowboard or skis as they move over snow can be treated as vectors.

SUGGESTED ASSESSMENT STRATEGIES

Vectors provide a clear method for students to model and solve 2-D and 3-D problems. Assessment focuses on students' abilities to use vectors to accurately represent physical situations, select appropriate problem-solving methods, and use trigonometry appropriately.

Observe

- As students conduct experiments or solve problems involving vectors, observe how effectively they can:
  - relate vector quantities to the physical world
  - use appropriate vector terminology and associated notation
  - use protractors and other measuring devices effectively
  - follow directions to measure angles and vector lengths
  - calculate vector lengths
  - select and use appropriate trigonometric concepts to solve problems
  - explain the problem-solving processes they use
  - communicate their conclusions
- Work with students to develop a table of specifications for a test on this organizer. Identify for them the content categories along a vertical axis and the intellectual levels along the horizontal axis. After the weighting is identified for each cell, have students work in groups to develop questions for each cell.

Self-Assessment

- Ask students to describe to the class ways in which a student might make errors when evaluating vectors. Ask them to then share how to circumvent those errors by using rules or reminder systems.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 12 available June 2001

Software

- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- use normal and binomial probability distributions to solve problems involving uncertainty.
- solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.
- model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

It is expected that students will:

- find the population standard deviation of a data set or a probability distribution, using technology
- use *z*-scores and the normal distribution to solve problems
- use the normal approximation to the binomial distribution to solve problems involving probability calculations for large samples (where $npq>10$)
- solve pathway problems, interpreting and applying any constraints
- use the fundamental counting principle to determine the number of different ways to perform multistep operations
- construct a sample space for two or three events
- classify events as independent or dependent
- solve problems, using the probabilities of mutually exclusive and complementary events

SUGGESTED INSTRUCTIONAL STRATEGIES

Decision-making situations that involve probability and uncertainty occur frequently in daily life. Students need opportunities to learn how to use the normal probability distribution as a tool when interpreting and solving such problems.

- In a large group discussion, point out that in problems involving probability, words often have a meaning different from their ordinary English usage. Help students distinguish between mathematical usage and common usage.
- Provide students with examples that do not fit the normal curve and ask them to determine why using the *z*-score is inappropriate, emphasizing that the *z*-score will not solve all probability problems. Some examples include:
  - a crown-and-anchor game in which every number is equally likely (i.e., no cluster about the mean)
  - the median age of teachers is 45 and their mean is 50 (i.e., not symmetric about the mean)
- Help students develop a list of important things to consider when calculating the probability of an event. The list could include:
  - trying to organize the events of the sample space using tables or tree diagrams when possible
  - recalling the difference between AND and OR and how it relates to union and intersection
  - determining if each part of the problem is a permutation or combination
  - trying to solve a simpler version of the problem by taking a small sample of the same event using the counting principle
- Use real-life problem situations to introduce new concepts. For example:
  - Monica and André play a tennis match in which the first person to win two games in a row (or a total of three games) wins the match. How many possibilities are there to a final outcome? (Using a tree diagram may be helpful.)
  - Is selecting three people from a class to be on the student council any different from picking a person from the class to be president, another to be treasurer, and a third to be secretary?
- Have students research the role of probability in the field of genetics (e.g., eye colour, blood type).

SUGGESTED ASSESSMENT STRATEGIES

The study of statistics relies heavily on the theory of probability. Probability provides concepts and methods for dealing with uncertainty and for interpreting predictions based on uncertainty. Evidence of students' abilities to use probability can best be collected in the context of experimental design and problem-solving activities.

Observe

- While students are working on problems involving uncertainty, look for evidence that they can:
  - use the appropriate terminology
  - distinguish between permutation and combination and provide an example of each
  - identify appropriate uses of the normal distribution
  - correctly calculate the mean and standard deviation of a data set or distribution
  - correctly use normal distribution and z-scores
  - make sound conclusions using data at hand
- To assess students' work on statistics projects, look for evidence that they can:
  - compile appropriate data for the project
  - identify the problem to be analysed and explain how the data contribute to the solution
  - calculate appropriate statistics from the following: mean, standard deviation, and *z*-score
  - summarize data using tables, diagrams, or graphs
  - draw conclusions and inferences based on analyses of the data

Question

- To assess students' understanding of basic concepts, pose questions such as the following:
  - What is the difference between possible outcomes and the probability of an outcome?
  - What is the difference in meaning between combination and permutation?
  - How does the use of AND and OR in the context of compound events affect the meaning?

RECOMMENDED LEARNING RESOURCES

Print Materials

- Applied Mathematics 12 available June 2001
- Exploring Advanced Algebra with the TI-83
- Exploring Statistics with the TI-82 Graphics Calculator
- Using the TI-81 Graphics Calculator to Explore Statistics

Software

- Secondary Math Lab Toolkit

Curriculum

Essentials of Mathematics 10

ESTIMATED INSTRUCTIONAL TIME

Essentials of Mathematics 10 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

ESSENTIALS OF MATHEMATICS 10

Organizer (Suborganizer)	% of Time
Problem Solving	**Integrated Throughout**
Personal Banking	**10 - 15**
Wages, Salaries and Expenses	**10 - 20**
Spreadsheets	**10 - 15**
Rate, Ratio and Proportion	**10 - 15**
Trigonometry	**10 - 15**
Geometry Project	**10 - 20**
Probability and Sampling	**15 - 25**

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

Prescribed Learning Outcomes

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- interpret their solutions by describing what the solution means within the context of the original problem
- use appropriate technology to assist in problem solving

Suggested Instructional Strategies

Problem solving is a key aspect of any mathematics course. Working on problems involving estimation, measurement, and constructing can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Essentials of Mathematics 10.

- Reinforce the concept that "problem solving" is more than just word problems and includes other aspects of mathematics than algebra.
- Introduce new types of problems directly to students (without demonstration) and play the role of facilitator as they attempt to solve such problems.
- Recognize when students use a variety of approaches; avoid becoming prescriptive about approaches to problem solving.
- Reiterate that problems might not be solved in one sitting and that "playing around" with the problem—revisiting it and trying again—is sometimes needed.
- Frequently engage small groups of students (three to five) in co-operative problem solving when introducing new types of problems.
- Have students or groups discuss their thought processes as they attempt a problem. Point out the strategies inherent in their thinking (e.g., guess and check, look for a pattern, make and use a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- Once students have arrived at solutions to particular problems, encourage them to generalize or extend the problem situation.
- See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These types of problems are indicated with an asterisk (*).

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe

- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question

- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their
  - own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work

Collect

- On selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not work as they solved particular problems.

Self-Assessment

- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set *Evaluating Problem Solving Across Curriculum* may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Problem Solving: What You Do When You Don't Know What To Do

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

Prescribed Learning Outcomes

It is expected that students will prepare bank forms including cheques, deposit slips, chequebook activity record and reconcilation statements

It is expected that students will:

- name and describe various types of commonly used consumer bank accounts
- complete various banking forms
- describe the use of a bank card for automated teller machines (ATMs) and debit payments
- identify different types of bank service charges and their relative costs
- reconcile financial statements, such as chequebooks and electronic bank transactions with bank statements

Suggested Instructional Strategies

Using the services and facilities of banks, trust companies, and credit unions involves important life skills. This unit will give students opportunities to gain familiarity with many available banking procedures and appropriate banking forms.

- Invite a bank employee as a guest speaker to visit the classroom to talk about basic banking services. Ask the guest speaker to discuss the importance of using banking forms correctly and accurately. He or she could also relate stories of common errors and their implications.
- Have students complete bank forms, such as cheques and deposit and withdrawal forms.
- Have students update a sample chequebook record that includes cheques, deposits, withdrawals, and ATM, Internet, and telephone banking transactions (e.g., bill payments, transfers).
- Have students complete a statement reconciliation for a given sample chequebook record and corresponding bank statement.
- Brainstorm and record a list of typical bank transactions that might occur over the period of a month. From the list, have students create a chequebook record and corresponding bank statement and statement reconciliation.
- Have students work in pairs to create a poster that shows the interrelationship of chequebooks, activity records, bank statements, and reconciliation documents.

Suggested Assessment Strategies

Being able to use bank forms correctly and knowing current chequing account balances are valuable skills. Assessment should focus on students' completion of various bank forms, noting the extent to which they pay attention to the details. Look for evidence of students' ability to effectively use and reconcile bank statements.

Observe

- As students complete forms, pay close attention to the completion of pertinent parts such as last name, date, and the listing of individual cheques on deposit slips.
- While students complete chequebook activity records, note the extent to which they are able to place items in the correct places. Also pay attention to students' ability to carry a running total.

Collect

- Collect students' individual best examples of each of the following:
  - cheque
  - deposit slip
  - chequebook activity record
  - reconciliation form

  Look for clarity, accuracy, and completeness.

Question

- Ask students to explain the benefits of accuracy and completeness when completing bank documents.

Self-Assessment

- Place examples of correctly completed bank forms in the classroom and encourage students to compare their work to the examples. Ask students to comment on any differences they observe (e.g., short forms of dates).

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Video

- The ABC's of Personal Finance

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve problems involving wage, salaries and expenses

It is expected that students will:

- calculate hours worked and gross pay
- calculate net income using deduction tables (focus on weekly) with different pay periods
- calculate changes in income
- develop a budget that matches predicted income

SUGGESTED INSTRUCTIONAL STRATEGIES

This unit may be immediately applicable for students holding part-time jobs. The ability to calculate gross pay, net pay, and percentage changes in income are important skills for everyone.

- Provide students with a range of hourly pay rates that they might expect to earn upon reaching their educational goals. Have them calculate monthly gross pay for several of the given rates.
- Ask students to list ways in which their incomes might change and to show how the changes would affect their monthly gross pay.
- Have students select two monthly incomes, one higher and one lower, then use deduction tables to find the take-home pay for each income level.
- Using newspaper classified ads for data on rent or leasing costs, have student produce appropriate monthly and/or yearly budgets for a variety of incomes.
- Set up a mock payroll for students, giving each student a weekly wage. Charge weekly expenses for missed time, desk rental, calculator or other equipment rental, and services such as sharpening pencils. Have students look up payroll deductions (i.e., taxes, EI, CPP) in tables to determine their net pay. Students record gross pay, deduction calculations, and net pay in deposit books. Students write "cheques" for the weekly expenses, deducting the cheque amount from their cumulative pay totals. At appropriate intervals, give students a pay raise and have them calculate the percentage increase in dollars.

SUGGESTED ASSESSMENT STRATEGIES

Understanding how income and expenses are used to develop budgets is a key life skill. Assessment should focus on students' understanding of the entire process.

Observe

- Have students present a budget and note to what extent they are able to:
  - justify the inclusion of budget items (expenses)
  - describe how they went about selecting the appropriate weighting of the items
  - present their data
- As students work on their net pay, note to what extent they
  - choose the appropriate table
  - accurately subtract the deductions from the gross pay
  - present their data

Collect

- On selected problems, have students explain by annotation how they filled in given tables.
- Collect student deposit books and check for accuracy, neatness, and completeness.

Self Assessment

- Provide a pay stub which included deductions and have students comment on whether deduction calculations are reasonable.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Software

- Green Globs & Graphing Equations

PRESCRIBED LEARNING OUTCOMES

It is expected that students will design and use a spreadsheet to make and justify decisions

It is expected that students will:

- create a spreadsheet using various formatting options
- use a spreadsheet template to solve problems
- create a spreadsheet using formulas and functions
- use a spreadsheet to answer "what-if" questions
- identify where spreadsheets could be effectively used

SUGGESTED INSTRUCTIONAL STRATEGIES

This unit is intended to introduce students to the "how" and "what-if" questions that spreadsheets may be used to answer. Students should be able to use pre-existing spreadsheets such as loan analysers. Students will also learn to create simple spreadsheets and analyse their usefulness.

- Give students a simple spreadsheet template, such as one that lists length and width of a rectangle and calculates area. Have students add a column that calculates perimeter. Show students how to make the value appear as a rounded number. Modify the spreadsheet and demonstrate how it could be used to calculate the cost of carpet given a unit price or baseboard quantity for a rectangular room. Ask students how the calculations would change if hardwood flooring were installed. Have them insert an appropriate new unit price and find the new costs.
- Ask students to brainstorm real-life circumstances where spreadsheets would be useful (e.g., budgets, tracking investments).
- Demonstrate how a spreadsheet might be used in the following circumstances:
  - pricing items in a store given wholesale cost
  - producing a household budget
  - tracking statistics on a sports team
- Give students a spreadsheet template to calculate weekly wages and EI, CPP, and taxes payable. Have them enter the appropriate data and complete the calculations.
- Have students use a spreadsheet to keep track of their marks for a set period of time.

Suggested Assessment Strategies

Assessment in this section should focus on students understanding of when spreadsheets are an appropriate tool to use, as well as their understanding of how to create and apply spreadsheets.

Observe
- Watch students as they create or use spreadsheets. Note the extent to which they are able to:
  - grasp how cells are filled using simple formulas
  - insert columns or rows
  - change values of cells
  - copy information from cell to cell
  - format the spreadsheet for better presentation

Collect
- Using selected print copies of student generated spreadsheets have students show how the cell values were obtained.

Peer Assessment
- Have peers access other students' spreadsheet solutions to individual problem. Have them share comments such as whether the answers make sense.

Self Assessment
- Have individual students:
  - describe benefits of using spreadsheets
  - brainstorm other uses for spreadsheets

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Software

- Hot Dog Stand: The Works
- Math Tools

PRESCRIBED LEARNING OUTCOMES

It is expected that students will apply the concepts of rate, ratio and proportion to solve problems

It is expected that students will:

- use the concept of unit rate to determine the best buy on a consumer item and justify the decision
- solve problems on the application of sales tax in Canada
- describe a variety of sales promotion techniques and their financial implications for the consumer
- solve rate, ratio, and proportion problems involving length, area, volume, time, mass, and rates derived from these

SUGGESTED INSTRUCTIONAL STRATEGIES

The ability to make financial decisions based on calculations of unit rate, percent taxes, and discounts, and to calculate rates such as fuel consumption are important real-life skills.

- Have students, using information in grocery flyers, calculate unit prices and make price comparisons in order to identify the best buy.
- Brainstorm items that students would be interested in purchasing. Have students choose an item from the list and collect advertising data on prices and features. Have them select what they believe is the best deal offered and justify their decision. Ask them to calculate the actual cost of the item, including sales tax. Using a given value for hourly take-home pay, students find the number of hours they would need to work to earn enough money to buy the item.
- Ask students to select a retail item to purchase (e.g., mountain bike, snowboard). Based on a reasonable hourly rate of pay, ask them to calculate, taking deductions into account, how many hours they would have to work to purchase the item.
- Using a recipe for waffles for four people, guide students in determining the amounts of ingredients that would be needed to make waffles for 20 people.

SUGGESTED ASSESSMENT STRATEGIES

As rate, ratio, and proportion are often difficult concepts, assessment should focus on students' abilities to apply appropriate strategies to solve problems.

Observe
- As students work in pairs on problems, observe the extent to which they show their understanding through their discussions on how to proceed to solve them.

Collect
- Have students make a poster showing a chart indicates the varying number of servings for waffles and the resulting changes in ingredient amounts.

Self Assessment
- Develop with students methods for testing their solutions by applying common sense, estimation and reasoning.

Peer Assessment
- Have students solve problems, showing all their work. Ask them to exchange solutions and use keys to mark each other's work. Students can identify the errors in their peers' solutions and explain the errors and how to fix them.

Discussion
- Provide a list of problems and ask students to identify which of them can be solved using rate, ratio, and/or proportion strategies.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Software

- Hot Dog Stand: The Works

PRESCRIBED LEARNING OUTCOMES

It is expected that students will demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles

It is expected that students will:

- apply ratio and proportion in similar triangles
- use the trigonometric ratios sine, cosine, and tangent in solving right triangles

SUGGESTED INSTRUCTIONAL STRATEGIES

The study of trigonometry, starting with similar triangles, enables students to solve many ratio, proportion, and distance problems as well as problems that require determining the lengths of sides of triangles and the measure of unknown angles. These skills are particularly useful for construction trades such as carpentry.

Similar Triangles

- Have students draw (possibly using dynamic geometry software) a triangle in which two of the angles are between 10° and 60°. Ask them to:
  - avoid using a 30°-60°-90° triangle
  - draw the base horizontally
  - label the base a, and the sides b and c (clockwise)
  - label the corresponding angles A, B, and C

  Have them draw a second triangle larger than the first, where the angles of the new triangle are equal to the angles of the previous triangle, and label the new triangle a′, b′, and c′. Have students draw a line segment, perpendicular to the bases, that ends at A and A′, and label it D.
- Have students fill in a data table comparing all aspects of the two triangles; the measures of the angles, ratio of corresponding angles, lengths of the sides and their corresponding ratios, and the ratio of the areas of the two triangles. Ask them to draw conclusions about the relationships between the two triangles.

Trigonometry

- Have students use calculators or tables to find sine, cosine, and tangent for angles from 5° to 85° in 5° jumps. Have them compare the values from their previous tables for the 30° and 60° measures. Point out that the ratios of the lengths of the sides are what make up the values found in trigonometry tables or calculators.
- Demonstrate how to find the length of an unknown side (when given an angle and the length of one side) using derivations of the basic trigonometry formulas.
- Have students solve problems in real-world contexts that illustrate how knowledge of similar triangles and solving right triangles are useful skills (e.g., simple navigation problems, calculating inaccessible measurements such as the width of a river).

Suggested Assessment Strategies

Assessment focuses on students' understanding of the basic properties of similar triangles and applications of their properties. Assessment of students' comprehension of trigonometry should focus on their ability to use sine, cosine, and tangent to solve simple right triangle problems.

Observe

- Review students' work as they draw, label, and measure the components of right triangles. Note their ability to label correctly and use protractor, calculator, and trigonometry table (or drawing utility) effectively. Look for proper use and function of calculators.

Question

- Ask students to explain or demonstrate the processes they used to solve similar triangle questions. Note the extent to which they are able to describe the ratios involved and their usefulness.

Collect

- Have students create tables of values that relate the lengths of sides of right triangles to angles for triangles they have drawn. Evaluate students' ability to find unknown lengths and angles using direct measure and to calculate using the basic trigonometry identities.
- Collect students' solutions to problems involving right triangles. Assess to what extent students are able to justify their answers.
- Ask students to solve right triangle problems and note the extent to which they are able to draw, label, and show the calculations leading to their answers.

Self Assessment

- Have students develop a study guide to solving problems using similar triangle ratios or trigonometry. Ask students to describe when and how to use the information in the guide (e.g., when to use sine or cosine when solving problems). Ask them to describe strategies that worked as well as those that did not.

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Exploring Trigonometry with the Geometer's Sketchpad

Software

- The Geometer's Sketchpad

Prescribed Learning Outcomes

It is expected that students will complete a project that includes a 2-D plan and a 3-D model of some physical structure

It is expected that students will:

- measure lengths using both SI Metric and Imperial units
- estimate measurements of objects in SI and Imperial systems including:
  - length
  - area
  - volume
  - mass
- draw top, front and side views for both 3-D rod or block objects and their sketches
- sketch and build 3-D designs using isometric dot paper
- determine the relationships among linear scale factors, areas, surface areas, and volumes of similar figures and objects
- enlarge or reduce a dimensioned object according to a specified scale
- solve problems involving linear dimensions, area, and volume
- interpret drawings and use the information to solve problems
- complete a project that includes a 2-D plan and a 3-D model of some physical structure

Suggested Instructional Strategies

The ability to make sense of 2-D and 3-D scale representations is an important skill for solving construction problems. Study of geometry gives students opportunities to acquire a sense of proportion and shape.

- Provide students with a 3-D model using straight lines, and ask students to produce top, front, and side views in 1:1 scale.
- Ask students to produce a 2:1 and a 1:2 version of the same view.
- Using examples of simple 3-D objects, ask students to produce 2-D representations to a simple scale.
- Ask students to find the surface areas and volumes of several objects (e.g., cuisinaire rods, dice, shoebox, blocks).
- Provide drawings of simple 3-D objects, and ask students to build models of the objects using blocks. Have them solve for missing measurements, either directly or by calculation.
- Provide students with the dimensions of a helium or hot-air balloon (e.g., the Hindenberg) and have them calculate the volume of gas needed to inflate it (volume).
- Provide students with the dimensions of a local landmark (e.g., the HR MacMillan Planetarium) and ask them to calculate the volume of paint that would be needed to cover it (surface area).
- Ask students to calculate the length of fencing (perimeter) that would be required to surround a well-known structure (e.g., Stonehenge), if the fence remains exactly 75m away from the outer perimeter of the structure.

SUGGESTED ASSESSMENT STRATEGIES

Being able to measure using instruments and modeling 2-D and 3-D objects is fundamental to applying mathematics practically. Students should be able to devise a plan to construct a 3-D object.

Observe
- Notice how students are able to grasp the concept of scale. Use simple 3-D objects such as small blocks or toys to observe if students are understanding view point and scale.

Collect
- Have students create a 3-D model from given plans. Use materials such as firm cardboard, wax, soap or modeling clay. Use a rubric to assess how well the students meet the criteria previously identified for spatial understanding.

Question
- Ask students to comment on advantages and difficulties of working with scale models and note to what extent they are able to list them. Ask them to describe how they overcame difficulties.

Self-Assessment
- Ask students to estimate and record how far they can throw an object. Then have students complete the activity and measure the distance. Repeat this activity with golf clubs and balls, hockey pucks, and empty boxes. Have students examine their recording and notice how the measurements are becoming more closely aligned.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Software

- 3-D Images
- Math Tools

PRESCRIBED LEARNING OUTCOMES

It is expected that students will develop and implement a plan for the collection, display and analysis of data using technology as required

It is expected that students will:

- read and interpret graphs
- discuss how collected data are affected by the nature of the sample, the method of collection, the sample size and biases
- describe issues to be considered when collecting data (e.g., appropriate language, ethics, cost, privacy, cultural sensitivity)
- select, defend and use appropriate methods of collecting data:
  - designing and using questionnaires
  - interviews
  - experiments
  - research
- determine and use measures of central tendency to support decisions
- use sample data to make predictions and decisions
- use suitable graph types to display data (by hand or using technology)
- critique ways in which statistical information and conclusions are presented by the media and other sources

SUGGESTED INSTRUCTIONAL STRATEGIES

In everyday life, we are frequently faced with important information that is survey-based and/or graphically presented. This unit gives students opportunities to select appropriate methods of data collection and to read, interpret, and analyse the validity of data presentations.

- Lead a class discussion on ethics of data collection and reporting of results, including experimental design and questionnaire design. Ensure that discussion is centered on issues of appropriate language, privacy, and cultural sensitivity.
- Have students create a survey for a restaurant owner who wants to offer three flavours of ice cream in an area that serves high school students. Ask students to decide which ice cream flavours to include in the survey and to choose a method for collecting and ranking the data.
- Students design a survey on an issue that interests them, drawing their sample from the school or community at large. Have students justify their survey content and sampling method. Ask students to draw valid, non-trivial conclusions from their data.

SUGGESTED ASSESSMENT STRATEGIES

Assessment should focus on students' ability to identify poor sampling techniques and to improve them. Students' work should also be assessed on appropriate choice of sampling technique.

Collect

- Ask students to collect data and draw conclusions from that data. Note the extent to which students are able to justify their conclusions.

Question

- Ask students to examine and comment on surveys provided by the teacher. Ask students to describe how they might be improved both in terms of data gathering and data analysis.

Peer Assessment

- Have students in small groups create and complete a simple survey. Have them report their results to the class. Ask the class to criticize their presentations.
- Have students design and conduct research projects requiring them to choose and apply sampling techniques. As students work on their projects, ask questions such as:
  - How did you choose your sampling technique?
  - Can you justify your choice?
  - Can you identify possible sources of bias or error in your sample?
  - When might someone want to bias a sample and how might they do it?
  - What makes the graphical representations you are using appropriate for your data?
  - How are your conclusions affected by your sample?

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Problem Solving: What You Do When You Don't Know What To Do
- Triple 'A' Mathematics Program: Data Management & Probability

Software

- Math Tools
- Probability Constructor

Games/Manipulatives

- D.I.M.E. Probability Pack A
- D.I.M.E. Probability Pack B

Curriculum

Essentials of Mathematics 11

ESTIMATED INSTRUCTIONAL TIME

Essentials of Mathematics 11 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

ESSENTIALS OF MATHEMATICS 11

Organizer (Suborganizer)	% of Time
Problem Solving	**Integrated Throughout**
Relations and Formulas	**10 - 15**
Income and Debt	**10 - 20**
Data Analysis and Interpretation	**10 - 15**
Measurement Technology	**10 - 15**
Owning and Operating a Vehicle	**10 - 15**
Personal Income Tax	**~ 5**
Applications of Probability	**10 - 15**
Business Plan	**15 - 20**

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

Prescribed Learning Outcomes

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- interpret their solutions by describing what the solution means within the context of the original problem
- use appropriate technology to assist in problem solving

Suggested Instructional Strategies

Problem solving is a key aspect of any mathematics course. Working on problems involving estimation, analysis, and measurement can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Essentials of Mathematics 11

- Reinforce the concept that "problem solving" is more than just word problems and includes other aspects of mathematics than algebra.
- Introduce new types of problems directly to students (without demonstration) and play the role of facilitator as they attempt to solve such problems.
- Recognize when students use a variety of approaches; avoid becoming prescriptive about approaches to problem solving.
- Reiterate that problems might not be solved in one sitting and that "playing around" with the problem—revisiting it and trying again—is sometimes needed.
- Frequently engage small groups of students (three to five) in co-operative problem solving when introducing new types of problems.
- Have students or groups discuss their thought processes as they attempt a problem. Point out the strategies inherent in their thinking (e.g., guess and check, look for a pattern, make and use a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- Once students have arrived at solutions to particular problems, encourage them to generalize or extend the problem situation.
- See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These types of problems are indicated with an asterisk (*).

Suggested Assessment Strategies

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe

- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question

- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work

Collect

- On selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not work as they solved particular problems.

Self-Assessment

- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set *Evaluating Problem Solving Across Curriculum* may be helpful in identifying such criteria.

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- The Multicultural Math Classroom: Bringing in the World
- Problem Solving: What You Do When You Don’t Know What To Do

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

Prescribed Learning Outcomes

It is expected that students will represent and interpret relations in a variety of contexts

It is expected that students will:

- express a linear relation of the form $y = mx + b$
  - in words
  - as a formula
  - with a table of values
  - as a graph
- interpolate and extrapolate values from the graph of a linear relation
- determine the slope of a linear relation and describe it in words and interpret its meaning in a problem context
- interpret the graph of a relation and describe it in words
- construct a graph of a relation from its description in words
- evaluate formulas

Suggested Instructional Strategies

Using graphs of linear relations to represent real-life situations, and interpreting graphs of relations are key components of many problem solving situations. It is important for students to be able to make appropriate conclusions and interpretations based on graphical data.

- Provide students with graphs showing the wages earned versus hours worked for jobs typically held by young temporary workers (e.g., cashier, server). Ask students the following questions:
  - What is the hourly wage?
  - Can the wages earned be predicted for a point off the graph?
  - How would the graph change if a raise were incorporated into the hourly wage?
  - What type of job would result in wages earned for 0 hours of work?
- Have students research in newspapers, trade reports, and the Internet to collect tables of data for graphing. Ask them to define the variables, determine the appropriate scaling of axes, plot the data, and draw conclusions. Then have students identify the linear relationships, identify the slope and y-intercepts, and develop equations to describe the relations.
- Give pairs of students two sets of data showing the relationship between two different base salaries and two different rates of commission. One partner graphs the first set of data, and the other graphs the second. Have students discuss their graphs in relation to slope and y-intercepts. Ask them to generate an equation for each graph to predict each income for a given value of sales. Ask students:
  - What effect will the change in commission rate have on the graph and equation?
  - Can you suggest other applications for this type of relationship?
- Have one group of students find car rental advertisements in newspapers or magazines, and graph the costs of renting a vehicle for one day for several distances. Have another group predict the costs of renting the same vehicle driven for longer distances or for distances in between the plotted points.

SUGGESTED ASSESSMENT STRATEGIES

By extracting meaning from linear relations, graphical representations and using graphs to represent data, students can use these skills throughout their lives. Assessment in this area should focus on the real-world applications of these skills.

Observe
- When students are solving problems involving real-life applications, note whether they:
  - choose appropriate axes for given quantities
  - scale and label axes appropriately, and title the graph
  - plot data correctly and dnØermine the slopes of linear relationships
  - can make predictions (e.g. extrapolate or interpolate) about other values based on their equations or graphs
  - correctly interpret the graph of a relation and describe its intent in words
  - make appropriate conclusions and interpretations of slope and y intercept in problem situations (e.g. car rental problems where there is an initial cost and a rate charge)

Collect
- Give students various linear graphs taken from newspapers, business magazines, the Internet and various government publications. Have students write the equations of the lines, defining the variables and identifying the units involved. Have the students provide feedback: can they provide reasonable interpretations of the data and make interpolations or extrapolations with the data?

Peer Assessment
- Have students develop criteria for evaluating graphs and then mark each others graphs.

Question
- Have students examine a prepared graph and explain why some information is not shown.
- Have students determine missing data on a graph (e.g., labels, y-intercept, slope).

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Models and Patterns: Experimenting With Linear Equations
- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator
- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Software

- Green Globs & Graphing Equations
- Math Tools

PRESCRIBED LEARNING OUTCOMES

It is expected that students will demonstrate an awareness of selected forms of personal income and debt

It is expected that students will:

- solve problems involving performance-based income, including commission sales, piece work, and salary plus commission
- use simple and compound interest calculations to solve problems
- solve consumer problems involving:
  - credit cards
  - exchange rates
  - personal loans

SUGGESTED INSTRUCTIONAL STRATEGIES

The ability to use skills involved in money management is needed throughout a person's lifetime. An understanding of procedures and applications of mathematics can help students make reasonable economic decisions.

- Invite a guest speaker (e.g., accountant, banker, financial planner) to address topics related to credit and borrowing. Students should prepare questions to focus on the underlying mathematics involved.
- Have students research and compare credit cards for interest rates charged and beneficial features (e.g., frequent-flyer miles or points, dividends, low interest rates, purchase insurance).
- Have students use examples of credit card statements to calculate interest owed and balance forward for a variety of scenarios, including partial payment and late payment of amount owing.
- Provide students with descriptions of various employment situations (e.g., pension plans, benefit plans, salaries and/or wages, taxes) and have them determine the effects of deductions on gross pay.
- Provide students with descriptions of various financial situations (e.g., inheritance in trust, retirement plans, appreciating assets, a first vehicle purchase). Have students use tables, calculators, and computer spreadsheets to compare the effects of various rates of compounding interest on personal investments or loans for each situation.
- Ask students to use information collected from advertisements to compare the cost of paying cash for an item to the cost of buying on a variety of credit plans.

SUGGESTED ASSESSMENT STRATEGIES

Earning, spending, and saving money are important life skills. To assess students' thinking and the strategies they are using, note their abilities to estimate, predict, calculate, make financial decisions, and verify the reasonableness of their conclusions.

Observe

- As students work on problems involving income and debt, observe their abilities to:
  - compare various pay scenarios using graphs and explain the advantages and disadvantages of straight wages, commission, salary, or graduated commissions
  - explain the advantages and disadvantages of using credit cards, installment plans, and bank loans (e.g., monthly payments, interest payments, principal)
- Observe how students perform basic calculations manually, and how effectively they use available technology to solve problems involving simple and compound interest. Use these observations to determine what types of learning experiences would benefit them (e.g. review sheets, re-teaching, peer teaching).

Question

- Ask each student to write a report summarizing the visit of a bank or financial institution employee. Review each report, looking for evidence that the student is able to:
  - identify all facets of the job
  - give information about the use of technology in the industry
  - generate a list of educational requirements
  - identify personal attributes and required skills for workers in the field

Collect

- Review written work for evidence of students' ability to explain the relative advantages of wage-earning options and incentives offered by the employer for each wage earning option.
- Have students complete loan applications.
- Review written work for evidence of students' abilities to explain the advantages and disadvantages of using credit cards and debit cards.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Math Projects: Organization, Implementation, and Assessment

Video

- The ABC's of Personal Finance

Prescribed Learning Outcomes

It is expected that students will analyse data with a focus on the validity of its presentation and the inferences made

It is expected that students will:

- display and analyze data on a line plot
- manipulate the presentation of data to stress a particular point of view

Suggested Instructional Strategies

The study of statistics enables students to extend and integrate previous knowledge by collecting and analyzing data from real-life situations.

- Have student research newspapers and magazines to collect two surveys, one that includes information about how the survey was conducted (e.g., sample size, margin of error), the other a popular culture survey. Have students analyse the merits of each survey, and determine whether or not decisions can be based on them.
- Ask students to record statistics on local weather characteristics over a period of time. In a class discussion ask questions such as:
  - Do you see trends in the data?
  - Can inferences be made from the data?

  Discuss the relevance of the data.
- Using the class as the population, have each student design and complete a survey, presenting the resulting data in a variety of formats (e.g., histograms, stacked graphs, line graphs, pie charts, tables). Have the class discuss the merits of various methods of organizing and presenting data.
- Have students research newspapers and magazines to find a survey question to apply to a random sample of students in the classroom or people in the local community. Ask students: Is this class or community sample a reliable representation of the whole school population? of the community? of Canada?
- Have students research statistics on selected segments of the population (e.g., age groups, ethnic groups, gender groups) and compare the data with data on other segments of the population or with the population as a whole. Ask questions such as the following:
  - Are there trends in the data?
  - What does this tell you about the data groups?

SUGGESTED ASSESSMENT STRATEGIES

Working with statistics allows students to demonstrate their understanding of the practical applications of mathematics. As students conduct data-analysis activities, teachers have an opportunity to observe the processes students use to organize and summarize data and to make predictions.

Observe

- While students are working with data, circulate, ask questions, observe, and check to see how effectively they are able to:
  - design and collect data from simple surveys
  - represent data effectively using tables, charts, plots, and graphs
  - make predictions and inferences based on graphs
  - identify, analyse, and explain the misuse of statistics
  - use appropriate terminology

Collect

- After students have had the opportunity to display information in a variety of ways, ask the class to make a list of decisions that need to be made when summarizing and displaying data.
- Give students examples of statistical information and conclusions taken from the media, the Internet, or professional journals. Have them write evaluations analysing how the data was gathered, and the validity of conclusions.

Question

- Ask students to explain why someone might intentionally manipulate the presentation of data and how this is done.
- Ask students if inferences can be made about general weather patterns based on one weeks' recorded temperature and precipitation.

Peer Assessment

- While students are presenting their analyses of data, have others assess the effectiveness of the presentations using criteria developed by the class as a whole.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Triple 'A' Mathematics Program: Data Management and Probability

Software

- Green Globs & Graphing Equations
- Hot Dog Stand: The Works

PRESCRIBED LEARNING OUTCOMES

It is expected that students will determine measurements in Systeme International (SI) and Imperial systems using different measuring devices

It is expected that students will:

- select and use appropriate measuring devices and measurement units in Imperial or SI systems
- perform basic conversions within and between the Imperial and SI systems, using technology as appropriate
- use measurement strategies to solve problems

SUGGESTED INSTRUCTIONAL STRATEGIES

Real-world activities involving measuring, converting between systems, and estimating enable students to explore, describe, and manipulate the physical world around them.

- Divide the class into small groups and provide students with a variety of measuring instruments (e.g., calipers, micrometers, scales, metre-sticks, measuring tapes, trundle wheels). Assign objects to be measured to each group and have students report to the class their choice of measuring device, the ease or difficulty of measurement, and the precision of the resulting measurement.
- Use models and manipulatives to demonstrate the multiplication of units in two and three dimensions (e.g., cm, cm^2, cm^3). Have students research relationships in the metric system (e.g., 1 cm^3 = 1 ml = 1 g of water).
- Have students create posters that display conversion factors or simple instructions for conversions between Imperial measures and corresponding SI measures, and vice versa. Compile the posters into a large book for classroom use, or display individual posters in a hall display. Have students use the posters to practice conversion problems.
- Have students measure the diameter of a round table in metres. Have them convert the distance to feet, then measure again in feet to check their work.

Suggested Assessment Strategies

Measurement experiences are a powerful application of mathematical theory to everyday phenomena. Students demonstrate their knowledge of measurement concepts by generalizing about strategies and performing basic conversions.

Observe
- While students are working on measurement activities, circulate and provide feedback on:
  - their abilities to use the correct measuring devices and measurement units
  - the extent to which they consider the reasonableness of their answers
  - their overall understanding of measurement concepts in solving problems

Collect
- Review students' written work in using measurement technology for evidence that they can:
  - give a situation in which using one particular measuring device is preferable to another
  - give examples of situations in which a most appropriate device for measurement exists
  - change units of linear measure from SI to Imperial and vice versa
- Have students research the history or development of a particular unit of measure.

Self Assessment
- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked, and those that did not. Have students keep a journal or scrapbook on the advantages of the SI system over the Imperial system.

Peer Assessment
- Have students check each other's work using appropriate measuring systems.

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

[no resources]

Prescribed Learning Outcomes

It is expected that students will analyse the cost of acquiring and operating a vehicle

It is expected that students will:

- solve problems involving the acquisition and operation of a vehicle, including
  - renting
  - leasing
  - buying
  - licensing
  - insuring
  - operating (e.g., fuel and oil)
  - maintaining (e.g., repairs, tune-ups)

Suggested Instructional Strategies

Learning to make sound decisions about acquiring a motor vehicle helps students to make connections between needs and personal budgets. Understanding the costs of vehicle acquisition and operation can help students make rational economic decisions.

- Provide students with newspapers or other advertisements that show the price of purchasing a vehicle and the cost of leasing the same vehicle. Have students calculate the costs of owning and of leasing a vehicle of a given price, and ask them to research the advantages and disadvantages of each method by interviewing a loan manager, car dealership employee, car owner, or accountant.
- Have students research the cost of operating a vehicle for a year. Ask students to prepare a report suitable for class presentation that includes all anticipated costs (e.g., fuel, oil, tires, tune-ups, insurance). Have students convert the overall cost to cost per kilometer.
- Have each student find a new or used vehicle listed in a newspaper classified section, on the Internet, or in a specialized "buy-and-sell" magazine. Ask students to research and prepare a report on the reliability of the vehicle, the vehicle condition, specifications (e.g., engine size, gas mileage), and options (e.g., power features, air conditioning, car alarm, stereo system), the depreciation amount, and the cost of financing.
- Ask students to contact insurance agents to determine costs of insurance for various classes of vehicle and types of driver (e.g., age of vehicle, age of driver, type of vehicle).

Suggested Assessment Strategies

Acquiring and operating a vehicle involves costs that will have an impact on the present and future budgets of students. Assessment should focus on students' abilities to estimate, predict, research, calculate (using appropriate technology), and compare costs in the acquisition and operation of a vehicle, and to verify the reasonableness of their conclusions.

Observe

- As students work on problems involving the costs of acquiring and operating a vehicle, observe their abilities to:
  - explain the advantages and disadvantages of leasing, renting, or buying a vehicle
  - estimate the cost of a vehicle given the monthly payments and other charges
  - estimate the operating expenses of a vehicle, given published mileage figures and a set yearly mileage
- Note the extent to which students can read, create, and display vehicle acquisition and operating costs using a variety of forms such as tables, graphs, and histograms.

Collect

- Have each student interview an automotive technician, an insurance agent, or a loan manager and develop a report outlining the acquisition and operating costs associated with a vehicle. Review each report, looking for evidence that the student is able to:
  - compare and contrast the maintenance costs of new and pre-owned vehicles
  - identify basic motor vehicle insurance requirements
  - identify other costs associated with leasing a vehicle
  - create a personal budget to operate a vehicle

Peer Assessment

- Have students enter their information on operating and maintenance on a table. Have them analyse each other's projects based on size and utility of vehicle.

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Video

- The ABC's of Personal Finance

Prescribed Learning Outcomes

It is expected that students will prepare a simple income tax form

It is expected that students will:

- prepare an income tax form for an individual who is single, employed, and without dependents

Suggested Instructional Strategies

A working knowledge of the skills involved in the preparation of an income tax return will be essential to most students.

- Ask each student to find a description, including salary, of a job that is of personal interest to them in the careers section of a newspaper or on the Internet. Have students prepare a basic income tax return for one year of employment in that job.
- Have groups of students report to the rest of the class on the effects of RRSPs, donations, and educational tuition fees on an income tax return for a fixed income.
- Have students research and create tables of the levels of taxation according to income amount and of the various rates of taxation according to province or territory of residence.
- Have students report on the effects of levels of taxation if a given income is increased or decreased substantially.
- Discuss with students provincial and federal taxes, including how one is based on the other and how they can change.

Suggested Assessment Strategies

Preparing an income tax return involves skills that students can use throughout life. As much as possible, assessment should relate to the real-world applications of this skill.

Collect

- Give students job descriptions, including wages, and have them choose an occupation and complete an income tax form for a single individual without dependents. Collect the forms, noting the extent to which students:
  - properly complete arithmetic operations
  - use tables and forms properly
  - properly calculate provincial taxes and surtaxes

Self-Assessment

- Ask students to note the effects on income tax of:
  - deductions, including RRSPs
  - the amount of income
  - taxable benefits

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

[no resources]

PRESCRIBED LEARNING OUTCOMES

It is expected that students will demonstrate an awareness of the applications of probability to real world situations

It is expected that students will:

- express probabilities as ratios, fractions, decimals, percents, and in words
- use probability to predict the result in a given situation
- determine the odds for and against a particular event occurring
- compare experimental observations with theoretical predictions
- use probabilities to calculate expected gains and losses
- communicate and justify solutions to probability problems

SUGGESTED INSTRUCTIONAL STRATEGIES

Applications of probability that connect the real world with students' mathematical knowledge will help students understand the importance of probability. The study of applications of probability enables students to explore how probability affects our daily decision making.

- Have students work in groups to develop hypotheses that they can test. Provide each group with coins, cards, or identically shaped objects of different colours (e.g., plastic "chips," marbles) to draw from a bag. Have the students calculate the theoretical probabilities of a given activity (e.g., coin toss, card match, object match), then conduct experiments to test their hypotheses.
- Ask students to search newspaper or magazine articles for words such as *chance*, *probability*, and *likelihood*. As a class, discuss the following questions:
  - How are these terms used?
  - Are they used "correctly" by the media?
  - Are they always used in relation to mathematical calculations?
  - In what ways are probabilities expressed, other than as a percent?
- Provide students with a table of odds for major catastrophic events (e.g., lightning striking a given location, earthquake occurrence, volcano eruption) or other natural phenomena (e.g., crop failure or success, meteor striking earth, weather prediction). Discuss the following questions as a class:
  - Does the event with the best odds always occur? Why or why not?
  - How are such odds calculated?
- Have students examine actuarial tables outlining various probability relationships (e.g., life expectancy of men and women in relation to country, relationship of nutrition to health, relationship of location to baby mortality rates).

SUGGESTED ASSESSMENT STRATEGIES

Probability provides concepts and methods for interpreting predictions based on uncertainty. Evidence of students' abilities to use probability formulae can best be assessed in the context of experimental design and problem-solving activities.

Observe
- While students are working with data, ask questions, observe, and note how effectively they are able to:
  - make predictions and inferences based on probabilities
  - compare collected data with expected results
  - use appropriate terminology

Collect
- Have students conduct a research project comparing the probabilities of different events occurring (e.g., lightning striking a given location, earthquake occurrence, volcano eruption). Ask students to present their findings to the class, explaining how they acquired their data and what it means in terms of risk.

Question
- To assess students' understanding of basic concepts, pose questions that require students to express probabilities as words.
- Ask students to identify situations in their lives and in society where it would be useful to be able to calculate probabilities (e.g., weather prediction, management of natural resources).

Self-Assessment
- At the start of the unit, have students note in their journals what they already know about this field of mathematics and what else they would like to know. After completing the organizer, ask them to reflect back on their earlier journal entries, identify any new knowledge they gained, and determine if their initial questions have been answered.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Triple 'A' Mathematics Program: Data Management and Probability

Software

- Math Tools
- Probability Constructor

Games/Manipulatives

- D.I.M.E. Probability Pack A
- D.I.M.E. Probability Pack B

PRESCRIBED LEARNING OUTCOMES

It is expected that students will prepare a plan and operate a successful business

It is expected that students will:

- prepare a business plan to own and operate a business that includes the following information as appropriate:
  - monthly cost of operation (e.g., inventory, rent, wages, insurance, advertising, loan payments, etc.)
  - hours of operation
  - estimated daily sales (average)
  - gross/net profit
  - hourly earnings
- draw a scale floor plan of the store

SUGGESTED INSTRUCTIONAL STRATEGIES

Realistic activities involving the research and design of a business plan enable students to recognize the connections between mathematical skills they have learned and the concrete situations these skills represent. An understanding of procedures and applications of mathematics can help students make reasonable business decisions.

- Have students choose a business in their community at which they might like to work, and interview the owner or manager. Ask them to report their findings to the class, including the following information:
  - size of the store including inventory storage
  - number of staff
  - hours of operation
  - type of clientele
  - special features of the store (e.g., service, price, inventory)
- Ask each student to choose the inventory for a self-owned business from a store catalogue. After choosing their inventories, students set criteria for their businesses, including retail prices and costs of:
  - purchasing the inventory
  - rental of floor space
  - insurance and utilities (rough estimates)
  - staffing
  - advertising and promotion
- Have students develop a business plan, including:
  - estimates of gross revenue and gross profit
  - estimates of daily profit and annual profit
- Discuss the following questions as a class:
  - What are the advantages and disadvantages of operating your own business?
  - What are the advantages and disadvantages of working for others?
  - What mathematical skills do you need to operate your own business?
- Invite a business owner and/or operator to discuss with the class how he or she manages the business. Have students prepare questions related to the topic beforehand.
- Have students investigate real-world situations by comparing their business plans with the plans of established businesses.

Suggested Assessment Strategies

The study of a business plan provides an excellent opportunity for students to demonstrate their mathematical knowledge in a meaningful fashion. The plan to own and operate a successful business can connect students' understanding of the underlying mathematics to relevant and practical applications.

Collect

- Have students identify factors that might contribute to the success of their model businesses.
- Work with students to develop criteria and use them to assess their plans for their businesses. Criteria might include:
  - Will the business have sufficient space for inventory?
  - Will the business have enough staff to operate properly?
  - Are all costs considered, including rent, utilities, wages, insurance and advertising?
  - Are the identified daily sales and profit figures reasonable?

Self-/Peer Assessment

- Have students assess peer business plans, to determine whether the factors considered are relevant and/or important, and to propose other factors.

Question

- Ask students to what extent they have considered non-inventory costs such as franchise, advertising costs, environmental factors, and business location.

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Math Projects: Organization, Implementation, and Assessment

Software

- Hot Dog Stand: The Works

Curriculum

Essentials of Mathematics 12

ESTIMATED INSTRUCTIONAL TIME

Essentials of Mathematics 12 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

ESSENTIALS OF MATHEMATICS 12

Organizer (Suborganizer)	% of Time
Problem Solving	**Integrated Throughout**
Personal Finance	**10 - 20**
Design and Measurement	**10 - 20**
Government Finances	**10 - 20**
Investments	**10 - 20**
Taxation	**10 - 15**
Variation and Formulas	**10 - 15**
Life/Career Project	**10 - 20**

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- interpret their solutions by describing what the solution means within the context of the original problem
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems involving finances, modeling, and analysis can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Essentials of Mathematics 12

- Reinforce the concept that "problem solving" is more than just word problems and includes other aspects of mathematics than algebra.
- Introduce new types of problems directly to students (without demonstration) and play the role of facilitator as they attempt to solve such problems.
- Recognize when students use a variety of approaches; avoid becoming prescriptive about approaches to problem solving.
- Reiterate that problems might not be solved in one sitting and that "playing around" with the problem—revisiting it and trying again—is sometimes needed.
- Frequently engage small groups of students (three to five) in co-operative problem solving when introducing new types of problems.
- Have students or groups discuss their thought processes as they attempt a problem. Point out the strategies inherent in their thinking (e.g., guess and check, look for a pattern, make and use a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- Once students have arrived at solutions to particular problems, encourage them to generalize or extend the problem situation.
- See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These types of problems are indicated with an asterisk (*).

SUGGESTED ASSESSMENT STRATEGIES

Students analyze problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe

- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question

- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their
  - own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work.

Collect

- On selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not work as they solved particular problems.

Self-Assessment

- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set *Evaluating Problem Solving Across Curriculum* may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Problem Solving: What You Do When You Don’t Know What To Do

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve consumer problems involving insurance, mortgages and loans

It is expected that students will:

- solve problems involving different types of insurance
- determine the costs involved in purchasing a home, including gross debt service ratio
- solve problems involving different types of mortgages

SUGGESTED INSTRUCTIONAL STRATEGIES

Purchasing insurance and acquiring a mortgage involve important decision-making skills that will pay an important role in every student's future. By analysing various types of insurance and mortgage scenarios and considering additional cost factors, students are better able to make informed financial decisions.

- Brainstorm a class list of types of insurance (e.g., renter's, home-owner's, life, health, dental, eye-care, long-term disability).
- Have students discuss in small groups, research, and report on the following questions:
  - Why purchase life insurance?
  - How much life insurance is needed in various situations?
  - What types of life insurance are available?
  - Why do smokers pay more for life insurance?
  - Why do people pay more for life insurance as they age?
- As a class, discuss home-owner's insurance, including:
  - deciding how much insurance is needed, including contents coverage
  - the importance of reviewing exactly what is covered by a particular life insurance plan
  - the difference between standard and comprehensive home-owner's insurance
  - the need for renter's insurance
- Have students research the following terms, and provide a brief definition for each: *down payment, inspection fee, mortgage, application fee, service charges, legal fee, closing cost, land transfer tax, assets, debt-equity ratio, liabilities*.
- Ask students to make a list of additional costs to a mortgage beyond the cost of the building. Have them include reasons for buying a home within their price range.
- Discuss gross debt service ratio and demonstrate how it is used by financial institutions to determine eligibility for loans:
  $$\frac{\text{monthly mortgage payment + prop. taxes + heating}}{\text{gross monthly income}}$$
- Conduct a class discussion on types of mortgages, including closed, open, convertible, fixed-rate, and variable-rate.
- Have students analyse the relationships between principal, length of term, and interest rate using a mortgage calculator.

SUGGESTED ASSESSMENT STRATEGIES

To make wise financial decisions when purchasing insurance or acquiring a mortgage, consumers must understand the mathematics involved. Assessment should center around real-life scenarios involving premium and amortization tables and the use of spreadsheets. Look for evidence of students' abilities to analyze situations so they can make informed decisions.

Observe

- While students are working on problems involving insurance or mortgages, look for evidence that they can:
  - use the appropriate terminology (e.g., standard and broad insurance, comprehensive coverage, guaranteed replacement cost, N/C [not covered], deductible, down payment, mortgage, equity, appraisal, principal, amortization period, gross debt service ratio, net worth)
  - distinguish between the various types of insurance
  - distinguish between closed, open, convertible, fixed-rate and variable-rate mortgages
  - correctly calculate gross debt service ratio and net worth
  - identify an appropriate insurance plan and mortgage plan
  - identify additional costs involved in purchasing a home
  - use amortization tables

Collect

- Have students investigate real-life situations in purchasing an insurance plan or a mortgage plan. Assess their work for the extent to which they:
  - accurately determine the factors involved
  - set up and fill a spreadsheet to determine insurance cost, additional costs, mortgage
  - interpret the results from the insurance and mortgage calculation

Question

- After students have analyzed a real-life situation, ask them to answer the following questions:
  - what would happen if _____?
  - what if _____ had happened instead?
  - what does this information suggest?

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Video

- The ABC's of Personal Finance

Software

- Math Tools

PRESCRIBED LEARNING OUTCOMES

It is expected that students will analyse objects, shapes and processes to solve cost and design problems

It is expected that students will:

- analyze objects shown in "exploded" format
- draw objects in "exploded" format
- solve problems involving estimation and costing for objects, shapes, or processes when a design is given
- design an object within a specified budget

SUGGESTED INSTRUCTIONAL STRATEGIES

In this unit, students will have opportunities to use geometric ideas in design and measurment of 3-D objects. Combined with their skills in using technology and problem solving, these design and measurement skills will have broad real-life applications.

- Ask students to bring to class examples of illustrations or diagrams in "exploded" format. (These are available from hardware stores, home improvement stores, or lumber yards.) Have them choose an object and illustrate it in the same format.
- Invite a builder, architect, interior designer, or other appropriate professional to visit the classroom. Have students prepare and ask questions about his or her measurement techniques, design and/or construction procedures, and "tricks of the trade."
- Have each student design a 3-D object (e.g., picnic table, bird house, dog house, item of clothing) within a specified budget. Have them consider the following steps in completing the project:
  - set a budget
  - make a design plan
  - compile a list of materials
  - perform a cost comparison of the material by researching at least two suppliers
  - if necessary, adjust materials list to lower costs within the proposed budget

  Ask students to present the design project to the rest of the class.

SUGGESTED ASSESSMENT STRATEGIES

Project design and modeling are a powerful application of mathematical theory to everyday phenomena. Students demonstrate their knowledge of geometric principles, scale factors, measuring devices and measurement concepts by solving problems involving scale drawing, area, perimeter, surface area, and volume.

Observe

- While students are working on measurement activities, circulate and provide feedback on:
  - their abilities to match the correct measurement formula to a given figure
  - the extent to which they consider the reasonableness of their answers
  - their overall understanding of measurement concepts in solving problems
  - their ability to provide an "exploded" view of the sketch of an object
  - their methods for calculating the cost of a design

Collect

- Review students' written work in solving measurement problems for evidence that they can explain the method used in finding the cost of a design, and solve various types of problems based on the interpretation of measurements from a blueprint or 3-D sketch.

Presentation

- As students complete a project to design and analyze the cost of an object given a specified budget and present results to the class, look for evidence that they can:
  - logically and clearly present their work
  - model an object accurately
  - provide calculations to support their solution to the problem

Question

- Ask students to critique the presentation of a guest speaker, comment on the usefulness of the "tricks of the trade" and analyse the degree to which they applied these tricks to their own projects.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Math Projects: Organization, Implementation, and Assessment

Software

- Math Tools

PRESCRIBED LEARNING OUTCOMES

It is expected that students will demonstrate an awareness of the income and expenditures of federal, provincial and municipal governments

It is expected that students will:

- describe government expenditures including the amounts spent on social welfare benefits, social security, education, health care, policing, armed forces, and employee wages and salaries
- solve problems involving the calculation of selected federal taxes (e.g., GST, excise tax and duties)
- calculate provincial taxes, (e.g., PST, corporation capital, licenses, gasoline)
- determine how selected municipal taxes are calculated (e.g., property)

SUGGESTED INSTRUCTIONAL STRATEGIES

Every student will one day be a taxpayer and as such will encounter a wide range of payroll deductions and taxes. It is important that students understand the motivation for these and how the money collected through them is used. This knowledge will help students to become informed citizens.

- Ask students to discuss in cooperative groups, the following questions:
  - Why do governments impose taxes?
  - Where does the money come from?
  - How is tax money distributed?
  - What are GST and PST and what are they used for?
  - What is the purpose of custom duties and excise taxes?

Bring the class together to discuss their answers.

- Provide students with examples of real-life tax assessments, such as property tax assessments. Have students analyse how the statements are calculated.
- Have students research and discuss the following questions on municipal taxes:
  - Where does the tax money come from?
  - How is the collected tax money used?
  - What is the major source of revenue for cities?
- Explain what an educational levy is and how it is used, then have students calculate municipal tax bills for a variety of real-life situations.
- Provide students with examples of the local community tax bill. In a class discussion, review the layout of the form and the types of information included. Ask students to locate the following items:
  - the tax rate
  - the municipal mill rate
  - the educational mill rate
- Have students examine a summary of a federal or provincial budget presented over the last four to five years, then discuss as a class.

SUGGESTED ASSESSMENT STRATEGIES

Reading tax assessments and payroll deductions can be a tedious job and requires an understanding of the underlying math. To assess students' knowledge of income and expenditures of federal, provincial and municipal taxes, note their ability to discuss these items, calculate taxes, and verify given tax statements.

Observe

- As students discuss taxes, observe their ability to:
  - correctly label taxes.
  - identify the tax rate involved
  - provide a reason for the tax
  - explain what the tax is used for
- Observe how completely students analyse tax statements and the extent to which they verify given calculations.

Presentation

- Have students analyse real-life situations that involve a variety of tax calculations and have them present their solutions to the class.

Collect

- Have students design a flow chart or pie chart showing how our tax dollars are distributed. Assess their work with respect to:
  - presentation
  - accuracy
  - completeness

Peer Assessment

- Have students calculate tax bills for realistic situations and swap their work with a peer to have it checked for accuracy.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

[no resources]

Prescribed Learning Outcomes

It is expected that students will demonstrate and recognize the differences concerning different types of financial investments

It is expected that students will:

- determine a financial plan to achieve personal goals
- describe different investment vehicles (e.g., GICs, bonds, mutual funds, stocks, and real estate)
- compare and contrast different investment vehicles in terms of risk factors, rates of return, costs, and lengths of term
- identify reasons for investing money in RRSPs and RESPs
- investigate how to purchase and sell stocks

Suggested Instructional Strategies

The ability to make sound financial decisions is a critical factor in financial independence. Financial decisions are influenced by an individual's values, goals, attitudes, age, level of education, and income.

- Ask students to produce point-form paragraphs on the following topics:
  - List three goals you hope to achieve this year. Provide reasons why they are important to you and how you plan on achieving them. Consider any financial implications these goals may have.
  - List the ten things you value most in life. Briefly describe why each item on your list is important to you.
  - List three financial goals for people at different stages of life (e.g., 20s single, 50s married).
- Have students in small groups discuss the following questions:
  - What is an acceptable level of income to support your current life style?
  - What is a desired level of income to support a predicted life style in your future?
  - What factors can you influence to increase future income and benefits?

  Discuss their answers as a class.
- Invite a guest speaker (e.g., investment agent, banker, financial advisor) to address topics related to investment. Have students prepare questions that address the underlying mathematics of savings and investment options.
- Provide students with information on RRSPs and RESPs (e.g., bank pamphlets, web sites). Working in small groups, students list pros and cons for each.
- Have each student, given a fixed amount of money to invest, research and select stocks, "sell" old stocks, and "buy" new stocks according to predetermined rules. Students keep track of the stocks over a fixed period of time using a spreadsheet. Periodically compare the results as a class.

SUGGESTED ASSESSMENT STRATEGIES

Students need experience in setting financial goals and making financial decisions. It is therefore important that students demonstrate an understanding of the underlying mathematics involved in investment options and provide evidence that they are able to research and contrast a variety of investments.

Observe
- Note the extent to which students are able to establish a financial plan.
- Look for evidence that students differentiate between the different types of investment during group discussions.
- Observe the students' ability to explain advantages and disadvantages of a variety of investments during the research on a variety of investments.

Collect
- Have students investigate in depth one type of investment vehicle, and write a report summarizing their findings. Look for evidence that:
  - a description of the investment vehicle is provided
  - a variety of pros and cons are listed
  - a spreadsheet is used to analyze the performance of the investment
  - a visual description (e.g., time graph, bar graph, pie graph) is given comparing a number of investments within that investment vehicle (e.g., mutual funds: resource, technology, European, Asian)
  - risk factors involved in making investments vs. potential returns or losses

Self-Assessment
- During the stock market competition, have students note in their journals the reasons for any action or non-action they are taking about their stock portfolio. After the competition, ask them to reflect back on their earlier journal entries, identify positive and negative decisions they made, and determine the factors that are important in making investment decisions.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Video

- The ABC's of Personal Finance

Software

- Math Tools

PRESCRIBED LEARNING OUTCOMES

It is expected that students will demonstrate an ability to fill out an income tax form

It is expected that students will:

- fill out income tax forms for:
  - a single parent with child
  - a married couple with one person earning an income
  - a married couple with child

SUGGESTED INSTRUCTIONAL STRATEGIES

Understanding the mathematics of a variety of income tax situations helps students prepare for a time when they may be acquiring property and investments, forming a family, and/or starting a business.

- Have students work in small groups to create mindmaps that illustrate what they already know about personal income tax. In a class discussion, pool the groups' information to create a class mindmap.
- Invite a guest speaker (e.g., taxation lawyer, tax consultant) to talk to students about important issues related to personal income tax. Have students prepare questions that identify and apply the underlying mathematics.
- Encourage students to include an income tax return in connection with their Life/Career Project discussion of budgets, life style, and personal finances.
- Provide students with blank income tax forms and appropriate financial data for the following lifestyle situations:
  - a single parent earning and income, with one child
  - a married couple, one person earning an income, with no children
  - a married couple, both earning an income, with one child

  Have students complete the forms and, as a class, discuss the tax implications of the various situations.

SUGGESTED ASSESSMENT STRATEGIES

Modeling a variety of living and working scenarios will provide a basis for students to fill out an income tax form. Assessment should focus on students' abilities to read and interpret income tax documents, perform the appropriate tax calculations, select appropriate deductions, and solve problems associated with filling out an income tax form.

Observe

- As students fill out an income tax form, look for evidence that they
  - use appropriate calculation techniques
  - are able to interpret documentation for the income tax form
  - refer to the relevant section in the documentation for specific lines on the income tax form
  - are able to follow the steps laid out in the income tax form

Poster Presentation

- Choosing one scenario, have small groups of students prepare a poster for each section of the income tax form. Have them annotate their work to explain how they arrived at each answer and what method they used. Assess their posters for:
  - presentation
  - completeness
  - clarity
  - annotation
  - correctness

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

[no resources]

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use algebraic and graphical models to generate patterns, make prediction and solve patterns

It is expected that students will:

- plot and analyse examples of direct variation, partial variation, and inverse variation
- given data, graph, or a situation, recognize the variation represented
- use formulas to solve problems

SUGGESTED INSTRUCTIONAL STRATEGIES

The ability to identify relationships between various quantities is essential to solving most problems. Students need to develop the ability to visualize and identify the appropriate variation given a variety of information and situations.

- In a class discussion, encourage students to determine the relationship between sets of data such as the following:
  - pay to hours worked
  - light intensity to water depth
  - profit to fixed and variable costs
  - electrical components for current, voltage, and resistance
- Have students work in groups to identify the type of variation represented in each situation (e.g., direct, inverse, partial) and develop an appropriate equation representing the variation. For example, the electrical resistance R of a wire varies directly as the length L and inversely as the square of the diameter D of the wire:
  $R = k\frac{L}{D^2}$ where k is a constant.
- Have students use appropriate technology to create graphs and tables of each variation.
- Provide students with data from modeling real-life situations (e.g., raising a flag on a pole, pouring water out of a container) and ask them to draw diagrams, find a formula, and solve by substitution for an unknown quantity.
- Provide students with several graphs, and ask them to describe the real-life scenarios that would have produced them.
- Have students design a puzzle work sheet of formulas and their answers, together with an answer key.

SUGGESTED ASSESSMENT STRATEGIES

As students work with algebraic and graphical representations, assessment should focus on their competency in solving problems involving several kinds of variation.

Observe

- As students work with graphing calculators, observe their abilities to:
  - set appropriate domain and range values
  - verify their results
  - generate examples that demonstrate an understanding of the different types of variation

Self-Assessment

- Assign experiments that require students to demonstrate the type of variation involved in real-life situations (e.g., intensity of light varies inversely as the square of the distance from the source). Let students check their own understanding and accuracy by:
  - using graphing calculators or software.
  - checking work against models that are provided

Peer Assessment

- Have students share the puzzle worksheets with their peers and have them assess the puzzle by completing it and checking it against the provided key.

RECOMMENDED LEARNING RESOURCES

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator
- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Software

- Green Globs & Graphing Equations

Prescribed Learning Outcomes

It is expected that students will research career choices and perform a comparative study

It is expected that students will:

- determine what factors are important in analysing careers
- describe two specific career opportunities
- identify mathematical educational requirements for two careers
- compare two careers in terms of salary, working hours, training time and cost, cost of living, and benefits

Suggested Instructional Strategies

It is expected that today's student will hold several different careers in his or her lifetime. To be able to make informed decisions about future employment, students need opportunities to practise job-search skills and to analyse career information objectively.

- Brainstorm with the whole class to find as many factors as possible that influence students' career opportunities and decisions (e.g., food, shelter, salary, education, transportation, insurance, recreation, job security).
- Have students research two careers in fields appropriate to their individual interests and abilities. This research should include:
  - a job description for each selected career
  - the educational background required and associated costs, including a detailed description of the mathematical educational background required
  - the salary or wage levels offered, including starting rates, potential income, and benefits available
  - an indication of the employment opportunities in the two fields
  - health issues, including stress, that might be relevant to each career
  - a description of the life style each career can support, with a supporting budgetary analysis
  - an individual statement of the long-term personal and educational requirements for advancement in each career
- Ask students to use spreadsheets, graphs, and/or diagrams to display their budgetary analysis of the two careers for the rest of the class.

Suggested Assessment Strategies

Analysing career choices is an important life skill and requires a working knowledge of the underlying mathematics. Assessment should focus on the students' ability to research, compile, interpret, and present career factors related to two types of careers.

Observe

- Observe the extent to which students:
  - use a variety of sources to research the two careers
  - collect information on areas necessary to make an informed decision (e.g., salary, educational background required)
  - use technology to display information

Collect

- Have students write-up the comparative study for the two careers. Assess their items for presentation and the extent to which their items demonstrate an analysis of:
  - job description
  - requirements: education, budgetary analysis, income
  - resume
  - salary factors
  - health issues
  - life style

Presentation

- Have students present the pros and cons of one or two careers to the class and have their peers assess the presentation according to criteria established as a class beforehand.

Recommended Learning Resources

Comprehensive learning resources for this course are currently under development. As an interim measure, schools are encouraged to use the teacher-developed learning resources distributed to schools (student and teacher resources). Please note that the student materials require photocopying for student use.

Print Materials

- Math Projects: Organization, Implementation, and Assessment

Video

- The ABC's of Personal Finance

Software

- Math Tools

CURRICULUM

Principles of Mathematics 10

ESTIMATED INSTRUCTIONAL TIME

Principles of Mathematics 10 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

PRINCIPLES OF MATHEMATICS 10

Organizer (Suborganizer)	% of Time
Problem Solving	Integrated Throughout
Number (Number Concepts)	5 - 15
Number (Number Operations)	10 - 20
Patterns and Relations (Variables and Equations)	10 - 20
Patterns and Relations (Relations and Functions)	10 - 20
Shape and Space (Measurement)	10 - 20
Shape and Space (3-D Objects and 2-D Shapes)	10 - 20
Statistics and Probability (Data Analysis)	10 - 20

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, probability
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly explain the solution to a problem and justify the processes used to solve it
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems involving a range of mathematical disciplines can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Principles of Mathematics 10.

- In a class discussion, define *problem solving* with students, pointing out that in mathematics problem solving involves many disciplines, including algebra, geometry, trigonometry, statistics, and probability.
- Introduce new types of problems directly to students (i.e., without demonstration), and facilitate as they attempt to solve them.
- Have students work in small co-operative groups of three to five when introducing new types of problems.
- Model and reinforce student use of a variety of approaches to problem solving (e.g., algebraic and geometric).
- Emphasize that many problems may not be solved in one try and that it is sometimes necessary to revisit the problem, start over, and try again.
- Have students or groups discuss their thought processes as they attempt to solve a problem. Point out the strategies inherent in their thinking (e.g., guessing and checking, looking for a pattern, making and using a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- When students arrive at solutions to particular problems, encourage them to generalize from or extend the problem situations.

  Note: See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These problems are indicated with an asterisk (*)

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe
- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question
- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work

Collect
- On selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not for particular problems.
- Have students use mind maps and flowcharts to describe and display problem solutions.

Self-Assessment
- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set, *Evaluating Problem Solving Across Curriculum*, may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises.

These resources, while not part of the Grade Collections, have Provincially Recommended status.

Appendix B includes an annotated bibliography of these resources for ordering convenience.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- analyse the numerical data in a table for trends, patterns and interrelationships.
- explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

It is expected that students will:

- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data)
- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data)
- classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are "nested" within the real number system

SUGGESTED INSTRUCTIONAL STRATEGIES

Using tables enhances students' ability to see data in a compact and easily readable format. Students can extract specific information from tables, and use them to do general comparisons and find relationships.

- Distribute single newspapers to groups of three or four students and have them browse through the paper to find the sections that contain the most tables. Ask the groups to give two reasons why those particular sections use tables extensively.
- Have each student find three specific pieces of information in a table from a newspaper, magazine, or the Internet and share with a partner. Have pairs share their findings with the rest of the class.
- Place a multi-column, non-recursive table (e.g., hockey standings/stock market data) on an overhead. In a class discussion, encourage students to compare and contrast the data in the table. As a class, produce a word list of connections between the rows and columns of data.
- Provide students with a recursive table (e.g., mortgage, credit card account, savings account). Point out that the closing balance for any month is the opening balance of the next month. Ask students to analyse the repetitive nature of the calculations. Have them calculate the balance after three payments, given an opening balance, a monthly rate of interest, and the payment amount.
- Discuss the definitions of the various number systems. Use a set of nested measuring cups with appropriate labels to illustrate how the number systems fit inside one another. Illustrate the anomaly of irrational numbers by slipping a piece of paper between the rational-number cup and the real-number cup. Have students transfer these concepts to Venn diagrams in their notebooks.
- Draw a number line on the board and have students place rational numbers in appropriate places, then ask them to place irrational numbers on the line. Point out that a decimal approximation of the number is necessary to allow for the proper placement and that more decimal places lead to greater accuracy.

SUGGESTED ASSESSMENT STRATEGIES

Tables are commonly used to represent real life data. Assess students' ability to use tables in a practical way. Students demonstrate their understanding of the structure and interrelationship within the number system when they explain the processes they use as they experiment with and manipulate numbers.

Observe
- While students are drawing meaning from tabular representations of data, note their ability to:
  - use words to describe data
  - identify specific facts from tables
  - determine trends where they exist
  - develop recursive tables in the form of spreadsheets
- Provide students with a nested Venn Diagram and have them place numbers appropriately. Observe students' abilities to place numbers in such a way that they fit all the appropriate numbers systems.

Collect
- Give students multi-column tables such as sports standings, and note the extent to which students are able to glean information. Have students write a newspaper article based on the information and note the extent to which they are able to interpret data.

Question
- Ask students to suppose that they are responsible for adding 10 new items to the inventory of a clothing store. Have them create a spreadsheet that shows price, GST, PST, and cost. Ask them to explain how the value for each cell in their spreadsheet is calculated.

Self/Peer Assessment
- Work with students to develop criteria they can use to assess their posters. Criteria might include:
  - it is easy to use for classifying numbers
  - it makes sense to others
  - it gives accurate examples

Students can use these criteria to assess their work, then compare their assessment to the teacher's assessment.

Presentation
- Have students gather similar information on each of five scientists and present it in tabular form to another student or class.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Pure Mathematics 10 (Distance Learning Package)

Multi-media

- The Learning Equation: Mathematics 10 Lessons 1 - 4
- Mathematics 10, Western Canadian Edition Ch. 1 (Sections 1.6, 1.7) Ch. 2 (Sections 2.1, 2.5)
- MATHPOWER 10, Western Edition Ch. 1 (Sections 1.1, 1.3) Ch. 2 (Sections 2.1, 2,2)

Software

- Understanding Math Series

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- use basic arithmetic operations on real numbers to solve problems.
- describe and apply arithmetic operations on tables to solve problems, using technology as required.
- use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

It is expected that students will:

- communicate a set of instructions used to solve an arithmetic problem
- perform arithmetic operations on irrational numbers, using appropriate decimal approximations
- create and modify tables from both recursive and nonrecursive situations
- use and modify a spreadsheet template to model recursive situations
- perform operations on irrational numbers of monomial and binomial form, using exact values
- explain and apply the exponent laws for powers of numbers and for variables with rational exponents

SUGGESTED INSTRUCTIONAL STRATEGIES

Many calculations in science, industry, and finance involve irrational numbers. Students need to be able to perform simple calculations with irrational numbers and properly interpret the results. Although many questions can be solved adequately using approximate values, students should be able to work with exact representations of irrational numbers where possible. Operations of radicals and exponents with rational exponents build on algebraic and exponential skills previously learned.

A table enables organized data to be recognized and facilitates calculations.

- Provide students with a descriptive paragraph or set of written instructions for guessing a secret number, and have them work out the series of mathematical steps required. Ask them to confirm their process with a partner to test its accuracy.
- Ask students to find the maximum length umbrella that can fit in a rectangular gift box, bottom corner to top corner, for a box with rational lengths and for one with irrational lengths.
- Provide students with a descriptive paragraph containing numeric data (e.g., mutual funds over a 10-year period) and ask them to organize the data into a table.
- Give students a collection of sale papers and have them comparison shop for specific items. Have them place the results of the comparison in a table and underline the best price.
- Provide students with a basic spreadsheet template for calculating the cost of building a house. Have them change the dimensions of the house and number of bathrooms and bedrooms to see how the construction, electrical, plumbing, and total costs change.
- Have students work in pairs to design a template to show balance owing on a sample credit card account at the end of one month after a payment has been made.
- Discuss with the class the historical necessity of operations on radicals (e.g., calculators not yet invented) and their place in the language of mathematics.
- Assign an activity sheet to parallel the operations on radicals with operations on variables (e.g., questions on both $2\sqrt{7}+3\sqrt{7}=5\sqrt{7}$ and $2x+3x=5x$).

SUGGESTED ASSESSMENT STRATEGIES

Number operations are the tools students use to solve problems. Students need to demonstrate a clear understanding of the processes involved in performing various operations involving irrational numbers and how the operations are used.

Observe

- Note students' abilities to use their calculators to approximate $\sqrt{2}\times\sqrt{7}\times\sqrt{5}$, have them find $\sqrt{70}$ and compare.
- Ask students to draw several regular polygons and have them count the number of sides, diagonals, and intersection points and prepare a table showing this data. Observe students' abilities to use the tables to describe how the values of each polygon can be predicted without counting.

Self-/ Peer Assessment

- Have each student create a question involving irrational numbers and submit the question and its solution. Display students' questions on the overhead for the class to answer. Ask the class to compare its solutions with those of the originators and determine whether students have correctly solved their own questions.
- Have students write out a description of how to produce Pascal's Triangle and have a classmate use their instructions to produce the first six rows.

Collect

- Ask students to create a spreadsheet that would show bills for a credit card account that would occur in a situation in which no payments are made. Assume the amount borrowed was $1000 and the annual interest rate is 18% compounded monthly. Have them extend the table to show 12 months of account balances. Ask students to write a letter from the bank explaining the table to the credit card holder.

Presentation

- Have students collect data from a school sports team (e.g., free-throw percentage, penalties or fouls vs. minutes played) and create a table that shows an analysis of calculated values. Have students present their results to the class.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Pure Mathematics 10 (Distance Learning Package)

Multi-media

- The Learning Equation: Mathematics 10 Lessons 5 - 11
- Mathematics 10, Western Canadian Edition
  Ch. 1 (Section 1.5 - 1.8)
  Ch. 2 (Sections 2.2 - 2.4, 2.6 - 2.10)
- MATHPOWER 10, Western Edition
  Ch. 1 (Sections 1.3 - 1.5, 1.7, 1.8)
  Ch. 2 (Section 2.1)

Software

- Secondary Math Lab Toolkit
- Understanding Math Series

PRESCRIBED LEARNING OUTCOMES

It is expected that students will generate and analyse number patterns.

It is expected that students will:

- generate number patterns exhibiting arithmetic growth
- use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems
- relate arithmetic sequences to linear functions defined over the natural numbers
- generate number patterns exhibiting geometric growth

SUGGESTED INSTRUCTIONAL STRATEGIES

Many natural phenomena and man-made processes grow and shrink according to easily defined mathematical rules. An understanding of arithmetic and geometric growth helps students to better understand how the world around them changes.

- Give students a collection of arithmetic or geometric sequences with blanks either between the numbers or at the end of the sequence for students to fill in. Have them state the common difference or ratio and give a word-based rule describing how one term is calculated from the previous term.
- Have students find three examples of arithmetic or geometric sequences found in nature or used by people in their day-to-day lives.
- Ask students to match a column of linear equations to a column of arithmetic sequences.
- Have students describe or model natural phenomena that incorporate geometric sequences in their structure (e.g., flowers, spiral shells, paper folding).
- Give students numerical and non-numerical patterns and ask them to define the rule(s) for each pattern. Have them use the rules to make predictions and classify the patterns.
- Have students use concepts of arithmetic and geometric growth to solve problems such as:
  - If $1000 is deposited each year into an account that earns 12% compounded annually, how much money will have accumulated after 25 years?

  Then have them:
  - generate a model of this situation
  - identify it as geometric or arithmetic
  - discuss the answer (Is it surprising? Reasonable?)

SUGGESTED ASSESSMENT STRATEGIES

Assessment strategies should demonstrate algebraic skills as well as holistic pattern recognition. Students should also be able to articulate methods to aid in pattern discovery.

Observe
- How quickly do students see the patterns in partially completed sequences:
  - with no hints (2, 5, 8…)
  - with hints (2, _, _, 16, _… - geometric)
- Can students develop a mathematical equation from a sequence?

Question
- Can students articulate what to look for in determining whether a collection of numbers is an arithmetic sequence, geometric sequence, or neither?

Collect
- Have students collect five arithmetic, five geometric, and five other sequences. Provide them with equations or descriptions of how to generate each sequence.

Peer/Self Assessment
- Have each student develop a connect-the-dots puzzle using a combination of sequences and linear equations. Ask them to have another student do the puzzle and check it for accuracy.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Exploring Advanced Algebra with the TI-83
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Pure Mathematics 10 (Distance Learning Package)
- A Visual Approach to Algebra

Multi-media

- The Learning Equation: Mathematics 10 Lessons 22 - 24
- Mathematics 10, Western Canadian Edition
  Ch. 1 (Sections 1.1, 1.2, 1.4)
  Ch. 5 (Section 5.5)
- MATHPOWER 10, Western Edition
  Ch. 2 (Sections 2.3, 2.5, 2.7)

Software

- Secondary Math Lab Toolkit
- Understanding Math Series

Games/Manipulatives

- Radical Math: Math Games Using Cards and Dice (Volume VII)

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will generalize operations on polynomials to include rational expressions.

It is expected that students will:

- factor polynomial expressions of the form $ax^2 + bx + c$ and $a^2x^2 - b^2x^2$
- find the product of polynomials
- divide a polynomial by a binomial and express the result in the forms:
  - $\frac{P}{D} = Q + \frac{R}{D}$
  - $P = DQ + R$
  - $P(x) = D(x)Q(x) + R$
- determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored
- determine the nonpermissible values for the variable in rational expressions
- perform the operations of addition, subtraction, multiplication, and division on rational expressions
- find and verify the solutions of rational equations that reduce to linear form

SUGGESTED INSTRUCTIONAL STRATEGIES

Extensive exposure to factoring will help students increase their skills in manipulating algebraic expressions and consolidate their understanding of algebraic ideas and procedures. Appropriate technology can enhance student learning and extend skill development.

- Model the use of algebra tiles to represent quadratic equations. Have students work in pairs to manipulate the tiles to do factoring.
- Demonstrate types and methods of factoring. Provide or have students create examples of factoring and ask them to create the types and therefore the processes to be used. Have students justify and discuss their solutions in groups.
- Discuss as a class the ideas that not all data in real-world problems fit a linear model and that there may be more than one solution to a problem. Point out that solutions for such problems are usually more difficult to calculate than those for problems that fit the linear model (e.g., $10x^2 + 11x - 6 = 0$). Present a quadratic that does not factor and discuss possible solutions with the class. Are there any solutions?
- Have students use available technology to compare graphs of the quadratic functions with the equation, and to relate the zeros to the factors.
- Ask students to suggest reasons for simplifying expressions (e.g., to solve complex problems involving polynomial functions and equations).
- Have students factor polynomial expressions (trinomials) using trial and error. Give examples of polynomial equations that students would have difficulty recognizing without the skills of factoring or simplifying.
- Encourage students to develop their skills in factoring perfect square trinomials, by practising squaring binomials by sight.
- Ask students to graph rational functions on their graphing calculator to link the excluded value with the asymptote (e.g., $y = \frac{x-3}{x+2}$).
- Shuffle a card deck consisting of factored and non-factored pairs of polynomials, give each student a card, and ask them to find their partners.

SUGGESTED ASSESSMENT STRATEGIES

In order to solve equations and problems based on quadratic and rational expressions, students must be able to factor polynomials and understand when a root is extraneous. Assessment strategies should focus on observing student proficiency of these two concepts.

Observe
- Develop a concentration card game using cards that consist of polynomials in factored and non-factored form. Observe the speed at which students can match cards and win the game.
- Have students model factored polynomials using algebra tiles.

Question
- Encourage students to verbalize the patterns involved in factoring various types of polynomials.
- Have students draw connections between restrictions for a rational expression and the asymptotes on its graph.

Collect
- Give students a worksheet of non-simplified rational expressions. Post simplified answers and restrictions around the room on posters and have students record the locations of correct answers.
- Have each student develop a one-page summary of skills involved in factoring. Alternatively, have them write the summaries in flow chart form.

Peer/Self Evaluation
- Have students work in pairs to develop a factoring puzzle or rational expression. Photocopy the work of each pair and give it to another pair to solve and assess by criteria supplied by the teacher or developed as a class activity.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Exploring Advanced Algebra with the TI-83
- Exploring Functions with the TI-82 Graphics Calculator
- Graphing Calculator Activities for Enriching Middle School Mathematics
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Pure Mathematics 10 (Distance Learning Package)
- A Visual Approach to Algebra

Multi-media

- The Learning Equation: Mathematics 10 Lessons 25 - 33
- Mathematics 10, Western Canadian Edition
  Ch. 6 (Sections 6.4 - 6.6, 6.10)
  Ch. 7 (Sections 7.1 - 7.7)
- MATHPOWER 10, Western Edition
  Ch. 3 (Sections 3.3, 3.4, 3.6, 3.8 - 3.10)
  Ch. 4 (Sections 4.2 - 4.9)

Software

- Green Globs & Graphing Equations
- Secondary Math Lab Toolkit
- Understanding Math Series

Games/Manipulatives

- Radical Math: Math Games Using Cards and Dice (Volume VII)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- examine the nature of relations with an emphasis on functions.
- represent data, using linear function models.

It is expected that students will:

- plot linear and nonlinear data, using appropriate scales
- represent data, using function models
- use a graphing tool to draw the graph of a function from its equation
- describe a function in terms of:
  - ordered pairs
  - a rule, in word or equation form
  - a graph
- use function notation to evaluate and represent functions
- determine the domain and range of a relation from its graph
- determine the following characteristics of the graph of a linear function, given its equation:
  - intercepts
  - slope
  - domain
  - range
- use partial variation and arithmetic sequences as applications of linear functions

SUGGESTED INSTRUCTIONAL STRATEGIES

Describing functional relationships is essential to interpreting and predicting the behavior of the world around us. Students will explore ways to translate between algebraic and graphical representations and use these skills to make inferences and solve problems.

- Ask students to explore non-linear functions with graphing technology such as calculators or computer software programs. Discuss the similarities and differences in the graphs and equations with the class.
- Have students use graphing calculators or computers to graph $y = x$ and investigate the graphic changes as a number is added (e.g., $y = x + 3$ and when is multiplied by a constant (e.g., $y = -2x$) to develop the concept $y = mx + b$. Have them record their findings in their journals, using their own words.
- Ask students to work in groups to determine techniques that will generate linear equations, given information other than slope and intercept.
- To explore the relationship of linear equations in computer programming, have students work in co-operative groups to generate short programs that produce linear drawings, then share these with the class.
- Organize a carousel activity in which students move in groups around various stations set up with activities to generate data. Students decide as a group how best to graph or display the data. Discuss as a class the similarities and differences in the solutions.
- Use a bakery model (ingredients in → pastry out) or a sawmill/pulp model (logs in → lumber pulp/paper out) to illustrate the concept of domain and range.
- Have students use a function machine to calculate the range of a function from its domain.
- Have students generate two data points in a linear function (e.g., kilometres driven and car rental cost) and graph them. Ask them to generate model functions to answer questions about the intercepts (fixed costs) and slope (variable costs).
- Ask students to use maps and scale diagrams to calculate distances and sizes.

SUGGESTED ASSESSMENT STRATEGIES

Plotting data, and the subsequent analysis of relations and functions, are key to student understanding of patterns. Assessment in this area should focus on students' ability to perform individual outcomes (such as graphing and analyzing functions derived from the physical world), and reflect students' understanding of the uses of graphs.

Collect

- Have students collect data on relations between pizza costs and diameters. This data could be represented as a list of ordered pairs, expressed as a rule, described in equation form, as a hand sketched graph, and finally drawn using a graphing calculator. Check particularly for students' ability to select scale, find slope in appropriate units, and select window parameters.

Question

- As students work on using function notation, evaluating functions and determining domain, range, intercepts and slope, check for their ability to work competently using both sketches and graphing tools. Note students' abilities to translate their graphing skills to understand the real-world implications of the data.

Presentation

- Have students conduct research into the cost of video rentals over time and present the data as a list of ordered pairs and the graph of the function. Note students' understanding of the use of the graph they have created (e.g., At what point has the rental cost exceeded purchase price?).

Peer Assessment

- Ask students to work in pairs using graphing calculators. Students can display graphs on their calculators, then challenge their partners to reproduce the graphs on their own calculators.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Exploring Functions with the TI-82 Graphics Calculator
- Graphing Calculator Activities for Enriching Middle School Mathematics
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Pure Mathematics 10 (Distance Learning Package)
- Using the TI-81 Graphics Calculator to Explore Functions
- A Visual Approach to Algebra
- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator
- What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Multi-media

- The Learning Equation: Mathematics 10 Lessons 34 - 42
- Mathematics 10, Western Canadian Edition Ch. 5 (Sections 5.1 - 5.7)
- MATHPOWER 10, Western Edition Ch. 5 (Sections 5.1 - 5.6) Ch. 6 (Sections 6.5 - 6.8)

Software

- GrafEq (Macintosh & Windows Version 2.09)
- Green Globs & Graphing Equations
- Secondary Math Lab Toolkit

Games/Manipulatives

- Radical Math: Math Games Using Cards and Dice (Volume VII)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.
- solve problems involving triangles, including those found in 3-D and 2-D applications.

It is expected that students will:

- calculate the volume and surface area of a sphere, using formulas that are provided
- determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects
- solve problems involving two right triangles
- extend the concepts of sine and cosine for angles through to 180˚
- apply the sine and cosine laws, excluding the ambiguous case, to solve problems

SUGGESTED INSTRUCTIONAL STRATEGIES

Many worthwhile and practical problems in mathematics involve shape and space and are most easily solved by labeling a drawing. Some drawings work best when reduced to scale while in other cases an annotated sketch is sufficient. Visualizing shape and space is a critical component of learning mathematics.

- Bring a collection of spherical balls related to various sports into the classroom. Have students find the circumference of each ball using a fabric tape, and, from the circumference, calculate radius-diameter-area of the greatest circle. Ask them to place the results in a table and find the relationship between each column and classify it as linear or non-linear.
- Discuss the limitations in manufacturing that large structures increase in weight faster than the surface area increases which results in the need for special bracing to keep them from collapsing.
- Discuss why smaller animals require faster metabolisms to maintain a constant body temperature, and require more food as volume decreases faster than the surface area decreases.
- Have students use equipment such as a parallax viewer (for range) and clinometer (for height) to analyse the motion of a ball thrown on a field, resolving the motion into horizontal and vertical components. Point out that the horizontal motion vs. time is linear while the vertical motion vs. time is described using a quadratic function (i.e., non-linear).
- Provide students with pictures of non-right triangles that show either two sides and one angle or two angles and one side. Have students calculate the missing side or angle. A practical example might involve using an orienteering map.
- Give instructions to students (in terms of angles and sides) to move to a certain point in two steps (two sides of a triangle), using Law of Cosines or Law of Sine to calculate how to return to the starting point.

SUGGESTED ASSESSMENT STRATEGIES

Assessment of students' measurement skills reflects their abilities to calculate using formulas, and to see and use the relationships that exist among scale factors, areas, and volumes. Assessment should focus on students' abilities to extract relevant triangle information from drawn or written problems, as well as their abilities to solve simple triangle problems.

Observe

- While students are solving problems involving scale, note their ability to transfer skills to new situations. Ask them, for example, to estimate the change in surface area that occurs as a result of increasing one of the dimensions of an object. Check their results and ask them to explain any differences.
- Note students' ability to translate written problems into sketches, including those whose angles range from 0° to 180°. Observe students' ability to use calculators and manipulate angle representations to solve problems.

Collect

- Check students' solutions to problems for clarity and presentation. Students should be able to explain the strategy they used to solve selected problems.
- Post a number of sketches around the classroom. Give students a worksheet with word problems on it. Have them match sketches to appropriate problems. To make it interesting, give out different sheets with the problems scrambled. Provide one or two problems with no sketch and post more sketches than problems.

Peer/Self-Assessment

- Have students create two or three problems involving sine and cosine laws, including solutions. Students should exchange problems with a partner and have the partner complete, and then comment on, the construction of the problem and its solution.

Presentation

- Have students work in pairs on problems from the construction or other industry, and have them write up and present solutions to the rest of the class.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Pure Mathematics 10 (Distance Learning Package)

Multi-media

- The Learning Equation: Mathematics 10 Lessons 12 - 16
- Mathematics 10, Western Canadian Edition
  Ch. 6 (Sections 6.1, 6.2)
  Ch. 8 (Sections 8.4 - 8.6)
- MATHPOWER 10, Western Edition
  Ch. 7 (Sections 7.2, 7.3, 7.5 - 7.8)

Software

- The Geometer's Sketchpad
- Understanding Math Series

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve coordinate geometry problems involving lines and line segments.

It is expected that students will:

- solve problems involving distances between points in the coordinate plane
- solve problems involving midpoints of line segments
- solve problems involving rise, run and slope of line segments
- determine the equation of a line, given information that uniquely determines the line
- solve problems using slopes of:
  - parallel lines
  - perpendicular lines

SUGGESTED INSTRUCTIONAL STRATEGIES

Being able to measure using instruments and understanding measurement limitation is fundamental to applying mathematics practically. Assessing students' abilities to solve formula problems, or drawing interpretation problems should be done in a variety of ways that reflect real world methodologies.

- Using a geoboard, have students stretch an elastic between two posts to form a line segment, then stretch one side of the elastic around the appropriate post to form a right triangle. Have them measure the orthogonal sides by counting the posts and use the Pythagorean Theorem to find the length of the original line segment (hypotenuse of triangle).
- Have students use a clinometer to measure the angle to the top of a building and connect the tangent of the angle to the slope of the line to the top of the building.
- Develop a five-station carousel where each station has a different type of word problem (e.g., distance, midpoint, slope, equations of lines, parallel and perpendicular lines). Design a treasure hunt or rally to send students from station to station to solve problems. Use a point system to award prizes for the most accurate solutions.
- Given an equation for a line, have students list five unique pieces or types of information that they could use to find that equation. For example, given $2x - 3y = 6$ students could list the following:
  - different points on the line (6,2),(9,4),...
  - slope of line is m=2/3
  - y-intercept is b = -2 or the point (0,-2)
  - x-intercept = 3 or the point (3,0)
  - standard form of equation, $2x - 3y = 6$
  - slope intercept form is: $y = \frac{2}{3}x - 2$
- Have students use a graphing calculator or graphing software to explore the similar attributes of parallel or perpendicular lines. Give students six lines to graph and have them note which lines are parallel or perpendicular, analysing the similarities in their equations.
- Have students use technology with dynamic geometry software to draw two lines, then rotate one line. Have them note how its slope changes as it first becomes parallel, then perpendicular to the other line.

SUGGESTED ASSESSMENT STRATEGIES

Solving problems on the coordinate plane is a natural extension of graphing, geometry, and algebraic skills. Assessment should focus students' conceptualization of the coordinate plane and its uses in understanding the world around them.

Observe
- Take note of students' approaches to word problems that require line and segment solutions. Students should be able to explain the steps they took to translate word problems into a coordinate rendering. Observe students' abilities to draw and label clearly.
- Have students model solutions to problems on a geoboard where appropriate.

Question
- For select problems, question students as to the method they used. For example, ask if the student can check a pen-and-paper solution using a graphing tool or vice versa.

Presentation
- Ask students to select a graphic representation of a solution to a problem they have created. Have them present the problem to members of a small group and then teach the problem solution. It is motivational and good assessment to use student created problems on teacher tests where appropriate.

Collect
- Place information about various lineal equations on file cards on a table. Give students one file card and have them list or collect cards with data related to the equation on their card.

Peer Assessment
- Ask students to work in pairs using graphing calculators. Students can display graphs on their calculators, then challenge their partners to reproduce the graphs on their own calculators.
- Have one student generate an equation in slope-intercept form and a partner change it to general form and vice versa. Students can check each other's work for accuracy.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Pure Mathematics 10 (Distance Learning Package)

Multi-media

- The Learning Equation: Mathematics 10
  Lessons 17 - 21
- Mathematics 10, Western Canadian Edition
  Ch. 3 (Sections 3.1 - 3.5)
  Ch. 4 (Sections 4.1 - 4.5)
- MATHPOWER 10, Western Edition
  Ch. 6 (Sections 6.1 - 6.7)

Software

- The Geometer's Sketchpad
- Green Globs & Graphing Equations
- Secondary Math Lab Toolkit

Games/Manipulatives

Radical Math: Math Games Using Cards and Dice (Volume VII)

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will implement and analyse sampling procedures, and draw appropriate inferences from the data collected.

It is expected that students will:

- choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population
- defend or oppose inferences and generalizations about populations, based on data from samples

SUGGESTED INSTRUCTIONAL STRATEGIES

Most data collection involves the use of samples rather than populations. Understanding the uses, advantages, and disadvantages of sampling is critical to interpreting statistical information.

- Ask students to identify ways of using sampling to deliberately skew survey results (e.g., survey of probable Stanley Cup winners by polling only fans of one team, survey of favourite musical groups by polling people from only one age group). Have students identify examples of biased or invalid samples on data reported in the media. Ask them to work in groups to analyse the data and determine what the sample population might have been. Discuss as a class.
- Have students suggest real-world applications for using distributions (e.g., knowing the size distribution, how many of each shoe size a shoe store should stock).
- Invite a pollster to speak to the class about polling as a sampling technique. Discuss various types of polls (e.g., random selection, target market, phone-in polls). Ask students: Are each of these sampling techniques unbiased? If they are biased, how? (e.g., a telephone sample would not include people who do not speak English well enough to answer the questions)
- Students could design a survey, drawing their sample from the school or community at large. Have students justify their survey content and sampling method. Ask students to draw valid non-trivial conclusions from their data.
- Give students a series of case studies involving survey information and have them critique the studies and support or reject conclusions in the case studies. Alternatively, limit the case study to the survey setup and data and then have students develop their own conclusions.

SUGGESTED ASSESSMENT STRATEGIES

Projects of both long and short duration provide a basis for ongoing assessment and summative evaluation. Assessment in this area should give students a chance to demonstrate their skills and knowledge while completing projects they find meaningful.

Collect

- Have students design and conduct research projects requiring them to choose and apply sampling techniques. As students work on their projects, ask questions such as:
  - How did you choose your sampling technique?
  - Can you justify your choice?
  - Can you identify possible sources of bias or error in your sample?
  - When might someone want to bias a sample and how might they do it?
  - What makes the graphical representations you are using appropriate for your data?
  - How are your conclusions affected by your sample?
- Work with students to develop criteria they can use to assess their projects. Students use the criteria to assess their own work, then describe how their projects could be changed to correct identified problems. Criteria might include:
  - a clear description of sampling procedures
  - justification for the sampling technique
  - accurate identification of possible reasons for bias or error
  - effective graphical representations of data summaries
  - appropriate references, clearly linked to the data, about the population from which the sample was taken

Self-/ Peer Assessment

- Provide students with materials necessary to develop study cards describing terms and concepts related to data analysis. Have students quiz one another using the cards. Small groups of students can use the cards to compose tests for other groups to take. Give students basic test requirements such as length and type of items. Students should be able to answer their own questions.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Exploring Statistics with the TI-82 Graphics Calculator
- Graphing Calculator Activities for Enriching Middle School Mathematics
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Pure Mathematics 10 (Distance Learning Package)
- What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Multi-media

- The Learning Equation: Mathematics 10 Lessons 43 & 44
- Mathematics 10, Western Canadian Edition Ch. 9 (Sections 9.1 - 9.4)
- MATHPOWER 10, Western Edition Ch. 8 (Sections 8.1 - 8.4)

Software

- Secondary Math Lab Toolkit

CURRICULUM

Principles of Mathematics 11

ESTIMATED INSTRUCTIONAL TIME

Principles of Mathematics 11 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

PRINCIPLES OF MATHEMATICS 11

Organizer (Suborganizer)	% of Time
Problem Solving	**Integrated Throughout**
Number (Number Operations)	**10 - 20**
Patterns and Relations (Patterns)	**5 - 15**
Patterns and Relations (Variables and Equations)	**15 - 25**
Patterns and Relations (Relations and Functions)	**20 - 25**
Shape and Space (Measurement)	**5 - 10**
Shape and Space (3-D Objects and 2-D Shapes)	**20 - 30**

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, probability
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly explain the solution to a problem and justify the processes used to solve it
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems involving a range of mathematical disciplines can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Principles of Mathematics 11.

- In a class discussion, define *problem solving* with students, pointing out that in mathematics problem solving involves many disciplines, including algebra, geometry, trigonometry, statistics, and probability.
- Introduce new types of problems directly to students (i.e., without demonstration), and facilitate as they attempt to solve them.
- Have students work in small co-operative groups of three to five when introducing new types of problems.
- Model and reinforce student use of a variety of approaches to problem solving (e.g., algebraic and geometric).
- Emphasize that many problems may not be solved in one try and that it is sometimes necessary to revisit the problem, start over, and try again.
- Have students or groups discuss their thought processes as they attempt to solve a problem. Point out the strategies inherent in their thinking (e.g., guessing and checking, looking for a pattern, making and using a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- When students arrive at solutions to particular problems, encourage them to generalize from or extend the problem situations.

Note: See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These problems are indicated with an asterisk (*)

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe

- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question

- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work

Collect

- For selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not for particular problems.

Self-Assessment

- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set, *Evaluating Problem Solving Across Curriculum*, may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises.

These resources, while not part of the Grade Collections, have Provincially Recommended status.

Appendix B includes an annotated bibliography of these resources for ordering convenience.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve consumer problems, using arithmetic operations.

It is expected that students will:

- solve consumer problems, including:
  - wages earned in various situations
  - property taxation
  - exchange rates
  - unit prices
- reconcile financial statements including:
  - cheque books with bank statements
  - cash register tallies with daily receipts
- solve budget problems, using graphs and tables to communicate solutions
- solve investment and credit problems involving simple and compound interest

SUGGESTED INSTRUCTIONAL STRATEGIES

A working knowledge of the skills involved in money management is of advantage to all students throughout their lifetimes. An understanding of procedures and applications of mathematics can help students make rational economic decisions.

- Explain various deductions from earnings, then have students determine the effects of these deductions on gross pay. Students who have jobs could use their pay slips as examples.
- Have students use tables, calculators, and computer spreadsheets to compare the effects of various rates of compounding interest on personal investments and loans.
- Invite a guest speaker (e.g., accountant, banker, financial planner) to talk about credit and loans. Have students prepare questions that focus on the underlying mathematics involved.
- Ask students to use information collected from advertising (e.g., print, television, on-line) to compare the cost of paying cash for an item to the cost of buying on a variety of credit plans.
- Have students use spreadsheet software to:
  - record and track expenditures
  - build mortgage and other repayment schedules
- Have each student use spreadsheet software to prepare a budget, given:
  - an income (e.g., salary, wages, commissions, investments, pension)
  - fixed expenses (e.g., utilities, rent, insurance, deductions, savings, investment)
  - variable expenses (e.g., clothes, entertainment)
- Ask students questions that require interpretation of the budget (e.g., How long would it take to save enough money to make a major purchase?).

SUGGESTED ASSESSMENT STRATEGIES

Consumer problems play an important role in the daily lives of students. To assess students' thinking and the strategies they are using, note their abilities to estimate, predict, calculate (using appropriate formulae and technology), make financial decisions, and verify the reasonableness of their conclusions. As they check their solutions and justify their mathematical thinking, look for evidence of developing abilities to communicate mathematically.

Observe

- As students work on problems involving money management, observe their abilities to:
  - accurately create and modify various tables.
  - explain the advantages and disadvantages of using credit cards, installment plans, and bank loans
- While students solve consumer problems, note the extent to which they:
  - estimate and verify their results using appropriate technology
  - apply a variety of strategies
  - generalize from their solutions
  - assess the usefulness of different Suggested Instructional Strategies
  - observe soundness of reasoning for financial decisions

Question

- Have each student interview someone who works in the financial field, and write a report summarizing the research. Review each report, looking for evidence that the student is able to:
  - identify all facets of the job
  - give information about the use of technology in the industry
  - identify personal attributes and required skills for workers in the field
  - general a list of educational requirements

Collect

- Have students create budgets of income and expenses for their personal situations using spreadsheet and graphics programs.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Mathematics 11, Western Canadian Edition
  Ch. 1 (Sections 1.1 - 1.8)
- MATHPOWER 11, Western Edition
  Ch. 9 (Sections 9.1 - 9.7)

Software

- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will apply the principles of mathematical reasoning to solve problems and to justify solutions.

It is expected that students will:

- differentiate between inductive and deductive reasoning
- explain and apply connecting words, such as *and, or*, and *not*, to solve problems
- use examples and counterexamples to analyse conjectures
- distinguish between an *if–then* proposition, its converse and its contrapositive
- prove assertions in a variety of settings, using direct and indirect reasoning

SUGGESTED INSTRUCTIONAL STRATEGIES

Students need to understand the language used in logic and reasoning. This understanding assists students in drawing conclusions distinguishing between facts and propaganda. These concepts of mathematical reasoning need to be reinforced in all areas of mathematical study.

- Give students examples of deductive and inductive reasoning and have students work in small groups to decide whether examples belong in either of two columns in a table, where one column will contain deductive examples and one will contain inductive examples.
- Have students use Venn diagrams to illustrate AND, OR, NOT and combinations of these. Use examples from law where deductive proof is used.
- Have students analyse the logic used in newspaper articles or videos of television news programs that report statistical results and that draw oversimplified conclusions.
- Provide examples of debatable or controversial issues argued in a short paragraph and have students find flaws in the arguments.

SUGGESTED ASSESSMENT STRATEGIES

Students should be able to recognize valid arguments and reject invalid arguments. These skills should be assessed in all areas of mathematics.

Observe

- While students are working in pairs and reading a newspaper article or report, observe the extent to which they are able to:
  - determine the valid arguments or facts
  - determine the point or conclusion of the argument
  - distinguish between points that are logically reached and conclusions that are sweeping statements or simply do not follow logically

Question

- Ask students to:
  - list examples of if-then propositions, converses and contrapositives
  - design an *if-then* game where, for example, a person's turn ends when a connecting statement cannot be made or when a converse statement is made
  - explain how they would prove that the sum of the angles in a polygon of n sides is $(n-2) \times 180^\circ$
  - prove that objects with the same perimeter may have different areas

Collect

- Ask students to collect newspaper articles that present good arguments, some that jump to conclusions, and some that contradict themselves.
- Ask students to collect cartoon strips that show causal relationships and give examples of *if-then* situations.

Self-Assessment

- Provide students with arguments that are either flawed or not flawed. Have a student try to convince the class of the conclusion and see if the group is correct in accepting or rejecting the arguments. This may lead to a class discussion (e.g., on how people can be swayed into believing others such as sales pitches).
- Have students list necessary criteria for a good argument.

RECOMMENDED LEARNING RESOURCES

Print Materials

- An Introduction to the TI-82 Graphing Calculator
- Mathematics 11, Western Canadian Edition
  Ch. 6 (Sections 6.1 - 6.7)
  Ch. 7 (Section 7.5)
  Ch. 9 (Sections 9.4, 9.5)
- MATHPOWER 11, Western Edition
  Ch. 6 (Sections 6.1 - 6.6)
- Modeling Motion: High School Math Activities with the CBR

Software

- Secondary Math Lab Toolkit

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will represent and analyse situations that involve expressions, equations and inequalities.

It is expected that students will:

- graph linear inequalities, in two variables
- solve systems of linear equations, in two variables:
  - algebraically (elimination and substitution)
  - graphically
- solve systems of linear equations, in three variables:
  - algebraically
  - with technology
- solve non-linear equations, using a graphing tool
- solve non-linear equations:
  - by factoring
  - graphically
- use the Remainder Theorem to evaluate polynomial expressions, the Rational Zeros Theorem, and the Factor Theorem to determine factors of polynomials
- determine the solution to a system of nonlinear equations, using technology as appropriate

SUGGESTED INSTRUCTIONAL STRATEGIES

Polynomial functions can be used to model many complex, real-world situations. Students can use these functions and the appropriate technology to solve problems that would otherwise be too difficult.

- Have students graph two equations on the same coordinate plane, identify points of intersection, and check whether these satisfy each equation. Discuss the limitations of graphing by referring to a system for which the solution values cannot be accurately determined by graphing.
- Discuss solving a system of two equations in two variables by reducing to one equation in one variable (start with simple examples).

Substitution Method	*Elimination Method*
$3x + y = 6$	$2x + y = 7$
$y = x + 2$	$2x - y = 3$

- As enrichment, discuss possible ways of solving systems of four equations in four variables, up to n equations in variables. Information Technology students could be encouraged to create programs that do this.
- Have students graph equation families such as:
  $y = 2x$, $y = 2x + 3$, and $y = 2x + 4$
  $x + 2y = 4$, $2x + 4y = 8$, and $3x + 6y = 12$
  Have students identify the patterns that group these functions into families. Ask them what relationships between coefficients and constants would result in systems having:
  - no solutions (parallel lines)
  - infinite solutions (concurrent lines)
  - a unique solution (intersecting lines)
- Have students use graphing calculators or computer software to introduce concepts related to polynomial functions, such as:
  - the effect on the graph when the sign of the leading term is changed
  - the effect on the graph when multiple factors are introduced
  - the relationship between factors of a polynomial and zeros of the associated polynomial function
  - the relationship between the degree of a polynomial and its general shape
  - the relationship of the constant term to the y-intercept of the graph
- Have students create functions that contain specified characteristics (e.g., tangent, multiple zeros). Use these examples to illustrate the connections between the function and its graph.

SUGGESTED ASSESSMENT STRATEGIES

In assessing student performance involving relations and functions, it is important to observe students' abilities to recognize patterns and functional relationships. In working with polynomial functions and equations, students demonstrate their reasoning through the application of algebraic skills and graphical skills and by making interpretations, generalizations, and representations.

Observe

- As students solve systems of linear equations, watch for and provide feedback on the extent to which they are able to:
  - use a variety of methods to solve problems
  - determine the domain and range for given relations
  - solve applied problems
- Note students' efforts to create functions or graphs that adhere to prescribed conditions, using paper and pencil or graphing technology. For example, have students write a polynomial function with a double zero at (-2,0), single zero at (30,0), and passing through the point (1,5)). Observe to what extent students give logical reasons for the choices they made.

Self-Assessment

- Pose questions such as the following to help students review what they learned:
  - What are the most important ideas you learned about solving linear equations?
  - How are these ideas connected to what you already knew and to the world around you?
- To assess students' command of nonlinear functions and equations, have them list common misconceptions or difficulties they encounter in solving the equations, or while graphing the functions.

Collect

- Discuss with students the relative merits of the various methods of analyzing and displaying solutions for nonlinear equations. Have them summarize in writing their understanding of the merits of each. Work with them to develop a set of criteria to assess the summaries.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Explore Quadratic Functions with the TI-83 or TI-82
- Exploring Functions with the TI-82 Graphics Calculator
- An Introduction to the TI-82 Graphing Calculator
- Mathematics 11, Western Canadian Edition
  Ch. 4 (Sections 4.1 - 4.8)
  Ch. 5 (Sections 5.1 - 5.8)
- MATHPOWER 11, Western Edition
  Ch. 1 (Sections 1.2 - 1.6)
  Ch. 2 (Sections 2.1, 2.3)
  Ch. 4 (Sections 4.1, 4.8 - 4.10)
- Modeling Motion: High School Math Activities with the CBR

Software

- Green Globs & Graphing Equations
- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- represent and analyse quadratic, polynomal and rational functions, using technology as appropriate
- examine the nature of relations with an emphasis on functions

It is expected that students will:

- determine the following characteristics of the graph of a quadratic function:
  - vertex
  - domain and range
  - axis of symmetry
  - intercepts
- perform operations on functions and compositions of functions
- determine the inverse of a function
- connect algebraic and graphical transformations of quadratic functions, using completing the square as required
- model real-world situations, using quadratic functions
- solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:
  - factoring
  - the quadratic formula
  - graphing
- determine the character of the real and non-real roots of a quadratic equation, using:
  - the discriminant in the quadratic formula
  - graphing
- describe, graph, and analyse polynomial and rational functions, using technology
- formulate and apply strategies to solve absolute value equations, radical equations, rational equations, and inequalities

SUGGESTED INSTRUCTIONAL STRATEGIES

Describing functional relationships is essential to interpreting and predicting the behaviour of the world around us. Students will explore how to translate between algebraic and graphical representations and use these skills to make inferences and solve problems.

- Demonstrate the concept of a function as a machine (e.g., $g(x) = x^2 + 3$, the "g" function always takes what you give it, squares it, and adds three to it).
- To demonstrate substitution of functions, always show the same function in the same color, especially when substituting into another.
- To develop the concept of the inverse of a function, provide five or six functions and have students:
  - make tables of values for them and graph them all in blue ink
  - switch x and y in the original equations and call the new equations the "inverse" equations
  - graph the new "inverse" equations on the same grid as the originals in red ink, pointing out that it may be easier to first solve the equations for y and then to make a table of values to graph the inverses
  - analyse the relationship between the original blue function and the new red function on each grid, describe the transformation,and draw the "mirror" line as a black dashed line
  - describe the "mirror" line as an algebraic equation
- Have students:
  - use programmable calculators or calculators with built-in programs to solve quadratic equations
  - use graphing calculators or computer graphing programs to illustrate the solutions of quadratic equations
- Suggest that students create a question comparing ticket price to revenue. Display the graph of the function on the graphing calculator and analyse key values (e.g., maximum value, intercepts, curve shape). As algebraic techniques are introduced, compare the algebraic results with the graphing calculator results.
- Obtain non-linear data through experiments or calculator based laboratory (e.g., heights of falling objects) and solve them for the roots, vertices, and time at a given height.

SUGGESTED ASSESSMENT STRATEGIES

Students should demonstrate the ability to describe functional relationships. Assessment focusses on students' abilities to move easily between graphical and algebraic representations.

Observe

- As students work with non-linear equations, circulate, asking questions and observing how effectively they are able to:
  - solve non-linear equations using a variety of methods
  - understand the concept of extraneous root
  - test for extraneous roots
  - determine if solutions are reasonable within the context of the problem

Question

- Probe students' problem-solving strategies as they work on quadratic equations by asking questions such as:
  - How did you decide which method to use to solve the equation?
  - How did you identify the proper form of the equation for each method?

Collect

- Ask students to find examples of real-world situations that involve quadratic relations. Have them present their situations, using some type of electronic technology. Observe the extent to which students:
  - make use of quadratic relations
  - use real-world situations
  - identify the correct quadratic to fit each situation
  - use research to verify their results

Self- and Peer Assessment

- Have students summarize and assess their results from a set of review exercises, noting what they do well and where they need to improve. Ask them to record how they plan to improve in the identified areas.

  Ask students to reflect on the various methods they use to solve quadratic equations:
  - Which methods do they clearly understand?
  - Which forms of the equations do they have trouble dealing with?
  - To what extent can they generalize from what they have learned by restating results in more widely applicable terms?

RECOMMENDED LEARNING RESOURCES

Print Materials

- Explore Quadratic Functions with the TI-83 or TI-82
- Exploring Functions with the TI-82 Graphics Calculator
- An Introduction to the TI-82 Graphing Calculator
- Mathematics 11, Western Canadian Edition
  Ch. 2 (Sections 2.1 - 2.7)
  Ch. 3 (Sections 3.1 - 3.9)
- MATHPOWER 11, Western Edition
  Ch. 3 (Sections 3.1, 3.3, 3.5)
  Ch. 4 (Sections 4.1, 4.2, 4.4, 4.6, 4.7)
  Ch. 5 (Sections 5.1 - 5.7)
- Modeling Motion: High School Math Activities with the CBR
- Using the TI-81 Graphics Calculator to Explore Functions
- What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Software

- GrafEq (Macintosh & Windows Version 2.09)
- Green Globs & Graphing Equations
- Secondary Math Lab Toolkit
- ZAP-A-GRAPH

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- solve problems involving triangles, including those found in 3-D and 2-D applications.
- solve coordinate geometry problems involving lines and line segments, and justify the solutions.

It is expected that students will:

- solve problems involving ambiguous case triangles in 3-D and 2-D
- solve problems involving distances between points and lines
- verify and prove assertions in plane geometry, using coordinate geometry

SUGGESTED INSTRUCTIONAL STRATEGIES

Students are surrounded by geometric forms in nature, architecture, technology, science, and visual arts. In their study of geometry, students develop skills in deductive reasoning, problem solving, and the visual representation of ideas.

- Have students use trigonometry to calculate heights or lengths of items around the school (e.g., lamppost, flagpole, totem pole).
- Include numerous non-traditional learning opportunities such as;
  - participating in field studies (e.g., a construction site, airport)
  - interviewing guest speakers (e.g., engineers, surveyors)
  - investigating the operation of tools such as the commercial clinometer, or compass
  - researching projects involving real-life applications such as navigation, surveying, and astronomy
- Provide students with several examples of ambiguous case triangles and ask them to identify why the ambiguous case triangle has two possibilities. Have students describe the one situation where there is no ambiguity.
- Encourage students to develop an appreciation of the use of coordinate geometry in many real-world applications.
- Demonstrate the definition of a circle by using a student volunteer as the center and another as the locus point (x, y). As the student moves around the classroom, point out that the (x, y) values are constantly changing. Show the same effect using the trace feature on the graphing calculator.
- Encourage students to draw detailed diagrams whenever attempting geometry problems.
- Demonstrate concepts by using paper folding or other physical examples.
- Whenever possible, model inductive reasoning, and have students discover rules and theorems through experiments.

SUGGESTED ASSESSMENT STRATEGIES

Students apply their understanding of terms and procedures when solving problems using geometric properties of angles, lines, triangles, polygons, and the distance formula. Assessment in this area can focus on the importance of problem-solving skills.

Observe
- Observe the extent to which students communicate with each other using appropriate mathematical language and descriptions.
- Have students complete problems involving non-right triangles. Observe:
  - their ability to recognize the ambiguous case
  - how accurately students use mathematical language and symbols
  - the extent to which they demonstrate appropriate knowledge and understanding of concepts and procedures
  - the extent to which they persevere with assigned task
  - how accurately they complete diagrams and calculations

Question
- Ask groups or pairs of students to list as many applications of geometry as they can.

Collect
- Check students' study sheets for accuracy, clarity, and the effectiveness of their illustrations.

Self-Assessment
- Have students make up a general marking rubric for geometry questions. Ask students to select a number of criteria that they need to improve.
- Give students a series of word problems and have them work in pairs. Ask pairs to compare their work with other pairs to verify their methods and solutions. Have them make individual action plans that include how they will overcome any weaknesses or how they will challenge themselves to learn more about the topic.

Presentation
- Have each students present an example of a problem from another discipline that requires a trigonometric solution. Examples could include physics (e.g., light refraction—Snell's law), chemistry (e.g., bond angles), geography (e.g., map projections). Prior to the presentations, develop assessment criteria with students.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Mathematics 11, Western Canadian Edition
  Ch. 9 (Sections 9.1 - 9.3)
- MATHPOWER 11, Western Edition
  Ch. 8 (Section 8.7)

Software

- The Geometer's Sketchpad

CD-ROM

- Geometry Blaster

PRESCRIBED LEARNING OUTCOMES

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

It is expected that students will:

- use technology with dynamic geometry software to confirm and apply the following properties:
  - the perpendicular from the center of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle, from any external point, are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is 180(n-2)
- prove the following general properties, using established concepts and theorems:
  - the perpendicular bisector of a chord contains the center of the circle
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the center of the circle is in the interior of the inscribed angle)
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle from any external point are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is 180(n-2)
- solve problems, using a variety of circle properties, and justify the solution strategy used

SUGGESTED INSTRUCTIONAL STRATEGIES

Students will investigate, hypothesize, and verify properties of circle geometry and apply these properties to solve problems. The deductive reasoning used to solve problems involving circle geometry can be employed to make decisions in a variety of fields e.g., landscape architecture, meteorology, and civil engineering.

Note:
Although the prescribed learning outcomes do not require students to use formal two-column proof, students are nevertheless expected to logically justify conjectures regarding geometric properties and explain their solutions to the problems.

- Have groups of students experiment (e.g., investigate, hypothesize, confirm, discuss, use inductive reasoning) to identify the properties of a circle. Encourage students to verify their results through measurements, calculations, and reasoning. Possible experiments include:
  - constructing a perpendicular bisector with compass and straightedge
  - drawing circles of varying sizes, and using chords of different length on these circles to construct the perpendicular bisectors

  Have students discuss the importance of accuracy while presenting their conclusions to the class.
- As enrichment, have students research and report on the life and work of a famous mathematician (historical or contemporary). Discuss with students the concept of mathematics as an evolving discipline and not as a static set of rules. For example Euclid's Elements is an in-depth set of rules that were deduced inductively.
- Have students write a logical-sequential verbal explanation of a solution to a problem or a proof.

SUGGESTED ASSESSMENT STRATEGIES

To enable them to work effectively with properties of circles, students need a solid understanding of the properties of parallel lines, similar and congruent figures, polygons, and angle relationships. Students demonstrate their learning when they solve geometric problems by applying circle properties and their converse statements.

Observe

- Before advancing in this topic, assess students' background knowledge of the properties of parallel lines, similar and congruent figures, polygons, and angle relationships previously learned. Look for evidence of students' recognition of properties and their abilities to solve problems using:
  - parallel lines
  - congruent triangles
  - basic geometric constructions
  - properties of polygons
- Ask students to perform experiments with circle properties, observe how effectively they can:
  - use a protractor and compass correctly and accurately
  - follow directions to draw the figures and measure the angles
  - explain orally the process of experimentation to derive conclusions
  - communicate their conclusions clearly in writing; explain the basis of their conclusions
  - identify the appropriate circle properties used to find the measurements of sides and angles

Self-/Peer Assessment

- Work with students to develop a table of specifications for a test on this unit of work. Identify for them the content categories along the vertical axis and the intellectual levels along the horizontal axis. After the weighing is identified for each cell, have students work in groups to develop questions for each cell.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Mathematics 11, Western Canadian Edition
  Ch. 7 (Sections 7.1 - 7.4)
  Ch. 8 (Sections 8.1 - 8.6)
- MATHPOWER 11, Western Edition
  Ch. 7 (Sections 7.2 - 7.7)
  Ch. 8 (Sections 8.1 - 8.5)

Software

- The Geometer's Sketchpad
- Green Globs & Graphing Equations
- Secondary Math Lab Toolkit

CD-ROM

- Geometry Blaster

Curriculum

Principles of Mathematics 12

ESTIMATED INSTRUCTIONAL TIME

Principles of Mathematics 12 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

PRINCIPLES OF MATHEMATICS 12

Organizer (Suborganizer)	% of Time
Problem Solving	**Integrated Throughout**
Patterns and Relations (Patterns)	**5 - 15**
Patterns and Relations (Variables and Equations)	**10 - 20**
Patterns and Relations (Relations and Functions)	**15 - 25**
Shape and Space (3-D Objects and 2-D Shapes)	**10 - 20**
Shape and Space (Transformations)	**10 - 20**
Statistics and Probability (Chance and Uncertainty)	**20 - 30**

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students willuse a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, probability
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly explain the solution to a problem and justify the processes used to solve it
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems involving a range of mathematical disciplines can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Multi-strand and interdisciplinary problems should be included throughout Principles of Mathematics 12.

- In a class discussion, define *problem solving* with students, pointing out that in mathematics problem solving involves many disciplines, including algebra, geometry, trigonometry, statistics, and probability.
- Introduce new types of problems directly to students (i.e., without demonstration), and facilitate as they attempt to solve them.
- Have students work in small co-operative groups of three to five when introducing new types of problems.
- Model and reinforce student use of a variety of approaches to problem solving (e.g., algebraic and geometric).
- Emphasize that many problems may not be solved in one try and that it is sometimes necessary to revisit the problem, start over, and try again.
- Have students or groups discuss their thought processes as they attempt to solve a problem. Point out the strategies inherent in their thinking (e.g., guessing and checking, looking for a pattern, making and using a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- When students arrive at solutions to particular problems, encourage them to generalize from or extend the problem situations.

Note: See Appendix G for examples of multi-strand and interdisciplinary problems that most students should be able to solve. These problems are indicated with an asterisk (*)

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe

- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question

- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in theirown words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work

Collect

- For selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not work for particular problems.

Self-Assessment

- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set, *Evaluating Problem Solving Across Curriculum,* may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises.

These resources, while not part of the Grade Collections, have Provincially Recommended status.

Appendix B includes an annotated bibliography of these resources for ordering convenience.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will generate and analyse exponential patterns.

It is expected that students will:

- derive and apply expressions to represent general terms and sums for geometric growth and to solve problems
- connect geometric sequences to exponential functions over the natural numbers
- estimate values of expressions for infinite geometric processes

SUGGESTED INSTRUCTIONAL STRATEGIES

Sequences and series can be used to model the patterns observed in phenomena such as a launched rocket, a bouncing ball, and plant or animal growth. Limit theory, a fundamental building block of calculus, extends from the concepts of sequences and series.

- Give students numerical and non-numerical patterns and ask them to define the rule(s) for each pattern. Have them use the rules to make predictions and classify the patterns (e.g., arithmetic, geometric, neither).
- Have students find examples and describe the type of pattern of geometric sequences and series occurring in real-world tools and scales (e.g., the f-stops on a camera lens, rule of five).
- Have students research and report on pattern-related topics such as;
  - recursive definitions of a sequence (e.g., computer programming)
  - development and significance of mathematical symbolism (e.g., summation notation)
  - the relationship between infinite geometric series and limits (e.g., Zeno's paradox, Koch's snowflake curve, fractal curves, the notion of infinity)
- Encourage students to use a variety of media and to refer to a range of real-life examples to show patterns (e.g., in history, fine arts, science, economics).
- Have students use concepts of sequence and series to solve compounding interest problems (e.g., If $1000.00 is deposited each year into an account that earns 8% compounded annually, how much money will have accumulated after 25 years?). Ask students to:
  - generate a sequence or series modeling each situation
  - use the appropriate format to answer the questions
  - discuss the reasonableness of the answers

SUGGESTED ASSESSMENT STRATEGIES

Students provide a variety of evidence of their ability to develop sequences and series and to apply these when they solve problems. Using a variety of assessment procedures will give the most complete picture of students' abilities to solve such problems.

Observe

- While students are working on problems involving patterns, look for evidence that they can:
  - recognize, classify, and make predictions and generalizations from functional relationships and patterns
  - recognize patterns in a sequence and generate successive terms of that sequence
  - classify different types of sequences and series

Question

- To discover how well students can recognize and explain a fallacy, give them a problem such as:

$$S = 1 + 2 + 4 + 8 + \ldots$$
$$S = 1 + 2(1 + 2 + 4 + \ldots)$$
$$\text{therefore } S = 1 + 2S$$
$$\text{and } S - 2S = 1$$
$$\text{so } S = -1$$

  In their analyses, look for evidence that they understand *infinite geometric series* when discussing the fallacy. For example, do they:
  - recognize that something is wrong
  - identify what it is that is wrong and explain why
- Have students brainstorm real world examples of geometric sequences and then encourage them to see that they are examples of exponential functions.

Collect

- Ask student to research fractal-like objects that are naturally occurring, such as broccoli or lung tissue. Ask them to explain, using geometric series, how lung tissue might benefit from a maximum surface area.

Collect/Present

- Have students state five geometric and five non-geometric sequences and defend why each is placed in a particular group. Alternatively, give students a list of sequences, and have them sort them and defend their choices.

RECOMMENDED LEARNING RESOURCES

Patterns and Relations - Patterns

Print Materials

- Exploring Advanced Algebra with the TI-83
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR

Multi-media

- Mathematics 12, Western Canadian Edition Ch. 2 (Sections 2.7 - 2.9)
- MATHPOWER 12, Western Edition Ch. 6 (Sections 6.3, 6.5, 6.6)

Software

- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve exponential, logarithmic and trigonometric equations and identities.

It is expected that students will:

- solve exponential equations having bases that are powers of one another
- solve and verify exponential and logarithmic equations and identities
- distinguish between degree and radian measure, and solve problems using both
- determine the exact and the approximate values of trigonometric ratios for any multiples of 0˚, 30˚, 45˚, 60˚ and 90˚ and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$
- solve first and second degree trigonometric equations over a domain of length 2π:
  - algebraically
  - graphically
- determine the general solutions to trigonometric equations where the domain is the set of real numbers
- analyse trigonometric identities:
  - graphically
  - algebraically for general cases
- use sum, difference,and double angle identities for sine and cosine to verify and simplify trigonometric expressions

SUGGESTED INSTRUCTIONAL STRATEGIES

Many real-world problems (e.g., in navigation, engineering, surveying, chemistry) require the application of exponential, logarithmic, or trigonometric equations. Trigonometry also links geometry and algebra (e.g., through its applications in matrix representations of rotations and direction angles of vectors).

- Provide students with a list of logarithmic equations. Have students categorize the equations in terms of the equation-solving technique required (e.g., $2\log 4x - \log 4(x+3) = 1$ would be solved by the power law and equating arguments).
- Have students generate multiple-choice questions with emphasis on creating good distracters. Use these questions created by the class as a review.
- Present examples illustrating the need for radian measure when modeling real-world situations (e.g., amount of air in lungs as a function of time, tide problems).
- As students simplify trigonometric expressions, encourage them to develop and state useful strategies such as:
  - express in terms of sine and cosine only
  - simplify fractions using conjugates, factoring, and common denominators
  - make substitutions using equivalent identities
- Relate solving trigonometric equations to solving algebraic equations with similar forms:

  Algebraic Equations

  $$2x^2 - 3x - 2 = 0$$
  $$(2x+1)(x-2) = 0$$
  $$x = \frac{1}{2} \quad or\ x = 2$$

  Trigonometric Equations

  Solve for θ to the nearest degree $(0° \le \theta < 360°)$.

  $$2\cos^2\theta - 3\cos\theta - 2 = 0$$
  $$(2\cos\theta + 1)(\cos\theta - 2) = 0$$
  $$\cos\theta = -\frac{1}{2} \ \ or\ \ \cos\theta = 2 \ \text{ no solution}$$
  $$\theta = 120^\circ \ or\ \theta = 240^\circ$$
- Have students analyse and discuss incorrect solutions to trigonometric equations.

SUGGESTED ASSESSMENT STRATEGIES

The ability to solve problems involving exponential, logarithmic, and trigonometric functions is important in fields such as engineering. Knowing how well students can manipulate equations and identities of these types is important. Assessment strategies should focus on the demonstration of algebraic skills and, to a lesser degree, the translation of word problems into algebraic equations.

Observe

- While students are working on trigonometric and logarithmic functions, observe that:
  - when solving bases other than 10, students convert them to base 10 or are comfortable with the base they are in
  - where necessary, students can convert all bases to a single appropriate base
  - students memorize tables of special angles or they can derive them quickly as needed
  - when proving trigonometric identities, students should apply appropriate techniques such as converting everything to sine and cosine
- Provide samples of student work on proofs that are not complete. Observe the extent to which students are able to:
  - determine the errors in the formal proof
  - correct the errors in the formal proof
  - assign a mark to the proof based on a criterion-based rubric

Question

- Can students differentiate between specific solutions and general solutions to trigonometric equations?

Self-/PeerAsssessment

- Design a puzzle sheet which requires the solution to many trigonometric or logarithmic equations to find the answer. Encourage students to have a peer solve their puzzle and check their work for accuracy.
- Ask students to select three proofs from their homework assignment and have partners mark each others' work using a criterion-based rubric.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Exploring Advanced Algebra with the TI-83
- Exploring Functions with the TI-82 Graphics Calculator
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR

Multi-media

- Mathematics 12, Western Canadian Edition
  Ch. 2 (Sections 2.1, 2.11, 2.12)
  Ch. 3 (Sections 3.2, 3.5)
  Ch. 5 (Section 5.1 - 5.6)
- MATHPOWER 12, Western Edition
  Ch. 2 (Sections 2.4, 2.7)
  Ch. 4 (Sections 4.1, 4.2)
  Ch. 5 (Sections 5.1 - 5.5)

Software

- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will represent and analyse exponential and logarithmic functions, using technology as appropriate.

It is expected that students will:

- model, graph, and apply exponential functions to solve problems
- change functions from exponential form to logarithmic form and vice versa
- model, graph, and apply logarithmic functions to solve problems
- explain the relationship between the laws of logarithms and the laws of exponents
- describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position
- draw (using technology), sketch, and analyse the graphs of sine, cosine, and tangent functions, for:
  - amplitude, if defined
  - period
  - domain and range
  - asymptotes, if any
  - behavior under transformations
- use trigonometric functions to model and solve problems

SUGGESTED INSTRUCTIONAL STRATEGIES

The inverse relationship between exponential and logarithmic functions makes possible the solution of a number of equations not readily solved algebraically (e.g., investment analysis, population growth, earthquake magnitudes, radioactive decay).

- Give students an exponential function and its inverse (logarithmic). Have them work in groups, using graphing calculators or computer software to identify the relationship between the two.
- Demonstrate for students the impossibility of graphing an exponential function with a negative base, using anything but integer values of x. Refer to the resulting difficulties to emphasize the need for domain and range restrictions when dealing with exponential and logarithmic functions. As a follow up, discuss base e and natural logarithms.
- Use the graphing calculator to discuss the effects of changing A, B, C, and D on the graph of $y = \mathrm{A}\sin\mathrm{B}(\theta - \mathrm{C}) + \mathrm{D}$
- Provide students with real sinusoidal data. Ask them to graph the data, estimate the amplitude and suggest possible equations to represent the data. The data can then be entered into the graphing calculator and the sinusoidal regression function can be applied. Students can compare the calculator results to their own predicted equation.
- Have students display the parallels of the log laws and exponent laws on an 81/2"x11" poster
- Develop a model of the wrapping function (e.g., sine, cosine) using a circle of cardboard and an acetate overlay to show how x, y and x/y change as the angle changes. Alternatively use geometry software to model the relationships of the inscribed right angle and the change in the central angle.
- Use a graphing calculator or graphing software to match a given graph to a trigonometric equation.
- Given a solution, have students write a problem that matches it.
- Play trigonometric graph bingo Have students choose a number of equations from a list to make up their card, then graph equations on overhead until someone gets 5 in a row.

SUGGESTED ASSESSMENT STRATEGIES

Trigonometric, exponential, and logarithmic functions enable students to solve more complex problems in areas such as science, engineering, and finance. Students should be able to demonstrate their knowledge of the relationship between logarithms and exponential functions, both in theory and in applications, in a variety of problem situations. Assessment of students' understanding of trigonometric concepts includes the observation of the processes they use when they relate the physical world to mathematical representations.

Observe

- Ask students to outline how they would teach a classmate:
  - the inverse relationship between exponential and logarithmic functions
  - the restrictions associated with exponential and logarithmic functions

  Note the extent to which the outlines:
  - include general steps to follow
  - use mathematical terms correctly
  - provide clear examples
  - describe common errors and how they can be avoided
- When reviewing students' work, note the extent to which they can:
  - generate several different equations to describe the graph of a given cosine or sine function
  - graph a particular sine or cosine function using pencil and paper
  - check their solutions using graphing technology

Collect

- Assign a series of problems that require students to apply their knowledge of the relationship between logarithms and exponential functions. Check their work for evidence that they:
  - clearly understood the requirements of the problem
  - used efficient strategies and procedures to solve the problem
  - recognized when a strategy or procedure was not appropriate
  - verified that their solutions were accurate and reasonable

RECOMMENDED LEARNING RESOURCES

Print Materials

- Exploring Functions with the TI-82 Graphics Calculator
- An Introduction to the TI-82 Graphing Calculator
- Modeling Motion: High School Math Activities with the CBR
- Using the TI-81 Graphics Calculator to Explore Functions

Multi-media

- Mathematics 12, Western Canadian Edition
  Ch. 2 (Sections 2.1 - 2.6, 2.10)
  Ch. 3 (Sections 3.1, 3.3 - 3.9)
  Ch. 4 (Sections 4.1, 4.4 - 4.6)
  Ch. 5 (Section 5.6)
- MATHPOWER 12, Western Edition
  Ch. 2 (Sections 2.2, 2.5, 2.6, 2.8)
  Ch. 4 (Sections 4.2 - 4.6)

Software

- GrafEq (Macintosh & Windows Version 2.09)
- Secondary Math Lab Toolkit
- ZAP-A-GRAPH

Prescribed Learning Outcomes

It is expected that students will classify conic sections, using their shapes and equations.

It is expected that students will:

- classify conic sections according to shape
- classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only)
- convert a given equation of a conic section from general to standard form and vice versa

Suggested Instructional Strategies

The study of quadratic relations helps students connect geometry and algebra. The theory of conic sections finds application in fields ranging from construction and telecommunications to aerospace and astronomy.

- Have students define quadratic relations in terms of their component functions so that they can use their graphing calculators. For example, a circle of radius 5 can be entered as:
  $y_1 = -\sqrt{25 - x^2}$
  $y_2 = \sqrt{25 - x^2}$ or $y_2 = -y_1$
  This allows students to explore domain, range, and limits as they relate to asymptotes.
- Ask students to collect examples of how quadratic relations are applied. Examples could include:
  - satellite dishes
  - jet engines
  - domed stadiums
  - car headlights
  - parabolic microphones
- Demonstrate, using a “take-apart” model, that conic sections result from the intersection of a plane and a cone.
- Discuss the advantages of writing equations in general or in standard form.

SUGGESTED ASSESSMENT STRATEGIES

As students explore problems involving quadratic functions and relations, they develop an understanding of how algebra and analytic geometry are applied in a real-world context. In assessing students' abilities to use variables, graphing techniques, and algebraic representations, the focus is on the presentations (in words, symbols, and graphs) and explanations they use to illustrate their understanding of the concepts.

Observe
- To check students' understanding of quadratic functions and relations, circulate through the classroom and note:
  - the extent to which students can analyse a particular quadratic relation given a graphing analysis outline
  - whether students can recognize the declining characteristics (locus) and properties of each type of quadratic relation, and can relate the type of conic to specific equations
  - whether students can explain the effect of an x^2 term in an equation — how does it affect the possible values of x (domain)? What about y^2?

Question
- Give students data or a list of measurements that describe various relationships or structures found in the real world. Have them develop an appropriate equation to describe each of these relationships. Ask students to predict other values, and answer associated questions based on their equations.

Collect
- Place a number of conics in graphic form or standard form and general form around the classroom. Have students sort them according to type and either
  - defend their choices
  - develop/present the criteria used in making their choices

Present
- Have students use a poster to summarize the characteristics of each conic as a subset of the general conic.

RECOMMENDED LEARNING RESOURCES

Multi-media

- Mathematics 12, Western Canadian Edition
  Ch. 9 (Sections 9.1 - 9.3, 9.5, 9.6)
- MATHPOWER 12, Western Edition
  Ch. 3 (Sections 3.1, 3.3 - 3.6)

Software

- Secondary Math Lab Toolkit

PRESCRIBED LEARNING OUTCOMES

It is expected that students will perform, analyse and create transformations of functions and relations that are described by equations or graphs

It is expected that students will:

- describe how various translations of functions affect graphs and their related equations:
  - $y = f(x-h)$
  - $y-k = f(x)$
- describe how various stretches of functions (compressions and expansions) affect graphs and their related equations:
  - $y = af(x)$
  - $y = f(kx)$
- describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations:
  - $y = f(-x)$
  - $y = -f(x)$
  - $y = f^{-1}(x)$
- using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$
- using the graph and/or the equation of $f(x)$, describe and sketch $|f(x)|$
- describe and perform single transformations and combinations of transformations on functions and relations

SUGGESTED INSTRUCTIONAL STRATEGIES

The transformation of functions helps to develop students' visualization skills and spatial sense. Practice in this area will help students answer questions about "how much" and "where" in other areas of mathematics. It will also help in the understanding of functions as a precursor to calculus.

- As transformations of functions are applicable for all functions studied in this course, review these with students whenever a new function is introduced.
- Students can use appropriate technology (e.g., graphing calculator, computer software) to investigate transformations. On the same coordinate plane, they could graph:

  $x^2 - y^2 = 4$

  $(x = 3)^2 - (y+2)^2 = 4$

  $\frac{x^2}{16} - 4y^2 = 4$

  Have them sketch each transformation on grid paper using the "trace" or "table" feature for accuracy. The first transformation (e.g., compression/expansion) might always be in red, the second reflection step always blue and the final translated function always in green, with the original basic function sketched accurately in pencil. Have students analyse the changes in the original graph as the equation is modified, and repeat this process for other transformations.
- Have students use the parabola or circle to play "function calisthenics". Ask students to model changes to functions using their body position in relation to others.

SUGGESTED ASSESSMENT STRATEGIES

Transforming functions requires both an understanding of algorithms and a developed spatial sense. Assessment strategies should focus on both abstract and concrete transformation skills.

Question

- To check students' understanding of transformations, have them work in reverse. Provide the graph of a transformed relation, and ask them to write or identify its equation. Look for evidence that the student can match a basic function equation to the graph and adjust the equation based on how the graph is transformed from the basic graph (e.g., 2 right, 3 down).

Collect

- Have students compile portfolios containing examples of their work modeling each basic type of relation studied. Ask them to give examples and explain how various transformations can be applied to each type. Assess portfolios with students for completeness, variety, organization, accuracy, presentation, and for specific criteria sets.
- Have students graph the transformations of the basic equations manually. Look for their abilities to draw the graphs with paper and pencil, without the use of a graphing calculator.

Self-Assessment

- Based on a previously developed, student-generated checklist of graphing criteria, students could summarize analyses of their own work by listing what they do well and where they need to improve.

RECOMMENDED LEARNING RESOURCES

Multi-media

- Mathematics 12, Western Canadian Edition
  Ch. 1 (Sections 1.1 - 1.6)
  Ch. 4 (Section 4.7)
  Ch. 9 (Sections 9.4 - 9.5)
- MATHPOWER 12, Western Edition
  Ch. 1 (Sections 1.1, 1.3, 1.4, 1.6)

Software

- Secondary Math Lab Toolkit
- ZAP-A-GRAPH

Games/Manipulatives

- String Invision Conics Model

PRESCRIBED LEARNING OUTCOMES

It is expected that students will:

- use normal and binomial probability distributions to solve problems involving uncertainty.
- solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.
- model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

It is expected that students will:

- find the standard deviation of a data set or a probability distribution, using technology
- use *z*-scores and the normal distribution to solve problems
- use the normal approximation to the binomial distribution to solve problems involving probability calculations for large samples where npq>10
- solve pathway problems, interpreting and applying any constraints
- use the fundamental counting principle to determine the number of different ways to perform multistep operations
- determine the number of permutations of n different objects taken r at a time, and use this to solve problems
- determine the number of combinations of n different objects taken r at a time, and use this to solve problems
- solve problems, using the binomial theorem where N belongs to the set of natural numbers
- construct a sample space for two or three events
- classify events as independent or dependent
- solve problems, using the probabilities of mutually exclusive and complementary events
- determine the conditional probability of two events
- solve probability problems involving permutations, and combinations and conditional probability

SUGGESTED INSTRUCTIONAL STRATEGIES

Decision-making situations that involve probability and uncertainty occur frequently in daily life. Students need opportunities to learn how to use the normal probability distribution as a tool when interpreting and solving such problems.

- In a large-group discussion, point out that in problems involving probability, words often have a meaning different from their ordinary English usage. Help students distinguish between mathematical usage and common usage (e.g., validity, percentage).
- Provide students with examples that do not fit the normal curve and ask them to determine why using the *z*-score is inappropriate, emphasizing that the *z*-score will not solve all probability problems. Some examples include:
  - a crown-and-anchor game in which every number is equally likely (i.e., no cluster about the mean)
  - the median age of teachers is 45 and their mean is 50 (i.e., not symmetric about the mean)
- Help students develop a list of important items to consider when calculating the probability of an event. The list could include:
  - trying to organize the events of the sample space using tables or tree diagrams when possible
  - recalling the difference between AND and OR and how it relates to union and intersection
  - determining if each part of the problem is a permutation or combination
  - trying to solve a simpler version of the problem by taking a small sample of the same event using the counting principle
- Use real-life problem situations to introduce new concepts. For example a tennis match in which the first person to win two games in a row wins the match. How many possibilities are there to a final outcome? (Using a tree diagram may be helpful).
- Have students research the role of probability in the field of genetics (e.g., eye colour, blood type).

SUGGESTED ASSESSMENT STRATEGIES

The study of statistics relies heavily on the theory of probability. Probability provides concepts and methods for dealing with uncertainty and for interpreting predictions based on uncertainty. Evidence of students' abilities to use probability can best be collected in the context of experimental design and problem-solving activities.

Observe
- While students are working on problems involving uncertainty, look for evidence that they can:
  - use the appropriate terminology
  - distinguish between permutation and combination and provide an example of each
  - correctly calculate the mean and standard deviation of a data set or distribution
  - correctly use normal distribution and z-scores
  - make sound conclusions using data at hand
- To assess students' work on statistics projects, look for evidence that they can:
  - compile appropriate data for the project
  - identify the problem to be analyzed and explain how the data contribute to the solution
  - calculate appropriate statistics from the following: mean, standard deviation, and *z*-score
  - summarize data using tables, diagrams, or graphs
  - draw conclusions and inferences based on analyses of the data

Question
- To assess students' understanding of basic concepts, pose questions such as the following:
  - What is the difference between possible outcomes and the probability of an outcome?
  - What is the difference between combination and permutation?
  - How does the use of AND and OR in the context of compound events affect the meaning?

RECOMMENDED LEARNING RESOURCES

Print Materials

- Exploring Advanced Algebra with the TI-83
- Exploring Statistics with the TI-82 Graphics Calculator
- Using the TI-81 Graphics Calculator to Explore Statistics

Multi-media

- Mathematics 12, Western Canadian Edition
  Ch. 6 (Sections 6.1 - 6.6)
  Ch. 7 (Sections 7.2 - 7.7)
  Ch. 8 (Sections 8.2 - 8.7)
- MATHPOWER 12, Western Edition
  Ch. 7 (Sections 7.1 - 7.5)
  Ch. 8 (Sections 8.1 - 8.4)
  Ch. 9 (Sections 9.1 - 9.5)

Software

- Secondary Math Lab Toolkit

Curriculum

Calculus 12

ESTIMATED INSTRUCTIONAL TIME

Calculus 12 has been developed assuming that teachers have 100 instructional hours available to them. The following chart shows the estimated instructional time for each curriculum suborganizer, expressed as a percentage of total time available to teach the course.

Calculus 12

Organizer (Suborganizer)	% of Time
Problem Solving	**Integrated Throughout**
Overview and History of Calculus	**Integrated Throughout**
Functions, Graphs, and Limits (Functions and their Graphs)(Limits)	**10 - 15**
The Derivative (Concept and Interpretations)	**10 - 15**
The Derivative (Computing Derivatives)	**15 - 20**
Applications of Derivatives (Derivatives and the Graph of the Function)	**15 - 20**
Applications of Derivatives (Applied Problems)	**20 - 25**
Antidifferentiation (Recovering Functions from their Derivatives)	**5 - 10**
Antidifferentiation (Applications of Antidifferentiation)	**10 - 15**

When delivering the prescribed curriculum, teachers may freely adjust the instructional time to meet their students' diverse needs. These estimated instructional times have been recommended by the IRP writers to assist their colleagues; they are suggestions only.

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

It is expected that students will:

- solve problems that involve a specific content area (e.g., geometry, algebra, trigonometry statistics, probability)
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and justify the process used to solve it
- use appropriate technology to assist in problem solving

SUGGESTED INSTRUCTIONAL STRATEGIES

Problem solving is a key aspect of any mathematics course. Working on problems can give students a sense of the excitement involved in creative and logical thinking. It can also help students develop transferable real-life skills and attitudes. Problems may come from various fields, including algebra, geometry, and statistics. Multi-strand and interdisciplinary problems should be included throughout Calculus 12.

- Introduce new types of problems directly to students (without demonstration) and play the role of facilitator as they attempt to solve such problems.
- Recognize when students use a variety of approaches (e.g., algebraic and geometric solutions); avoid becoming prescriptive about approaches to problem solving.
- Reiterate that problems might not be solved in one sitting and that "playing around" with the problem-revisiting it and trying again-is sometimes needed.
- Frequently engage small groups of students (three to five) in co-operative problem solving when introducing new types of problems.
- Have students or groups discuss their thought processes as they attempt a problem. Point out the strategies inherent in their thinking (e.g., guess and check, look for a pattern, make and use a drawing or model).
- Ask leading questions such as:
  - What are you being asked to find out?
  - What do you already know?
  - Do you need additional information?
  - Have you ever seen similar problems?
  - What else can you try?
- Once students have arrived at solutions to particular problems, encourage them to generalize or extend the problem situation.

SUGGESTED ASSESSMENT STRATEGIES

Students analyse problems and solve them using a variety of approaches. Assessment of problem-solving skills is made over time, based on observations of many situations.

Observe
- Have students present solutions to the class individually, in pairs, or in small groups. Note the extent to which they clarify their problems and how succinctly they describe the processes used.

Question
- To check the approaches students use when solving problems, ask questions that prompt them to:
  - paraphrase or describe the problem in their
  - own words
  - explain the processes used to derive an answer
  - describe alternative methods to solve a problem
  - relate the strategies used in new situations
  - link mathematics to other subjects and to the world of work

Collect
- On selected problems, have students annotate their work to describe the processes they used. Alternatively, have them provide brief descriptions of what worked and what did not work as they solved particular problems.

Self-Assessment
- Ask students to keep journals to describe the processes they used in dealing with problems. Have them include descriptions of strategies that worked and those that did not.
- Develop with students a set of criteria to self-assess problem-solving skills. The reference set Evaluating Problem Solving Across Curriculum may be helpful in identifying such criteria.

RECOMMENDED LEARNING RESOURCES

Please see the introduction to Appendix B for a list of suggested utility software that supports this course.

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises.

These resources, while not part of the Grade Collections, have Provincially Recommended status.

Appendix B includes an annotated bibliography of these resources for ordering convenience.

Prescribed Learning Outcomes

It is expected that students will understand that calculus was developed to help model dynamic situations.

It is expected that students will:

- distinguish between static situations and dynamic situations
- identify the two classical problems that were solved by the discovery of calculus:
  - the tangent problem
  - the area problem
- describe the two main branches of calculus:
  - differential calculus
  - integral calculus
- understand the limit process and that calculus centers around this concept

Suggested Instructional Strategies

Students will quickly come to realize that calculus is very different from the mathematics they have previously studied. Of greatest importance is an understanding that calculus is concerned with change and motion. It is a mathematics of change that enables scientists, engineers, economists, and many others to model real-life, dynamic situations.

- The concepts of average and instantaneous velocity are a good place to start. These concepts could be related to the motion of a car. Ask students if the displacement of the car is given by $f(t)$, what is the average velocity of the car between times $t = t_1$ and $t = t_2$, and the instantaneous velocity at $t = t_1$? Point out that calculus is not needed to determine the average velocity $\frac{f(t_2) - f(t_1)}{t_2 - t_1}$, but is needed to determine the velocity at t_1 (the speedometer reading).
- As a way of solving the tangent problem (i.e., the need for two points and the fact that only one point is available), create a secant line that can be moved toward the tangent line. Ask students if they can move the secant line towards the tangent line using algebraic techniques.
- Ask students to determine the area under a curve by estimating the answer using rectangles. Introduce the concept of greater accuracy when the rectangles are of smaller and smaller widths (the idea of limit).
- Present the following statement for discussion:

  $$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots = 2$$

  Use geometry to illustrate the relationship between the left-hand and right-hand sides.

SUGGESTED ASSESSMENT STRATEGIES

To demonstrate their achievement of the outcomes for this organizer, students need opportunities to engage in open-ended activities that allow for a range of responses and representations.

Although formative assessment of students' achievement of the outcomes related to this organizer should be ongoing, summative assessment can only be carried out effectively toward the end of the course, when students have sufficient understanding of the details of calculus to appreciate the "big picture."

Self-Assessment

- Work with the students to develop criteria and rating systems they can use to assess their own descriptions of the two main branches of calculus. They can represent their progress with symbols to indicate when they have addressed a particular criterion in their work. Appropriate criteria might include the extent to which they:
  - make connections to other branches of mathematics
  - demonstrate understanding of the classical problems
  - consider the different attempts to solve the classical problem(s)
  - explain the significance of limits with reference to specific examples
  - accurately and comprehensively describe the relationship between integral and differential calculus (methodology and purpose)

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  pp. 83-85, 95, 190, 262, 272-273,375
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  pp. 57 - 61, 63
- Single Variable Calculus Early Transcendentals, Fourth Edition
  Ch. 2 (Section 2.2)
  pp. 3-4, 6-9, 85, 351

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 2, 112, 114, 375
- Calculus of a Single Variable, Sixth Edition
  Ch. 1 (Section 1.1)
  Ch. 4 (Section 4.2, 4.3
  pp. 41-45, 91, 265

PRESCRIBED LEARNING OUTCOMES

It is expected that students will understand the historical background and problems that lead to the development of calculus.

It is expected that students will:

- describe the contributions made by various mathematicians and philosophers to the development of calculus, including:
  - Archimedes
  - Fermat
  - Descartes
  - Barrow
  - Newton
  - Leibniz
  - Jakob and Johann Bernoulli
  - Euler
  - L'Hospital

SUGGESTED INSTRUCTIONAL STRATEGIES

Students gain a better understanding and appreciation for this field of mathematics by studying the lives of principal mathematicians credited for the invention of calculus, including the period in which they lived and the significant mathematical problems they were attempting to solve.

- Conduct a brief initial overview of the historical development of calculus, then deal with specific historical developments when addressing related topics (e.g., the contributions of Fermat and Descares to solving the tangent line problem can be covered when dealing with functions, graphs, and limits). For additional ideas, see the suggested instructional strategies for other organizers.
- Encourage students to access the Internet for information on the history of mathematics.
- Ask students to investigate various mathematicians and the associated periods of calculus development and to present their findings to the class.
- Point out the connection between integral and differential calculus when students are working on the "area under the curve" problem.
- Conduct a class discussion about the contributions of Leibniz and Newton when the derivative is being introduced (e.g., the notation $\frac{dy}{dx}$ for the derivative and $\int y\,dx$ for the integral are due to Leibniz; the $f'(x)$ notation is due to Lagrange).
- Have students research the historical context of the applications of antidifferentiation. For example:
  - The area above the x axis, under $y = x^2$, from $x = 0$ to $x = b$ is $b^3/3$. Archimedes was able to show this without calculus, using a difficult argument.
  - The area problem for $y = \frac{1}{x}$ was still unsolved in the early 17th century. Using antiderivatives, the area under the curve $y = \frac{1}{t}$ from $t = 1$ to $t = x$ is $\ln x$.
  - The area under $y = \sin x$, from $x = 0$ to $x = \frac{\pi}{2}$, was calculated by Roberval, without calculus, using a difficult argument. It can be solved easily using antiderivatives.

SUGGESTED ASSESSMENT STRATEGIES

When students are aware of the outcomes they are responsible for and the criteria by which their work will be assessed, they can represent their comprehension of the historical development of calculus more creatively and effectively.

Observe

- Have students construct and present a calculus timeline, showing important discoveries and prominent mathematicians. Check the extent to which students' work is:
  - accurate
  - complete
  - clear

Collect

- Ask students to write a short article about one mathematician's contributions to calculus. The article should describe the mathematical context within which the person lived and worked and the contributions as seen in calculus today.

Research

- Use a research assignment to assess students' abilities to synthesize information from more than one source. Have each student select a topic of personal interest, develop a list of three to five key questions, and locate relevant information from at least three different sources. Ask students to summarize what they learn by responding to each of the questions in note form, including diagrams if needed. Look for evidence that they are able to:
  - combine the information, avoiding duplications or contradictions
  - make decisions about which points are most important

Peer Assessment

- To check on students' knowledge of historical figures, form small groups and ask each group to prepare a series of three to five questions about the contribution of a particular mathematician. Have groups exchange questions, then discuss, summarize, and present their answers. For each presentation, the group that designed the questions offers feedback on the extent to which the answers are thorough, logical, relevant, and supported by specific explanation of the mathematics involved.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  Integrated Throughout
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  Integrated Throughout
- Single Variable Calculus Early Transcendentals, Fourth Edition
  Integrated Throughout

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 10-11, 15, 19, 99, 225, 237, 278, 352, 378, 638, A3
- Calculus of a Single Variable, Sixth Edition
  Integrated Throughout

Prescribed Learning Outcomes

It is expected that students will represent and analyse rational, inverse trigonometric, base e exponential, natural logarithmic, elementary implicit, and composite functions, using technology as appropriate.

It is expected that students will:

- model and apply inverse trigonometric, base e exponential, natural logarithmic, elementary implicit and composite functions to solve problems
- draw (using technology), sketch and analyse the graphs of rational, inverse trigonometric, base e exponential, natural logarithmic, elementary implicit and composite functions, for:
  - domain and range
  - intercepts
- recognize the relationship between a base a exponential function ($a > 0$) and the equivalent base e exponential function (convert $y = a^x$ to $y = e^{x(1na)}$)
- determine, using the appropriate method (analytic or graphing utility) the points where $f(x) = 0$

Suggested Instructional Strategies

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

- Prior to undertaking work on functions and their graphs, conduct review activities to ensure that students can:
  - perform algebraic operations on functions and compute composite functions
  - use function and inverse function notation appropriately
  - determine the inverse of a function and whether it exists
  - describe the relationship between the domain and range of a function and its inverse
- Give students an exponential function and its inverse logarithmic function. Have them work in groups, using graphing calculators or computer software, to identify the relationship between the two. (e.g., compare and contrast the graphs of $\log_2 x$ and 2^x).
- Emphasize the domain and range restrictions as students work with exponential and natural logarithmic functions.
- Have students use technology (e.g., graphing calculators) to compare the graphs of functions such as:

  $y = e^x$ and $y = \ln x$

  $y = e^{\ln x}$ and $y = \ln e^x$

  $y = 3^x$ and $y = e^{x \ln 3}$
- Similarly apply this procedure to trigonometric and inverse trigonometric graphs including: sine and arcsine, cosine and arccosine, and tangent and arctangent. Emphasize that arcsine $x = \sin^{-1} x$ and does not mean the reciprocal of sin x.

SUGGESTED ASSESSMENT STRATEGIES

The exponential and natural logarithmic functions enable students to solve more complex problems in areas such as science, engineering, and finance. Students should be able to demonstrate their knowledge of the relationship between natural logarithms and exponential functions, both in theory and in application, in a variety of problem situations.

Observe

- Ask students to outline how they would teach a classmate to explain:
  - the inverse relationship between base *e* exponential and natural logarithmic functions
  - the restrictions associated with base *e* exponential and natural logarithmic, and trigonometric and inverse trigonometric functions

  Note the extent to which the outlines:
  - include general steps to follow
  - use mathematical terms correctly
  - provide clear examples
  - describe common errors and how they can be avoided

Collect

- Assign a series of problems that require students to apply their knowledge of the relationship between logarithms and exponential functions. Check their work for evidence that they:
  - clearly understood the requirements of the problem
  - used efficient strategies and procedures to solve the problem
  - recognized when a strategy or procedure was not appropriate
  - verified that their solutions were accurate and reasonable

Self-Assessment

- Discuss with students the criteria for assessing graphing skills. Show them how to develop a rating scale. Have them use the scale to assess their own graphing skills.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  pp. 12, 16, 23, 36-37, 45-51, 165
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  Ch. 1 (Sections 1.4, 1.5, 1.6)
  Ch. 4 (Section 4.6)
  pp. 5, 20, 26-27, 41, 51-52, 63, 158, 182
- Single Variable Calculus Early Transcendentals, Fourth Edition
  pp. 43-46, 61-63, 69-72, A31-A34

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 49, 57, 240, 242-243, 246, 254, 261, 308
- Calculus of a Single Variable, Sixth Edition
  Ch. 5 (Section 5.1, 5.4, 5.5, 5.8)
  pp. 20, 26, 27, 75, 134, 380

PRESCRIBED LEARNING OUTCOMES

It is expected that students will understand the concept of limit of a function and the notation used, and be able to evaluate the limit of a function.

It is expected that students will:

- demonstrate an understanding of the concept of limit and notation used in expressing the limit of a function $f(x)$ as x approaches a: $\lim_{x \to a} f(x)$
- evaluate the limit of a function
  - analytically
  - graphically
  - numerically
- distinguish between the limit of a function as x approaches a and the value of the function at $x = a$
- demonstrate an understanding of the concept of one-sided limits and evaluate one-sided limits
- determine limits that result in infinity (infinite limits)
- evaluate limits of functions as x approaches infinity (limits at infinity)
- determine vertical and horizontal asymptotes of a function, using limits
- determine whether a function is continuous at $x = a$

SUGGESTED INSTRUCTIONAL STRATEGIES

Students require a firm understanding of limits in order to fully appreciate the development of calculus.

- Although the concept of limit should be introduced at the beginning of the course, particular limits (e.g., $\lim_{x \to 0} \frac{\sin x}{x}$) need only be introduced as needed to deal with new derivatives.
- Using diagrams, introduce students in a general way to the two classic problems in calculus: the **tangent line problem** and the **area problem** and explain how the concept of limit is used in the analysis.
- Give students a simple function such as $f(x) = x^2$ and have them:
  - determine the slope of the tangent line to $f(x) = x^2$ at the point (2,4) by evaluating the slope of the secant lines through (2,4) and (2.5, f(2.5)), (2.1, f(2.1)), (2.05, f(2.05)), (2.01, f(2.01)), then draw conclusions about the value of the slope of the tangent line
  - determine the area under $y = x^2$ above the x axis from $x = 0$ to $x = 2$ by letting the number of rectangles be 4, 8, and 16, then inferring the exact area under the curve.
- Give students limit problems that call for analytic, graphical, and/or numerical evaluation, as in the following examples:
  - $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ (evaluate analytically)
  - $\lim_{x \to 0} \frac{\sin x}{x}$ (evaluate numerically, geometrically, and using technology)
  - $\lim_{x \to 0} \frac{3x^2 + 5}{4 - x^2}$ (evaluate analytically and numerically)
  - $\lim_{x \to 0} \frac{1}{x^2} = \infty$ (draw conclusions numerically)
- When introducing students to one-sided limits that result in infinity, have them draw conclusions about the vertical asymptote to $y = f(x)$, as in the following example: from $\lim_{x \to 3^+} \frac{1}{x - 3}$ and $\lim_{x \to 3^-} \frac{1}{x - 3}$, conclude that $x = 3$ is the vertical asymptote for $f(x) = \frac{1}{x - 3}$.
- Have students explore the limits to infinity of a function and draw conclusions about the horizontal asymptote, as in the following example: because $\lim_{x \to \infty} \frac{e^x}{e^x + 1} = 1$ and because $\lim_{x \to -\infty} \frac{e^x}{e^x + 1} = 0$, y=1 and y=0 are horizontal asymptotes for $f(x) = \frac{e^x}{e^x + 1}$.

SUGGESTED ASSESSMENT STRATEGIES

Limits form the basis of how calculus can be used to solve previously unsolvable problems. Students should be able to demonstrate their knowledge of both the theory and application of limits in a variety of problem situations.

Observe

- While students are working on problems involving limits, look for evidence that they can:
  - distinguish between the limit of a function and the **value** of a function
  - Identify and evaluate one-sided limits
  - recognize and evaluate infinite limits and limits at infinity
  - determine any vertical or horizontal asymptote of a function
  - determine whether a function is continuous over a specified range or point
- To check students' abilities to reason mathematically, have them describe orally the characteristics of limits in relation to one-sided limits, infinite limits, limits at infinity, asymptotes, and continuity of a function. To what extent do they give explanations that are mathematically correct, logical, and clearly presented.

Collect

- Assign a series of problems that require students to apply their knowledge of limits. Check their work for evidence that they:
  - clearly understood the requirements of the problem
  - used the appropriate method to evaluate the limit in the problem
  - verified that their solutions were accurate and reasonable

Self-Assess/Peer Assess

- Have students explain concepts such as "limit" and "continuity" to each other in their own words.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic pp. 55-61, 65-77
- Calculus of a Single Variable Early Transcendental Functions, Second Edition pp. 63, 72, 75, 84, 96-97, 113, 227-228
- Single Variable Calculus Early Transcendentals, Fourth Edition Ch. 2 (Sections 2.2, 2.5, 2.6)

Multimedia

- Calculus: A New Horizon, Sixth Edition pp. 115, 118, 120, 122-124, 131, 148, 161
- Calculus of a Single Variable, Sixth Edition Ch. 1 (Section 1.2, 1.4, 1.5) Ch. 3 (Section 3.5)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will understand the concept of a derivative and evaluate derivatives of a function using the definition of derivative.

It is expected that students will:

- describe geometrically a secant line and a tangent line for the graph of a function at $x = a$
- define and evaluate the derivative at $x = a$ as: $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ and $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
- define and calculate the derivative of a function using:
  $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or
  $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ or
  $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$
- use alternate notation interchangeably to express derivatives (i.e., $f'(x), \frac{dy}{dx}, y'$, etc.)
- compute derivatives using the definition of derivative
- distinguish between continuity and differentiability of a function at a point
- determine when a function is non-differentiable, and explain why
- determine the slope of a tangent line to a curve at a given point
- determine the equation of the tangent line to a curve at a given point
- for a displacement function $s = s(t)$, calculate the average velocity over a given time interval and the instantaneous velocity at a given time
- distinguish between average and instantaneous rate of change

SUGGESTED INSTRUCTIONAL STRATEGIES

The concept of the derivative can be used to calculate instantaneous rate of change. Students can best understand this concept if it is presented to them geometrically, analytically, and numerically. This allows them to make connections between the various forms of presentation.

- Present an introduction to the "tangent line" problem by discussing the contribution made by Fermat, Descartes, Newton, and Leibniz.
- Use a graphing utility to graph $f(x) = 2x^3 - 4x^2 + 3x - 5$.
  On the same screen, add the graphs of $y = x - 5$, $y = 2x - 5$, $y = 3x - 5$. Have students identify which of these lines appears to be tangent to the graph at the point (0, -5). Have them explain their answers.
- Sketch a simple parabola $y = x^2$ on the board. Attach a string at a point and show how it can be moved from a given secant line to a tangent line at a different point on the graph. Show how the slope of the secant line approaches that of the tangent line as $\Delta x \to 0$.
- Have students calculate the slope of a linear function at a point $x = a$ (e.g., $f(x) = 3x + 2$ at $x = 1$), using the formula $\lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$. Have students use the same formula to find the slope of the tangent lines to the graph of a non-linear function such as $f(x) = x^2 - 4$ at $a = 1$, and then find the equation of the tangent lines.
- Ask students to use technology to:
  - reinforce the result found when using the definition to calculate the derivative of a function
  - verify that the tangent line as calculated, appears to be the tangent to the curve. As an extension, have students zoom in at the point of tangency and describe the relationship between the tangent line and the curve
- Introduce the differentiability of a function by generating problems that require students to create, using technology, graphs with sharp turns and vertical tangent lines.
- Have students work in groups to prepare a presentation on tangent lines and the use of mathematics by the ancient Greeks (circle, ellipse, parabola). Ask them to include an explanation of how the initial work of the Greeks was surpassed by the work of Fermat and Descartes and how previously difficult arguments were made into essentially routine calculations.

SUGGESTED ASSESSMENT STRATEGIES

The concept of the derivative facilitates the solving of complex problems in fields such as science, engineering, and finance. Students should be able to demonstrate knowledge of the derivative in relation to problems involving instantaneous rate of change.

Observe

- As students work, circulate through the classroom and note:
  - the extent to which they relate their algebraic work to their graphing work
  - whether they can recognize the declining nature of the x values
- Have students develop a table of values to calculate a "slope predictor" in the parabola $y = x^2$ at a point. In small groups they can justify the concept of a limit using technology. Higher magnifications will eliminate the difference in the slope values of secant and tangent lines at a point. Ask one student from each group to explain this phenomenon, using the group's 'unique' sample slope calculation. Note how succinctly they describe the processes used.
- When students work with technology (e.g., graphing calculator), check the extent to which they:
  - select appropriate viewing windows
  - enter the functions correctly
  - interpret the results correctly

Question

- Have students explain in their own words the concepts of average and instantaneous rates of change.

Research

- When students report on the contributions of various mathematicians to the tangent line problem, check the extent to which they:
  - clearly address key concepts (e.g., explain how Fermat found the length of the subtangent; how Descartes found the slope of the normal)
  - include careful graphic illustrations

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic pp. 83-85, 95-97, 105-107, 122-124, 152
- Calculus of a Single Variable Early Transcendental Functions, Second Edition pp. 60, 110, 112-117, 123, 125-129, 137, 142, 174, 662, 675
- Single Variable Calculus Early Transcendentals, Fourth Edition Ch. 2 (Sections 2.7, 2.8, 2.9)

Multimedia

- Calculus: A New Horizon, Sixth Edition Ch. 3 pp. 170, 172, 175, 178-180, 186, 187, 353
- Calculus of a Single Variable, Sixth Edition Ch. 2 (Section 2.1) pp. 92-97, 108-109, 166

PRESCRIBED LEARNING OUTCOMES

It is expected that students will determine derivatives of functions using a variety of techniques.

It is expected that students will:

- compute and recall the derivatives of elementary functions including:
  - $\frac{d}{dx}(x^r) = rx^{r-1}$, r is real
  - $\frac{d}{dx}(e^x) = e^x$
  - $\frac{d}{dx}(\ln x) = \frac{1}{x}$
  - $\frac{d}{dx}(\cos x) = -\sin x$
  - $\frac{d}{dx}(\sin x) = \cos x$
  - $\frac{d}{dx}(\tan x) = \sec^2 x$
  - $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
  - $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- use the following derivative formulas to compute derivatives for the corresponding types of functions:
  - constant times a function $\frac{d}{dx}cu = c\frac{du}{dx}$:
  - $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$ (sum rule)
  - $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ (product rule)
  - $\frac{d}{dx}(\frac{u}{v}) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (quotient rule)
  - $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$ (power rule)
- use the Chain Rule to compute the derivative of a composite function: $\frac{dy}{dx} = \frac{du}{dx}\frac{dy}{du}$ or $\frac{d}{dx}(F(g(x))) = g'(x)F'(g(x))$
- compute the derivative of an implicit function
- use the technique of logarithmic differentiation
- compute higher order derivatives

SUGGESTED INSTRUCTIONAL STRATEGIES

Knowledge of a variety of techniques for computing the derivatives of different types of functions allows students to solve an ever-increasing range of problems. To become efficient problem solvers students need to understand when and how to use derivative formulas, but will also find it useful to memorize some basic derivatives.

- Point out to students that the derivatives of a few functions (e.g., x^2, $\frac{1}{x}$, $\sqrt{x}$) will already have been obtained using the definition of a derivative.
- To help students connect the limit concept to the derivative of sin *x*, demonstrate the details of $\frac{d}{dx}(\sin x) = \cos x$, using the known fact that $\sin(x + h) = \cos x \sin h + \sin x \cos h$ and that $\lim_{h \to 0} \frac{\sin h}{h} = 1$. Alternatively, you can provide these known facts to students and have them work in groups to develop the solution.
- Have students work in groups to differentiate $f(x) = \frac{(1+x)^5(1-2x)^7}{x^3}$, using two different methods:
  1) the chain, product, and quotient rules
  2) logarithmic differentiation
  Ask students to compare the two methods.
- Demonstrate that the derivatives of functions such as ln *x*, $\sin^{-1} x$, and $\cos^{-1} x$ can be found using implicit differentiation. For example, given $y = \sin^{-1} x$, $\frac{dy}{dx}$ can be determined implicitly, as follows:
  - Since $y = \sin^{-1} x$, then $\sin y = x$
  - Thus $\cos y \frac{dy}{dx} = x$, which means $\frac{dy}{dx} = \frac{1}{\cos y}$
  - Since $\cos^2 y = 1 - \sin^2 y$, then $\cos y = \sqrt{1-\sin^2 y}$
  - Substituting $\sin y = x$, we get $\cos y = \sqrt{1-x^2}$.
  - Now substitute this into $\frac{dy}{dx} = \frac{1}{\cos y}$ to get $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$
- Although full proof of the chain rule is uninformative, a graphing calculator can be used to show quite persuasively that for example $\frac{d}{dx}(\sin 3x)$ ought to be 3 cos 3x at any particular place *x* [given that $\frac{d}{dx}\sin x = \cos x$].

SUGGESTED ASSESSMENT STRATEGIES

As students develop confidence and proficiency computing the derivative of elementary functions, they are better prepared to solve more complex problems that involve derivative formulas. Assessment should focus on students' ability to recall derivatives of elementary functions and apply this knowledge appropriately when computing the derivative of a more complex function (using formulas, implicit differentiation, or logarithmic differentiation).

Observe

- As students are working on problems, circulate and provide feedback on their notation use. Have students verify their work using a graphing calculator.

Collect

- For a question where $f(x) = (x^2 + x)\ 3$ ask students to debate the effectiveness of finding the derivative using expansion, the chain rule, the product rule, and logarithmic differentiation. To what extent do their arguments illustrate their ability to expand on current mathematical idea?
- Have students present to the class research on some elementary functions and their derivatives.
- Discuss with students the merits of the various methods of taking derivatives. Have them summarize in writing their understanding of the merits of each. Work with them to develop a set of criteria to assess the summaries.

Self-Assessment

- To assess students' command of the chain rule, have them list common difficulties they encounter in taking derivatives. Let them work in pairs to develop checklists of reminders to use when they're checking their work.
- Ask students to summarize derivatives of elementary functions, making notes on notation errors. Allow students to complete assignments using their summaries.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  Ch. 3 (Sections 3.3, 3.5, 3.6, 3.7, 3.8, 3.9)
  pp. 119, 151-154, 169
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  pp. 121, 125-126, 133-134, 137, 139, 143, 149, 158, 163, 168, 170
- Single Variable Calculus Early Transcendentals, Fourth Edition
  Ch. 3 (Sections 3.1, 3.2, 3.4, 3.5, 3.6, 3.7 3.8)

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 189, 191-196, 200, 204, 246, 257, 258, 261
- Calculus of a Single Variable, Sixth Edition
  Ch. 2 (Section 2.5)
  pp. 103, 105-107, 114-120,125-126, 315-316, 340, 380

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use the first and second derivatives to describe the characteristic of the graph of a function.

It is expected that students will:

- given the graph of $y = f(x)$:
  - graph $y = f'(x)$ and $y = f''(x)$
  - relate the sign of the derivative on an interval to whether the function is increasing or decreasing over that interval.
  - relate the sign of the second derivative to the concavity of a function
- determine the critical numbers and inflection points of a function
- determine the maximum and minimum values of a function and use the first and/or second derivative test(s) to justify their solutions
- use Newton's iterative formula (with technology) to find the solution of given equations, $f(x) = 0$
- use the tangent line approximation to estimate values of a function near a point and analyse the approximation using the second derivative

SUGGESTED INSTRUCTIONAL STRATEGIES

The first and second derivatives of a function provide a great deal of information concerning the graph of the function. This information is critical to solving problems involving calculus and helps students understand what the graph of a function represents.

- Ask students to compare the questions, "Where is $-x^3 + 14x^2 + 20x$ increasing most rapidly? Where is $-x^3 + 14x^2 + 20x$ increasing?" Have students develop similar questions.
- Have students work in teams of two to determine why the calculator is not helpful in questions such as: "let $f(x) = ax^3 - 5x^2$. Describe in terms of the parameter a, where $f(x)$ reaches a maximum or minimum."
- Have students use technology to explore features of functions such as the following:
  - when a function has maximum/minimum points or neither
  - where the inflection points occur
  - where curves are concave up or down
  - vertical or horizontal tangent lines
  - the relationship among graphs of the 1st, 2nd derivatives of a function and the function
  - the impact that endpoints have on the maximum/minimum
- Use a summary chart to demonstrate increasing and decreasing aspects of a function. Have students describe how this relates to the 1st derivative.
- Have groups of students learn particular concepts, (e.g., Newton's method) and teach these to their peers. Have them examine situations where it works very efficiently, situations where it works very slowly (e.g., $x^5 = 0$), and situations where it does not work (e.g., $x^{\frac{1}{3}} = 0$). Challenge them to develop hypotheses as to why it does or does not work.
- Have students research and report on the history of Newton's method for determining square roots (e.g., mentioning Heron of Alexandria, and the work of mathematicians in Babylon, India).
- Discuss with students the relative merits of using or not using a graphing calculator "to do" calculus. Present them with questions such as $g(x) = 8x^3 - 5x^2 + x - 3$ contrasted with $f(x) = \frac{1}{3}x^3 - 10{,}000x + 100$. Note that the calculator does not always give the necessary detail.

SUGGESTED ASSESSMENT STRATEGIES

Students demonstrate their understanding of first and second derivatives by relating the information they obtain from these to the graph of a function (i.e., critical numbers, inflection points, maximum and minimum values, and concavity).

Observe

- When reviewing students work, note the extent to which they can:
  - accurately determine the 1st and 2nd derivative
  - apply the respective tests
  - recognize and describe the relationship among the graphs of f(x), f′(x), f″(x)

Collect

- Ask the students to sketch a cubic graph and determine what the slopes of the tangent line would be at the local maximum and minimum points. Have them present their summary to the class. Consider the extent to which they can explain why the derivative is 0 at a local maximum/minimum.

Peer assessment

- Have students critique each others' graphs using criteria generated by the class. These criteria could include the extent to which:
  - a graph is appropriate to the function it is supposed to represent
  - the axes are accurately labelled
  - appropriate scales have been chosen for the axes
  - the graphs present smooth curves
  - domain, range, asymptotes, intercepts, and vertices have been correctly determined
  - inflection points, maximum and minimum points, and region of concavity have been correctly identified

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  Ch. 4 (Sections 4.4, 4.6)
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  pp. 82, 127-128, 173, 247
- Single Variable Calculus Early Transcendentals, Fourth Edition
  Ch. 3 (Section 3.10)
  Ch. 4 (Sections 41, 4.7)

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 172, 270, 329
  Calculus of a Single Variable, Sixth Edition
  Ch. 2 (Section 2.6)
  Ch. 3 (Section 3.7)

PRESCRIBED LEARNING OUTCOMES

It is expected that students will solve applied problems from a variety of fields including the Physical and Biological Sciences, Economics, and Business.

It is expected that students will:

- solve problems involving displacement, velocity, acceleration
- solve related rates problems
- solve optimization problems (applied maximum/minimum problems)

SUGGESTED INSTRUCTIONAL STRATEGIES

Calculus was developed to solve problems that had previously been difficult or impossible to solve. Such problems include related rate and optimization problems, which arise in a variety of fields that students may be studying (e.g., the physical and biological sciences, economics, and business).

- Have a student demonstrate a "student trip" in front of the class — constant speed, accelerate, stop, slow down, stop, back up. Have students sketch a graph of displacement against time for the movements.
- Discuss average velocity over a specified interval, instantaneous velocity at specified times. Have students in pairs create a graph and have their partner perform the motion.
- Have students brainstorm examples of the need for calculus in the real world:
  - population growths of bacteria
  - the optimum shape of a container
  - water draining out of a tank
  - the path that requires the least time to travel
  - marginal cost and profit
- Challenge students to create their own "new" problems, which they must then try to solve. These problems could be used to develop tests or unit reviews.
- Have students use technology (e.g., graphing calculators) to investigate problems and confirm their analytical solutions graphically.
- Discuss with students the merits of using versus not using a graphing calculator "to do" calculus.

SUGGESTED ASSESSMENT STRATEGIES

Students develop their knowledge of derivatives by solving problems involving rates of change, maximum, and minimum. When assessing student performance in relation to these problems, it is important to consider students' abilities to make generalizations and predictions about how calculus is used in the real world.

Observe

- While students are working on problems involving derivatives, look for evidence they:
  - clearly understand the requirements of the problem
  - recognized when a strategy was not appropriate
  - can explain the process used to determine their answers
  - used a graphing calculator where appropriate to help visualize their solution
  - verified that their solutions where correct and reasonable
- Provide a number of problems where students are required to use average velocity **or** instantaneous velocity. Observe the extent to which students are able to:
  - determine whether the situation calls for the calculation of average velocity or instantaneous velocity
  - explain the differences between the two
  - provide other examples

Collect

- Assign a series of problems that require students to apply their knowledge of the dynamics of change: how, at certain time the speed of an object is related to its height, and how, at a certain time the speed of an object is related to the change in velocity.
- To discover how well students can recognize and explain the concepts of change and rapid change give them a problem such as:

  A bacterial colony grows at a rate $\frac{dy}{dt} = y(C - y)$.
  How large is the colony when it is growing most rapidly? In their analysis look for evidence that they understand the rate of change of "the rate of change."

Question

- While students are working on simple area problems based on real-life applications, ask them to explain the relationship between the graph's units (on each axis) and the units of area under the curve.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  Ch. 4 (Sections 4.3, 5.4)
  pp. 97-98, 172, 180, 198, 202
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  pp. 182, 184, 195, 197, 209, 219-221, 247, 257
- Single Variable Calculus Early Transcendentals, Fourth Edition
  Ch. 2 (Section 2.9)
  Ch. 3 (Section 3.11)
  Ch. 4 (Sections 4.2, 4.3, 4.9)

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 211, 290, 299, 363
- Calculus of a Single Variable, Sixth Edition
  Ch. 3 (Sections 3.2, 3.3, 3.8, 3.9)
  pp. 157, 171-175, 182-183

PRESCRIBED LEARNING OUTCOMES

It is expected that students will recognize antidifferentiation (indefinite integral) as the reverse of the differentiation process.

It is expected that students will:

- explain the meaning of the phrase "F(x) is an antiderivative (or indefinite integral) of f(x)"
- use antiderivative notation appropriately (i.e., $\int f(x)dx$ for the antiderivative of $f(x)$)
- compute the antiderivatives of linear combinations of functions whose individual antiderivatives are known including:
  - $\int k\,dx = kx + C$
  - $\int x^r dx = \dfrac{x^{r+1}}{r+1} + C$ if $r \neq -1$
  - $\int \dfrac{dx}{x} = \ln|x| + C$
  - $\int e^x dx = e^x + C$
  - $\int \sin x\,dx = -\cos x + C$
  - $\int \cos x\,dx = \sin x + C$
  - $\int \sec^2 x\,dx = \tan x + C$
  - $\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
  - $\int \dfrac{dx}{1+x^2} = \tan^{-1} x + C$
- compute $\int f(ax+b)\,dx$ if $\int f(u)\,du$ is known
- create integration formulas from the known differentiation formulas
- solve initial value problems using the concept that if $F'(x) = G'(x)$ on an interval, then $F(x)$ and $G(x)$ differ by a constant on that interval

SUGGESTED INSTRUCTIONAL STRATEGIES

Students need to recognize that antidifferentiation is the reverse of the differentiation process. This understanding will enable them to calculate or verify.

- Use a variety of methods to explain the need for the constant, "C":
  - computational: $\dfrac{dy}{dx}(C) = 0$
  - geometrical: if we draw curves y = F(x), y = F(x) + C, one curve is obtained from the other by simply lifting, so slopes of tangent lines match
  - kinematic: if v(t) (velocity) is specified, then $\int v(t)dt$ represents all possible position (displacement) functions (we can't reconstruct position from velocity unless we know where we were at some time)
- To ensure students understand the arbitrary nature of assigning letters to variables, use a reasonable variety of letters in doing integration problems. For example, $\int \sin 2u\,du$, $\int e^{at}dt$.
- Integrals such as $\int (1+2x)^9 dx$ or $\int 7e^{x/4} dx$ can be calculated by a "guess and check" process; there is no need at this stage for a substitution rule of integrals. For example, to find $\int 5\sin 6x\,dx$, a student might "guess" that an answer is cos 6x, check by differentiating to get -6 sin 6x, then adjust to determine that the indefinite integral is $-\dfrac{5}{6}\cos 6x + C$.
- Initial value problems can be treated very informally. For example, suppose that f′(x) = e^{2x} and f(1) = 3. What is f(x)? One antiderivative of e^{2x} is $\dfrac{e^{2x}}{2}$ (check by differentiating), but this function does not satisfy the initial condition. To fix it, students can simply remember that the general antiderivative is $\dfrac{e^{2x}}{2} + C$, then choose C appropriately.
- Play a game by challenging students to write down as fast as possible antiderivatives of simple functions, such as $\int$ f(au + b)du where $\int$ f(x)dx is one of the standard integrals obtained by reversing familiar differentiation results.
- Introduce students to integration tables or software applications for symbolic integration.

SUGGESTED ASSESSMENT STRATEGIES

In making connections between antidifferentiation and differentiation, students should relate the physical world to abstract calculus representations. To assess students' learning, it is appropriate to consider their oral and written explanations of the meaning of antidifferentiation as well as evidence from their computation, their drawing, and their problem-solving activities.

Peer Assessment

- Have students work in pairs to find antidifferentiation problems on the Internet (e.g., at university mathematics department web pages), try to solve the problems they find, and verify their solutions.

Observe/Question

- Ensure students' notation is correct at all times (e.g., $\int e^{3x}dx = \frac{e^{3x}}{3} + C$). It is not necessary to try to justify the dx part of the notation, but the C (constant) part should be understood. Use questions to check understanding, such as "Can you think of a function whose derivative is x^2?"
- Divide the students into small groups, and give them a problem whose conventional solution uses a technique that they have not learned, (e.g., general substitution or integration by parts). For example, ($\int xe^{3x}$ dx, or $\int x^2 \ln x\, dx$. Observe the degree of working control students have of antiderivatives (and derivatives) by seeing how efficiently they can experiment their way to an answer (i.e., generate a variety of plausible ideas).

Collect

- Collect samples of students' worksheets dealing with recovery of functions from their derivatives. Assess the extent to which students are able to compute the antiderivation of functions and determine the value of C, when given initial conditions.

Self/Peer Assessment

- Have students make up tests or quizzes on antidifferentiation techniques and exchange their tests with partners. The partners should check each others' work. Allow the partners time to discuss the work and help each other formulate a plan to address areas of weakness.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  Ch. 6 (Sections 6.1, 6.2)
  pp. 190, 304 - 307
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  pp. 278, 279-282, 319, 325, 342, 347, 374, 490, 495
- Single Variable Calculus Early Transcendentals, Fourth Edition
  Ch. 4 (Section 4.10)
  Ch. 5 (Section 5.4, 5.5)
  p. 585

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 382, 384, 388, 392, 581
  Calculus of a Single Variable, Sixth Edition
  pp. 241, 243, 246, 290-291, 366, 385

PRESCRIBED LEARNING OUTCOMES

It is expected that students will use antidifferentiation to solve a variety of problems.

It is expected that students will:

- use antidifferentiation to solve problems about motion of a particle along a line that involve:
  - computing the displacement given initial position and velocity as a function of time
  - computing velocity and/or displacement given suitable initial conditions and acceleration as a function of time
- use antidifferentiation to find the area under the curve $y = f(x)$, above the x-axis, from $x = a$ to $x = b$
- use differentiation to determine whether a given function or family of functions is a solution of a given differential equation
- use correct notation and form when writing the general and particular solution for differential equations
- model and solve exponential growth and decay problems using a differential equation of the form: $\frac{dy}{dt} = ky$
- model and solve problems involving Newton's Law of Cooling using a differential equation of the form: $\frac{dy}{dt} = ay + b$

SUGGESTED INSTRUCTIONAL STRATEGIES

Antidifferentiation enables students to solve problems that are otherwise unsolvable (e.g., problems related to exponential growth and decay). These problems are found in a variety of contexts, from science to business.

- Demonstrate how physics formulae about motion under constant acceleration can be justified using antidifferentiation. If a(t) = -g, then v(t) = -gt + C for some C, and therefore $s(t) = -\frac{1}{2}gt^2 + Ct + D$ for some D. The two constants of integration can be found by using initial conditions. Given g = 9.81 and a rock thrown upward at a given speed from a 100 m tower, it is possible to calculate the maximum height reached and the time it takes for the rock to hit the ground. There are many variations on this problem.
- To explain why antiderivatives can be used to solve area problems, let f(x) be some given function, and let A(u) be the area under y = f(x), above the x axis, from x=a to x=u. By looking at $\frac{A(u+h) - A(u)}{h}$, we can argue reasonably that A′(u) = f(u), at the same time reinforcing the concept of derivative from the definition. So A(u) is an antiderivative of f(u). A(a)=0.
- Many calculators have a numerical integration feature that approximates $\int_a^b f(x)dx$. Have students find the area from a to b under a curve y=f(x) for which they can find $\int$ f(x)dx. Compare this solution with the calculator's approximation.
- Give students a function such as f(x) = ln x and ask them to make a rough sketch of an antiderivative F(x) of f(x) such that F(1) = 0.
- Illustrate the wide applicability of the concepts by using examples that are not from the physical sciences. For example, let C(x) be the cost of producing x tons of a certain fertilizer. Suppose that (for reasonable x), C′(x) is about 30 - 0.02x. The cost of producing 2 tons is $5000. What is the cost of producing 100 tons?
- Have students generate and maintain a list of phenomena other than radioactive decay that are described by the same differential equation. For example, the illumination I(x) that reaches x metres below the surface of the water can be described by $\frac{dI}{dx} = -kI$. Under constant inflation, the buying power V(t) of a dollar t years of now can be described by $\frac{dV}{dt} = -kV$.

SUGGESTED ASSESSMENT STRATEGIES

An understanding of antidifferentiation and its applications is essential to the solution of the "area under the curve" problems. Students can demonstrate their understanding of antidifferentiation ideas and skills through their problem-solving work.

- To check whether the meaning of the phrase "solution of a differential equation" is fully understood, have students work in groups to discuss and solve problems such as the following:
  - show that $y = x^8$ is a solution of the differential equation $x\frac{dy}{dx} = 8y$
  - find a solution of the above differential equation with $y(2) = 16$
- When students are asked to solve problems such as the following, verify that they are able to identify sin kt and cos kt as solutions of the differential equation and that they can find other solutions as well:
  - A weight hangs from an ideal spring. It is pulled down a few centimetres then released. If y is the displacement of the weight from its rest position, it turns out that $\frac{d^2y}{dt^2} = -k^2y$ for some constant k
- Take a problem (e.g., an exponential decay problem with several parts) and ask for the solution to be written up as a prose report, in complete sentences, with the reasons for each step clearly explained. Criteria for assessment include clarity and grammatical correctness.
- Pose the problem, "In how many different ways can we find the area under $y = x^2$, above the x axis, from $x = 0$ to $x = a$, exactly or approximately?" Students can write a report on this. Students should be able to identify
  - Archimedes' method
  - standard antiderivative method
  - approximation techniques of their own devising
- Ask students to use standard Internet resources to find information about differential equations in various areas of application, and to report on an area of interest to them. Assess the extent to which they are able to express their findings coherently and in their own words.

RECOMMENDED LEARNING RESOURCES

Print Materials

- Calculus: Graphical, Numerical, Algebraic
  Ch. 6 (Section 6.1, 6.4)
  pp. 262 - 263, 311, 333-334
- Calculus of a Single Variable Early Transcendental Functions, Second Edition
  pp. 278, 290, 373-374, 377, 381-382, 385, 409
- Single Variable Calculus Early Transcendentals, Fourth Edition
  Ch. 4 (Section 4.10)
  Ch. 5 (Sections 5.1, 5.4)
  pp. 586, 603, 611

Multimedia

- Calculus: A New Horizon, Sixth Edition
  pp. 328, 408, 433, 580, 601, 611
- Calculus of a Single Variable, Sixth Edition
  Ch. 4 (Section 4.4)
  Ch. 5 (Section 5.5)
  pp. 242, 362

APPENDICES

Mathematics 10 to 12

Appendix A

Prescribed Learning Outcomes

Prescribed Learning Outcomes	
▶ PROBLEM SOLVING It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.	***It is expected that students will:*** • solve problems that involve a specific content area such as, geometry, algebra, trigonometry, statistics, probability, etc. • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • use appropriate technology to assist in problem solving
▶ NUMBER ***(Number Concepts)*** It is expected that students will analyse the numerical data in a table for trends, patterns and interrelationships.	***It is expected that students will:*** • use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data) • use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data)

Prescribed Learning Outcomes

▶ **NUMBER**
(Number Operations)

It is expected that students will:

- use basic arithmetic operations on real numbers to solve problems.
- describe and apply arithmetic operations on tables to solve problems, using technology as required.

It is expected that students will:

- communicate a set of instructions used to solve an arithmetic problem
- perform arithmetic operations on irrational numbers, using appropriate decimal approximations.
- create and modify tables from both recursive and nonrecursive situations
- use and modify a spreadsheet template to model recursive situations
- solve problems involving combinations of tables, using:
  - addition or subtraction of two tables
  - multiplication of a table by a real number
  - spreadsheet functions and templates

▶ **PATTERNS AND RELATIONS**
(Relations and Functions)

It is expected that students will :

- examine the nature of relations with an emphasis on functions.
- represent data, using function models.

It is expected that students will:

- plot linear and nonlinear data, using appropriate scales
- represent data, using function models
- use a graphing tool to draw the graph of a function from its equation
- describe a function in terms of:
  - ordered pairs
  - a rule, in word or equation form
  - a graph
- use function notation to evaluate and represent functions
- determine the domain and range of a relation from its graph
- determine the following characteristics of the graph of a linear function, given its equation:
  - intercepts
  - slope
  - domain
  - range
- use partial variation and arithmetic sequences as applications of linear functions

Prescribed Learning Outcomes

▶ SHAPE AND SPACE *(Measurment)*

It is expected that students will:

- demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects
- solve problems involving triangles, including those found in 3-D and 2-D applications

It is expected that students will:

- calculate the volume and surface area of a sphere, using formulas that are provided
- determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects
- solve problems involving two right triangles
- extend the concepts of sine and cosine for angles through to 180°
- apply the sine and cosine laws, excluding the ambiguous case, to solve problems
- select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes
- analyse the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy
- solve problems involving length, area, volume, time, mass and rates derived from these
- interpret drawings, and use the information to solve problems

▶ SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will solve coordinate geometry problems involving lines and line segments.

It is expected that students will:

- solve problems involving distances between points in the coordinate plane
- solve problems involving midpoints of line segments
- solve problems involving rise, run and slope of line segments
- determine the equation of a line, given information that uniquely determines the line
- solve problems using slopes of:
  - parallel lines
  - perpendicular lines

Prescribed Learning Outcomes

▶ STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will implement and analyse sampling procedures, and draw appropriate inferences from the data collected.

It is expected that students will:

- choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population
- defend or oppose inferences and generalizations about populations, based on data from samples
- determine the equation of a line of best fit, using:
  - estimate of slope and one point
  - least squares method with technology
- use technological devices to determine the correlation coefficient *r*
- interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots

Prescribed Learning Outcomes

▶ PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, or probability
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solvingIt is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

▶ NUMBER
(Number Operations)

It is expected that students will solve consumer problems, using arithmetic operations.

It is expected that students will:

- solve consumer problems, including:
  - wages earned in various situations
  - property taxation
  - exchange rates
  - unit prices
- reconcile financial statements including:
  - cheque books with bank statements
  - cash register tallies with daily receipts
- solve budget problems, using graphs and tables to communicate solutions
- solve investment and credit problems involving simple and compound interestIt is expected that students will analyse the numerical data in a table for trends, patterns and interrelationships.

Prescribed Learning Outcomes

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PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will:

- represent and analyse situations that involve expressions, equations and inequalities.
- use linear programming to solve optimization problems.

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It is expected that students will:

- graph linear inequalities, in two variables
- solve systems of linear equations, in two variables:
  - algebraically (elimination and substitution)
  - graphically
- solve nonlinear equations, using a graphing tool.
- solve systems of linear inequalities in two variables using graphing technology
- design and solve linear and nonlinear systems, in two variables, to model problem situations
- apply linear programming to find optimal solutions to decision-making problems

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PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.

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It is expected that students will:

- determine the following characteristics of the graph of a quadratic function:
  - vertex
  - domain and range
  - axis of symmetry
  - intercepts

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Prescribed Learning Outcomes

▶ **SHAPE AND SPACE *(Measurment)*** It is expected that students will: • demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. • use measuring devices to make estimates and to perform calculations in solving problems.	***It is expected that students will:*** • enlarge or reduce a dimensioned object, according to a specified scale • calculate maximum and minimum values, using tolerances, for lengths, areas and volumes • solve problems involving percentage error when input variables are expressed with percentage errors • design an appropriate measuring process or device to solve a problem
▶ **SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*** It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.	***It is expected that students will:*** • use technology with dynamic geometry software to confirm and apply the following properties: - the perpendicular from the centre of a circle to a chord bisects the chord - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc - the inscribed angles subtended by the same arc are congruent - the angle inscribed in a semicircle is a right angle - the opposite angles of a cyclic quadrilateral are supplementary - a tangent to a circle is perpendicular to the radius at the point of tangency - the tangent segments to a circle, from any external point, are congruent - the sum of the interior angles of an n-sided polygon is equal to $(2n - 4)$ right angles • use properties of circles and polygons to solve design and layout problems

Prescribed Learning Outcomes

▶ STATISTICS AND PROBABILITY ***(Data Analysis)*** It is expected that students will analyse graphs or charts of given situations to derive specific information.	***It is expected that students will:*** • extract information from given graphs of discrete or continuous data, using: - time series - continuous data - contour lines • draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data • design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation • collect experimental data and use best-fit exponential and quadratic functions, to make predictions and solve problems

Prescribed Learning Outcomes

▶ A: Problem Solving

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

A1. solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, probability, etc.
A2. solve problems that involve more than one content area
A3. solve problems that involve mathematics within other disciplines
A4. analyse problems and identify the significant elements
A5. develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
- guess and check
- look for a pattern
- make a systematic list
- make and use a drawing or model
- eliminate possibilities
- work backward
- simplify the original problem
- develop alternative original approaches
- analyse keywords

A6. demonstrate the ability to work individually and co-operatively to solve problems
A7. determine that their solutions are correct and reasonable
A8. clearly communicate a solution to a problem and the process used to solve it
A9. use appropriate technology to assist in problem solving

▶ B: Number *(Number Operations)*

It is expected that students will describe and apply operations on matrices to solve problems, using technology as required.

It is expected that students will:

B1. model and solve problems, including those solved previously, using technology to perform matrix operations of addition, subtraction, and scalar multiplication as required
B2. model and solve consumer and network problems using technology to perform matrix multiplication as required

Prescribed Learning Outcomes

▶ **C: PATTERNS AND RELATIONS** ***(Variables and Equations)*** It is expected that students will design or use a spreadsheet to make and justify financial decisions.	***It is expected that students will:*** **C1.** design a financial spreadsheet template to allow users to input their own variables **C2.** analyse the costs and benefits of renting or buying an increasing asset, such as land or property, under various circumstances **C3.** analyse the costs and benefits of leasing or buying a decreasing asset, such as a vehicle or computer, under various circumstances **C4.** analyse an investment portfolio applying such concepts as interest rate, rate of return and total return
▶ **D: PATTERNS AND RELATIONS** ***(Patterns)*** It is expected that students will generate and analyze cyclic, recursive, and fractal paterns	***It is expected that students will:*** **D1.** describe periodic events, including those represented by sinusoidal curves, using the terms amplitude, period, maximum and minimum values, vertical and horizontal shift **D2.** collect sinusoidal data; graph the graph using technology, and, represent the data with a best fit equation of the form: - $y = a \sin (bx + c) + d$ **D3.** use best fit sinusoidal equations, and their associated graphs, to make predictions (interpolation, extraction) **D4.** use technology to generate and graph sequences that model real-life phenomena **D5.** use technology to contruct a fractal pattern by repeatedly applying a procedure to a geometric figure **D6.** use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns

Prescribed Learning Outcomes	
E: SHAPE AND SPACE ***(Measurment)*** It is expected that students will analyse objects, shapes and processes to solve cost and design problems.	***It is expected that students will:*** **E1.** use dimensions and unit prices to solve problems involving perimeter, area and volume **E2.** solve problems involving estimation and costing for objects, shapes or processes when a design is given **E3.** design an object, shape, layout or process within a specified budget **E4.** use simplified models to estimate the solutions to complex measurement problems
F: SHAPE AND SPACE ***(3-D Objects and 2-D Shapes)*** It is expected that students will solve problems involving polygons and vectors, including both 3-D and 2-D applications.	***It is expected that students will:*** **F1.** use appropriate terminology to describe: - vectors (i.e., direction, magnitude) - scalar quantities (i.e., magnitude) **F2.** assign meaning to the multiplication of a vector by a scalar **F3.** determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods **F4.** model and solve problems in 2-D and simple 3-D, using vector diagrams and technology

Prescribed Learning Outcomes

G: STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will:

- use normal and binomial probability distributions to solve problems involving uncertainty.
- solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.
- model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

It is expected that students will:

G1. find the population standard deviation of a data set or a probability distribution, using technology

G2. use z-scores and the normal distribution to solve problems

G3. use the normal approximation to the binomial distribution to solve problems involving probability calculations for large samples (where $npq>10$)

G4. solve pathway problems, interpreting and applying any constraints

G5. use the fundamental counting principle to determine the number of different ways to perform multistep operations

G6. construct a sample space for two or three events

G7. classify events as independent or dependent

G8. solve problems, using the probabilities of mutually exclusive and complementary events

Prescribed Learning Outcomes	
▶ **PROBLEM SOLVING** It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems	*It is expected that students will:* • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving
▶ **PERSONAL BANKING** It is expected that students will prepare bank forms including cheques, deposit slips, chequebook activity record and reconcilation statements	*It is expected that students will:* • name and describe various types of commonly used consumer bank accounts • complete various banking forms • describe the use of a bank card for automated teller machines (ATMs) and debit payments • identify different types of bank service charges and their relative costs • reconcile financial statements, such as chequebooks and electronic bank transactions with bank statements

Prescribed Learning Outcomes	
▶ **WAGES, SALARIES AND EXPENSES** It is expected that students will solve problems involving wage, salaries and expenses	*It is expected that students will:* • calculate hours worked and gross pay • calculate net income using deduction tables (focus on weekly) with different pay periods • calculate changes in income • develop a budget that matches predicted income
▶ **SPREADSHEETS** It is expected that students will design and use a spreadsheet to make and justify decisions	*It is expected that students will:* • create a spreadsheet using various formatting options • use a spreadsheet template to solve problems • create a spreadsheet using formulas and functions • use a spreadsheet to answer "what-if" questions • identify where spreadsheets could be effectively used
▶ **RATES, RATIO AND PROPORTION** It is expected that students will apply the concepts of rate, ratio and proportion to solve problems	*It is expected that students will:* • use the concept of unit rate to determine the best buy on a consumer item and justify the decision • solve problems on the application of sales tax in Canada • describe a variety of sales promotion techniques and their financial implications for the consumer • solve rate, ratio, and proportion problems involving length, area, volume, time, mass, and rates derived from these

Prescribed Learning Outcomes	
▶ **TRIGONOMETRY** It is expected that students will demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles	***It is expected that students will:*** • apply ratio and proportion in similar triangles • use the trigonometric ratios sine, cosine, and tangent in solving right triangles

▶ **GEOMETRY PROJECT**
 	
It is expected that students will complete a project that includes a 2-D plan and a 3-D model of some physical structure
	It is expected that students will:
	
	• measure lengths using both SI Metric and Imperial units
	• estimate measurements of objects in SI and Imperial systems including:
	  - length
	  - area
	  - volume
	  - mass
	• draw top, front and side views for both 3-D rod or block objects and their sketches
	• sketch and build 3-D designs using isometric dot paper
	• determine the relationships among linear scale factors, areas, surface areas, and volumes of similar figures and objects
	• enlarge or reduce a dimensioned object according to a specified scale
	• solve problems involving linear dimensions, area, and volume
	• interpret drawings and use the information to solve problems
	• complete a project that includes a 2-D plan and a 3-D model of some physical structure

Prescribed Learning Outcomes

▶ PROBABILITY AND SAMPLING

It is expected that students will develop and implement a plan for the collection, display and analysis of data using technology as required

It is expected that students will:

- read and interpret graphs
- discuss how collected data are affected by the nature of the sample, the method of collection, the sample size and biases
- describe issues to be considered when collecting data (e.g., appropriate language, ethics, cost, privacy, cultural sensitivity)
- select, defend and use appropriate methods of collecting data:
  - designing and using questionnaires
  - interviews
  - experiments
  - research
- determine and use measures of central tendency to support decisions
- use sample data to make predictions and decisions
- use suitable graph types to display data (by hand or using technology)
- critique ways in which statistical information and conclusions are presented by the media and other sources

	Prescribed Learning Outcomes

▶ **Problem Solving**
 	
It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems
	It is expected that students will:
	
	• solve problems that involve a specific content area
	• solve problems that involve more than one content area
	• solve problems that involve mathematics within other disciplines
	• analyse problems and identify the significant elements
	• develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
	  - guess and check
	  - look for a pattern
	  - make a systematic list
	  - make and use a drawing or model
	  - eliminate possibilities
	  - work backward
	  - simplify the original problem
	  - develop alternative original approaches
	  - analyse keywords
	• demonstrate the ability to work individually and co-operatively to solve problems
	• determine that their solutions are correct and reasonable
	• clearly communicate a solution to a problem and the process used to solve it
	• interpret their solutions by describing what the solution means within the context of the original problem
	• use appropriate technology to assist in problem solving
	• determine that their solutions are correct and reasonable
	• clearly communicate a solution to a problem and the process used to solve it
	• interpret their solutions by describing what the solution means within the context of the original problem
	• use appropriate technology to assist in problem solving

▶ **Relations and Formulas**
 	
It is expected that students will represent and interpret relations in a variety of contexts
	It is expected that students will:
	
	• express a linear relation of the form $y = mx + b$
	  - in words
	  - as a formula
	  - with a table of values
	  - as a graph
	• interpolate and extrapolate values from the graph of a linear relation
	• determine the slope of a linear relation and describe it in words and interpret its meaning in a problem context
	• interpret the graph of a relation and describe it in words
	• construct a graph of a relation from its description in words
	• evaluate formulas

Prescribed Learning Outcomes	

▶ **INCOME AND DEBT**
 	
It is expected that students will demonstrate an awareness of selected forms of personal income and debt
	It is expected that students will:
	
	• solve problems involving performance-based income, including commission sales, piece work, and salary plus commission
	• use simple and compound interest calculations to solve problems
	• solve consumer problems involving:
	  - credit cards
	  - exchange rates
	  - personal loans

▶ **DATA ANALYSIS AND INTERPRETATION** It is expected that students will analyse data with a focus on the validity of its presentation and the inferences made	*It is expected that students will:* • display and analyze data on a line plot • manipulate the presentation of data to stress a particular point of view
▶ **MEASUREMENT TECHNOLOGY** It is expected that students will determine measurements in Systeme International (SI) and Imperial systems using different measuring devices	*It is expected that students will:* • select and use appropriate measuring devices and measurement units in Imperial or SI systems • perform basic conversions within and between the Imperial and SI systems, using technology as appropriate • use measurement strategies to solve problems

▶ **OWNING AND OPERATING A VEHICLE**
 	
It is expected that students will analyse the cost of acquiring and operating a vehicle
	It is expected that students will:
	
	• solve problems involving the acquisition and operation of a vehicle, including
	  - renting
	  - leasing
	  - buying
	  - licensing
	  - insuring
	  - operating (e.g., fuel and oil)
	  - maintaining (e.g., repairs, tune-ups)

Prescribed Learning Outcomes	
▶ **PERSONAL INCOME TAX** It is expected that students will prepare a simple income tax form	*It is expected that students will:* • prepare an income tax form for an individual who is single, employed, and without dependents
▶ **APPLICATIONS OF PROBABILITY** It is expected that students will demonstrate an awareness of the applications of probability to real world situations	*It is expected that students will:* • express probabilities as ratios, fractions, decimals, percents, and in words • use probability to predict the result in a given situation • determine the odds for and against a particular event occurring • compare experimental observations with theoretical predictions • use probabilities to calculate expected gains and losses • communicate and justify solutions to probability problems
▶ **BUSINESS PLAN** It is expected that students will prepare a plan and operate a successful business	*It is expected that students will:* • prepare a business plan to own and operate a business that includes the following information as appropriate: - monthly cost of operation (e.g., inventory, rent, wages, insurance, advertising, loan payments, etc.) - hours of operation - estimated daily sales (average) - gross/net profit - hourly earnings • draw a scale floor plan of the store

Prescribed Learning Outcomes	
▶ **A: PROBLEM SOLVING** It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems	*It is expected that students will:* • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving
▶ **B: PERSONAL FINANCE** It is expected that students will solve consumer problems involving insurance, mortgages and loans	*It is expected that students will:* • solve problems involving different types of insurance • determine the costs involved in purchasing a home, including gross debt service ratio • solve problems involving different types of mortgages

Prescribed Learning Outcomes	
▶ **C: DESIGN AND MEASUREMENTS** It is expected that students will analyse objects, shapes and processes to solve cost and design problems	*It is expected that students will:* • analyze objects shown in "exploded" format • draw objects in "exploded" format • solve problems involving estimation and costing for objects, shapes, or processes when a design is given • design an object within a specified budget
▶ **D: GOVERNMENT FINANCES** It is expected that students will demonstrate an awareness of the income and expenditures of federal, provincial and municipal governments	*It is expected that students will:* • describe government expenditures including the amounts spent on social welfare benefits, social security, education, health care, policing, armed forces, and employee wages and salaries • solve problems involving the calculation of selected federal taxes (e.g., GST, excise tax and duties) • calculate provincial taxes, (e.g., PST, corporation capital, licenses, gasoline) • determine how selected municipal taxes are calculated (e.g., property)
▶ **E: INVESTMENTS** It is expected that students will demonstrate and recognize the differences concerning different types of financial investments	*It is expected that students will:* • determine a financial plan to achieve personal goals • describe different investment vehicles (e.g., GICs, bonds, mutual funds, stocks, and real estate) • compare and contrast different investment vehicles in terms of risk factors, rates of return, costs, and lengths of term • identify reasons for investing money in RRSPs and RESPs • investigate how to purchase and sell stocks
▶ **F: TAXATION** It is expected that students will demonstrate an ability to fill out an income tax form	*It is expected that students will:* • fill out income tax forms for: - a single parent with child - a married couple with one person earning an income - a married couple with child

Prescribed Learning Outcomes	
▶ G: VARIATION AND FORMULAS It is expected that students will use algebraic and graphical models to generate patterns, make prediction and solve patterns	*It is expected that students will:* • plot and analyse examples of direct variation, partial variation, and inverse variation • given data, graph, or a situation, recognize the variation represented • use formulas to solve problems
▶ H: LIFE/CAREER PROJECT It is expected that students will research career choices and perform a comparative study	*It is expected that students will:* • determine what factors are important in analysing careers • describe two specific career opportunities • identify mathematical educational requirements for two careers • compare two careers in terms of salary, working hours, training time and cost, cost of living, and benefits

Prescribed Learning Outcomes

▶ PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, probability
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly explain the solution to a problem and justify the processes used to solve it
- use appropriate technology to assist in problem solving

▶ NUMBER
(Number Concepts)

It is expected that students will:

- analyse the numerical data in a table for trends, patterns and interrelationships.
- explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

It is expected that students will:

- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data)
- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data)
- classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are “nested” within the real number system

Prescribed Learning Outcomes

▶ **NUMBER** *(Number Operations)* It is expected that students will: • use basic arithmetic operations on real numbers to solve problems. • describe and apply arithmetic operations on tables to solve problems, using technology as required. • use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.	*It is expected that students will:* • communicate a set of instructions used to solve an arithmetic problem • perform arithmetic operations on irrational numbers, using appropriate decimal approximations • create and modify tables from both recursive and nonrecursive situations • use and modify a spreadsheet template to model recursive situations • perform operations on irrational numbers of monomial and binomial form, using exact values • explain and apply the exponent laws for powers of numbers and for variables with rational exponents
▶ **PATTERNS AND RELATIONS** *(Patterns)* It is expected that students will generate and analyse number patterns. It is expected that students will generate number patterns exhibiting arithmetic growth	• use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems • relate arithmetic sequences to linear functions defined over the natural numbers • generate number patterns exhibiting geometric growth

Prescribed Learning Outcomes

▶ **PATTERNS AND RELATIONS** ***(Variables and Equations)***
 	
It is expected that students will generalize operations on polynomials to include rational expressions.
	It is expected that students will:
	
	• factor polynomial expressions of the form $ax^2 + bx + c$ and $a^2x^2 - b^2x^2$
	• find the product of polynomials
	• divide a polynomial by a binomial and express the result in the forms:
	  - $\frac{P}{D} = Q + \frac{R}{D}$
	  - $P = DQ + R$
	  - $P(x) = D(x)Q(x) + R$
	• determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored
	• determine the nonpermissible values for the variable in rational expressions
	• perform the operations of addition, subtraction, multiplication, and division on rational expressions
	• find and verify the solutions of rational equations that reduce to linear form

▶ **PATTERNS AND RELATIONS** ***(Relations and Functions)***
 	
It is expected that students will:
 	
• examine the nature of relations with an emphasis on functions.
• represent data, using linear function models.
	It is expected that students will:
	
	• plot linear and nonlinear data, using appropriate scales
	• represent data, using function models
	• use a graphing tool to draw the graph of a function from its equation
	• describe a function in terms of:
	  - ordered pairs
	  - a rule, in word or equation form
	  - a graph
	• use function notation to evaluate and represent functions
	• determine the domain and range of a relation from its graph
	• determine the following characteristics of the graph of a linear function, given its equation:
	  - intercepts
	  - slope
	  - domain
	  - range
	• use partial variation and arithmetic sequences as applications of linear functions

Prescribed Learning Outcomes	
▶ **SHAPE AND SPACE *(Measurement)*** It is expected that students will: • demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. • solve problems involving triangles, including those found in 3-D and 2-D applications.	***It is expected that students will:*** • calculate the volume and surface area of a sphere, using formulas that are provided • determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects • solve problems involving two right triangles • extend the concepts of sine and cosine for angles through to 180° • apply the sine and cosine laws, excluding the ambiguous case, to solve problems
▶ **SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*** It is expected that students will solve coordinate geometry problems involving lines and line segments.	***It is expected that students will:*** • solve problems involving distances between points in the coordinate plane • solve problems involving midpoints of line segments • solve problems involving rise, run and slope of line segments • determine the equation of a line, given information that uniquely determines the line • solve problems using slopes of: - parallel lines - perpendicular lines

Prescribed Learning Outcomes

STATISTICS AND PROBABILITY *(Data Analysis)* It is expected that students will implement and analyse sampling procedures, and draw appropriate inferences from the data collected.	***It is expected that students will:*** • choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population • defend or oppose inferences and generalizations about populations, based on data from samples

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*
 	
It is expected that students will solve coordinate geometry problems involving lines and line segments.
	It is expected that students will:
	• solve problems involving distances between points in the coordinate plane
	• solve problems involving midpoints of line segments
	• solve problems involving rise, run and slope of line segments
	• determine the equation of a line, given information that uniquely determines the line
	• solve problems using slopes of:
	  - parallel lines
	  - perpendicular lines

Prescribed Learning Outcomes

▶ Problem Solving

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems

It is expected that students will:

- solve problems that involve a specific content area such as geometry, algebra, trigonometry, statistics, probability
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly explain the solution to a problem and justify the processes used to solve it
- use appropriate technology to assist in problem solving

▶ Number *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

It is expected that students will:

- solve consumer problems, including:
  - wages earned in various situations
  - property taxation
  - exchange rates
  - unit prices
- reconcile financial statements including:
  - cheque books with bank statements
  - cash register tallies with daily receipts
- solve budget problems, using graphs and tables to communicate solutions
- solve investment and credit problems involving simple and compound interest

Prescribed Learning Outcomes

▶ PATTERNS AND RELATIONS *(Patterns)* It is expected that students will apply the principles of mathematical reasoning to solve problems and to justify solutions.	***It is expected that students will:*** • differentiate between inductive and deductive reasoning • explain and apply connecting words, such as *and*, *or*, and *not*, to solve problems • use examples and counterexamples to analyse conjectures • distinguish between an *if–then* proposition, its converse and its contrapositive • prove assertions in a variety of settings, using direct and indirect reasoning

▶ PATTERNS AND RELATIONS *(Variables and Equations)*
 	
It is expected that students will represent and analyse situations that involve expressions, equations and inequalities.
	It is expected that students will:
	
	• graph linear inequalities, in two variables
	• solve systems of linear equations, in two variables:
	  - algebraically (elimination and substitution)
	  - graphically
	• solve systems of linear equations, in three variables:
	  - algebraically
	  - with technology
	• solve non-linear equations, using a graphing tool
	• solve non-linear equations:
	  - by factoring
	  - graphically
	• use the Remainder Theorem to evaluate polynomial expressions, the Rational Zeros Theorem, and the Factor Theorem to determine factors of polynomials
	• determine the solution to a system of nonlinear equations, using technology as appropriate

Prescribed Learning Outcomes

▶ PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will:

- represent and analyse quadratic, polynomal and rational functions, using technology as appropriate
- examine the nature of relations with an emphasis on functions

It is expected that students will:

- determine the following characteristics of the graph of a quadratic function:
  - vertex
  - domain and range
  - axis of symmetry
  - intercepts
- perform operations on functions and compositions of functions
- determine the inverse of a function
- connect algebraic and graphical transformations of quadratic functions, using completing the square as required
- model real-world situations, using quadratic functions
- solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:
  - factoring
  - the quadratic formula
  - graphing
- determine the character of the real and non-real roots of a quadratic equation, using:
  - the discriminant in the quadratic formula
  - graphing
- describe, graph, and analyse polynomial and rational functions, using technology
- formulate and apply strategies to solve absolute value equations, radical equations, rational equations, and inequalities

▶ SHAPE AND SPACE *(Measurement)*

It is expected that students will:

- solve problems involving triangles, including those found in 3-D and 2-D applications.
- solve coordinate geometry problems involving lines and line segments, and justify the solutions.

It is expected that students will:

- solve problems involving ambiguous case triangles in 3-D and 2-D
- solve problems involving distances between points and lines
- verify and prove assertions in plane geometry, using coordinate geometry

Prescribed Learning Outcomes

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

It is expected that students will:

- use technology with dynamic geometry software to confirm and apply the following properties:
  - the perpendicular from the center of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle, from any external point, are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is 180(n-2)
- prove the following general properties, using established concepts and theorems:
  - the perpendicular bisector of a chord contains the center of the circle
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the center of the circle is in the interior of the inscribed angle)
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle from any external point are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is 180(n-2)
- solve problems, using a variety of circle properties, and justify the solution strategy used

Prescribed Learning Outcomes

▶ A: PROBLEM SOLVING It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems	***It is expected that students will:*** **A1.** solve problems that involve a specific content area such as, geometry, algebra, trigonometry, statistics, probability, etc. **A2.** solve problems that involve more than one content area **A3.** solve problems that involve mathematics within other disciplines **A4.** analyse problems and identify the significant elements **A5.** develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords **A6.** demonstrate the ability to work individually and co-operatively to solve problems **A7.** determine that their solutions are correct and reasonable **A8.** clearly communicate a solution to a problem and the process used to solve it **A9.** use appropriate technology to assist in problem solving
▶ B: PATTERNS AND RELATIONS *(Patterns)* It is expected that students will generate and analyse exponential patterns.	***It is expected that students will:*** **B1.** derive and apply expressions to represent general terms and sums for geometric growth and to solve problems **B2.** connect geometric sequences to exponential functions over the natural numbers **B3.** estimate values of expressions for infinite geometric processes

Prescribed Learning Outcomes

▶ **C: PATTERNS AND RELATIONS *(Variables and Equations)*** It is expected that students will solve exponential, logarithmic and trigonometric equations and identities.	*It is expected that students will:* **C1.** solve exponential equations having bases that are powers of one another **C2.** solve and verify exponential and logarithmic equations and identities **C3.** distinguish between degree and radian measure, and solve problems using both **C4.** determine the exact and the approximate values of trigonometric ratios for any multiples of 0˚, 30˚, 45˚, 60˚ and 90˚ and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ **C5.** solve first and second degree trigonometric equations over a domain of length 2π: - algebraically - graphically **C6.** determine the general solutions to trigonometric equations where the domain is the set of real numbers **C7.** analyse trigonometric identities: - graphically - algebraically for general cases **C8.** use sum, difference,and double angle identities for sine and cosine to verify and simplify trigonometric expressions
▶ **D: PATTERNS AND RELATIONS *(Relations and Functions)*** It is expected that students will represent and analyse exponential and logarithmic functions, using technology as appropriate.	*It is expected that students will:* **D1.** model, graph, and apply exponential functions to solve problems **D2.** change functions from exponential form to logarithmic form and vice versa **D3.** model, graph, and apply logarithmic functions to solve problems **D4.** explain the relationship between the laws of logarithms and the laws of exponents **D5.** describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position **D6.** draw (using technology), sketch, and analyse the graphs of sine, cosine, and tangent functions, for: - amplitude, if defined - period - domain and range - asymptotes, if any - behavior under transformations **D7.** use trigonometric functions to model and solve problems

Prescribed Learning Outcomes	
▶ **E: SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*** It is expected that students will classify conic sections, using their shapes and equations.	***It is expected that students will:*** **E1** classify conic sections according to shape **E2.** classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only) **E3.** convert a given equation of a conic section from general to standard form and vice versa
▶ **F: SHAPE AND SPACE *(Transformations)*** It is expected that students will perform, analyse and create transformations of functions and relations that are described by equations or graphs	***It is expected that students will:*** **F1.** describe how various translations of functions affect graphs and their related equations: - $y = f(x-h)$ - $y-k = f(x)$ **F2.** describe how various stretches of functions (compressions and expansions) affect graphs and their related equations: - $y = af(x)$ - $y = f(kx)$ **F3.** describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations: - $y = f(-x)$ - $y = -f(x)$ - $y = f^{-1}(x)$ **F4.** using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$ **F5.** using the graph and/or the equation of $f(x)$, describe and sketch $\lvert f(x)\rvert$ **F6.** describe and perform single transformations and combinations of transformations on functions and relations

Prescribed Learning Outcomes

G: STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will:

- use normal and binomial probability distributions to solve problems involving uncertainty.
- solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.
- model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

It is expected that students will:

G1. find the standard deviation of a data set or a probability distribution, using technology

G2. use z-scores and the normal distribution to solve problems

G3. use the normal approximation to the binomial distribution to solve problems involving probability calculations for large samples

G4. solve pathway problems, interpreting (where npq>10) and applying any constraints

G5. use the fundamental counting principle to determine the number of different ways to perform multistep operations

G6. determine the number of permutations of n different objects taken r at a time, and use this to solve problems

G7. determine the number of combinations of n different objects taken r at a time, and use this to solve problems

G8. solve problems, using the binomial theorem where N belongs to the set of natural numbers

G9. construct a sample space for two or three events

G10. classify events as independent or dependent

G11. solve problems, using the probabilities of mutually exclusive and complementary events

G12. determine the conditional probability of two events

G13. solve probability problems involving permutations, and combinations and conditional probability

Prescribed Learning Outcomes

▶ **PROBLEM SOLVING** It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.	*It is expected that students will:* • solve problems that involve a specific content area (e.g., geometry, algebra, trigonometry statistics, probability) • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and justify the process used to solve it • use appropriate technology to assist in problem solving
▶ **OVERVIEW AND HISTORY OF CALCULUS** ***(Overview of Calculus)*** It is expected that students will understand that calculus was developed to help model dynamic situations.	*It is expected that students will:* • distinguish between static situations and dynamic situations. • identify the two classical problems that were solved by the discovery of calculus: - the tangent problem - the area problem • describe the two main branches of calculus: - differential calculus - integral calculus • understand the limit process and that calculus centers around this concept

Prescribed Learning Outcomes

▶ OVERVIEW AND HISTORY OF CALCULUS *(Historical Development of Calculus)*

It is expected that students will understand the historical background and problems that led to the development of calculus.

It is expected that students will:

- describe the contributions made by various mathematicians and philosophers to the development of calculus, including:
  - Archimedes
  - Fermat
  - Descartes
  - Barrow
  - Newton
  - Leibniz
  - Jakob and Johann Bernoulli
  - Euler
  - L'Hospital

▶ FUNCTIONS, GRAPHS, AND LIMITS *(Functions and their Graphs)*

It is expected that students will represent and analyze rational, inverse trigonometric, base e exponential, natural logarithmic, elementary implicit, and composite functions, using technology as appropriate.

It is expected that students will:

- model and apply inverse trigonometric, base e exponential, natural logarithmic, elementary implicit and composite functions to solve problems
- draw (using technology), sketch and analyze the graphs of rational, inverse trigonometric, base *e* exponential, natural logarithmic, elementary implicit, and composite functions for:
  - domain and range
  - intercepts
- recognize the relationship between a base a exponential function ($a > 0$) and the equivalent base *e* exponential function (convert $y = a^x$ to $y = e^{x(\ln a)}$)
- determine, using the appropriate method (analytic or graphing utility) the points where $f(x) = 0$

Prescribed Learning Outcomes

► FUNCTIONS, GRAPHS, AND LIMITS *(Limits)*

It is expected that students will understand the concept of a limit of a function, notation used, and be able to evaluate the limit of a function.

It is expected that students will:

- demonstrate an understanding of the concept of limit and notation used in expressing the limit of a function as x approaches a: $\lim_{x \to a} f(x)$
- evaluate the limit of a function
  - analytically
  - graphically
  - numerically
- distinguish between the limit of a function $f(x)$ as x approaches a and the value of the function at $x = a$
- demonstrate an understanding of the concept of one-sided limits and evaluate one-sided limits
- determine limits that result in infinity (infinite limits)
- evaluate limits of functions as x approaches infinity (limits at infinity)
- determine vertical and horizontal asymptotes of a function, using limits
- determine whether a function is continuous at $x = a$

► THE DERIVATIVE *(Concept and Interpretations)*

It is expected that students will understand the concept of a derivative and evaluate derivatives of a function using the definition of derivative.

It is expected that students will:

- describe geometrically a secant line and a tangent line for the graph of a function at $x = a$.
- define and evaluate the derivative at $x = a$ as: $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ and $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
- define and calculate the derivative of a function using: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ or $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$
- use alternate notation interchangeably to express derivatives (i.e., $f'(x), \frac{dy}{dx}, y'$, etc.)
- compute derivatives using the definition of derivative
- distinguish between continuity and differentiability of a function at a point
- determine when a function is non-differentiable, and explain why
- determine the slope of a tangent line to a curve at a given point
- determine the equation of the tangent line to a curve at a given point
- for a displacement function $s = s(t)$, calculate the average velocity over a given time interval and the instantaneous velocity at a given time
- distinguish between average and instantaneous rate of change

Prescribed Learning Outcomes

▶ THE DERIVATIVE *(Computing Derivatives)*

It is expected that students will determine derivatives of functions using a variety of techniques.

It is expected that students will:

- compute and recall the derivatives of elementary functions including:
  - $\frac{d}{dx}(x^r)=rx^{r-1}$, r is real
  - $\frac{d}{dx}(e^x)=e^x$
  - $\frac{d}{dx}(\ln x)=\frac{1}{x}$
  - $\frac{d}{dx}(\cos x)=-\sin x$
  - $\frac{d}{dx}(\sin x)=\cos x$
  - $\frac{d}{dx}(\tan x)=\sec^2 x$
  - $\frac{d}{dx}(\sin^{-1} x)=\frac{1}{\sqrt{1-x^2}}$
  - $\frac{d}{dx}(\tan^{-1} x)=\frac{1}{1+x^2}$
- use the following derivative formulas to compute derivatives for the corresponding types of functions:
  - constant times a function: $\frac{d}{dx}cu=c\frac{du}{dx}$
  - $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$ (sum rule)
  - $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$ (product rule)
  - $\frac{d}{dx}(\frac{u}{v})=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$ (quotient rule)
  - $\frac{d}{dx}(u^n)=nu^{n-1}\frac{du}{dx}$ (power rule)
- use the Chain Rule to compute the derivative of a composite function:

  $\frac{dy}{dx}=\frac{du}{dx}\frac{dy}{du}$ or $\frac{d}{dx}(F(g(x)))=g'(x)F'(g(x))$
- compute the derivative of an implicit function.
- use the technique of logarithmic differentiation.
- compute higher order derivatives

Prescribed Learning Outcomes	
► **APPLICATIONS OF DERIVATIVES** *(Applied Problems)* It is expected that students will solve applied problems from a variety of fields including the Physical and Biological Sciences, Economics, and Business.	*It is expected that students will:* • solve problems involving displacement, velocity, acceleration • solve related rates problems • solve optimization problems (applied maximum / minimum problems)

► **APPLICATIONS OF DERIVATIVES** *(Derivatives and the Graph of the Function)*
 	
It is expected that students will use the first and second derivatives to describe the characteristic of the graph of a function.
	It is expected that students will:
	
	• given the graph of $y = f(x)$:
	  - graph $y = f'(x)$ and $y = f''(x)$
	  - relate the sign of the derivative on an interval to whether the function is increasing or decreasing over that interval.
	  - relate the sign of the second derivative to the concavity of a function.
	• determine the critical numbers and inflection points of a function.
	• determine the maximum and minimum values of a function and use the first and / or second derivative test(s) to justify their solutions
	• use Newton's iterative formula (with technology) to find the solution of given equations, $f(x) = 0$.
	• use the tangent line approximation to estimate values of a function near a point and analyze the approximation using the second derivative.

Prescribed Learning Outcomes

► ANTIDIFFERENTIATION *(Recovering Functions from their Derivatives)*

It is expected that students will recognize antidifferentiation (indefinite integral) as the reverse of the differentiation process.

It is expected that students will:

- explain the meaning of the phrase "$F(x)$ is an antiderivative (or indefinite integral) of f(x)."
- use antiderivative notation appropriately $\int f(x)dx$ (i.e., for the antiderivative of $f(x)$).
- compute the antiderivatives of linear combinations of functions whose individual antiderivatives are known including:
  - $\int k\,dx = kx + C$
  - $\int x^r dx = \frac{x^{r+1}}{r+1} + C$ if $r \neq -1$
  - $\int \frac{dx}{x} = \ln|x| + C$
  - $\int e^x dx = e^x + C$
  - $\int \sin x\,dx = -\cos x + C$
  - $\int \cos x\,dx = \sin x + C$
  - $\int \sec^2 x\,dx = \tan x + C$
  - $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
  - $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
- compute $\int f(ax+b)\,dx$ if $\int f(u)\,du$ is known.
- create integration formulas from the known differentiation formulas
- solve initial value problems using the concept that if $F'(x) = G'(x)$ on an interval, then $F(x)$ and $G(x)$ differ by a constant on that interval.

Prescribed Learning Outcomes

► ANTIDIFFERENTIATION *(Applications of Antidifferentiation)*

It is expected that students will use antidifferentiation to solve a variety of problems.

It is expected that students will:

- use antidifferentiation to solve problems about motion of a particle along a line that involve:
  - computing the displacement given initial position and velocity as a function of time
  - computing velocity and/or displacement given suitable initial conditions and acceleration as a function of time
- use antidifferentiation to find the area under the curve $y = f(x)$, above the x-axis, from $x = a$ to $x = b$.
- use differentiation to determine whether a given function or family of functions is a solution of a given differential equation
- use correct notation and form when writing the general and particular solution for differential equations.
- model and solve exponential growth and decay problems using a differential equation of the form: $\frac{dy}{dt} = ky$
- model and solve problems involving Newton's Law of Cooling using a differential equation of the form: $\frac{dy}{dt} = ay + b$.

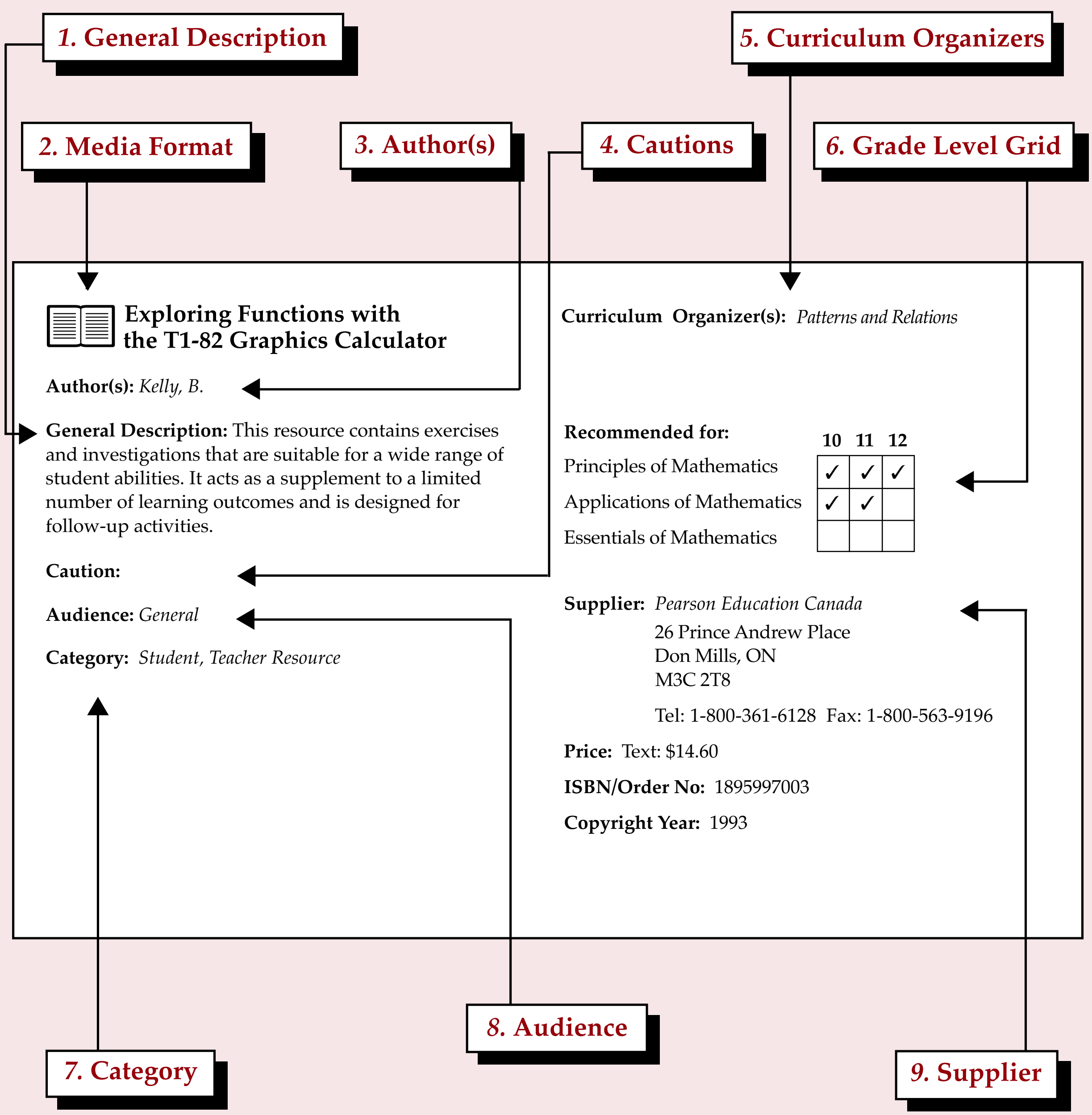
Appendix B

Learning Resources

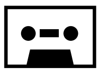
WHAT IS APPENDIX B?

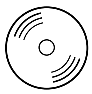
Appendix B for this IRP includes Grade Collection resources for Grades 10 to 12 Mathematics courses selected in 1998. These resources are all provincially recommended. It is the minstry's intention to add additional resources to these Grade Collections as they are evaluated.
The titles are listed alphabetically and each resource is annotated. In addition, Appendix B contains information on selecting learning resources for the classroom.

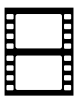
What information does an annotation provide?

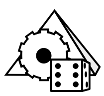


1. ***General Description:*** This section provides an overview of the resource.
2. ***Media Format:*** This part is represented by an icon next to the title. Possible icons include:

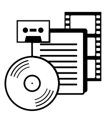

Audio Cassette


CD-ROM

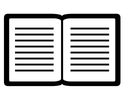

Film


Games/Manipulatives

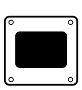

Laserdisc/Videodisc

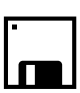

Multimedia

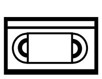

Music CD


Print Materials


Record


Slides


Software


Video

3. ***Author(s):*** Author or editor information is provided where it might be of use to the teacher.
4. ***Cautions:*** This category is used to alert teachers about potentially sensitive issues.
5. ***Curriculum Organizers:*** This category helps teachers make links between the resource and the curriculum.
6. ***Grade Level Grid:*** This category indicates the suitable age range for the resource.
7. ***Category:*** This section indicates whether it is a student and teacher resource, teacher resource, or professional reference.
8. ***Audience:*** This category indicates the suitability of the resource for different types of students. Possible student audiences include the following:
   - general
   - English as a second language (ESL)
   - *Students who are:*
     - gifted
     - blind or have visual impairments
     - deaf or hard of hearing
   - *Students with:*
     - severe behavioural disorders
     - dependent handicaps
     - physical disabilities
     - autism
     - learning disabilities (LD)
     - mild intellectual disabilities (ID-mild)
     - moderate to severe/profound disabilities (ID-moderate to severe/profound)
9. ***Supplier:*** The name and address of the supplier are included in this category. Prices shown here are approximate and subject to change. Prices should be verified with the supplier.

What about the videos?

The ministry attempts to obtain rights for most provincially recommended videos. Negotiations for the most recently recommended videos may not be complete. For these titles, the original distributor is listed in this document, instead of British Columbia Learning Connection Inc. Rights for new listings take effect the year implementation begins. Please check with British Columbia Learning Connection Inc. before ordering new videos.

SELECTING LEARNING RESOURCES FOR THE CLASSROOM

Selecting a learning resource for Grades 10 to 12 Mathematics means choosing locally appropriate materials from the Grade Collection or other lists of evaluated resources. The process of selection involves many of the same considerations as the process of evaluation, though not to the same level of detail. Content, instructional design, technical design, and social considerations may be included in the decision-making process, along with a number of other criteria.

The selection of learning resources should be an ongoing process to ensure a constant flow of new materials into the classroom. It is most effective as an exercise in group decision making, co-ordinated at the school, district, and ministry levels. To function efficiently and realize the maximum benefit from finite resources, the process should operate in conjunction with an overall district and school learning resource implementation plan.

Teachers may choose to use provincially recommended resources to support provincial or locally developed curricula; choose resources that are not on the ministry's list; or choose to develop their own resources. Resources that are not on the provincially recommended list must be evaluated through a local, board-approved process.

CRITERIA FOR SELECTION

There are a number of factors to consider when selecting learning resources.

Content

The foremost consideration for selection is the curriculum to be taught. Prospective resources must adequately support the particular learning outcomes that the teacher wants to address. Teachers will determine whether a resource will effectively support any given learning outcomes within a curriculum organizer. This can only be done by examining descriptive information regarding that resource; acquiring additional information about the material from the supplier, published reviews, or colleagues; and by examining the resource first-hand.

Instructional Design

When selecting learning resources, teachers must keep in mind the individual learning styles and abilities of their students, as well as anticipate the students they may have in the future. Resources have been recommended to support a variety of special audiences, including gifted, learning disabled, mildly intellectually disabled, and ESL students. The suitability of a resource for any of these audiences has been noted in the resource annotation. The instructional design of a resource includes the organization and presentation techniques; the methods used to introduce, develop, and summarize concepts; and the vocabulary level. The suitability of all of these should be considered for the intended audience.

Teachers should also consider their own teaching styles and select resources that will complement them. Lists of provincially recommended resources contain materials that range from prescriptive or self-contained resources to open-ended resources that require considerable teacher preparation. There are provincially recommended materials for teachers with varying levels of experience with a particular subject, as well as those that strongly support particular teaching styles.

Technology Considerations

Teachers are encouraged to embrace a variety of educational technologies in their classrooms. To do so, they will need to ensure the availability of the necessary equipment and familiarize themselves with its operation. If the equipment is not currently available, then the need must be incorporated into the school or district technology plan.

Social Considerations

All resources on the ministry's provincially recommended lists have been thoroughly screened for social concerns from a provincial perspective. However, teachers must consider the appropriateness of any resource from the perspective of the local community.

Media

When selecting resources, teachers should consider the advantages of various media. Some topics may be best taught using a specific medium. For example, video may be the most appropriate medium when teaching a particular, observable skill, since it provides a visual model that can be played over and over or viewed in slow motion for detailed analysis. Video can also bring otherwise unavailable experiences into the classroom and reveal "unseen worlds" to students. Software may be particularly useful when students are expected to develop critical-thinking skills through the manipulation of a simulation, or where safety or repetition is a factor. Print resources or CD-ROM can best be used to provide extensive background information on a given topic. Once again, teachers must consider the needs of their individual students, some of whom may learn better from the use of one medium than another.

Funding

As part of the selection process, teachers should determine how much money is available to spend on learning resources. This requires an awareness of school and district policies, and procedures for learning resource funding. Teachers will need to know how funding is allocated in their district and how much is available for their needs. Learning resource selection should be viewed as an ongoing process that requires a determination of needs, as well as long-term planning to co-ordinate individual goals and local priorities.

Existing Materials

Prior to selecting and purchasing new learning resources, an inventory of those resources that are already available should be established through consultation with the school and district resource centres. In some districts, this can be facilitated through the use of district and school resource management and tracking systems. Such systems usually involve a database to help keep track of a multitude of titles. If such a system is available, then teachers can check the availability of a particular resource via a computer.

SELECTION TOOLS

The Ministry of Education has developed a variety of tools to assist teachers with the selection of learning resources.

These include:

- Integrated Resource Packages (IRPs) that contain curriculum information, teaching and assessment strategies, and provincially recommended learning resources, including Grade Collections
- resource databases on disks or on-line
- sets of the most recently recommended learning resources (provided each year to a number of host districts throughout the province to allow teachers to examine the materials first-hand at regional displays)
- sample sets of provincially recommended resources (available on loan to districts on request)

A MODEL SELECTION PROCESS

The following series of steps is one way a school resource committee might go about selecting learning resources:

1. Identify a resource co-ordinator (for example, a teacher-librarian).
2. Establish a learning resources committee made up of department heads or lead teachers.
3. Develop a school vision and approach to resource-based learning.
4. Identify existing learning resource and library materials, personnel, and infrastructure.
5. Identify the strengths and weaknesses of existing systems.
6. Examine the district Learning Resources Implementation Plan.
7. Identify resource priorities.
8. Apply criteria such as those found in *Evaluating, Selecting, and Managing Learning Resources: A Guide* to shortlist potential resources.
9. Examine shortlisted resources first-hand at a regional display or at a publishers' display, or borrow a set by contacting either a host district or the Curriculum Branch.
10. Make recommendations for purchase.

FURTHER INFORMATION

For further information on evaluation and selection processes, catalogues, CD-ROM catalogues, annotation sets, or resource databases, please contact the Curriculum Branch of the Ministry of Education.

INDUSTRY STANDARD SOFTWARE

It is expected that students in Grades 10 to 12 Mathematics will have access to grade-level appropriate productivity tools including spreadsheets, database packages, word processors, drawing and painting tools, and so on. Use of industry standard software is encouraged.

Reviews of appropriate software are regularly published in a variety of computer and trade magazines. Selection of a particular application should consider:
- existing hardware and upgrade path
- cross-platform capability
- instructor training requirements
- time spent on student skill development versus curricular intent
- cross-curriculum applicability
- general flexibility and utility

Software for Grades 10 to 12 Mathematics

Software that supports Grades 10 to 12 Mathematics includes but is not limited to:

Title	Function/Purpose
Excel	Spreadsheet
Lotus 123	Spreadsheet
Supercalc	Spreadsheet

Inclusion in this list does not constitute recommended status or endorsement of the product.

Appendix B

Learning Resources
Grade Collections

MATHEMATICS 10 - 12 GRADE COLLECTIONS INTRODUCTION

The majority of the learning resources in these Grade Collections have been evaluated through the Western Canadian Protocol Learning Resource Evaluation Process and subsequently have been given Provincially Recommended status by the Ministry of Education.

Grade Collections are not prescriptive; they are intended to provide assistance and advice only. Teachers are encouraged to use existing resources that match the learning outcomes and to select additional resources to meet their specific classroom needs. It is recommended that teachers use the Mathematics 10 - 12 IRP when making resource decisions.

Resources that are identified through the Western Canadian Protocol Evergreening process as having strong curriculum match will be added to the Collections as they become available.

Categories of Resources

Learning resources selected for the Grade Collection have been categorized as either *comprehensive* , or *additional*.

- *Comprehensive resources* tend to provide a broad coverage of the learning outcomes for most curriculum organizers.
- *Additional resources* are more topic specific and support individual curriculum organizers or clusters of outcomes. They are recommended as valuable support or extension for specific topics.

In many cases, Grade Collections provide more than one resource to support specific outcomes, enabling teachers to select resources that best match different teaching and learning styles.

Other Recommended Resources

The Western Canadian Protocol Learning Resource Evaluation Process has also identified numerous *teacher resources* and *professional references*. These are generally cross-grade planning resources that include ideas for a variety of activities and exercises which may complement the comprehensive resources. These resources, while not part of the Grade Collections, have Provincially Recommended status. An alphabetical list indicating suggested course is included in this package, along with an annotated bibliography for ordering convenience.

Appendix B includes annotations for other Recommended resources not in the Grade Collections. While these resources meet only a limited number of outcomes, teachers are encouraged to consider them for different audience needs, teaching and learning styles, theme development, in-depth research, and so on.

Grade Collection Information

The following pages begin with an overview of the comprehensive resources for this curriculum, then present Grade Collection charts for each course. These charts list both *comprehensive* and *additional* resources for each curriculum organizer for the course. Each chart is followed by an annotated bibliography. There is an alphabetical list of the *teacher* resources, indicating course. This list is followed by an annotated bibliography. Please confirm with suppliers for complete and up-to-date price information. There is also a chart that shows the alphabetical list of

Grade Collection titles for each course and a blank template that can be used by teachers to record their individual choices.

Overview of Comprehensive Resources for Mathematics 10 - 12

- *Applied Mathematics, Western Canadian Edition* (Applications of Mathematics 10, Applications of Mathematics 11, Applications of Mathematics 12)

  Problem solving through extensive project work is the central focus of this program. Each developmental unit emphasizes at least one significant project. Connections are made to other curricular areas, and real-life contexts and applications. The use of technology, and the use and development of natural and mathematical language are emphasized. Components include student project book and source book, a teachers' resource book, and optional technology components. This resource requires the use of computer spreadsheets and graphing calculators, as well as some science laboratory equipment generally found in schools.

- *The Learning Equation* (Principles of Mathematics 10)

  Interactive CD-ROMs present lessons in the consistent format of introduction, tutorial example, summary of concepts, practice problems, extra practice, and a self-check. Students are able to monitor their own progress through the various components of each lesson. Print support consists of a Student Refresher and a Teacher's Manual.

- Mathematics, Western Canadian Edition (Principles of Mathematics 10, Principles of Mathematics 11, Principles of Mathematics 12)

  Program closely maps the content of the IRP. Concepts and skills are presented in problem-solving contexts and open-ended questions are designed to help students express their ideas about mathematics orally and in writing. The mathematical processes are highlighted through the use of icons in examples, explanations, and activities. Components include a student text, teacher's resource book, and an optional template and data kit and independent study guide. Requires the use of manipulatives.

- *MATHPOWER, Western Edition* (Principles of Mathematics 10, Principles of Mathematics 11, Principles of Mathematics 12)

  Program presents concepts and skills in a problem-solving context. It closely maps the content and philosophy of the IRP. The framework's mathematical processes are woven throughout the resource. Components include a student text, teacher's guide and blackline masters. Requires the use of manipulatives, and integrates the use of appropriate technology. Components include an optional computer databank and computerized assessment bank. The introductory pages of the student resource, which outline the mathematical processes and the NCTM Standards, are an essential part of the resource.

Appendix B

Grade Collections

Applications of Mathematics 10 Grade Collection

	Numbers		Patterns and Relations			Shape and Space			Statistics and Probability	
	Number Concepts	Number Operations	Patterns	Variables & Equations	Relations & Functions	Measure-ment	3-D Objects & 2-D Shapes	Transform-ations	Data Analysis	Chance & Uncertainty
Comprehensive Resources										
Applied Mathematics 10, Western Canadian Edition	✔	✔			✔	✔	✔		✔	
Additional Resources – Print										
Exploring Functions with the TI-82 Graphics Calculator				✔	✔					
Exploring Statistics with the TI-82 Graphics Calculator									✔	✔
Graphing Calculator Activities for Enriching Middle School Mathematics					✔				✔	
An Introduction to the TI-82 Graphing Calculator					✔				✔	
Modeling Motion: High School Math Activities with the CBR					✔				✔	
Using the TI-81 Graphics Calculator to Explore Functions										
Using the TI-81 Graphics Calculator to Explore Statistics					✔				✔	✔
A Visual Approach to Algebra					✔					
What If...?:The Straight Line: Investigations with the TI-81 Graphics Calculator					✔					
What If...?:The Straight Line: Investigations with the TI-82 Graphics Calculator					✔				✔	
Additional Resources – Games/Manipulatives										
Radical Math: Math Games Using Cards and Dice (Volume VII)			✔	✔	✔		✔			
Additional Resources – Software/CD-ROM										
The Geometer's Sketchpad						✔	✔	✔		
Geometry Blaster			✔			✔	✔	✔		
GrafEq				✔						

Applied Mathematics 10, Western Canadian Edition

Author(s): *Alexander, R. ... (et al.)*

General Description: Inquiry-approach resource consists of a Source Book, a Project Book, and a Teacher's Resource Book. The Source Book includes hands-on activities, research projects, tutorials, and review exercises. Applications and problem-solving questions use real-life examples. The Project Book comprises 20 additional open-ended projects that provide opportunities to apply mathematics in a new context.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Numbers*
Patterns and Relations
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics	✓		
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: Project Book and Source Book: $62.95
Teacher's Resource Book: $149.95

ISBN/Order No: Project Book: 0-201-39585-1
Source Book: 0-201-39586-X
Teacher's Resource Book: 0-201-39587-6

Copyright Year: 1999

Exploring Functions with the TI-82 Graphics Calculator

Author(s): *Kelly, B.*

General Description: This resource contains exercises and investigations that are suitable for a wide range of student abilities. It acts as a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 1895997003

Copyright Year: 1993

Exploring Statistics with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Statistics and Probability*

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-82 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Exercises allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 189599702X

Copyright Year: 1994

The Geometer's Sketchpad

Curriculum Organizer(s): *Shape and Space*

General Description: Software package for Macintosh or Windows consists of disks, a user's guide and reference manual, teaching notes, a quick reference, and an introductory video. Designed as a tool for teaching and exploring geometrical concepts, it allows students to manipulate geometrical figures with a hands-on approach. Sketches depict concrete geometry and emphasize spatial reasoning, while scripts describe constructions verbally and abstractly. The program allows for the construction, labelling, measurement, and manipulation of any geometric figure, as well as the exploration of geometry concepts. The teaching notes and sample activities provide suggestions for classroom use. Several sample investigations, explorations, demonstrations, and construction activities on a wide range of geometry topics are included.

System requirements for Macintosh: Macintosh Plus or later; 1Mb RAM; System 6 or later.
System requirements for Windows: 4Mb RAM; 3.5" floppy disk; hard drive.
System requirements for Windows 3.1 in enhanced mode.

Caution: *The program defaults to inches, but it can be set to centimetres.*

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics	✓		

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: $289.95 each version

ISBN/Order No: Macintosh: 1-55953-098-7
Windows: 1-55953-099-5

Copyright Year: 1995

Geometry Blaster

General Description: Interactive CD-ROM package for Macintosh or Windows includes lessons, activities, and games to teach and reinforce geometric concepts. It uses a three-level approach, allowing a student to progress at a beginning, an intermediate, or a mastery level. The lessons, designed for individual student use, are in three sections: teach, practise, and apply. They explore the main topics of points and lines, triangles, polygons and quadrilaterals, similarity, circles, perimeter and area, solids in 3D, coordinate geometry, transformational geometry and logical reasoning and proof. Requires decision making and creative thinking by students.

System requirements for Macintosh: System 7.0 or later, 8Mb RAM.
System requirements for Windows 3.1: 386/33 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD-ROM drive, and mouse. Printer recommended.
System requirements for Windows 95: 486/50 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD-ROM drive, and mouse. Printer is recommended.

Audience: *General*

Category: *Student*, *Teacher Resource*

Curriculum Organizer(s): *Shape and Space*
Patterns and Relations

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Davidson & Associates, Inc.*
19840 Pioneer Avenue
P.O. Box 2961
Torrance, CA
90503

Tel: 1-800- 545-7677 Fax: (310) 793-0601

Price: $79.95

ISBN/Order No: 0784910634

Copyright Year: 1996

GrafEq (Macintosh/Windows Version 2.09)

General Description: Relation plotting tool enables students to graph functions from implicit and explicit definitions, and from relations as well as inequalities. There is the ability to save and print graphs, and to integrate these graphs into other documents. Includes a manual.

System requirements for Macintosh: System 6.5 or higher (System 7 or higher recommended), 2 Mb RAM (4 Mb recommended).
System requirements for Windows: Windows 3.1 with win32's, Windows 95, Windows 98, or Windows NT 4; 8Mb RAM.

Audience: *General*

Category: *Student*, *Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pedagoguery Software*
4446 Lazelle Avenue
Terrace, BC
V8G 1R8

Tel: (250) 638-8606 Fax: (250) 638-8606

Price: Single User: $94.50
Site Licence (up to 30): $300.00

ISBN/Order No: (not available)

Copyright Year: 1998

Graphing Calculator Activities for Enriching Middle School Mathematics

Author(s): *Browning, C.A.; Channell, D.E.*

General Description: Twelve activities for using the TI-80, -82 and -83 graphing calculators require students to gather and organize data and explain their understanding of processes. Instructions for the use of the calculator accompany every step.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓		
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $24.48

ISBN/Order No: 1886309159

Copyright Year: 1997

Green Globs & Graphing Equations

General Description: User-friendly software for Macintosh or Windows graphs different forms of linear, quadratic, polynomial, and trigonometric relations. A second program provides a graph and requires the student to enter the equation. There are also two games that act as motivational tools. Five graphs can be displayed simultaneously. The accompanying teacher's guide provides ideas and worksheets.

System requirements for Macintosh: System 6.0.5 or higher (System 7 recommended), 1 Mb RAM.
System requirements for Windows 3.1: Windows 3.1, 486/25 MHz or better, 2 Mb RAM; 256 colour monitor.
System requirements for Windows 95: 486DX/50 MHz or higher (Pentium recommended), 4 Mb RAM, 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓		
Essentials of Mathematics		✓	✓

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $129.95 each version

ISBN/Order No: (not available)

Copyright Year: 1996

An Introduction to the TI-82 Graphing Calculator

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Author(s): *Govatos, M.*

General Description: Reproducible blackline masters give clear examples and steps for using the TI-82 graphing calculator in a variety of ways. Can also be used with the TI-83 graphing calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Frank Schaffer Publications Inc.*
23740 Howthorne Boulevard
Torrance, CA
90505

Tel: 1-800-421-5533 Fax: (310) 375-5090

Price: $18.95

ISBN/Order No: 0768200067

Copyright Year: 1998

Modeling Motion: High School Math Activities with the CBR

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Author(s): *Antinone, L.; Gough, S.*

General Description: This resource includes investigation activities that use the calculator-based ranger (CBR) to obtain data from physical phenomena. Using a TI-82 or TI-83 graphing calculator, students obtain a mathematical equation to describe the physical event(s). Topics include arithmetic and geometric growth, systems of linear equations in two variables, quadratic functions, and exponential functions. The resource provides practice data to enable students to complete the investigations without a CBR, if necessary.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $16.83

ISBN/Order No: 1886309140

Copyright Year: 1997

Radical Math: Math Games Using Cards and Dice (Volume VII)

Author(s): *Currah; Felling; Lachance*

General Description: This resource offers some unique activities that encourage the processes of communication, estimation and mental mathematics, problem solving, reasoning, and visualization. Many of the activities can be used to help students achieve fluency in such topics as polynomial manipulations and the use of equivalent forms of expressions containing radicals.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓		
Essentials of Mathematics			

Supplier: *Box Cars & One-Eyed Jacks*
3930-78 Avenue
Edmonton, AB
T6B 2W4

Tel: (780) 440-6284 Fax: (780) 440-1619

Price: $36.00

ISBN/Order No: 0968161308

Copyright Year: 1996

Using the TI-81 Graphics Calculator to Explore Functions

Author(s): *Kelly, B.*

General Description: This resource manual includes exercises and investigations using the TI-81 graphics calculator and is suitable for a wide range of student abilities. It provides a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524404

Copyright Year: 1991

Using the TI-81 Graphics Calculator to Explore Statistics

Curriculum Organizer(s): *Statistics and Probability*

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-81 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Student exercises, most of which are not calculator specific, allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics			✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524412

Copyright Year: 1992

A Visual Approach to Algebra

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Van Dyke, F.*

General Description: Supplementary activity book uses an investigative approach to introduce concepts related to the relationships among graphing, scale, data, and algebra. Student activity worksheets, accompanied by teacher notes, can be used as introductory or extension material. Includes the use of a graphing calculator for some activities. Imperial measurement is used in problem solving situations.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓		
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $20.92

ISBN/Order No: 1572324414

Copyright Year: 1998

What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-81 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698100

Copyright Year: 1993

What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-82 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes. This resource also introduces the concept of the line of best fit.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698135

Copyright Year: 1994

Applications of Mathematics 11 Grade Collection

	Numbers		Patterns and Relations			Shape and Space			Statistics and Probability	
	Number Concepts	Number Operations	Patterns	Variables & Equations	Relations & Functions	Measure-ment	3-D Objects & 2-D Shapes	Transform-ations	Data Analysis	Chance & Uncertainty
Comprehensive Resources										
Applied Mathematics 11, Western Canadian Edition		✔		✔	✔	✔	✔			✔
Additional Resources – Print										
Explore Quadratic Functions with the TI-83 or TI-82				✔	✔					
Exploring Functions with the TI-82 Graphics Calculator					✔					
Exploring Statistics with the TI-82 Graphics Calculator										✔
An Introduction to the TI-82 Graphing Calculator				✔	✔					✔
Modeling Motion: High School Math Activities with the CBR				✔	✔					✔
Using the TI-81 Graphics Calculator to Explore Functions					✔					
Using the TI-81 Graphics Calculator to Explore Statistics										✔
What If...?:The Straight Line: Investigations with the TI-81 Graphics Calculator					✔					
What If...?:The Straight Line: Investigations with the TI-82 Graphics Calculator					✔					✔
Additional Resources – Software/CD-ROM										
The Geometer's Sketchpad						✔	✔			
Geometry Blaster						✔	✔			
GrafEq					✔					

Applied Mathematics 11, Western Canadian Edition

Author(s): *Alexander, R. ... (et al.)*

General Description: Inquiry-approach resource consists of a Source Book, a Project Book, and a Teacher's Resource Book. The Source Book includes hands-on activities, research projects, tutorials, and review exercises. Applications and problem-solving questions use real-life examples. The Project Book comprises 24 additional open-ended projects that provide opportunities to apply mathematics in a new context.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Numbers*
Patterns and Relations
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics		✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: Source Book and Project Book: $65.95
Teacher's Resource Book: $159.95

ISBN/Order No: Source Book and Project Book: 0-201-39590-8
Teacher's Resource Book: 0-201-39593-2

Copyright Year: 2000

Explore Quadratic Functions with the TI-83 or TI-82

Author(s): *Alexander, B.*

General Description: Manual can be used for applying graphing calculator technology to the exploration of quadratic functions and equations. It contains concise and precise instructions on relevant keying sequences. There are many "window" diagrams showing input and output screens. Problems posed relate to real-world situations and provide ample opportunities for discussion, exploration, and extension of a variety of situations beyond the routine. The resource uses keying sequences that are specific to the TI-83 and TI-82 models of calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics		✓	
Applications of Mathematics		✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $17.95

ISBN/Order No: 0969698143

Copyright Year: 1997

Exploring Functions with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Kelly, B.*

General Description: This resource contains exercises and investigations that are suitable for a wide range of student abilities. It acts as a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 1895997003

Copyright Year: 1993

Exploring Statistics with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Statistics and Probability*

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-82 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Exercises allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 189599702X

Copyright Year: 1994

The Geometer's Sketchpad

General Description: This software package for Macintosh or Windows consists of disks, a user's guide and reference manual, teaching notes, a quick reference, and an introductory video. It is designed as a tool for teaching and exploring geometrical concepts and allows students to manipulate geometrical figures with a hands-on approach. The sketchpad models geometry in two linked views. Sketches depict concrete geometry and emphasize spatial reasoning, while scripts describe constructions verbally and abstractly. The teaching notes and sample activities provide suggestions for classroom use. Several sample investigations, explorations, demonstrations, and construction activities on a wide range of geometry topics are included. The instructional video supports both Macintosh and Windows.

System requirements for Macintosh: Macintosh Plus or later; 1Mb RAM; System 6 or later.
System requirements for Windows: 4Mb RAM; 3.5" floppy disk; hard drive.
System requirements for Windows 3.1 in enhanced mode.

Caution: *The program defaults to inches, but it can be set to centimetres.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Shape and Space*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics	✓		

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: Each version: $289.95

ISBN/Order No: Macintosh: 1-55953-098-7
Windows: 1-55953-099-5

Copyright Year: 1995

Geometry Blaster

General Description: Interactive CD-ROM package for Macintosh or Windows includes lessons, activities, and games to teach and reinforce geometric concepts. It uses a three-level approach, allowing a student to progress at a beginning, an intermediate, or a mastery level. The lessons, designed for individual student use, are in three sections: teacher, practise, and apply. They explore the main topics of points and lines, triangles, polygons and quadrilaterals, similarity, circles, perimeter and area, solids in 3D, coordinate geometry, transformational geometry and logical reasoning and proof. The program is interactive and requires decision making and creative thinking by students.

System Requirements:
Macintosh: System 7.0 or later, 8Mb RAM.
Windows 3.1: 386/33 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD drive, and mouse. Printer recommended.
Windows 95: 486/50 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD drive, and mouse. Printer is recommended.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Shape and Space*
Patterns and Relations

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Davidson & Associates, Inc.*
19840 Pioneer Avenue
P.O. Box 2961
Torrance, CA
90503

Tel: 1-800- 545-7677 Fax: (310) 793-0601

Price: $79.95

ISBN/Order No: 0784910634

Copyright Year: 1996

GrafEq (Macintosh/Windows Version 2.09)

Curriculum Organizer(s): *Patterns and Relations*

General Description: Relation plotting tool enables students to graph functions from implicit and explicit definitions, and from relations as well as inequalities. There is the ability to save and print graphs, and to integrate these graphs into other documents. Includes a manual.

System requirements for Macintosh: System 6.5 or higher (System 7 or higher recommended), 2 Mb RAM (4 Mb recommended). System requirements for Windows: Windows 3.1 with win32's, Windows 95, Windows 98, or Windows NT 4; 8Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pedagoguery Software*
4446 Lazelle Avenue
Terrace, BC
V8G 1R8

Tel: (250) 638-8606 Fax: (250) 638-8606

Price: Single User: $94.50
Site Licence (up to 30): $300.00

ISBN/Order No: (not available)

Copyright Year: 1998

An Introduction to the TI-82 Graphing Calculator

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Author(s): *Govatos, M.*

General Description: Reproducible blackline masters give clear examples and steps for using the TI-82 graphing calculator in a variety of ways. Can also be used with the TI-83 graphing calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Frank Schaffer Publications Inc.*
23740 Howthorne Boulevard
Torrance, CA
90505

Tel: 1-800-421-5533 Fax: (310) 375-5090

Price: $18.95

ISBN/Order No: 0768200067

Copyright Year: 1998

Modeling Motion: High School Math Activities with the CBR

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Author(s): *Antinone, L; Gough, S.*

General Description: This resource includes investigation activities that use the calculator-based ranger (CBR) to obtain data from physical phenomena. Using a TI-82 or TI-83 graphing calculator, students obtain a mathematical equation to describe the physical event(s). Topics include arithmetic and geometric growth, systems of linear equations in two variables, quadratic functions, and exponential functions. The resource provides practice data to enable students to complete the investigations without a CBR, if necessary.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $16.93

ISBN/Order No: 1886309140

Copyright Year: 1997

Using the TI-81 Graphics Calculator to Explore Functions

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Kelly, B.*

General Description: This resource manual includes exercises and investigations using the TI-81 graphics calculator and is suitable for a wide range of student abilities. It provides a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524404

Copyright Year: 1991

Using the TI-81 Graphics Calculator to Explore Statistics

Curriculum Organizer(s): *Statistics and Probability*

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-81 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Student exercises, most of which are not calculator specific, allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics			✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524412

Copyright Year: 1992

What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-81 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698100

Copyright Year: 1993

What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-82 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes. This resource also introduces the concept of the line of best fit.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698135

Copyright Year: 1994

Applications of Mathematics 12 Grade Collection

	Numbers		Patterns and Relations			Shape and Space			Statistics and Probability	
	Number Concepts	Number Operations	Patterns	Variables & Equations	Relations & Functions	Measure-ment	3-D Objects & 2-D Shapes	Transform-ations	Data Analysis	Chance & Uncertainty
Comprehensive Resources										
Applied Mathematics 12, Western Canadian Edition (Available June 2001)	✔		✔		✔	✔	✔			✔
Additional Resources – Print										
Exploring Advanced Algebra with the TI-83			✔							✔
Exploring Statistics with the TI-82 Graphics Calculator										✔
Using the TI-81 Graphics Calculator to Explore Statistics										✔
Additional Resources – Software/CD-ROM										
GrafEq			✔							
Secondary Math Lab Toolkit	✔		✔				✔			✔

Exploring Advanced Algebra with the TI-83

General Description: Resource contains exercises and investigations that supplement the following topics: sequences, fractals, combinatorics, probability, linear systems, matrices, and linear programming. The keying sequences are specific to TI-83 model of calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics			✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $15.71

ISBN/Order No: 1895997100

Copyright Year: 1998

Exploring Statistics with the TI-82 Graphics Calculator

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-82 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Exercises allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Statistics and Probability*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 189599702X

Copyright Year: 1994

GrafEq (Macintosh/Windows Version 2.09)

Curriculum Organizer(s): *Patterns and Relations*

General Description: Relation plotting tool enables students to graph functions from implicit and explicit definitions, and from relations as well as inequalities. There is the ability to save and print graphs, and to integrate these graphs into other documents. Includes a manual.

System requirements for Macintosh: System 6.5 or higher (System 7 or higher recommended), 2 Mb RAM (4 Mb recommended). System requirements for Windows: Windows 3.1 with win32's, Windows 95, Windows 98, or Windows NT 4; 8Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pedagoguery Software*
4446 Lazelle Avenue
Terrace, BC
V8G 1R8

Tel: (250) 638-8606 Fax: (250) 638-8606

Price: Single User: $94.50
Site Licence (up to 30): $300.00

ISBN/Order No: (not available)

Copyright Year: 1998

An Introduction to the TI-82 Graphing Calculator

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Govatos, M.*

General Description: Reproducible blackline masters give clear examples and steps for using the TI-82 graphing calculator in a variety of ways. Can also be used with the TI-83 graphing calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Frank Schaffer Publications Inc.*
23740 Howthorne Boulevard
Torrance, CA
90505

Tel: 1-800-421-5533 Fax: (310) 375-5090

Price: $18.95

ISBN/Order No: 0768200067

Copyright Year: 1998

Secondary Math Lab Toolkit

Author(s): *Bedoya, J.Z. ... (et al.)*

General Description: Software package for Macintosh or Windows consists of a CD-ROM and a user's guide. The toolkit contains graphical tools for algebra, geometry, data analysis, and probability. A text editor, function editor, spreadsheet editor, and matrix editor are linked. The graphs are clear and may be transformed in a variety of ways. Probability models use dice and spinners. Includes a visual representation of algebra tiles.

System requirements for Macintosh: Macintosh LC II or later, 6Mb RAM and System 7.0 or higher. PowerMac system with 8Mb RAM and System 7.5 or higher recommended.
System requirements for Windows 3.1: Windows 3.1 in enhanced mode, 486/33 MHz, 12Mb RAM, hard drive.
System requirements Windows 95: 486/50 MHz, 12Mb RAM, hard drive. Pentium processor with 16Mb RAM recommended.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics			✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: Each version $124.95

ISBN/Order No: Macintosh: 0134330439
Windows: 0134330447

Copyright Year: 1998

Using the TI-81 Graphics Calculator to Explore Statistics

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-81 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Student exercises, most of which are not calculator specific, allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Statistics and Probability*

Recommended for:

	10	11	12
Principle of Mathematics			✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524412

Copyright Year: 1992

Principles of Mathematics 10 Grade Collection

	Numbers		Patterns and Relations			Shape and Space			Statistics and Probability	
	Number Concepts	Number Operations	Patterns	Variables & Equations	Relations & Functions	Measure-ment	3-D Objects & 2-D Shapes	Transform-ations	Data Analysis	Chance & Uncertainty
Comprehensive Resources										
The Learning Equation, Mathematics 10	✔	✔	✔	✔	✔	✔	✔		✔	
Mathematics 10, Western Canadian Edition	✔	✔	✔	✔	✔	✔	✔		✔	
MATHPOWER 10, Western Edition	✔	✔	✔	✔	✔	✔	✔		✔	
Pure Mathematics 10 (Distance Learning Package)	✔	✔	✔	✔	✔	✔	✔		✔	
Additional Resources – Print										
Exploring Advanced Algebra with the TI-83			✔	✔						
Exploring Functions with the TI-82 Graphics Calculator				✔	✔					
Exploring Statistics with the TI-82 Graphics Calculator									✔	
Graphing Calculator Activities for Enriching Middle School Mathematics				✔	✔				✔	
An Introduction to the TI-82 Graphing Calculator			✔	✔	✔				✔	
Modeling Motion: High School Math Activities with the CBR			✔	✔	✔				✔	
Using the TI-81 Graphics Calculator to Explore Functions					✔					
A Visual Approach to Algebra			✔	✔	✔					
What If...?: The Straight Line: Investigations with the TI-81 Graphics Calculator					✔					
What If...?: The Straight Line: Investigations with the TI-82 Graphics Calculator					✔				✔	
Additional Resources – Games/Manipulatives										
Radical Math: Math Games Using Cards and Dice (Volume VII)			✔	✔			✔			
Additional Resources – Software/CD-ROM										
The Geometer's Sketchpad						✔	✔			
Geometry Blaster			✔			✔	✔			
GrafEq					✔					
Green Globs & Graphing Equations				✔	✔		✔			
Secondary Math Lab Toolkit		✔	✔	✔	✔		✔		✔	
Understanding Math Series	✔	✔	✔	✔		✔				

Exploring Advanced Algebra with the TI-83

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Author(s): *Kelly, B.*

General Description: Resource contains exercises and investigations that supplement the following topics: sequences, fractals, combinatorics, probability, linear systems, matrices, and linear programming. The keying sequences are specific to TI-83 model of calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics			✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $15.71

ISBN/Order No: 1895997100

Copyright Year: 1998

Exploring Functions with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Kelly, B.*

General Description: This resource contains exercises and investigations that are suitable for a wide range of student abilities. It acts as a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 1895997003

Copyright Year: 1993

Exploring Statistics with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Statistics and Probability*

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-82 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Exercises allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 189599702X

Copyright Year: 1994

The Geometer's Sketchpad

Curriculum Organizer(s): *Shape and Space*

General Description: Software package for Macintosh or Windows consists of disks, a user's guide and reference manual, teaching notes, a quick reference, and an introductory video. Designed as a tool for teaching and exploring geometrical concepts it allows students to manipulate geometrical figures with a hands-on approach. Sketches depict concrete geometry and emphasize spatial reasoning, while scripts describe constructions verbally and abstractly. The program allows for the construction, labelling, measurement, and manipulation of any geometric figure, as well as the exploration of geometry concepts. The teaching notes and sample activities provide suggestions for classroom use. Several sample investigations, explorations, demonstrations, and construction activities on a wide range of geometry topics are included.

System requirements for Macintosh: Macintosh Plus or later; 1Mb RAM; System 6 or later.
System requirements for Windows: 4Mb RAM; 3.5" floppy disk; hard drive.
System requirements for Windows 3.1 in enhanced mode.

Caution: *The program defaults to inches, but it can be set to centimetres.*

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics	✓		

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: $289.95 each version

ISBN/Order No: Macintosh version: 1-55953-098-7
Windows version: 1-55953-099-5

Copyright Year: 1995

Geometry Blaster

General Description: Interactive CD-ROM package for Macintosh or Windows includes lessons, activities, and games to teach and reinforce geometric concepts. It uses a three-level approach, allowing a student to progress at a beginning, an intermediate, or a mastery level. The lessons, designed for individual student use, are in three sections: teach, practise, and apply. They explore the main topics of points and lines, triangles, polygons and quadrilaterals, similarity, circles, perimeter and area, solids in 3D, coordinate geometry, transformational geometry and logical reasoning and proof. Requires decision making and creative thinking by students.

System requirements for Macintosh: System 7.0 or later, 8Mb RAM.
System requirements forWindows 3.1: 386/33 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD-ROM drive, and mouse. Printer recommended.
System requirements for Windows 95: 486/50 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD-ROM drive, and mouse. Printer is recommended.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Davidson & Associates, Inc.*
19840 Pioneer Avenue
P.O. Box 2961
Torrance, CA
90503

Tel: 1-800- 545-7677 Fax: (310) 793-0601

Price: $79.95

ISBN/Order No: 0784910634

Copyright Year: 1996

GrafEq (Macintosh/Windows Version 2.09)

General Description: Relation plotting tool enables students to graph functions from implicit and explicit definitions, and from relations as well as inequalities. There is the ability to save and print graphs, and to integrate these graphs into other documents. Includes a manual.

System requirements for Macintosh: System 6.5 or higher (System 7 or higher recommended), 2 Mb RAM (4 Mb recommended).
System requirements for Windows: Windows 3.1 with win32's, Windows 95, Windows 98, or Windows NT 4; 8Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pedagoguery Software*
4446 Lazelle Avenue
Terrace, BC
V8G 1R8

Tel: (250) 638-8606 Fax: (250) 638-8606

Price: Single User: $94.50
Site Licence (up to 30): $300.00

ISBN/Order No: (not available)

Copyright Year: 1998

Graphing Calculator Activities for Enriching Middle School Mathematics

Author(s): *Browning, C.A.; Channell, D.E.*

General Description: Twelve activities for using the TI-80, -82 and -83 graphing calculators require students to gather and organize data and explain their understanding of processes. Instructions for the use of the calculator accompany every step.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓		
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $24.48

ISBN/Order No: 1886309159

Copyright Year: 1997

Green Globs & Graphing Equations

General Description: User-friendly software for Macintosh or Windows graphs different forms of linear, quadratic, polynomial, and trigonometric relations. A second program provides a graph and requires the student to enter the equation. There are also two games that act as motivational tools. Five graphs can be displayed simultaneously. The accompanying teacher's guide provides ideas and worksheets.

System requirements for Macintosh: System 6.0.5 or higher (System 7 recommended), 1 Mb RAM.
System requirements for Windows 3.1: Windows 3.1, 486/25 MHz or better, 2 Mb RAM; 256 colour monitor.
System requirements for Windows 95: 486DX/50 MHz or higher (Pentium recommended), 4 Mb RAM, 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓		
Essentials of Mathematics		✓	✓

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $129.95 each version

ISBN/Order No: (not available)

Copyright Year: 1997

An Introduction to the TI-82 Graphing Calculator

Author(s): *Govatos, M.*

General Description: Reproducible blackline masters give clear examples and steps for using the TI-82 graphing calculator in a variety of ways. Can also be used with the TI-83 graphing calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Frank Schaffer Publications Inc.*
23740 Howthorne Boulevard
Torrance, CA
90505

Tel: 1-800-421-5533 Fax: (310) 375-5090

Price: $18.95

ISBN/Order No: 0768200067

Copyright Year: 1998

The Learning Equation: Mathematics 10

General Description: Interactive program for Macintosh or Windows consists of three CD-ROMs, a student refresher, and a teacher's guide. All lessons follow the pattern of an introduction, followed by a tutorial example for students, a summary of concepts, practice problems, extra practice, and a self check. Students are able to monitor their own progress through the various components of each lesson. Also included on the CD-ROMs is a glossary which students can access as a pull-down menu. The student refresher follows the organization of the CD-ROMs and provides additional examples for students as well as answers. The teacher's manual includes installation notes, instructional summaries, prerequisite skills, and cross-references to the CD-ROMs.

System requirements for Macintosh: System 7.1 or later, 12Mb RAM (16Mb preferred), VGA 256-colour monitor, 40Mb hard disk space, double speed CD-ROM drive, 8 bit sound support for audio.
System requirements for Windows 3.1: MPC2 compliant computer, 486SX 25 Mhz (33 or better preferred), 8Mb RAM, VGA 256-colour monitor, 40Mb hard disk space, double speed CD-ROm drive, soundblaster or 100& support for audio.
System requirements for Windows 95: MPC2 compliant computer, 486SX 25 MHz (33 or better preferred), 8Mb RAM, VGA 256-colour monitor, 40Mb hard disk space, double speed CD-ROM drive, soundblaster or 100% support for audio.

Performance is considerably improved with faster computers.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space
Statistics and Probability,

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Nelson Thomson Learning*
1120 Birchmount Road
Scarborough, ON
M1K 5G4

Tel: 1-800-268-2222 Fax: (416) 752-9365 or 752-9646

Price: CD-ROMs 1, 2, and 3: $150.00
Teacher's Manual: $74.00
Student Refresher: $8.50

ISBN/Order No: CD-ROMs 1, 2, and 3: 0-17-605959-8
Teacher's Manual: 0-17-605973-3
Student Refresher: 0-17-605969-5

Copyright Year: 1998

Mathematics 10, Western Canadian Edition

Author(s): *Alexander, R.; ... (et al.)*

General Description: This resource package closely maps the content of the Common Curriculum Framework. A student text, teacher's resource book, an optional template and data kit, an optional exercise database, an optional exercise and problem bank, and an optional independent study guide use a problem-solving approach. Open-ended questions are designed to help students express their ideas about mathematics orally and in writing. Group work, manipulatives, and technology are utilized where appropriate. Student text uses an inquiry approach and incorporates graphing calculators and software use. The teacher's resource book includes comprehensive teaching notes, blackline masters, additional exercises, a graphing handbook with reproducible graphing calculator activities, and a CD-ROM with solutions. The optional templates and data kit includes four databases, spreadsheet templates, and a teacher's guide with tutorials and additional technology activities. The exercise database provides 2400 electronic multiple choice questions. The independent study guide provides supplementary explanations, examples, and self-tests. The exercise and problem bank was reviewed for technical considerations only. Manipulatives, such as algebra tiles and suitable software, necessary for using the resource effectively, are not supplied with the resource. References are made in both the student and teacher materials to specific Internet web sites which have not been evaluated.

System requirements for software components:

Macintosh: 68020 or greater processor, Mac OS 7.0 or later, 7 MB RAM minimum, 5 MB hard drive space plus 7 MB temporary drive space available during installation, 256 colour 13-inch monitor, 2X CD drive or better.
Windows 3.1: 386DX/33 MHz minimum, 4 MB RAM minimum, 5 MB hard drive space plus 7 MB temporary drive space available during installation, 256 colour 13-inch monitor, 2X CD drive or better.
Windows 95: 486DX/50 MHz (Pentium recommended), 4 MB RAM minimum, 5 MB hard drive space plus 7 MB temporary drive space available during installation, 256 colour 13-inch monitor, 2X CD drive or better.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: Student Text: $49.95
Teacher's Res Bk/CD-ROM: $119.95

ISBN/Order No: Student Text: 0201346192
Teacher's Res Bk/CD-ROM: 0201346214

Copyright Year: 1998

MATHPOWER 10, Western Edition

Author(s): *Angel, P.; ... (et al.)*

General Description: This resource package, consisting of a student text, teacher's resource, and an optional computer data bank, computerized assessment bank, set of blackline masters, and solutions manual, closely maps the content and philosophy of the Common Curriculum Framework. The Framework's mathematical processes are woven throughout the resource. Mathematical concepts relate to real-world situations. Concepts and skills are presented in problem-solving contexts, and questions are designed to help students express their ideas about mathematics orally and in writing. Group work, manipulatives, and technology are utilized where appropriate. Student text uses an inquiry approach and incorporates graphing calculators and software use. The teacher's resource contains extensive teaching notes. The computer databank comprises five databases together with a teacher's resource. A Windows Test Bank provides about 50 questions per chapter, but does not include a tracking system. The blackline masters supplement the student text and the teacher's resource. The solutions manual gives step-by-step solutions to every question from the student text. Manipulatives, such as algebra tiles and suitable software, are necessary for using the resource effectively; these are not supplied with the resource. References are made in both the student and teacher materials to specific Internet websites which have not been evaluated.

System requirements for Windows 95: 486DX/66 MHz, Pentium processor recommended, 8Mb RAM, mouse, either Claris Works or Microsoft Works. System requirements for Windows 3.1: 486DX/66 MHz, Pentium processor recommended, 5Mb RAM (8Mb recommended), Microsoft Works. Windows 95 recommended. For those using the 486DX chip, a math coprocessor is required to use the computerized assessment bank. The bank can be operated with all Pentium chips.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *McGraw-Hill Ryerson Ltd. (Ontario)*
300 Water Street
Whitby, ON
L1N 9B6

Tel: 1-800-565-5758 (orders only) Fax: 1-800-463-5885

Price: Student Text: $51.38
Teacher's Resource: $115.50

ISBN/Order No: Student Text: 0075525968
Teacher's Resource: 0075525976

Copyright Year: 1998

Modeling Motion: High School Math Activities with the CBR

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Author(s): *Antinone, L.; Gough, S.*

General Description: This resource includes investigation activities that use the calculator-based ranger (CBR) to obtain data from physical phenomena. Using a TI-82 or TI-83 graphing calculator, students obtain a mathematical equation to describe the physical event(s). Topics include arithmetic and geometric growth, systems of linear equations in two variables, quadratic functions, and exponential functions. The resource provides practice data to enable students to complete the investigations without a CBR, if necessary.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $16.83

ISBN/Order No: 1886309140

Copyright Year: 1997

Pure Mathematics 10 (Distance Learning Package)

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space
Statistics and Probability

General Description: This resource, designed for distance education, includes five modules with assignements, as well as a learning facilitator's manual. The materials are designed to be used in conjunction with the *MATHPOWER 10* text. Manipulatives must be purchased separately.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Learning Resources Distributing Centre*
12360-142 Street
Edmonton, AB
T5L 4X9

Tel: (780) 427-5775 Fax: (780) 422-9750

Price: Modules: $18.60 each
Assignment Booklets: $10.00
Learning Facilitator's Manual: $60.00

ISBN/Order No: (not available)

Copyright Year: 1999

Radical Math: Math Games Using Cards and Dice (Volume VII)

Author(s): *Currah, J.; Felling, J.; Lachance, N.*

General Description: This resource offers some unique activities that encourage the processes of communication, estimation and mental mathematics, problem solving, reasoning, and visualization. Many of the activities can be used to help students achieve fluency in such topics as polynomial manipulations and the use of equivalent forms of expressions containing radicals. Includes ten special dice.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓		
Essentials of Mathematics			

Supplier: *Box Cars & One-Eyed Jacks*
3930-78 Avenue
Edmonton, AB
T6B 2W4

Tel: (780) 440-6284 Fax: (780) 440-1619

Price: $36.00

ISBN/Order No: 0075525992

Copyright Year: 1996

Secondary Math Lab Toolkit

Author(s): *Bedoya, J.Z.; ... (et al.)*

General Description: Software package for Macintosh or Windows consists of a CD-ROM and a user's guide. The toolkit contains graphical tools for algebra, geometry, data analysis, and probability. A text editor, function editor, spreadsheet editor, and matrix editor are linked. The graphs are clear and may be transformed in a variety of ways. Probability models use dice and spinners. Includes a visual representation of algebra tiles.

System requirements for Macintosh: Macintosh LC II or later, 6Mb RAM and System 7.0 or higher. PowerMac system with 8Mb RAM and System 7.5 or higher recommended.
System requirements for Windows 3.1: Windows 3.1 in enhanced mode, 486/33 MHz, 12Mb RAM, hard drive.
System requirements for Windows 95: 486/50 MHz, 12Mb RAM, hard drive. Pentium processor with 16Mb RAM recommended.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics			✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $124.95 each version

ISBN/Order No: Macintosh version: 0134330439
Windows version: 0134330447

Copyright Year: 1998

Understanding Math Series

General Description: CD-ROM series for Macintosh or Windows assists students in developing an understanding of mathematics concepts based on a solid foundation of concrete activities. Topics include fractions, integers, exponents, algebra, equations, coordinate geometry and graphing, and percent. For each topic the program contains concept sections with explanation, example sections, practice sections, and cumulative checks. The user is in control through pull-down menus and proceed and go-back buttons. Each program includes a teacher manual.

System requirements for Macintosh: 68020 or faster processor, including Power Macintosh, Macintosh 7.5.1 or later, 40 Mb hard drive space or CD-ROM drive.
System requirements for Windows: Windows 3.1, Windows 95, Windows NT3.51 or 4.0; 486DX2-66 (Pentium recommended); 640x480 with 256 colour display (or higher); 40 Mb hard drive space or CD-ROM drive.

Audience: *General*
LD - can be used for remediation

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $79.95 each

ISBN/Order No: (not available)

Copyright Year: 1996

Using the TI-81 Graphics Calculator to Explore Functions

Author(s): *Kelly, B.*

General Description: This resource manual includes exercises and investigations using the TI-81 graphics calculator and is suitable for a wide range of student abilities. It provides a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524404

Copyright Year: 1991

A Visual Approach to Algebra

Author(s): *Van Dyke, F.*

General Description: Supplementary activity book uses an investigative approach to introduce concepts related to the relationships among graphing, scale, data, and algebra. Student activity worksheets, accompanied by teacher notes, can be used as introductory or extension material. Includes the use of a graphing calculator for some activities. Imperial measurement is used in problem solving situations.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓		
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $20.92

ISBN/Order No: 1572324414

Copyright Year: 1998

What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-81 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698100

Copyright Year: 1993

What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-82 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes. This resource also introduces the concept of the line of best fit.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698135

Copyright Year: 1994

Principles of Mathematics 11 Grade Collection

	Numbers		Patterns and Relations			Shape and Space			Statistics and Probability	
	Number Concepts	Number Operations	Patterns	Variables & Equations	Relations & Functions	Measure-ment	3-D Objects & 2-D Shapes	Transform-ations	Data Analysis	Chance & Uncertainty
Comprehensive Resources										
Mathematics 11, Western Canadian Edition		✔	✔	✔	✔	✔	✔			
MATHPOWER 11, Western Edition		✔	✔	✔	✔	✔	✔			
Additional Resources – Print										
Explore Quadratic Functions with the TI-83 or TI-82				✔	✔					
Exploring Functions with the TI-82 Graphics Calculator				✔	✔					
An Introduction to the TI-82 Graphing Calculator			✔	✔	✔					
Modeling Motion: High School Math Activities with the CBR			✔	✔	✔					
Using the TI-81 Graphics Calculator to Explore Functions					✔					
What If...?:The Straight Line: Investigations with the TI-81 Graphics Calculator					✔					
Additional Resources – Software/CD-ROM										
The Geometer's Sketchpad						✔	✔			
Geometry Blaster			✔			✔	✔			
GrafEq					✔					
Green Globs & Graphing Equations				✔	✔		✔			
Secondary Math Lab Toolkit		✔	✔	✔	✔		✔			
ZAP-A-GRAPH					✔					

Explore Quadratic Functions with the TI-83 or TI-82

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Alexander, B.*

General Description: Manual can be used for applying graphing calculator technology to the exploration of quadratic functions and equations. It contains concise and precise instructions on relevant keying sequences. There are many "window" diagrams showing input and output screens. Problems posed relate to real-world situations and provide ample opportunities for discussion, exploration, and extension of a variety of situations beyond the routine. The resource uses keying sequences that are specific to the TI-83 and TI-82 models of calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics		✓	
Applications of Mathematics		✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $17.95

ISBN/Order No: 0969698143

Copyright Year: 1997

Exploring Functions with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Kelly, B.*

General Description: This resource contains exercises and investigations that are suitable for a wide range of student abilities. It acts as a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 1895997003

Copyright Year: 1993

The Geometer's Sketchpad

Curriculum Organizer(s): *Shape and Space*

General Description: This software package for Macintosh or Windows consists of disks, a user's guide and reference manual, teaching notes, a quick reference, and an introductory video. It is designed as a tool for teaching and exploring geometrical concepts and allows students to manipulate geometrical figures with a hands-on approach. The sketchpad models geometry in two linked views. Sketches depict concrete geometry and emphasize spatial reasoning, while scripts describe constructions verbally and abstractly. The teaching notes and sample activities provide suggestions for classroom use. Several sample investigations, explorations, demonstrations, and construction activities on a wide range of geometry topics are included. The instructional video supports both Macintosh and Windows.

System requirements for Macintosh: Macintosh Plus or later; 1Mb RAM; System 6 or later.
System requirements for Windows: 4Mb RAM; 3.5" floppy disk; hard drive. Windows 3.1 in enhanced mode.

Caution: *The program defaults to inches, but it can be set to centimetres.*

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics	✓		

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: $289.95 each version

ISBN/Order No: Macintosh version: 1-55953-098-7
Windows version: 1-55953-099-5

Copyright Year: 1995

Geometry Blaster

Curriculum Organizer(s): *Shape and Space*
Patterns and Relations

General Description: Interactive CD-ROM package for Macintosh or Windows includes lessons, activities, and games to teach and reinforce geometric concepts. It uses a three-level approach, allowing a student to progress at a beginning, an intermediate, or a mastery level. The lessons, designed for individual student use, are in three sections: teach, practise, and apply. They explore the main topics of points and lines, triangles, polygons and quadrilaterals, similarity, circles, perimeter and area, solids in 3D, coordinate geometry, transformational geometry and logical reasoning and proof. The program is interactive and requires decision making and creative thinking by students.

System requirements for Macintosh: System 7.0 or later, 8Mb RAM.
System requirements for Windows 3.1: 386/33 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD drive, and mouse. Printer recommended.
System requirements for Windows 95: 486/50 MHz, 8Mb RAM, Pentium processor recommended, 256 super VGA or better colour monitor, sound and graphics cards, CD drive, and mouse. Printer is recommended.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Davidson & Associates, Inc.*
19840 Pioneer Avenue
P.O. Box 2961
Torrance, CA
90503

Tel: 1-800- 545-7677 Fax: (310) 793-0601

Price: $79.95

ISBN/Order No: 0784910634

Copyright Year: 1996

GrafEq (Macintosh/Windows Version 2.09)

Curriculum Organizer(s): *Patterns and Relations*

General Description: Relation plotting tool enables students to graph functions from implicit and explicit definitions, and from relations as well as inequalities. There is the ability to save and print graphs, and to integrate these graphs into other documents. Includes a manual.

System requirements for Macintosh: System 6.5 or higher (System 7 or higher recommended), 2 Mb RAM (4 Mb recommended).
System requirements for Windows: Windows 3.1 with win32's, Windows 95, Windows 98, or Windows NT 4; 8Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pedagoguery Software*
4446 Lazelle Avenue
Terrace, BC
V8G 1R8

Tel: (250) 638-8606 Fax: (250) 638-8606

Price: Single User: $94.50
Site Licence (up to 30): $300.00

ISBN/Order No: (not available)

Copyright Year: 1998

Green Globs & Graphing Equations

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space

General Description: User-friendly software for Macintosh or Windows graphs different forms of linear, quadratic, polynomial, and trigonometric relations. A second program provides a graph and requires the student to enter the equation. There are also two games that act as motivational tools. Five graphs can be displayed simultaneously. The accompanying teacher's guide provides ideas and worksheets.

System requirements for Macintosh: System 6.0.5 or higher (System 7 recommended), 1 Mb RAM.
System requirements for Windows 3.1: Windows 3.1, 486/25 MHz or better, 2 Mb RAM; 256 colour monitor.
System requirements for Windows 95: 486DX/50 MHz or higher (Pentium recommended), 4 Mb RAM, 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics	✓		
Essentials of Mathematics		✓	✓

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $129.95 each version

ISBN/Order No: (not available)

Copyright Year: 1997

An Introduction to the TI-82 Graphing Calculator

Author(s): *Govatos, M.*

General Description: Reproducible blackline masters give clear examples and steps for using the TI-82 graphing calculator in a variety of ways. Can also be used with the TI-83 graphing calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Frank Schaffer Publications Inc.*
23740 Howthorne Boulevard
Torrance, CA
90505

Tel: 1-800-421-5533 Fax: (310) 375-5090

Price: $18.95

ISBN/Order No: 0768200067

Copyright Year: 1998

Mathematics 11, Western Canadian Edition

Author(s): *Alexander, R. et al.*

General Description: Resource package, consisting of a student text, a teacher's resource book, an optional independent study guide, and an optional Template and Data Kit, closely maps the curriculum and uses a problem-solving approach. The student text uses an inquiry approach and incorporates graphing calculators and software use. The independent study guide provides supplementary explanations, examples, and self-tests. The teacher's resource book includes comprehensive teaching notes, blackline masters, additional exercises, a graphing handbook with reproducible graphing calculator activities, and a CD-ROM with solutions. The optional template and data kit includes five databases, spreadsheet templates, and a teacher's guide with tutorials and additional technology activities. Manipulatives, such as algebra tiles and suitable software, necessary for using the resource effectively, are not supplied with the resource. References are made in both the student and teacher materials to specific Internet websites which have not been evaluated.

System requirements for Teacher's Resource Book CD-ROM:

Macintosh: 68020 or greater processor, Mac OS 7.0 or later, 7 Mb RAM minimum, 5 Mb hard drive space plus 7 Mb temporary drive space available during installation, 256 colour 13-inch monitor, 2X CD drive or better.
Windows 3.1: 386DX/33 MHz minimum, 4 Mb RAM minimum, 5 Mb hard drive space plus 7 Mb temporary drive space available during installation, 256 colour 13-inch monitor, 2X CD drive or better.
Windows 95: 486DX/50 MHz (Pentium recommended), 4 Mb RAM minimum, 5 Mb hard drive space plus 7 Mb temporary drive space available during installation, 256 colour 13-inch monitor, 2X CD drive or better.

System requirements for Template and Data Kit:

Macintosh: System 6.0.5 or higher, 4 Mb RAM, mouse, ClarisWorks.
Windows 3.1: 386DX/33 MHz minimum, 4 Mb RAM (8 Mb recommended), either Microsoft Works or ClarisWorks. Windows 95 recommended.
Windows 95: 486/50 MHz (Pentium recommended), 8 Mb RAM (16 Mb recommended), mouse, either ClarisWorks, Microsoft Works or Microsoft Excel/Acces.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relation
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics		✓	
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: Student Text: $51.95
Teacher's Resource Book: $121.64
Independent Study Guide: $5.67
Template and Data Kit: $131.15

ISBN/Order No: Student Text: 0201346249
Teacher's Resource Book: 0201346265
Independent Study Guide: 0201346257
Template and Data Kit: 0201395495

Copyright Year: 1998

MATHPOWER 11, Western Edition

Author(s): *Knill, G. ... (et al.)*

General Description: Resource closely maps the content and philosophy of the curriculum. The Framework's Mathematical processes are woven throughout. Mathematical concepts relate to real-world situations. Concepts and skills are presented in problem-solving contexts, and questions are designed to help students express their ideas about mathematics orally and in writing. Utilizes groupwork, manipulatives, and technology where appropriate. The student text uses an inquiry approach and includes practice exercises. The Teacher's Resource manual contains prerequisite skills, outcomes, suggestions for lessons and activities, answers to textbook questions, and instructions on the use of the TI-83 graphing calculator. Optional components include Blackline masters (which may be used regardless of the text being used by the class), a computer testbank, and a Solutions manual. Manipulatives must be purchased separately. References are made to specific Internet websites which have not been evaluated.

System requirements for Windows: Windows 95 minimum, 486 processor or later, 8 Mb RAM, 4X CD-ROM drive, ClarisWorks 4.0 for Windows or higher. For use on ClarisWorks 4.0 (Windows 950; Microsoft Works 4.0 (Windows 95); ClarisWorks 5.0 (Macintosh OS 7.0); Microsoft Access 97 (Windows 95).

Caution: *The teacher's Resource Manual contains few solutions to C level questions in the student text.*
Teachers may find that the time required to become familiar with the Data Bank software makes it difficult to supplement.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics		✓	
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *McGraw-Hill Ryerson Ltd. (Ontario)*
300 Water Street
Whitby, ON
L1N 9B6

Tel: 1-800-565-5758 (orders only) Fax: 1-800-463-5885

Price: Student Text: $51.95
Teacher's Edition: $54.57

ISBN/Order No: Student Text: 0075525984
Teacher's Edition: 0075525992

Copyright Year: 1999

Modeling Motion: High School Math Activities with the CBR

Author(s): *Antinone, L.; Gough, S.*

General Description: This resource includes investigation activities that use the calculator-based ranger (CBR) to obtain data from physical phenomena. Using a TI-82 or TI-83 graphing calculator, students obtain a mathematical equation to describe the physical event(s). Topics include arithmetic and geometric growth, systems of linear equations in two variables, quadratic functions, and exponential functions. The resource provides practice data to enable students to complete the investigations without a CBR, if necessary.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $16.83

ISBN/Order No: 1886309140

Copyright Year: 1997

Pure Mathematics 20 (Distance Learning Package)

General Description: This resource, designed for distance education, includes six modules with assignements, as well as a learning facilitator's manual. The materials are designed to be used in conjunction with the *MATHPOWER 11* text. Manipulatives must be purchased separately.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics		✓	
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Learning Resources Distributing Centre*
12360-142 Street
Edmonton, AB
T5L 4X9

Tel: (780) 427-5775 Fax: (780) 422-9750

Price: Modules: $16.10
Assignment Booklets: $12.00
Learning Facilitator's Manual: $60.00

ISBN/Order No: (not available)

Copyright Year: 1999

Secondary Math Lab Toolkit

Author(s): *Bedoya, J.Z. ... (et al.)*

General Description: Software package for Macintosh or Windows consists of a CD-ROM and a user's guide. The toolkit contains graphical tools for algebra, geometry, data analysis, and probability. A text editor, function editor, spreadsheet editor, and matrix editor are linked. The graphs are clear and may be transformed in a variety of ways. Probability models use dice and spinners. Includes a visual representation of algebra tiles.

System requirements for Macintosh: Macintosh LC II or later, 6Mb RAM and System 7.0 or higher. PowerMac system with 8Mb RAM and System 7.5 or higher recommended.
System requirements for Windows 3.1: Windows 3.1 in enhanced mode, 486/33 MHz, 12Mb RAM, hard drive.
System requirements for Windows 95: 486/50 MHz, 12Mb RAM, hard drive. Pentium processor with 16Mb RAM recommended.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Number*
Patterns and Relations
Shape and Space

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics			✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $124.95 each version

ISBN/Order No: Macintosh version: 0134330439
Windows version: 0134330447

Copyright Year: 1998

Using the TI-81 Graphics Calculator to Explore Functions

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Kelly, B.*

General Description: This resource manual includes exercises and investigations using the TI-81 graphics calculator and is suitable for a wide range of student abilities. It provides a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524404

Copyright Year: 1991

What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-81 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698100

Copyright Year: 1993

ZAP-A-GRAPH

General Description: Graphing program for Macintosh or Windows can be used to support learning and reviewing graphs of relations and functions. It uses transformations, derived functions, and composition of functions. Relations can be input without recasting into function form. Manual included.

System requirements for Macintosh: System 6.0 or higher, 1 Mb RAM.
System requirements for Windows: Windows 3.1, 3.11 or Windows 95, 1 Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Shape and Space*

Recommended for:

	10	11	12
Principle of Mathematics		✓	✓
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Brain Waves Software Inc.*
RR #1
Fitzroy Harbor, ON
K0A 1X0

Tel: (613) 623-8686 Fax: (613) 623-8686

Price: $79.00

ISBN/Order No: (not available)

Copyright Year: 1996

Principles of Mathematics 12 Grade Collection

	Numbers		Patterns and Relations			Shape and Space			Statistics and Probability	
	Number Concepts	Number Operations	Patterns	Variables & Equations	Relations & Functions	Measure-ment	3-D Objects & 2-D Shapes	Transform-ations	Data Analysis	Chance & Uncertainty
Comprehensive Resources										
Mathematics 12, Western Canadian Edition			✔	✔	✔		✔	✔		✔
MATHPOWER 12, Western Edition			✔	✔	✔		✔	✔		✔
Additional Resources – Print										
Exploring Advance Algebra with the TI-83			✔	✔						✔
Exploring Functions with the TI-82 Graphics Calculator				✔	✔					
Exploring Statistics with the TI-82 Graphics Calculator										✔
An Introduction to the TI-82 Graphing Calculator			✔	✔	✔					
Modeling Motion: High School Math Activities with the CBR			✔	✔	✔					
Using the TI-81 Graphics Calculator to Explore Functions					✔					✔
Additional Resources – Games/Manipulatives										
String Invision Conics Model								✔		
Additional Resources – Software/CD-ROM										
GrafEq					✔					
Secondary Math Lab Toolkit			✔	✔	✔		✔	✔		✔
ZAP-A-GRAPH					✔			✔		

Exploring Advanced Algebra with the TI-83

Curriculum Organizer(s): *Patterns and Relations*
Statistics and Probability

Author(s): *Kelly, B.*

General Description: Resource contains exercises and investigations that supplement the following topics: sequences, fractals, combinatorics, probability, linear systems, matrices, and linear programming. The keying sequences are specific to TI-83 model of calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics			✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $15.71

ISBN/Order No: 1895997100

Copyright Year: 1998

Exploring Functions with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Patterns and Relations*

Author(s): *Kelly, B.*

General Description: This resource contains exercises and investigations that are suitable for a wide range of student abilities. It acts as a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 1895997003

Copyright Year: 1993

Exploring Statistics with the TI-82 Graphics Calculator

Curriculum Organizer(s): *Statistics and Probability*

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-82 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Exercises allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓		✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $14.60

ISBN/Order No: 189599702X

Copyright Year: 1994

GrafEq (Macintosh/Windows Version 2.09)

Curriculum Organizer(s): *Patterns and Relations*

General Description: Relation plotting tool enables students to graph functions from implicit and explicit definitions, and from relations as well as inequalities. There is the ability to save and print graphs, and to integrate these graphs into other documents. Includes a manual.

System requirements for Macintosh: System 6.5 or higher (System 7 or higher recommended), 2 Mb RAM (4 Mb recommended).
System requirements for Windows: Windows 3.1 with win32's, Windows 95, Windows 98, or Windows NT 4; 8Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pedagoguery Software*
4446 Lazelle Avenue
Terrace, BC
V8G 1R8

Tel: (250) 638-8606 Fax: (250) 638-8606

Price: Single User: $94.50
Site Licence (up to 30): $300.00

ISBN/Order No: (not available)

Copyright Year: 1998

Green Globs & Graphing Equations

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space

General Description: User-friendly software for Macintosh or Windows graphs different forms of linear, quadratic, polynomial, and trigonometric relations. A second program provides a graph and requires the student to enter the equation. There are also two games that act as motivational tools. Five graphs can be displayed simultaneously. The accompanying teacher's guide provides ideas and worksheets.

System requirements for Macintosh: System 6.0.5 or higher (System 7 recommended), 1 Mb RAM.
System requirements for Windows 3.1: Windows 3.1, 486/25 MHz or better, 2 Mb RAM; 256 colour monitor.
System requirements for Windows 95: 486DX/50 MHz or higher (Pentium recommended), 4 Mb RAM, 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓		
Essentials of Mathematics		✓	✓

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $129.95 each version

ISBN/Order No: (not available)

Copyright Year: 1997

An Introduction to the TI-82 Graphing Calculator

Curriculum Organizer(s): *Patterns and Relations*

General Description: Reproducible blackline masters give clear examples and steps for using the TI-82 graphing calculator in a variety of ways. Can also be used with the TI-83 graphing calculator.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Frank Schaffer Publications Inc.*
23740 Howthorne Boulevard
Torrance, CA
90505

Tel: 1-800-421-5533 Fax: (310) 375-5090

Price: $18.95

ISBN/Order No: 0768200067

Copyright Year: 1998

Mathematics 12, Western Canadian Edition

Author(s): *LeBlanc, D.*

General Description: This resource closely maps the content of the curriculum and uses a problem solving approach. Components consist of a student text, Teacher's Resource Book, an optional Independent Study Guide, and an optional Template and Data Kit. Open-ended questions are designed to help students express their ideas about mathematics orally and in writing. Utilizes group work, manipulatives, and technology where appropriate. The student text uses an inquiry approach and incorporates graphing calculators and software use. The Teacher's Resource Book provides a correlation of outcomes with text pages, and each section supplies supplementary examples and assessment examples which the teacher can use during lessons. The Independent Study Guide covers each of the student text chapters and correlates directly with each section. The Template and Data CD-ROM includes four databases, spreadsheet templates, and a teacher's guide with tutorials, answers, and additional technology activities. Manipulatives must be purchased separately. References are made to specific Internet websites which have not been evaluated.

System requirements for Template and Data Kit:

Macintosh: System 6.0.5 or higher, 4 Mb RAM, mouse, ClarisWorks.
Windows 3.1: 386DX/33 MHz minimum, 4 Mb RAM (8 Mb recommended), either Microsoft Works or ClarisWorks. Windows 95 recommended.
Windows 95: 486/50 MHz (Pentium recommended), 8 Mb RAM (16 Mb recommended), mouse, either ClarisWorks, Microsoft Works or Microsoft Excel/Access.

Caution: *Standard deviation of a sample mean for large samples is not addressed.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics			✓
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: Student Text: $51.75
Teacher's Resource Book: $147.75

ISBN/Order No: Student Text: 0201346629X
Teacher's Resource Book: 0201346311

Copyright Year: 1999

MATHPOWER 12, Western Edition

Author(s): *Angel, P.; ... (et al.)*

General Description: The resource package closely maps the content and philosophy of the curriculum. Mathematical processes are woven throughout the resource. A conscientious effort was made to relate the mathematical concepts to real-world situations. The nine-chapter student text makes good use of visual presentations, and the format is consistent throughout. Each section includes a presentation of the concept, an explore and inquire section to develop the concept, examples with worked solutions and many exercises. An optional Computer Data Bank has been reviewed for technical considerations only and may be used in conjunction with various sections of the text.

System requirements for Windows: Windows 95 minimum, 486 processor or higher, 8 Mb RAM, 4X CD-ROM drive, ClarisWorks 5.0 for Windows or higher. For use on ClarisWorks 5.0 (Windows 95); Microsoft Works 4.0 (Windows 95); ClarisWorks 5.0 (Macintosh OS 7.0); Microsoft Access 97 (Windows 95).

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics			✓
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *McGraw-Hill Ryerson Ltd. (Ontario)*
300 Water Street
Whitby, ON
L1N 9B6

Tel: 1-800-565-5758 (orders only) Fax: 1-800-463-5885

Price: Student Text: $53.95
Teacher's Resource: $110.00

ISBN/Order No: Student Text: 007552600X
Teacher's Resource: 0075600218

Copyright Year: 2000

Modeling Motion: High School Math Activities with the CBR

Author(s): *Antinone, L.; Gough, S.*

General Description: This resource includes investigation activities that use the calculator-based ranger (CBR) to obtain data from physical phenomena. Using a TI-82 or TI-83 graphing calculator, students obtain a mathematical equation to describe the physical event(s). Topics include arithmetic and geometric growth, systems of linear equations in two variables, quadratic functions, and exponential functions. The resource provides practice data to enable students to complete the investigations without a CBR, if necessary.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $16.83

ISBN/Order No: 1886309140

Copyright Year: 1997

Secondary Math Lab Toolkit

Author(s): *Bedoya, J.Z. ... (et al.)*

General Description: Software package for Macintosh or Windows consists of a CD-ROM and a user's guide. The toolkit contains graphical tools for algebra, geometry, data analysis, and probability. A text editor, function editor, spreadsheet editor, and matrix editor are linked. The graphs are clear and may be transformed in a variety of ways. Probability models use dice and spinners. Includes a visual representation of algebra tiles.

System requirements for Macintosh: Macintosh LC II or later, 6Mb RAM and System 7.0 or higher. PowerMac system with 8Mb RAM and System 7.5 or higher recommended.
System requirements forWindows 3.1: Windows 3.1 in enhanced mode, 486/33 MHz, 12Mb RAM, hard drive.
System requirements for Windows 95: 486/50 MHz, 12Mb RAM, hard drive. Pentium processor with 16Mb RAM recommended.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*
Shape and Space
Statistics and Probability

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics			✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $124.95 each version

ISBN/Order No: Macintosh version: 0134330439
Windows version: 0134330447

Copyright Year: 1998

String Invision Conics Model

General Description: Resource consists of a model made of string, metal, and wood; a teacher's guide; and overhead materials. The model transfers a beam of light from the overhead projector to the visual representation of a double-napped cone, creating a conic. It is particularly effective in demonstrating the cases involving degenerate conics, rather than as a student exploration tool.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Shape and Space*

Recommended for:

	10	11	12
Principle of Mathematics			✓
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Belle Vue Software*
25 Windsor Street
Spruce Grove , AB
T7X 1L6

Tel: (780) 962-9275

Price: $80.00

ISBN/Order No: (not available)

Copyright Year: (not available)

Using the TI-81 Graphics Calculator to Explore Functions

Author(s): *Kelly, B.*

General Description: This resource manual includes exercises and investigations using the TI-81 graphics calculator and is suitable for a wide range of student abilities. It provides a supplement to a limited number of learning outcomes and is designed for follow-up activities.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Patterns and Relations*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓	✓	
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524404

Copyright Year: 1991

Using the TI-81 Graphics Calculator to Explore Statistics

Author(s): *Kelly, B.*

General Description: This resource manual uses the TI-81 graphics calculator to explore and develop applications in statistics and probability, through the use of investigations and worked examples. These examples reflect current and real-world situations. There are numerous references to historical influences on various developments in statistics and probability. Student exercises, most of which are not calculator specific, allow ample scope for development and extension of the main mathematical processes.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Statistics and Probability*

Recommended for:

	10	11	12
Principle of Mathematics			✓
Applications of Mathematics	✓	✓	✓
Essentials of Mathematics			

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $8.95

ISBN/Order No: 0969524412

Copyright Year: 1992

ZAP-A-GRAPH

Curriculum Organizer(s): *Shape and Space*

General Description: Graphing program for Macintosh or Windows can be used to support learning and reviewing graphs of relations and functions. It uses transformations, derived functions, and composition of functions. Relations can be input without recasting into function form. Manual included.

System requirements for Macintosh: System 6.0 or higher, 1 Mb RAM. System requirements for Windows: Windows 3.1, 3.11 or Windows 95, 1 Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics		✓	✓
Applications of Mathematics			
Essentials of Mathematics			

Supplier: *Brain Waves Software Inc.*
RR #1
Fitzroy Harbor, ON
K0A 1X0

Tel: (613) 623-8686 Fax: (613) 623-8686

Price: $79.00

ISBN/Order No: (not available)

Copyright Year: 1996

3-D Images

Curriculum Organizer(s): *Geometry Project*

General Description: Macintosh software enables students to investigate and create geometrical transformations. Using drawing and modifier tools, students draw and then re-form 2-D figures into related 3-D figures. Includes a user's manual and cardboard 3-D glasses.

System requirements: Macintosh Plus or later; System 6.0.3 or later; 1 Mb RAM; colour monitor optional.

Audience: *General*
ESL - minimal language required
Gifted - visuals allow open-ended exploration and divergent thinking
LD - visual focus

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics		✓	✓
Applications of Mathematics		✓	
Essentials of Mathematics	✓		

Supplier: *Westworld Computers Limited*
10333 - 170 Street
Edmonton, AB
T5P 4V4

Tel: 1-888-622-4280 Fax: 1-800-929-5630

Price: $132.00

ISBN/Order No: 3194

Copyright Year: 1991

The ABC's of Personal Finance

Curriculum Organizer(s): *Personal Banking*

Author(s): *Pool; Frick*

General Description: Twenty-five-minute American video emphasizes the importance of financial planning in order to achieve goals. It outlines the importance of budgeting, setting goals, and collecting interest as opposed to paying interest. Effects of compound interest versus inflation in money growth are discussed. Teachers will need to develop their own pre- and post-viewing activities.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics		✓	
Essentials of Mathematics	✓	✓	✓

Supplier: *B.C. Learning Connection Inc.*
#4-8755 Ash Street
Vancouver, BC
V6P 6T3

Tel: 1-800-884-2366 Fax: (604) 324-1844

Price: $26.00

ISBN/Order No: MA0003

Copyright Year: 1995

D.I.M.E. Probability Pack A

Author(s): *Giles, Geoff*

General Description: Set of activity cards features various experiments for exploring probability. Includes dice, disks, a teacher's guide, and a chart for recording and graphing results.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Probability and Sampling*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: 1-800-668-0600 Fax: (905) 727-6265

Price: $59.50

ISBN/Order No: 17262

Copyright Year: 1988

D.I.M.E. Probability Pack B

Author(s): *Giles, Geoff*

General Description: Four sets of activity cards feature experiments to explore probability and problem solving. Also includes sets of beads, a teaching guide, and a chart for recording and graphing results.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Probability & Sampling*
Problem Solving

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: $59.50

ISBN/Order No: 17264

Copyright Year: 1988

Exploring Trigonometry with the Geometer's Sketchpad

Curriculum Organizer(s): *Trigonometry*

Author(s): *Shaffer, David*

General Description: Supplementary blackline activity masters are for use with the software program *The Geometer's Sketchpad*, which is also recommended. Activities employ traditional geometric construction techniques and modelling, and emphasize the relationships among angles, triangles, and circles. Teacher's notes section is included. Accompanying software disks, for Macintosh and Windows, provide sample sketches and scripts.

Audience: *General*
Gifted - open-ended explorations

Category: *Student, Teacher Resource*

Recommended for:	10	11	12
Principle of Mathematics		✓	✓
Applications of Mathematics		✓	
Essentials of Mathematics	✓		

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: $41.55

ISBN/Order No: 1-55953-075-8/13798

Copyright Year: 1995

The Geometer's Sketchpad

Curriculum Organizer(s): *Trigonometry*

General Description: This resource consists of discs, a user guide and reference manual, teaching notes and sample activities, a quick reference, and an introductory video. The resource models geometry in two linked views, sketches and scripts. Sketches depict concrete geometry and emphasize spatial reasoning, while scripts describe constructions verbally and abstractly. The program allows for the construction, labelling, measurement and manipulation of any geometric figure, as well as the exploration of geometry concepts taught in Grade 7 through Grade 9. The user guide and reference manual provide instruction for learning to use the program. The teaching notes and sample activities provide suggestions for classroom use. Several sample investigations, explorations, demonstration and construction activities, on a wide range of geometry topics, are included.

Macintosh: minimum of a Macintosh Plus, with 1 MB RAM and System 6 or higher. A videocassette supplied with the Windows version supports both Macintosh and Windows.
Windows 3.1: Windows 3.1 in enhanced mode, 386/33 MHz, 4MB RAM, hard drive. The videocassette supports both Macintosh and Windows versions.
Windows 95: 486/50 MHz (Pentium recommended), 8MB RAM, hard drive. The videocassette supports both Macintosh and Windows versions.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics	✓	✓	
Essentials of Mathematics	✓		

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: $289.95 each version

ISBN/Order No: Macintosh Version: 1-55953-098-7
Windows Version: 1-55953-099-5

Copyright Year: 1995

Hot Dog Stand: The Works

General Description: This software for Macintosh or Windows simulates running a hot dog stand. Students gather information, keep records, plan inventory, decide on pricing and look at selling strategies. There are three levels of operation. At the intermediate and advanced leves, students also deal with the unexpected events of running a business, such as weather, unreliable suppliers, and machine breakdown. Students estimate, calculate, read graphs, and keep track of their progress in a journal - incorporated as a mini-word processor. In the teacher's guide there are follow-up activities relating to starting and running a business.

System requirements for Macintosh: System 7.0 or later; 68040 or later processor; Mb RAM; 31Mb hard drive space; double speed CD-ROM drive; and 640x480, 256 colour monitor.
System requirements for Windows: Windows 3.1 or Windows 95, 8Mb RAM, 30Mb hard drive, double speed CD-ROM drive, 640x480 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Rate, Ratio and Porportion*
Spreadsheets

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $89.95 each version

ISBN/Order No: (not available)

Copyright Year: 1996

Math Tools

General Description: This package is a set of Windows software utilities that can share data. The tools include spreadsheet, calculation, geometry, graphs, manipulatives, fraction strips, and probability. A colour linking feature between two or more windows allows data to be shared and represented in a variety of formats. Includes information on installing and using the software and chapters specific to each math tool. Provides a glossary and index.

System requirements for Windows 3.1: 386/33 MHz, 4Mb of RAM, 13" VGA monitor, 2Mb hard disk space, PKZIP.
System requirements for Windows 95: 486/50 MHz, 8Mb of RAM, 13" VGA monitor, 2Mb hard disk space, PKZIP.

Caution: *The algebra tile manipulatives have limited usage (first-degree variables only.) The spreadsheet uses radians as units for sine, cosine, and tangent calculations.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Geometry Project*
Probability and Sampling
Spreadsheets

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $58.94

ISBN/Order No: 0134109783

Copyright Year: 1995

Probability Constructor

Curriculum Organizer(s): *Probability and Sampling*

General Description: Interactive Macintosh computer program allows students to conduct probability experiments using six models: coins; dice; marbles in a jar; spinners; random digits; and plotting random points in areas of a plane. Accompanying manual provides opportunities for further exploration.

System requirements: 4 MB RAM; 13" colour monitor; System 7.0 or higher. A Windows version is available, but was not evaluated.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Tangent Scientific Supply Inc.*
23 Hannover Drive, Unit 2
St. Catherines, ON
L2W 1A3

Tel: 1-800-363-2908 Fax: (905) 704-1555

Price: $199.00

ISBN/Order No: 99-3108-01

Copyright Year: 1996

Problem Solving: What You Do When You Don't Know What To Do

Curriculum Organizer(s): *Probability & Sampling*
Problem Solving

Author(s): *Jones, G.*

General Description: Binder describes the processes and strategies used to solve problems, including trial and error, making a drawing or sketch, logical thinking, finding a pattern, working backward, elimination, and making a list. Explains each strategy and includes over 200 challenging activities.

Audience: *General*
Gifted - opportunities for critical thinking

Category: *Teacher Resource*

Recommended for:	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	✓

Supplier: *Exclusive Educational Products (Ontario)*
243 Saunders Road
Barrie, ON
L4M 6E7

Tel: 1-800-563-1166 Fax: (705) 725-1167

Price: $31.46

ISBN/Order No: (not available)

Copyright Year: 1994

Triple 'A' Mathematics Program: Data Management & Probability

Author(s): *Birse ... (et al.)*

General Description: Binder of activities, projects, and assessment items for data analysis and probability includes extensive teacher support in terms of suggested teaching strategies, sequencing, and scoring rubrics. Activities include both routine calculation and open-ended projects. Some activities involve critical thinking. Provides random number tables, but other manipulatives must be provided by the teacher.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Probability & Sampling*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Exclusive Educational Products (Ontario)*
243 Saunders Road
Barrie, ON
L4M 6E7

Tel: 1-800-563-1166 Fax: (705) 725-1167

Price: $33.26

ISBN/Order No: (not available)

Copyright Year: 1995

The ABC's of Personal Finance

Author(s): *Pool; Frick*

General Description: Twenty-five-minute American video emphasizes the importance of financial planning in order to achieve goals. It outlines the importance of budgeting, setting goals, and collecting interest as opposed to paying interest. Effects of compound interest versus inflation in money growth are discussed. Teachers will need to develop their own pre- and post-viewing activities.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Income and Debt*
Owning and Operating a Vehicle

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics		✓	✓

Supplier: *B.C. Learning Connection Inc.*
#4-8755 Ash Street
Vancouver, BC
V6P 6T3

Tel: 1-800-884-2366 Fax: (604) 324-1844

Price: $26.00

ISBN/Order No: MA0003

Copyright Year: 1995

D.I.M.E. Probability Pack A

Author(s): *Giles, Geoff*

General Description: Set of activity cards features various experiments for exploring probability. Includes dice, disks, a teacher's guide, and a chart for recording and graphing results.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Applications of Probability*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: 1-800-668-0600 Fax: (905) 727-6265

Price: $59.50

ISBN/Order No: 17262

Copyright Year: 1988

D.I.M.E. Probability Pack B

Curriculum Organizer(s): *Applications of Probability*

Author(s): *Giles, Geoff*

General Description: Four sets of activity cards feature experiments to explore probability and problem solving. Also includes sets of beads, a teaching guide, and a chart for recording and graphing results.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Spectrum Educational Supplies Ltd.*
125 Mary St.
Aurora, ON
L4G 1G3

Tel: (905) 841-0600 Fax: (905) 727-6265

Price: $59.50

ISBN/Order No: 17264

Copyright Year: 1988

Green Globs & Graphing Equations

Curriculum Organizer(s): *Data Analysis and Interpretation*
Relations and Formulas

General Description: User-friendly software for Macintosh or Windows graphs different forms of linear, quadratic, polynomial, and trigonometric relations. A second program provides a graph and requires the student to enter the equation. There are also two games that act as motivational tools. Five graphs can be displayed simultaneously. The accompanying teacher's guide provides ideas and worksheets.

System requirements for Macintosh: System 6.0.5 or higher (System 7 recommended), 1 Mb RAM.
System requirements for Windows 3.1: Windows 3.1, 486/25 MHz or better, 2 Mb RAM; 256 colour monitor.
System requirements for Windows 95: 486DX/50 MHz or higher (Pentium recommended), 4 Mb RAM, 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓		
Essentials of Mathematics		✓	✓

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $129.95 eah version

ISBN/Order No: (not available)

Copyright Year: 1996

Hot Dog Stand: The Works

General Description: This software for Macintosh or Windows simulates running a hot dog stand. Students gather information, keep records, plan inventory, decide on pricing and look at selling strategies. There are three levels of operation. At the intermediate and advanced leves, students also deal with the unexpected events of running a business, such as weather, unreliable suppliers, and machine breakdown. Students estimate, calculate, read graphs, and keep track of their progress in a journal - incorporated as a mini-word processor. In the teacher's guide there are follow-up activities relating to starting and running a business.

System requirements for Macintosh: System 7.0 or later; 68040 or later processor; Mb RAM; 31Mb hard drive space; double speed CD-ROM drive; and 640x480, 256 colour monitor.
System requirements for Windows: Windows 3.1 or Windows 95, 8Mb RAM, 30Mb hard drive, double speed CD-ROM drive, 640x480 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Business Plan*
Data Analysis and Interpretation

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $89.95 each version

ISBN/Order No: (not available)

Copyright Year: 1996

Math Projects: Organization, Implementation, and Assessment

Author(s): *DeMeulemeester, Katie*

General Description: Resource uses specific examples to explain how to organize student projects. The book describes nine different presentation formats: videos, skits, photo albums, graphics portfolios, class books, games, posters, models, and business presentations.

Audience: *General*

Category: *Teacher Resource*

Curriculum Organizer(s): *Business Plan*
Income and Debt

Recommended for:

	10	11	12
Principle of Mathematics		✓	✓
Applications of Mathematics		✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $18.15

ISBN/Order No: 0-86651-836-3

Copyright Year: 1995

Math Tools

General Description: This package is a set of Windows software utilities that can share data. The tools include spreadsheet, calculation, geometry, graphs, manipulatives, fraction strips, and probability. A colour linking feature between two or more windows allows data to be shared and represented in a variety of formats. Includes information on installing and using the software and chapters specific to each math tool. Provides a glossary and index.

System requirements for Windows 3.1: 386/33 MHz, 4Mb of RAM, 13" VGA monitor, 2Mb hard disk space, PKZIP.
System requirements for Windows 95: 486/50 MHz, 8Mb of RAM, 13" VGA monitor, 2Mb hard disk space, PKZIP.

Caution: *The algebra tile manipulatives have limited usage (first-degree variables only.) The spreadsheet uses radians as units for sine, cosine, and tangent calculations.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Applications of Probability*
Relations and Formulas

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $58.94

ISBN/Order No: 0134109783

Copyright Year: 1997

Models and Patterns: Experimenting With Linear Equations

Author(s): *Montesanto, Ralph; Zimmer, David*

General Description: Student text and teacher's guide outline hands-on activities for gathering and using data and statistics with the aid of a scatter plot. Explores linear graphing and slope/intercept.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Relations and Formulas*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics		✓	

Supplier: *Nelson Thomson Learning*
1120 Birchmount Road
Scarborough, ON
M1K 5G4

Tel: 1-800-268-2222 Fax: (416) 752-9365 or 752-9646

Price: Softcover Student Book: $12.95
Hardcover Student Book: $17.65
Teacher's Guide: $24.95

ISBN/Order No: Softcover Student Book: 0-669-95475-6
Hardcover Student Book: 0-669-95482-9
Teacher's Guide: 0-669-95479-9

Copyright Year: 1994

The Multicultural Math Classroom: Bringing in the World

General Description: Book introduces a multicultural perspective to elementary and middle grade mathematics, revealing how such a perspective can enrich the learning of all students, whatever their gender, ethnic/racial heritage, or socio-economic status. Students learn that mathematics was created by real people attempting to solve real problems. The collection of open-ended activities addresses a range of mathematical topics and cultural connections, and promotes problem solving and critical and creative thinking.

Audience: *General*

Category: *Teacher Resource*

Curriculum Organizer(s): *Problem Solving*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics		✓	

Supplier: *Irwin Publishing*
325 Humber College Blvd.
Toronto, ON
M9W 7C3

Tel: 1-800-387-0172 (orders only) Fax: (416) 798-1384

Price: $36.20

ISBN/Order No: 0435083732

Copyright Year: 1996

Probability Constructor

General Description: Interactive Macintosh computer program allows students to conduct probability experiments using six models: coins; dice; marbles in a jar; spinners; random digits; and plotting random points in areas of a plane. Accompanying manual provides opportunities for further exploration.

System requirements: 4 MB RAM; 13" colour monitor; System 7.0 or higher. A Windows version is available, but was not evaluated.

Audience: *General*

Category: *Student*, *Teacher Resource*

Curriculum Organizer(s): *Applications of Probability*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Tangent Scientific Supply Inc.*
23 Hannover Drive, Unit 2
St. Catherines, ON
L2W 1A3

Tel: 1-800-363-2908 Fax: (905) 704-1555

Price: $199.00

ISBN/Order No: 99-3108-01

Copyright Year: 1994

Problem Solving: What You Do When You Don't Know What To Do

Curriculum Organizer(s): *Problem Solving*

Author(s): *Jones, G.*

General Description: Binder describes the processes and strategies used to solve problems, including trial and error, making a drawing or sketch, logical thinking, finding a pattern, working backward, elimination, and making a list. Explains each strategy and includes over 200 challenging activities.

Audience: *General*
Gifted - opportunities for critical thinking

Category: *Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	✓

Supplier: *Exclusive Educational Products (Ontario)*
243 Saunders Road
Barrie, ON
L4M 6E7

Tel: 1-800-563-1166 Fax: (705) 725-1167

Price: $31.46

ISBN/Order No: (not available)

Copyright Year: 1991

Statistics Workshop

Curriculum Organizer(s): *Data Analysis and Interpretation*

General Description: Macintosh software provides a set of computer tools for understanding and exploring fundamental concepts of data analysis. Students can explore the underlying meaning of abstract statistical concepts and processes. Provides an easy-to-use system for entering, manipulating, and displaying data. Includes a binder with support materials.

System requirements: Macintosh Plus or later (Classic or later recommended) 1Mb RAM.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics		✓	✓
Applications of Mathematics		✓	✓
Essentials of Mathematics		✓	

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $129.95

ISBN/Order No: (not available)

Copyright Year: 1991

Triple 'A' Mathematics Program: Data Management & Probability

Curriculum Organizer(s): *Applications of Probability*
Data Analysis and Interpretation

Author(s): *Birse ... (et al.)*

General Description: Binder of activities, projects, and assessment items for data analysis and probability includes extensive teacher support in terms of suggested teaching strategies, sequencing, and scoring rubrics. Activities include both routine calculation and open-ended projects. Some activities involve critical thinking. Provides random number tables, but other manipulatives must be provided by the teacher.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	

Supplier: *Exclusive Educational Products (Ontario)*
243 Saunders Road
Barrie, ON
L4M 6E7

Tel: 1-800-563-1166 Fax: (705) 725-1167

Price: $33.26

ISBN/Order No: (not available)

Copyright Year: 1995

What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Curriculum Organizer(s): *Relations and Formulas*

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-81 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics		✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698100

Copyright Year: 1993

What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-82 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes. This resource also introduces the concept of the line of best fit.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Relations and Formulas*

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics		✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698135

Copyright Year: 1993

The ABC's of Personal Finance

Author(s): *Pool; Frick*

General Description: Twenty-five-minute American video emphasizes the importance of financial planning in order to achieve goals. It outlines the importance of budgeting, setting goals, and collecting interest as opposed to paying interest. Effects of compound interest versus inflation in money growth are discussed. Teachers will need to develop their own pre- and post-viewing activities.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Life/Career Project*
Personal Finance

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics		✓	
Essentials of Mathematics	✓	✓	✓

Supplier: *B.C. Learning Connection Inc.*
#4-8755 Ash Street
Vancouver, BC
V6P 6T3

Tel: 1-800-884-2366 Fax: (604) 324-1844

Price: $26.00

ISBN/Order No: MA0003

Copyright Year: 1995

Green Globs & Graphing Equations

General Description: User-friendly software for Macintosh or Windows graphs different forms of linear, quadratic, polynomial, and trigonometric relations. A second program provides a graph and requires the student to enter the equation. There are also two games that act as motivational tools. Five graphs can be displayed simultaneously. The accompanying teacher's guide provides ideas and worksheets.

System requirements for Macintosh: System 6.0.5 or higher (System 7 recommended), 1 Mb RAM.
System requirements for Windows 3.1: Windows 3.1, 486/25 MHz or better, 2 Mb RAM; 256 colour monitor.
System requirements for Windows 95: 486DX/50 MHz or higher (Pentium recommended), 4 Mb RAM, 256 colour monitor.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Variation and Formulas*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	✓
Applications of Mathematics	✓		
Essentials of Mathematics		✓	✓

Supplier: *Gage Educational Publishing Co. (Scarborough)*
164 Commander Boulevard
Scarborough, ON
M1S 3C7

Tel: 1-800-667-1115 Fax: (416) 293-9009

Price: $129.95 each version

ISBN/Order No: (not available)

Copyright Year: 1996

Math Projects: Organization, Implementation, and Assessment

Author(s): *DeMeulemeester, Katie*

General Description: Resource uses specific examples to explain how to organize student projects. The book describes nine different presentation formats: videos, skits, photo albums, graphics portfolios, class books, games, posters, models, and business presentations.

Audience: *General*

Category: *Teacher Resource*

Curriculum Organizer(s): *Design and Measurements*
Life/Career Projects

Recommended for:

	10	11	12
Principle of Mathematics		✓	✓
Applications of Mathematics		✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $18.15

ISBN/Order No: 0-86651-836-3

Copyright Year: 1995

Math Tools

General Description: This package is a set of Windows software utilities that can share data. The tools include spreadsheet, calculation, geometry, graphs, manipulatives, fraction strips, and probability. A colour linking feature between two or more windows allows data to be shared and represented in a variety of formats. Includes information on installing and using the software and chapters specific to each math tool. Provides a glossary and index.

System requirements for Windows 3.1: 386/33 MHz, 4Mb of RAM, 13" VGA monitor, 2Mb hard disk space, PKZIP.
System requirements for Windows 95: 486/50 MHz, 8Mb of RAM, 13" VGA monitor, 2Mb hard disk space, PKZIP.

Caution: *The algebra tile manipulatives have limited usage (first-degree variables only.) The spreadsheet uses radians as units for sine, cosine, and tangent calculations.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Design and Measurements*
Investments
Life/Career Project
Personal Finance

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $58.94

ISBN/Order No: 0134109783

Copyright Year: 1997

Problem Solving: What You Do When You Don't Know What To Do

Curriculum Organizer(s): *Problem Solving*

Author(s): *Jones, G.*

General Description: Binder describes the processes and strategies used to solve problems, including trial and error, making a drawing or sketch, logical thinking, finding a pattern, working backward, elimination, and making a list. Explains each strategy and includes over 200 challenging activities.

Audience: *General*
Gifted - opportunities for critical thinking

Category: *Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics			
Applications of Mathematics			
Essentials of Mathematics	✓	✓	✓

Supplier: *Exclusive Educational Products (Ontario)*
243 Saunders Road
Barrie, ON
L4M 6E7

Tel: 1-800-563-1166 Fax: (705) 725-1167

Price: $31.46

ISBN/Order No: (not available)

Copyright Year: 1991

What If ...?: The Straight Line: Investigations with the TI-81 Graphics Calculator

Curriculum Organizer(s): *Variation and Formulas*

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-81 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes.

Audience: *General*

Category: *Student, Teacher Resource*

Recommended for:

	10	11	12
Principle of Mathematics	✓	✓	
Applications of Mathematics		✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698100

Copyright Year: 1993

What If ...?: The Straight Line: Investigations with the TI-82 Graphics Calculator

Author(s): *Alexander, B.*

General Description: This resource explores the straight line, dependent upon using the TI-82 graphics calculator. It shows students how to plot linear equations, how to make changes to the variable, and uses the question "what if ...?" to pose multiple problems within a single context. Students are encouraged to compare their answers and to predict outcomes. This resource also introduces the concept of the line of best fit.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Variation and Formulas*

Recommended for:

	10	11	12
Principle of Mathematics	✓		
Applications of Mathematics	✓	✓	
Essentials of Mathematics		✓	✓

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $12.30

ISBN/Order No: 0969698135

Copyright Year: 1994

Calculus: A New Horizon, Sixth Edition

Author(s): *Anton, Howard*

General Description: Comprehensive resource package consisting of a student text and a student resource manual with sample tests covers all aspects of the Calculus 12 curriculum, with the exception of quadratic approximations. Text includes chapter and section overviews, section and chapter exercieses, answer key, and appendices. Student resource manual provides detailed solutions to odd-numbered exercises, and sample tests. A test bank, Student Resource and Survival CD, and an electronic version of the teacher's manual are optional resources.

Caution: *Software is not well implemented and does not add significantly to text. U.S. bias - little to no Canadian content. Exploration style questions weak. Little to no group work or exercises exercising student creativity.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Antidifferentiation*
Applications of Derivatives
The Derivative
Functions, Graphs and Limits
Overview and History of Calculus

Supplier: *John Wiley & Sons Canada Ltd.*
22 Worcester Road
Etobicoke, ON
M9W 1L1

Tel: 1-800-567-4797 Fax: 1-800-565-6802

Price: Textbook: $62.80
Student Resource Manual: $30.80

ISBN/Order No: Textbook: 0-471-15307-9
Student Resource Manual: 0-471-246239

Copyright Year: 1999

Calculus: Graphical, Numerical, Algebraic

Author(s): *Finney ... (et al.)*

General Description: This resource covers all topics in Calculus 12 with the exception of quadratic approximations of curves. The resource is well laid out and makes good use of technology to aid student learning. This resource makes a good attempt to allow for alternate teaching and learning styles through group work and journal writing assignments. Features include a balanced approach; applications to real-world problems; explorations that provide guided investigations; and exercise sets of various kinds. Selected answers and solutions are provided.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Antidifferentiation*
Applications of Derivatives
The Derivative
Functions, Graphs and Limits
Overview and History of Calculus

Supplier: *Pearson Education Canada*
26 Prince Andrew Place
Don Mills, ON
M3C 2T8

Tel: 1-800-361-6128 Fax: 1-800-563-9196

Price: $99.25

ISBN/Order No: 0-201-32445-8

Copyright Year: 1999

Calculus of a Single Variable Early Transcendental Functions, Second Edition

Author(s): *Larson; Hostetler; Edwards*

General Description: Well-laid out, visually appealing resource that integrates the historical development of calculus, comprises nine chapters, appendices, solutions, and an index. The text encourages lecture or discovery approaches, pencil and paper skills or technology, and supports a variety of presentation styles. Concepts are clearly introduced and developed. Includes exercise sets, real-world explorations, and problems. An Interactive Calculus CD-ROM is available to support the text.

System Requirements for Windows: Windows '95; CD-ROM drive; 8Mb RAM ; 1Mb VRAM.

Caution: *Chapter motivators do include significant reference to commercial products. Teaching strategies are not laid out. Few references to First Nations people.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Applications of Derivatives*
The Derivative
Functions, Graphs and Limits
Overview and History of Calculus

Supplier: *Nelson Thomson Learning*
1120 Birchmount Road
Scarborough, ON
M1K 5G4

Tel: 1-800-268-2222 Fax: (416) 752-9365 or 752-9646

Price: $105.00

ISBN/Order No: 0-395-93321-8

Copyright Year: 1999

Calculus of a Single Variable, Sixth Edition

Author(s): *Larson; Hostetler; Edwards*

General Description: Resource package consisting of a student text, instructor's resource guide, two-volume solutions guide, and optional Interactive Calculus CD-ROM integrates technology and real-world problems in a ten chapter format. The readable text provides motivational problems and explanations, and well-written examples take a variety of solution approaches, follow clearly defined strategies, and build in checking for reasonability of the solution. Exercises vary in level of difficulty and incorporate writing and thinking activities. The instructor's resource guide provides teaching strategies, tests and answer keys, and the complete solutions guides contain detailed exercise solutions. An optional Interactive Calculus CD-ROM provides an electronic version of the text.

Caution: *The CD-ROM package does not cover the whole curriculum and is only an electronic version of the text.*

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Antidifferentiation*
Applications of Derivatives
The Derivative
Functions, Graphs and Limits
Overview and History of Calculus

Supplier: *Nelson Thomson Learning*
1120 Birchmount Road
Scarborough, ON
M1K 5G4

Tel: 1-800-268-2222 Fax: (416) 752-9365 or 752-9646

Price: Textbook: $72.50
Solutions Guide Volume 1: $27.00
Solutions Guide Volume 2: $27.00
Instructor's Resource Guide: $15.50

ISBN/Order No: Textbook: 0-395-88578-7
Solutions Guide Volume 1: 0-395-88769-0
Solutions Guide Volume 2: 0-395-88770-4
Instructor's Resource Guide: 0-395-88766-6

Copyright Year: 1998

Single Variable Calculus Early Transcendentals, Fourth Edition

Author(s): *Stewart, James*

General Description: Comprehensive resource package consisting of a student text with demo CD-ROM learning tool, student solutions manual, and a study guide focuses on conceptual understanding of problems with an application to real-world situations. Text includes explanations and examples of the conceptual ideas, projects, graded exercises, answer key, and a demo CD-ROM that presents an interactive challenge to students. The student solutions manual presents problem solving strategies, and completely worked-out solutions. The study guide offers additional explanations and worked-out examples.

System Requirements for Windows: Windows '95; CD-ROM drive; 8Mb RAM ; 1Mb VRAM.

Audience: *General*

Category: *Student, Teacher Resource*

Curriculum Organizer(s): *Antidifferentiation*
Applications of Derivatives
The Derivative
Functions, Graphs and Limits
Overview and History of Calculus

Supplier: *Nelson Thomson Learning*
1120 Birchmount Road
Scarborough, ON
M1K 5G4

Tel: 1-800-268-2222 Fax: (416) 752-9365 or 752-9646

Price: Textbook: $68.00
Study Guide: $29.00
Student Solutions Manual: $32.00

ISBN/Order No: Textbook: 0-534-35563-3
Study Guide: 0-534-36820-4
Student Solutions Manaul: 0-534-36301-6

Copyright Year: 1999

Appendix C

Cross-Curricular Interests

The three principles of learning stated in the introduction of this Integrated Resource Package (IRP) support the foundation of The Kindergarten to Grade 12 Education Plan. They have guided all aspects of the development of this document, including the curriculum outcomes, instructional strategies, assessment strategies, and learning resource evaluations.

In addition to these three principles, the Ministry of Education wants to ensure that education in British Columbia is relevant, equitable, and accessible to all learners. In order to meet the needs of all learners, the development of each component of this document has been guided by a series of cross-curricular reviews. This appendix outlines the key aspects of each of these reviews. The information here is intended to guide the users of this document as they engage in school and classroom organization and instructional planning and practice.

The areas of cross-curricular interest are:

- Applied Focus in Curriculum
- Career Development
- English as a Second Language (ESL)
- Environment and Sustainability
- Aboriginal Studies
- Gender Equity
- Information Technology
- Media Education
- Multiculturalism and Anti-Racism
- Science-Technology-Society
- Special Needs

Applied Focus in Curriculum

An applied focus combines the following components in curriculum development, consistent with the nature of each subject area:

Learning Outcomes—expressed as observable, measurable, and reportable abilities or skills

Employability Skills—inclusion of outcomes or strategies that promote skills that will enable students to be successful in the workplace (e.g., literacy, numeracy, critical and creative thinking, problem solving, technology, and information management)

Contextual Learning—an emphasis on learning by doing; the use of abstract ideas and concepts, including theories, laws, principles, formulae, rules, or proofs in a practical context (e.g., home, workplace, community)

Interpersonal Skills—inclusion of strategies that promote co-operative activities and teamwork

Career Development—inclusion of appropriate connections to careers, occupations, entrepreneurship, or the workplace

An applied focus in all subjects and courses promotes the use of practical applications to demonstrate theoretical knowledge. Using real-world and workplace problems and situations as a context for the application of theory makes school more relevant to students' needs and goals. An applied focus strengthens the link between what students need to know to function effectively in the workplace or in postsecondary education and what they learn in Kindergarten through Grade 12.

Some examples of an applied focus in different subjects are:

English Language Arts—increasing emphasis on language used in everyday situations and in the workplace, such as for job interviews, memo and letter writing, word processing, and technical communications (including the ability to interpret technical reports, manuals, tables, charts, and graphics)

Mathematics—more emphasis on skills needed in the workplace, including knowledge of probability and statistics, logic, measurement theory, and problem solving

Science—more practical applications and hands-on experience of science, such as reducing energy waste in school or at home, caring for a plant or animal in the classroom, and using computers to produce tables and graphs and for spreadsheets

Business Education—more emphasis on real-world applications such as preparing résumés and personal portfolios, participating in groups to solve business communication problems, using computer software to keep records, and using technology to create and print marketing material

Visual Arts—applying visual arts skills to real-world design, problem solving, and communications; exploring career applications of visual arts skills; experimenting with a variety of new technologies to create images; and a new emphasis on creating and understanding images of social significance to the community

This summary is derived from *The Kindergarten to Grade 12 Education Plan* (September 1994), and curriculum documents from British Columbia and other jurisdictions.

CAREER DEVELOPMENT

Career development is an ongoing process through which learners integrate their personal, family, school, work, and community experiences to facilitate career and lifestyle choices.

Students develop:

- an open attitude toward a variety of occupations and types of work
- an understanding of the relationship between work and leisure, work and the family, and work and one's interests and abilities
- an understanding of the role of technology in the workplace and in daily life
- an understanding of the relationship between work and learning
- an understanding of the changes taking place in the economy, society, and the job market
- an ability to construct learning plans and reflect on the importance of lifelong learning
- an ability to prepare for multiple roles throughout life

The main emphases of career development are career awareness, career exploration, career preparation, career planning, and career work experience.

In the Primary Years

Career awareness promotes an open attitude toward a variety of career roles and types of work. Topics include:

- the role of work and leisure
- relationships among work, the family, one's personal interests, and one's abilities

A variety of careers can be highlighted through the use of in-class learning activities that focus on the students themselves and on a range of role models, including non-traditional role models.

In Grades 4 to 8

The emphasis on self-awareness and career awareness is continued. Topics include:

- interests, aptitudes, and possible future goals
- technology in the workplace and in our daily lives
- social, family, and economic changes
- future education options
- career clusters (careers that are related to one another)
- lifestyles
- external influences on decision making

Games, role-playing, drama, and appropriate community volunteer experience can be used to help students actively explore the world of work. Field experiences in which students observe and interview workers in their occupational environments may also be appropriate. These learning activities will facilitate the development of interpersonal communications and group problem-solving skills needed in the workplace and in other life situations.

In Grades 9 and 10

The emphasis is on providing students with opportunities to prepare for and make appropriate and realistic decisions. In developing their student learning plans, they will relate self-awareness to their goals and aspirations. They will also learn many basic skills and attitudes that are required for an effective transition into adulthood. This will assist in preparing them to be responsible and self-directed throughout their lives. Topics include:

- entrepreneurial education
- employability skills (e.g., how to find and keep a job)
- the importance of lifelong education and career planning
- involvement in the community
- the many different roles that an individual can play throughout life
- the dynamics of the working world (e.g., unions, unemployment, supply and demand, Pacific Rim, free trade)

The examination of personal interests and skills through a variety of career exploration opportunities (e.g., job shadowing) is emphasized at this level. Group discussion and individual consultation can be used to help students examine and confirm their personal values and beliefs.

In Grades 11 and 12

Career development in these grades is focussed more specifically on issues related to the world of work. These include:

- dynamics of the changing work force and changing influences on the job market (e.g., developing technology and economic trends)
- job-keeping and advancement skills (interpersonal skills needed in the workplace, employment standards)
- occupational health issues and accessing health support services
- funding for further education
- alternative learning strategies and environments for different life stages
- mandatory work experience (minimum 30 hours)

Work Experience

Work experience provides students with opportunities to participate in a variety of workplace situations to help prepare them for the transition to a work environment. Work experience also provides students with opportunities to:

- connect what they learn in school with the skills and knowledge needed in the workplace and society in general
- experience both theoretical and applied learning, which is part of a broad liberal education
- explore career directions identified in their Student Learning Plans

Descriptions of career development are drawn from the ministry's *Career Developer's Handbook, Guidelines for the Kindergarten to Grade 12 Education Plan, Implementation Resource, Part 1*, and the *Career and Personal Planning 8 to 12 IRP* (1997).

ENGLISH AS A SECOND LANGUAGE (ESL)

ESL assistance is provided to students whose use of English is sufficiently different from standard English to prevent them from reaching their potential. Many students learning English speak it quite fluently and seem to be proficient. School, however, demands a more sophisticated version of English, both in reading and writing. Thus even fluent speakers might require ESL to provide them with an appropriate language experience that is unavailable outside the classroom. ESL is a transitional service rather than a subject. Students are in the process of learning the language of instruction and, in many cases, the content matter of subjects appropriate to their grade level. Thus ESL does not have a specific curriculum. The provincial curriculum is the basis of much of the instruction and is used to teach English as well as individual subject areas. It is the methodology, the focus, and the level of engagement with the curriculum that differentiates ESL services from other school activities.

Students in ESL

Nearly 10% of the British Columbia school population is designated as ESL students. These students come from a diversity of backgrounds. Most are recent immigrants to British Columbia. Some are Canadian-born but have not had the opportunity to learn English before entering the primary grades. The majority of ESL students have a well-developed language system and have had similar schooling to that of British Columbia-educated students. A small number, because of previous experiences, are in need of basic support such as literacy training, academic upgrading, and trauma counselling.

Teachers may have ESL students at any level in their classes. Many ESL students are placed in subject-area classes primarily for the purpose of contact with English-speaking peers and experience with the subject and language. Other ESL students are wholly integrated into subject areas. A successful integration takes place when the student has reached a level of English proficiency and background knowledge in a subject to be successful with a minimum of extra support.

Optimum Learning Environment

The guiding principle for ESL support is the provision of a learning environment where the language and concepts can be understood by students.

Good practices to enhance learning include:

- using real objects and simple language at the beginning level
- taking into consideration other cultural backgrounds and learning styles at any level
- providing adapted (language-reduced) learning materials
- respecting a student's "silent period" when expression does not reflect the level of comprehension
- allowing students to practise and internalize information before giving detailed answers
- differentiating between form and content in student writing
- keeping in mind the level of demand placed on students

This summary is drawn from *Supporting Learners of English: Information for School and District Administrators*, RB0032, 1993, and *ESL Policy Discussion Paper (Draft)*, Social Equity Branch, December 1994.

ENVIRONMENT AND SUSTAINABILITY

Environmental education is defined as a way of understanding how humans are part of and influence the environment. It involves:

- students learning about their connections to the natural environment through all subjects
- students having direct experiences in the environment, both natural and human-built
- students making decisions about and acting for the environment

The term *sustainability* helps to describe societies that "promote diversity and do not compromise the natural world for any species in the future."

Value of Integrating Environment and Sustainability Themes

Integrating "environment and sustainability" themes into the curriculum helps students develop a responsible attitude toward caring for the earth. Students are provided with opportunities to identify their beliefs and opinions, reflect on a range of views, and ultimately make informed and responsible choices.

Some guiding principles that support the integration of "environment and sustainability" themes in subjects from Kindergarten to Grade 12 include:

- Direct experience is the basis of learning.
- Responsible action is integral to, and a consequence of, environmental education.
- Life on Earth depends on, and is part of, complex systems.
- Human decisions and actions have environmental consequences.
- Environmental awareness enables students to develop an aesthetic appreciation of the environment.
- The study of the environment enables students to develop an environmental ethic.

This summary is derived from *Environmental Concepts in the Classroom: A Guide for Teachers,* Ministry of Education, 1995.

ABORIGINAL STUDIES

Aboriginal studies focus on the richness and diversity of Aboriginal cultures and languages. These cultures and languages are examined within their own unique contexts and within historical, contemporary, and future realities. Aboriginal studies are based on a holistic perspective that integrates the past, present, and future. Aboriginal peoples are the original inhabitants of North America and live in sophisticated, organized, and self-sufficient societies. The First Nations constitute a cultural mosaic as rich and diverse as that of Western Europe, including different cultural groups (e.g., Nisga'a, KwaKwaka'Wakw, Nlaka'pamux, Secwepemc, Skomish, Tsimshian). Each is unique and has a reason to be featured in the school system. The First Nations of British Columbia constitute an important part of the historical and contemporary fabric of the province.

Value of Integrating Aboriginal Studies

- First Nations values and beliefs are durable and relevant today.
- There is a need to validate and substantiate First Nations identity.
- First Nations peoples have strong, dynamic, and evolving cultures that have adapted to changing world events and trends.
- There is a need to understand similarities and differences among cultures to create tolerance, acceptance, and mutual respect.

- There is a need for informed, reasonable discussion and decision making regarding First Nations issues, based on accurate information (for example, as modern treaties are negotiated by Canada, British Columbia, and First Nations).

In studying First Nations, it is expected that students will:

- demonstrate an understanding and appreciation for the values, customs, and traditions of First Nations peoples
- demonstrate an understanding of and appreciation for unique First Nations communications systems
- demonstrate a recognition of the importance of the relationship between First Nations peoples and the natural world
- recognize dimensions of First Nations art as a total cultural expression
- give examples of the diversity and functioning of the social, economic, and political systems of First Nations peoples in traditional and contemporary contexts
- describe the evolution of human rights and freedoms as they pertain to First Nations peoples

Some examples of curriculum integration include:

Visual Arts—comparing the artistic styles of two or more First Nations cultures

English Language Arts—analysing portrayals and images of First Nations peoples in various works of literature

Home Economics—identifying forms of food, clothing, and shelter in past and contemporary First Nations cultures

Technology Education—describing the sophistication of traditional First Nations technologies (e.g., bentwood or kerfed boxes, weaving, fishing gear)

Physical Education—participating in and developing an appreciation for First Nations games and dances

This summary is derived from *First Nations Studies: Curriculum Assessment Framework (Primary Through Graduation)*, Aboriginal Education Branch, 1992, and *B.C. First Nations Studies 12 Curriculum*, Aboriginal Education Branch, 1994.

GENDER EQUITY

Gender-equitable education involves the inclusion of the experiences, perceptions, and perspectives of girls and women, as well as boys and men, in all aspects of education. It will initially focus on girls in order to redress historical inequities. Generally, the inclusive strategies, which promote the participation of girls, also reach boys who are excluded by more traditional teaching styles and curriculum content.

Principles of Gender Equity in Education

- All students have the right to a learning environment that is gender equitable.
- All education programs and career decisions should be based on a student's interest and ability, regardless of gender.
- Gender equity incorporates a consideration of social class, culture, ethnicity, religion, sexual orientation, and age.
- Gender equity requires sensitivity, determination, commitment, and vigilance over time.
- The foundation of gender equity is co-operation and collaboration among students, educators, education organizations, families, and members of communities.

General Strategies for Gender-Equitable Teaching

- Be committed to learning about and practising equitable teaching.
- Use gender-specific terms to market opportunities—for example, if a technology fair has been designed to appeal to girls, mention girls clearly and specifically. Many girls assume that gender-neutral language in non-traditional fields means boys.
- Modify content, teaching style, and assessment practices to make non-traditional subjects more relevant and interesting for female and male students.
- Highlight the social aspects and usefulness of activities, skills, and knowledge.
- Comments received from female students suggest that they particularly enjoy integrative thinking; understanding context as well as facts; and exploring social, moral, and environmental impacts of decisions.
- When establishing relevance of material, consider the different interests and life experiences that girls and boys may have.
- Choose a variety of instructional strategies such as co-operative and collaborative work in small groups, opportunities for safe risk taking, hands-on work, and opportunities to integrate knowledge and skills (e.g., science and communication).
- Provide specific strategies, special opportunities, and resources to encourage students to excel in areas of study in which they are typically under-represented.
- Design lessons to explore many perspectives and to use different sources of information; refer to female and male experts.
- Manage competitiveness in the classroom, particularly in areas where male students typically excel.
- Watch for biases (e.g., in behaviour or learning resources) and teach students strategies to recognize and work to eliminate inequities they observe.
- Be aware of accepted gender-bias practices in physical activity (e.g., in team sport, funding for athletes, and choices in physical education programs).
- Do not assume that all students are heterosexual.
- Share information and build a network of colleagues with a strong commitment to equity.
- Model non-biased behaviour: use inclusive, parallel, or gender-sensitive language; question and coach male and female students with the same frequency, specificity, and depth; allow quiet students sufficient time to respond to questions.
- Have colleagues familiar with common gender biases observe your teaching and discuss any potential bias they may observe.
- Be consistent over time.

This summary is derived from the preliminary *Report of the Gender Equity Advisory Committee*, received by the Ministry of Education in February 1994, and from a review of related material.

INFORMATION TECHNOLOGY

Information technology is the use of tools and electronic devices that allow us to create, explore, transform, and express information.

Value of Integrating Information Technology

As Canada moves from an agricultural and industrial economy to the information age, students must develop new knowledge, skills, and attitudes. The information technology curriculum has been developed to be integrated into all new curricula to ensure that students know how to use computers and gain the technological literacy demanded in the workplace.

In learning about information technology, students acquire skills in information analysis and evaluation, word processing, database analysis, information management, graphics, and multimedia applications. Students also identify ethical and social issues arising from the use of information technology.

With information technology integrated into the curriculum, students will be expected to:

- demonstrate basic skills in handling information technology tools
- demonstrate an understanding of information technology structure and concepts
- relate information technology to personal and social issues
- define a problem and develop strategies for solving it
- apply search criteria to locate or send information
- transfer information from external sources
- evaluate information for authenticity and relevance
- arrange information in different patterns to create new meaning
- modify, revise, and transform information
- apply principles of design affecting the appearance of information
- deliver a message to an audience using information technology

The curriculum organizers are:

- ***Foundations***—provides the basic physical skills and intellectual and personal understanding required to use information technology, as well as self-directed learning skills and socially responsible attitudes

- ***Process***—allows students to select, organize, and modify information to solve problems

- ***Presentation***—provides students with an understanding of how to communicate ideas effectively using a variety of information technology tools

This information is derived from the Information Technology K to 12 curriculum.

MEDIA EDUCATION

Media education is a multidisciplinary and interdisciplinary approach to the study of media. Media education deals with key media concepts and focusses on broad issues such as the history and role of media in different societies and the social, political, economic, and cultural issues related to the media. Instead of addressing the concepts in depth, as one would in media studies, media education deals with most of the central media concepts as they relate to a variety of subjects.

Value of Integrating Media Education

Popular music, TV, film, radio, magazines, computer games, and information services—all supplying media messages—are pervasive in the lives of students today. Media education develops students' abilities to think critically and independently about issues that affect them. Media education encourages students to identify and examine the values contained in media messages. It also cultivates the understanding that these messages are produced by others to inform, persuade, and entertain for a variety of purposes. Media education helps students understand the distortions that may result from the use of particular media practices and techniques.

All curriculum areas provide learning opportunities for media education. It is not taught as a separate curriculum.

The key themes of media education are:

- media products (purpose, values, representation, codes, conventions, characteristics, production)
- audience interpretation and influence (interpretation, influence of media on audience, influence of audience on media)
- media and society (control, scope)

Examples of curriculum integration include:

English Language Arts—critiquing advertising and examining viewpoints

Visual Arts—analysing the appeal of an image by age, gender, status, and other characteristics of the target audience

Personal Planning—examining the influence of the media on body concepts and healthy lifestyle choices

Drama—critically viewing professional and amateur theatre productions, dramatic films, and television programs to identify purpose

Social Studies—comparing the depiction of First Nations in the media over time

This summary is derived from *A Cross-Curricular Planning Guide for Media Education*, prepared by the Canadian Association for Media Education for the Curriculum Branch in 1994.

MULTICULTURALISM AND ANTI-RACISM EDUCATION

Multiculturalism Education

Multiculturalism education stresses the promotion of understanding, respect, and acceptance of cultural diversity within our society.

Multiculturalism education involves:

- recognizing that everyone belongs to a cultural group
- accepting and appreciating cultural diversity as a positive feature of our society
- affirming that all ethnocultural groups are equal within our society
- understanding that multiculturalism education is for all students
- recognizing that similarities across cultures are much greater than differences and that cultural pluralism is a positive aspect in our society
- affirming and enhancing self-esteem through pride in heritage, and providing opportunities for individuals to appreciate the cultural heritage of others
- promoting cross-cultural understanding, citizenship, and racial harmony

Anti-Racism Education

Anti-racism education promotes the elimination of racism through identifying and changing institutional policies and practices as well as identifying individual attitudes and behaviours that contribute to racism.

Anti-racism education involves:

- proposing the need to reflect on one's own attitudes about race and anti-racism
- understanding what causes racism in order to achieve equality
- identifying and addressing racism at both the personal and institutional level
- acknowledging the need to take individual responsibility for eliminating racism
- working toward removing systemic barriers that marginalize groups of people
- providing opportunities for individuals to take action to eliminate all forms of racism, including stereotypes, prejudice, and discrimination

Value of Integrating Multiculturalism and Anti-Racism Education

Multiculturalism and anti-racism education provides learning experiences that promote strength through diversity and social, economic, political, and cultural equity. Multiculturalism and anti-racism education gives students learning experiences that are intended to enhance their social, emotional, aesthetic, artistic, physical, and intellectual development. It provides learners with the tools of social literacy and skills for effective cross-cultural interaction with diverse cultures. It also recognizes the importance of collaboration between students, parents, educators, and communities working toward social justice in the education system.

The key goals of multiculturalism and anti-racism education are:

- to enhance understanding of and respect for cultural diversity
- to increase creative intercultural communication in a pluralistic society
- to provide equal opportunities for educational achievement by all learners, regardless of culture, national origin, religion, or social class
- to develop self-worth, respect for oneself and others, and social responsibility
- to combat and eliminate stereotyping, prejudice, discrimination, and other forms of racism
- to include the experiences of all students in school curricula

Examples of curriculum integration include:

Fine Arts—identifying ways in which the fine arts portray cultural experiences

Humanities—identifying similarities and differences within cultural groups' lifestyles, histories, values, and beliefs

Mathematics or Science—recognizing that individuals and cultural groups have used both diverse and common methods to compute, to record numerical facts, and to measure

Physical Education—developing an appreciation of games and dances from diverse cultural groups

This summary is derived from *Multicultural and Anti-Racism Education—Planning Guide (Draft)*, developed by the Social Equity Branch in 1994.

SCIENCE-TECHNOLOGY-SOCIETY

Science-Technology-Society (STS) addresses our understanding of inventions and discoveries and of how science and technology affect the well-being of individuals and our global society.

The study of STS includes:

- the contributions of technology to scientific knowledge and vice versa
- the notion that science and technology are expressions of history, culture, and a range of personal factors
- the processes of science and technology such as experimentation, innovation, and invention
- the development of a conscious awareness of ethics, choices, and participation in science and technology

Value of Integrating STS

The aim of STS is to enable learners to investigate, analyse, understand, and experience the dynamic interconnection of science, technology, and human and natural systems.

The study of STS in a variety of subjects gives students opportunities to:

- discover knowledge and develop skills to foster critical and responsive attitudes toward innovation
- apply tools, processes, and strategies for actively challenging emerging issues
- identify and consider the evolution of scientific discovery, technological change, and human understanding over time, in the context of many societal and individual factors

- develop a conscious awareness of personal values, decisions, and responsible actions about science and technology
- explore scientific processes and technological solutions
- contribute to responsible and creative solutions using science and technology

The organizing principles of STS are: Human and Natural Systems, Inventions and Discoveries, Tools and Processes, Society and Change. Each organizer may be developed through a variety of contexts, such as the economy, the environment, ethics, social structures, culture, politics, and education. Each context provides a unique perspective for exploring the critical relationships that exist and the challenges we face as individuals and as a global society.

Examples of curriculum integration include:

Visual Arts—recognizing that demands generated by visual artists have led to the development of new technologies and processes (e.g., new permanent pigments, fritted glazes, drawing instruments)

English Language Arts—analysing the recent influence of technologies on listening, speaking, and writing (e.g., CDs, voice mail, computer-generated speech)

Physical Education—studying how technology has affected our understanding of the relationship between activity and well-being

This summary is derived from *Science-Technology-Society—A Conceptual Framework*, Curriculum Branch, 1994.

SPECIAL NEEDS

Students with special needs have disabilities of an intellectual, physical, sensory, emotional, or behavioural nature; or have learning disabilities; or have exceptional gifts or talents.

All students can benefit from an inclusive learning environment that is enriched by the diversity of the people within it. Opportunities for success are enhanced when provincial learning outcomes and resources are developed with regard for a wide range of student needs, learning styles, and modes of expression.

Educators can assist in creating more inclusive learning environments by introducing the following:

- activities that focus on development and mastery of foundational skills (basic literacy)
- a range of co-operative learning activities and experiences in the school and community, including the application of practical, hands-on skills in a variety of settings
- references to specialized learning resources, equipment, and technology
- ways to accommodate special needs (e.g., incorporating adaptations and extensions to content, process, product, pacing, and learning environment; suggesting alternative methodologies or strategies; making references to special services)
- a variety of ways, other than through paper-and-pencil tasks, for students to demonstrate learning (e.g., dramatizing events to demonstrate understanding of a poem, recording observations in science by drawing or by composing and performing a music piece)
- promotion of the capabilities and contributions of children and adults with special needs
- participation in physical activity

All students can work toward achievement of the provincial learning outcomes. Many students with special needs learn what all students are expected to learn. In some cases the student's needs and abilities require that education programs be adapted or modified. A student's program may include regular instruction in some subjects, modified instruction in others, and adapted instruction in still others. Adaptations and modifications are specified in the student's Individual Education Plan (IEP).

Adapted Programs

An adapted program addresses the learning outcomes of the prescribed curriculum but provides adaptations so the student can participate in the program. These adaptations may include alternative formats for resources (e.g., braille, books-on-tape), instructional strategies (e.g., use of interpreters, visual cues, learning aids), and assessment procedures (e.g., oral exams, additional time). Adaptations may also be made in areas such as skill sequence, pacing, methodology, materials, technology, equipment, services, and setting. Students on adapted programs are assessed using the curriculum standards and can receive full credit.

Modified Programs

A modified program has learning outcomes that are substantially different from the prescribed curriculum and specifically selected to meet the student's special needs. For example, a Grade 5 student in language arts may be working on recognizing common signs and using the telephone, or a secondary student could be mapping the key features of the main street between school and home. A student on a modified program is assessed in relation to the goals and objectives established in the student's IEP.

Appendix D

Assessment and Evaluation

ASSESSMENT AND EVALUATION IN GRADE 10 TO 12 MATHEMATICS COURSES

The samples in this appendix illustrate procedures teachers might use in assessing student understanding of content and processes in grade 10 to 12 mathematics courses. The focus of the samples is ongoing assessment and evaluation as a way of supporting learning in the classroom. Ongoing assessment includes observation, questioning, oral and written communication and peer and self-assessment. The assessment activities in each sample focuses on an understanding of mathematical processes, communication of ideas, emerging understanding of concepts, abilities to apply concepts and attitudes toward learning as well as on the end product.

Teacher-constructed tests and quizzes are recognized as a valid means of assessing certain aspects of student learning. The samples presented here represent alternatives that teachers might use to support and supplement traditional tests on a particular unit of work. Some samples include assessment ideas teachers might want to use to look at their students' communication skills or their abilities to work effectively and productively in groups. Following the samples is information about various assessment and evaluation practices, including test construction that teachers will find useful as a resource when developing classroom tests.

Using a variety of assessment techniques provides students with opportunities to demonstrate what they have learned more readily and gives teachers a broader base of information on students' mathematical development.

The assessment practices described in the samples are supported by current theories of assessment, including the Evaluation Standards outlined by the National Council of Teachers of Mathematics:

- Methods and tasks for assessing students' learning should be aligned with the curriculum's goals, objectives and mathematical content.
- Decisions concerning students' learning should be made on the basis of a convergence of information obtained from a variety of sources.
- Assessment methods and instruments should be selected on the basis of the type of information sought, the use to which the information will be put and the developmental level and maturity of the student.

The assessment practices described in the following samples include those that focus on assessment and evaluation of:

- Specific mathematics content
- Problem solving processes
- Students' development of group and communication skills
- Students' attitudes toward learning mathematics

Samples of student performance should reflect learning outcomes and identified criteria. The samples clarify and make explicit the link between evaluation and learning outcomes, criteria, and assessment.

Where a student's performance is not a product, and therefore not reproducible, a description of the performance sample should be provided.

Criterion-referenced evaluation may be based on these steps:

Step 1 ▶ Identify the expected learning outcomes (as stated in this Integrated Resource Package).

Step 2 ▶ Identify the key learning objectives for instruction and learning.

Step 3 ▶ Establish and set criteria. Involve students, when appropriate, in establishing criteria.

Step 4 ▶ Plan learning activities that will help students gain the knowledge or skills outlined in the criteria.

Step 5 ▶ Prior to the learning activity, inform students of the criteria against which their work will be evaluated.

Step 6 ▶ Provide examples of the desired levels of performance.

Step 7 ▶ Implement the learning activities.

Step 8 ▶ Use various assessment methods based on the particular assignment and student.

Step 9 ▶ Review the assessment data and evaluate each student's level of performance or quality of work in relation to criteria.

Step 10 ▶ Where appropriate or necessary, assign a letter grade that indicates how well the criteria are met.

Step 11 ▶ Report the results of the evaluations to students and parents.

Prescribed learning outcomes, expressed in measurable terms, provide the basis for the development of learning activities, and assessment and evaluation strategies. After a general discussion of assessment and evaluation, this appendix uses sample evaluation plans to show how activities, assessment, and evaluation might come together in a particular mathematics program. The final section, Assessment Practices, provides some general guidance for classroom assessment and evaluation.

ASSESSMENT AND EVALUATION

Assessment is the systematic gathering of information about what students know, are able to do, and are working toward. Assessment methods and tools include: observation, student self-assessments, daily practice assignments, quizzes, samples of student work, pencil-and paper tests, holistic rating scales, projects, oral and written reports, performance reviews, and portfolio assessments.

Students performance is evaluated from the information collected through assessment activities. Teachers use their insight, knowledge about learning, and experience with students, along with the specific criteria they establish, to make judgements about student performance in relation to prescribed learning outcomes.

Students benefit most when evaluation is provided on a regular, ongoing basis. When evaluation is seen as an opportunity to promote learning rather than as a final judgment, it shows learners their strengths and suggests how they can develop further. Students can use this information to redirect efforts, make plans, and establish future learning goals.

Evaluation may take different forms, depending on the purpose.

- Criterion-referenced evaluation should be used to evaluate student performance in classrooms. It is referenced to criteria based on learning outcomes described in the provincial curriculum. The criteria reflect a student's performance based on specific learning activities. When a student's program is substantially modified, evaluation may be referenced to individual goals. These modifications are recorded in an Individual Education Plan (IEP).
- Norm-referenced evaluation is used for large-scale system assessments; it is not to be used for a classroom assessment. A classroom does not provide a large enough reference group for a norm-referenced evaluation system. Norm-referenced evaluation compares student achievement to that of other, rather that comparing how well a student meets the criteria of a specified set of learning outcomes.

Criterion-Referenced Evaluation

In criterion-referenced evaluation, a student's performance is compared to established criteria rather that to the performance of other students. Evaluation referenced to prescribed curriculum requires that criteria are established based on the learning outcomes listed under the curriculum organizers for grade 11 and 12 mathematics courses.

Criteria are the basis of evaluating student progress. They identify the critical aspects of a performance of product that describe, in specific terms, what is involved in meeting the learning outcomes. Criteria can be used to evaluated student performance in relation to learning outcomes. For example, weighting criteria, using rating scales, or performance rubrics (reference sets) are three ways that student performance can be evaluated using criteria.

Appendix D

Assessment and Evaluation Samples

The samples in this section show how a teacher might link criteria to learning outcomes. Each sample is based on prescribed learning outcomes taken from one or more organizers. The samples provide background information to explain the classroom context; suggested instruction tasks and strategies; the tools and methods used to gather assessment information; and the criteria used to evaluate student performance.

How the Samples are Organized

There are six parts to each sample:

- identification of the prescribed learning outcomes
- unit focus
- planning the unit
- the unit
- defining the criteria
- assessing and evaluating student performance

Prescribed Learning Outcomes

This part identifies the organizer or organizers and the specific prescribed learning outcomes selected for the sample.

Unit Focus

This is a summary of the key features of the sample.

Planning the Unit

This part describes how the teacher prepared for the unit.

The Unit

This part outlines:

- instructional tasks
- the opportunities that students were given to practise learning
- the feedback and support that was offered students by the teacher
- the ways in which the teacher prepared students for the assessment

Defining the Criteria

This part illustrates the specific criteria, which are based on the prescribed learning outcomes, the assessment task, and various reference sets.

Assessing and Evaluating Student Performance

This part includes:

- assessment tasks or activities
- the support that the teacher offered students
- tools and methods used to gather the assessment information
- the way the criteria were used to evaluate student performance

EVALUATION SAMPLES

The samples on the following pages illustrate how a teacher might apply criterion-referenced evaluation in grade 10, 11, and 12 mathematics courses.

- Sample 5: Essentials of Mathematics 11
  Owning and Operating a Vehicle
  (Page D-31)
- Sample 6: Essentials of Mathematics 12
  Comparative Study of Career Choices
  (Page D-36)
- Sample 7: Principles of Mathematics 10
  Radicals
  (Page D-41)
- Sample 8: Principles of Mathematics 11
  Graphing Techniques
  (Page D-47)
- Sample 9: Principles of Mathematics 12
  Logarithms
  (Page D-53)
- Sample 10: Calculus 12
  Limits
  (Page D-60)

▼ SAMPLE 1: APPLICATIONS OF MATHEMATICS 10

Topic: *Data Tables*

Prescribed Learning Outcomes:

Problem Solving

It is expected that students will:

- analyse problems and identify significant elements
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

Number (Number Concepts)

It is expected that students will:

- analyze the numerical data in a table for trends, patterns and relationships
- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively

UNIT FOCUS

The overall goal of the unit was to help students understand how they can use tables to represent data and to use tables to solve real world problems. Assessment required students to demonstrate their ability to create tables that represented data, and processes, that they used to solve problems. In as much as possible, students were assessed in terms of their strategic approach to solving problems as well as their ability use strategies to resolve issues relevant to their project work.

PLANNING THE UNIT

To develop the unit, the teacher:

- determined the overall goals of the unit
- identified the related IRP outcomes to be targeted in the unit
- identified the prerequisite knowledge and skills students needed to achieve the targeted outcomes included in this unit, and planned for review
- looked for ways to connect students' learning with other desirable outcomes, including those associated with students' attitudes
- looked for ways to encourage students' understanding of the ways in which practical problems are solved mathematically
- planned a variety of integrated instructional and assessment activities to help students achieve identified outcomes
- determined criteria with which to evaluate students' learning for marking and reporting purposes. Whenever appropriate, the teacher involved the students in setting evaluation criteria.
- developed, with students, a rubric that would be used to mark the unit project and its' presentation. The development of the rubric's criteria was done as early as possible, and with the purpose of promoting students' understanding of the work they were expected to accomplish.

THE UNIT

Recalling, Reviewing and Extending Relevant Concepts

- To review and evaluate students' understanding of the use of tables in financial situations, students were given a mail order catalogue and assigned the task of spending $1000 on at least 10 different items. Cost was to include taxes, and the students were not to go over $1000 in total spending. Students prepared a table that listed the items, their price, GST, PST, and total cost. The teacher asked students to compare the production of such a table with the production of a spreadsheet.

Peer Assessment — Specific areas, such as the calculation of percentages and the use of formulas, were highlighted as students checked their peers' work on a sample basis. The teacher occasionally asked students to comment on how errors might arise and how to correct them when they did arise.

Assessment Strategies — The teacher assessed the extent of students' development of tables, providing assistance as needed. Particular attention was paid to checking the first few entries to ensure students were able to calculate taxes and complete each row.

Demonstrating How to Create A Spreadsheet

- As students worked through the shopping exercise, the teacher explained how a spreadsheet could be used to produce a dynamic table.
- The teacher also pointed out that buying selections made by the students represented a simple market survey. The teacher then introduced to the class the opening of a store in a mall as a way of meeting the shopping choices as expressed by the class.
- The teacher used an example of the opening of a store to encourage the development of relevant mathematical models.
- The teacher asked students to brainstorm for ideas on what steps a potential business owner could take before they were prepared to actively open a business. These steps, with teacher input, became the basis of a unit project.
- The teacher had students develop a flow chart that showed the steps necessary to produce a business plan, and ensured the steps made practical sense.
- Students brainstormed ideas on the types of information and decisions an entrepeneur might make to determine what to stock in the store, how to set up the store, and how to estimate sales and net and gross profit.
- The teacher developed parameters for the store such as using the shape and size of the classroom to represent the store area. This helped students to determine the theme for their store and to be realistic in terms of the inventory. The teacher told students to assume they had some stock storage space in addition to the area represented by the classroom.
- Students developed inventory lists for the store and estimated the wholesale cost of items by dividing the retail costs by 2. Students determined the number of each item to stock and the price that the store would charge for each item.
- The teacher checked students' tables for accuracy, and frequently asked them to explain their choices.
- Students were asked to develop tables that showed inventories, estimated daily sales, costs such as labour and rent, and estimated gross and net profit.

Self-Assessment — Students were encouraged to reflect on their choices in terms of stores that currently exist. They were asked to explain why customers would go to their store rather than to the competition.

Teacher Review — As students developed spreadsheets that represented process in the development of the simple business plan, the teacher reviewed their work both for accuracy and reasonableness. For example, the labour cost for a staff person working in a computer store could be significantly different than for a person working in a clothing store. Salary rates should represent expertise and be reasonable for the type of work expected.

Peer, Small Group and Teacher Assessment — Students were asked at various development stages to report their concepts in a small group setting. Members of the group were asked to provide feedback to the reporter.

In as much as possible, the teacher created an atmosphere that encouraged students to become engaged with the development of their business. Peer input with regard to marketing, advertising, and the way in which products could be displayed to encourage sales was encouraged as way of replicating market surveys.

The demonstration of students' ability to meet the prescribed outcomes, in individual terms, was encouraged through the students' production of a personalized business plan. While the plans were influenced by peer review and teacher input, each student was assessed and evaluated on their able to explain and replicate the production of the tables/spreadsheets they produced. Evaluations by the teacher were done in such a way that each student was held accountable for the meeting of outcomes, regardless of the amount of group, peer, or teacher input that led to its development.

Using Performance Activities in Instruction

- To help students understand the filling of the cells of a spreadsheet, the teacher allowed students to experiment with various simple applications of spreadsheet. Students were encouraged build spreadsheets to find the final costs of items such as snowboards, cars, CDs, etc. with a view to increasing enthusiasm. Students were asked to demonstrate the use of their spreadsheets. The spreadsheets calculated final costs from price by creating a cell that multiplied a price cell by 1.14. As the spreadsheets became more complicated the teacher asked students to test their viability and encouraged peers to assist in trouble shooting.

DEFINING THE CRITERIA

Mathematical Thinking

To what extent did students:

- demonstrate an understanding of table interrelationships, students were asked to explain how each row was calculated, and what formula might have been used in the context of a spreadsheet
- Use problem solving elements to trouble shoot the develop of their tables and or spreadsheets

Unit Test

Students were given a business plan created by the teacher. The plan had errors in the form of simple tabular mistakes, formulae errors, and unreasonable estimates. The teacher paid close attention to ensure that students could easily find simple errors and yet more able students might be able to offer quite sophisticated suggestions on how to improve the plan.

The plans themselves were marked on a rubric that was developed by the class near the beginning of the project process. The teacher ensured the rubric represented accurately the intended outcomes.

Self-Assessment

Students' assessment of the business plan and related tables were based on accuracy and presentation. Students were asked to include a statement of predicted hourly earnings for the owner of the store based on their estimates of sales and costs. Clearly most students would either rework their plans if the predicted hourly wage was unreasonable or need explain why they might abandon the business if it was deemed unworkable in its present form.

▼ SAMPLE 2: APPLICATIONS OF MATHEMATICS 11

Topic: *Probability and Statistics*

Prescribed Learning Outcomes:

Problem Solving

It is expected that the student will:

- solve problems that involve mathematics within other disciplines.
- use appropriate technology to assist in problem solving

Patterns and Relations

It is expected that the student will:

- solve nonlinear equations using a graphing tool.
- design and solve linear and nonlinear systems, in two variables, to model problem situations.

Statistics and Probability

It is expected that the student will:

- extract information from given graphs of discrete or continuous data.
- draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data.
- collect experimental data and use best fit exponential and quadratic functions, to make predictions and to solve problems.

UNIT FOCUS

The goal in this unit is to have students appreciate that the world around them can often be modeled using strict mathematical relationships. Although many of these relationships can be complex, much of the interaction that they observe between objects around them can be modeled simply and elegantly using mathematics well in within their grasp. In this unit we will study the motion of projectiles and discover how any object thrown by hand or shot out of a gun obeys simple mathematical laws.

PLANNING THE UNIT

To develop the unit, the teacher:

- identified the prescribed learning outcomes for the unit.
- examined the specific prerequisite knowledge and skills needed to achieve these outcomes.
- determined which prerequisites were in place and which should be reviewed
- planned a variety of activities to help students achieve the outcomes
- looked for ways to connect students' learning to other desirable outcomes
- identified criteria to evaluate students' learning
- designed assessment that was an integral part of the learning process

THE UNIT

Introduction

The teacher began the unit with a discussion about how many of the phenomena we observe in the world around us can be described using mathematics. The teacher introduced the example of projectiles and the many different objects that fit the description of projectiles and discussed with students how the computer and graphing calculators make it possible to use powerful analysis tools to discover the mathematical relationships that describe projectile motion.

Review

- Students reviewed basics of data gathering (tables) and graphing techniques.

- Students reviewed (or were introduced to) how to enter equations and individual data points into a graphing calculator or graphing software.

Demonstrating Understanding of Basic Concepts

- Students solved a short problem set involving simple physics problems that can be solved by graphing equations and analysing the results. Analysis of some of the questions involved regression analysis of individual data points.

The Activities

- Students worked on a problem set involving basic projectile motion problems to give them opportunities to practice manipulating the basic formulas associated with projectile motion.
- Introductory exercises were designed to teach students how to do a quadratic curve fit on projectile data.
- The teacher presented a video lab involving analysis of two projectiles: a ball rolled off a table and a thrown ball.
- Students then worked on a final problem set involving more complex projectile questions.
- Students filled out a questionnaire designed to reflecttheir competencies and attitudes toward the material covered in this unit.

LAB ACTIVITY
CURVE FITTING DATA

1. The following data was collected when a ball was rolled off a high shelf in a classroom.

   A. Enter the points (t, d_x) in either a graphing calculator or some type of graphing software.

   B. Use the capabilities of the technology you have chosen to fit a curve to the data on the graph. For this set of data what is the best shape to try fitting?

   C. The key descriptors for a line are its slope and y-intercept. What do the slope and the y-intercept stand for in the context of our projectile? What are their values? Explain those values in the context of the question. (Why does the line have the value it does? etc.)

   D. Now plot the data (t, d_y) and do a curve fit on this data. What shape does the data seem to take?

   E. For a simple projectile problem like this the equation that relates the distance a projectile travels to time is $d_y = \frac{1}{2} g t^2$ where g = 9.80 m/s². Is the regression constant for your curve ~ 4.9? If so this data follows a classic projectile curve.

t (sec)	d_x (m)	d_y (m)
0.0	0.000	0
0.1	0.072	0.049
0.2	0.144	0.196
0.3	0.216	0.441
0.4	0.288	0.784
0.5	0.360	1.225
0.6	0.432	1.764
0.7	0.504	2.401

2. Spaceman Spiff recently crash landed on the planet Blarg. Do a curve fit on his data, recorded in the following table. Find key information about the planet from your analysis of the data.

   A. What was the horizontal component of the initial velocity at which Spiff threw the rock?

   B. What was the vertical component of the initial velocity?

   C. What is the acceleration due to gravity on the Planet Blarg?

t (sec)	d_x (m)	d_y (m)
0.0	0.000	0
0.5	1.75	0.41
1.0	3.50	1.65
1.5	5.25	3.71
2.0	7.00	6.60
2.5	8.75	10.31
3.0	10.50	14.85
3.5	12.25	20.21
4.0	14.00	26.40

3. Robin Hood shot an arrow into the air. Use the data recorded in the table to answer some key questions about the arrow's flight.

   A. Find the arrow's maximum height.

   B. Find the arrow's range (how far it traveled horizontally).

   C. What was the arrow's initial velocity? (Combine the horizontal and vertical components of the initial velocity.)

   D. At what angle was it shot? (Use trigonometry.)

t (sec)	d_x (m)	d_y (m)
0.00	0.00	0.00
0.25	4.91	3.14
0.50	9.82	5.66
0.75	14.74	7.57
1.00	19.66	8.87
1.25	24.57	9.55
1.50	29.49	9.62
1.75	34.40	9.08
2.00	39.32	7.93
2.25	44.23	6.17
2.50	49.14	3.78
2.75	54.06	0.80

LAB ACTIVITY
PROJECTILE MOTION

Purpose: To analyze how a projectile behaves when it is:
a) rolled off the end of a table; and,
b) thrown upwards at an angle to the horizontal.

We hypothesize that as gravity is the only thing affecting a projectile's motion once it is no longer being supported/propelled, it should not accelerate at all horizontally and it should accelerate at 9.80 m/s^2 vertically. In other words $d_x = v_x t$ and $d_y = \frac{1}{2} g t^2$

Materials:

- Two videos, one of a ball rolling off a table and one of a ball being thrown. It is necessary to know the real size of some element on the video so that you can figure out the scale of the video.
- A video player with the ability to advance one frame at a time.
- A clear sheet of acetate.

Procedure:

1. Place acetate over the television screen.
2. Play the video frame by frame and mark the center of the ball in each second frame.
3. Measure the distances both horizontally and vertically from the first mark to each mark on the video.
4. Multiply these distances by the scale factor for the video (the real length of some object on the screen divided by the length in video).
5. Record data in the table below.

Data:

Image #	Time (s)	Horizontal Change Image (cm)	Vertical Change Image (cm)	Horizontal Change Real (m)	Vertical Change Real (m)
1	0.0667				
2	0.1333				
3	0.2000				
4	0.2667				
5	0.3333				
6	0.4000				
7	0.4667				
8	0.5333				
9	0.6000				
10	0.6667				
11	0.7333				

Analysis:

1. Convert distances to velocities by finding the change in distance in data points on either side of a given point and dividing it by the time elapsed for that data point. (Example: $v_2 = (d_3 - d_1)/t_2$). Fill completed values in the table below.

2. Plot v_x vs *t* on one graph and vy vs t on a second graph.

3. What does v_x vs t look like? It should be fairly linear and close to horizontal, so use a linear regression to find the line of best fit and find the slope of the regression line. What is the slope?

4. What does v_y vs t look like? It should also look linear but have an upward slant to it so use a linear regression to find the line of best fit and find the slope of the regression line. What is the slope?

5. In both cases above classical physics tells us that the values we have found should be the horizontal and vertical components of the ball's acceleration. Assuming gravity is the only force acting on the ball then the horizontal acceleration should be zero and the vertical acceleration should be -9.80 m/s^2. How close did you come? Do you feel that you have proved you hypothesis?

Conclusion:

Summarize the purpose of the experiment and your findings and state whether you achieved the purpose of the experiment.

Image #	Average Horizontal Velocity (m/s)	Average Vertical Velocity (m/s)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

LAB ACTIVITY (ALTERNATIVE)
PROJECTILE MOTION

Purpose:

To analyze how a projectile behaves when it is:
a) rolled off a table; and,
b) thrown upwards at an angle to the horizontal.

We hypothesize that as gravity is the only thing affecting the projectiles motion once it leaves the table, it should not accelerate at all horizontally and it should accelerate at 9.80 m/s2 vertically.

Materials:

- Video E-1710
- World in Motion software

Procedure:

1. Load the software and video E-1710.
2. Play the video frame by frame and mark the center of the ball in each frame.
3. Go to the analysis menu and choose the v_x vs t, v_y vs t graph.
4. Find the slope of each graph. (Use a mouse to mark the endpoints of the line. Slope will appear in the box below the graph.)
5. Print each graph.

Analysis:

1. The slope of each velocity-time graph is acceleration.
2. Plot v_x vs t on one graph and v_y vs t on a second graph.
3. What does v_x vs t look like? It should be fairly linear and close to horizontal so use a linear regression to find the line of best fit and find the slope of the regression line. What is the slope?
4. What does v_y vs t look like? It should also look linear but have an upward slant to it so use a linear regression to find the line of best fit and find the slope of the regression line. What is the slope?
5. In both cases above classical physics tells us that the values we have found should be the horizontal and vertical components of the ball's acceleration. Assuming gravity is the only force acting on the ball then the horizontal acceleration should be zero and the vertical acceleration should be -9.80 m/s^2. How close did you come? Do you feel that you have proved you hypothesis?

Conclusion: Summarize the purpose of the experiment and your findings and state whether you achieved the purpose of the experiment.

'BASIC' PROJECTILES
PROBLEM SET

Key Formulas

1. A ball travelling at 0.65 m/s rolls off a table 1.25 m high. How long does it take to hit the ground? (Assume no friction in all of these problems.)
2. A diver runs straight off the end of a 5.0 m diving tower at 4.5 m/s.

   A. How long does it take for her to reach the water below?

   B. How far does she travel horizontally in her flight?
3. In a crash scene for an adventure movie a car is wired to speed horizontally off a cliff at 38.0 m/s. If the car lands 75.0 m from the base of the cliff:

   A. How long was the car in the air?

   B. How high is the cliff?

'ADVANCED' PROJECTILES PROBLEM SET

Key Formulas

1. A bullet is fired from a rifle pointing horizontally, 1.66 m above the ground. The rifle's muzzle velocity is 294 m/s.

   A. How long is the bullet?

   A. How far does the bullet travel? in the air? (What is its range?)

2. Louie punts the football with an initial velocity of 28.8 m/s at an angle of above the horizontal.

   A. How long is the ball in the air?

   A. How high does the ball get?

   A. How far does the ball travel?

3. Marvelous Mark throws a baseball from the outfield towards home plate with an initial velocity of 14.0 m/s at an angle of 42° above the horizontal. (Ignore Mark's height for this question.)

   A. How long is the ball in the air?

   A. How high does the ball get?

   A. How far does the ball travel?

   A. At what angle does the ball hit the ground?

4. A cannon sits on top of a 6.5 m high cliff and is aimed at an angle of 48° to the horizontal. A daredevil is then shot out of the cannon at 75.0 m/s toward a target net on the ground below. Where should the net be placed so that she doesn't crash?

5. Racing Rory has decided to attempt to jump the Rushing Rapids Gorge. He builds a ramp at an angle of 30.0° to the horizontal at the edge of the gorge and plans to accelerate up the ramp and over the gorge to the other side. If his supercharged motorcycle will achieve a velocity of 35.0 m/s at the top of the ramp, and the bank on the other side of the gorge is 3.5 m lower than on the ramp side, will he successfully cross the 120 m gorge?

6. If the height of the ramp is 16.0 m above the river, what is the maximum height that the landing side could be in order for Rory to have a chance of making the leap?

EXPERIMENT CHECK LIST AND GRADING FORM

Experimenter:________________________ Lab Title: ________________ Date: ____________

	5 - Outstanding	4 - Competent	3 - Improving	2 - Emerging	1 -Undevelopedng		
Purpose		Clearly stated in write up. reflects discussion on the purpose for performing the experiment.		Purpose in writeup does not accurately reflect discussions about purpose for performing experiment.		____ $(x \frac{1}{2})$	____ (2)
Procedure		Clearly listed in sufficient detail. Instructions allow student to perform lab unaided by teacher.	Details complete but poorly or vaguely worded. Limited teacher intervention may be necessary to interpret what students have written.	Details lacking. How to gather data or what data to gather may not be obvious. May require teacher intervention to keep lab on track.	Instructions listed will not allow student to carry out lab without complete teacher intervention.	____ (x 1)	____ (4)
Data		Necessary data has been collected and clearly recorded in a chart, table, etc.	Necessary data has been collected but not displayed in a clear and easily usable manner.	Key data missing.		____ (x 1)	____ (4)
Analysis	Analysis carried out according to previous instructions. Findings clearly summarized. Insightful comments present.	Analysis carried our according to previous instructions. Findings clearly summarized				____ (x 1)	____ (5)
Conclusion	Purpose and findings of lab clearly summarized and connected so as to show the level of success achieved in performing the lab. Insightful comments about either shortcomings, important occurrences or out of the ordinary occurrences show a complete understanding of the purpose of the lab and its significance.	Purpose and findings of lab clearly summarized and connected so as to show the level of success achieved in performing the lab. Experimenter answers many of the questions that he/she set out to answer and backs up answers with his/her findings.	Purpose and findings of lab summarized. An attempt has been made to connect the two. Experimenter answers many of the questions that he/she set out to answer.	Purpose and findings of lab summarized. An attempt has been made to connect the two.	Vague statement made to finish off the lab that doesn't answer any of the questions that the experimenter set out to answer.	____ (x 2)	____ (10)

Comments____

____

Over-all Mark ____ (10)

EXPLORING SCIENCE AND MATHEMATICS USING STATISTICS AND TECHNOLOGY

Student Questionnaire

Indicate your understanding of the concepts explored in this unit as well as your comfort level with the statistics and technology used/introduced by circling the appropriate number on the scale for each question.

Connections	**Strongly** Disagree				**Strongly** Agree
1. Mathematics is a necessary component of science.	1	2	3	4	5
2. Mathematics is useful in the following ways:	1	2	3	4	5
a) Designing an experiment	1	2	3	4	5
b) Finding connections and relationships in the data collected	1	2	3	4	5
c) Supporting conclusions drawn from the data	1	2	3	4	5
Technology					
3. I can use graphing calculators/graphing software to find relationships in experimental data	1	2	3	4	5
4. I know how to use a graphing calculator/graphing software to perform linear and quadratic regressions on experimental data.	1	2	3	4	5
Problem Solving					
5. I can match basic nonlinear equations from physics to specific problem involving projectiles.	1	2	3	4	5
6. I can model problems involving projectiles (nonlinear data and/or nonlinear equations) using a graphing calculator or graphing software.	1	2	3	4	5
Statistics and Probability					
7. I can collect data according to the requirements laid out in an experiment.	1	2	3	4	5
8. I know how to do a 'best-fit' curve on experimental data.	1	2	3	4	5
9. I can extract needed information from a graph of nonlinear data.	1	2	3	4	5
10. I can draw conclusions and validate predictions from data presented in tables or in graphs.	1	2	3	4	5

SAMPLE 3: APPLICATIONS OF MATHEMATICS 12

Topic: *Vectors*

Prescribed Learning Outcomes:

Problem Solving

It is expected that students will:

- solve problems that involve a specific content area
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyze problems and identify the significant elements
- demonstrate the ability to work individually and cooperatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

SHAPE AND SPACE

It is expected that students will:

- use appropriate technology to describe vector and scalar quantities
- assign meaning to the multiplication of a vector by a scalar
- determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods
- model and solve problems in 2-D and simple 3-D using vector diagrams and technology

UNIT FOCUS

The overall goal of the unit was to familiarize students with the use of vector analysis to solve physical problems. Assessment required students demonstrate their ability to read and comprehend problems whose components contain vector and scalar quantities, that they could translate those quantities into vector representations and that they could resolve the key resultant components in such a way that the solution to the problem(s) was effectively communicated.

PLANNING THE UNIT

To develop the unit, the teacher:

- determined the overall goals of the unit
- identified the related IRP outcomes to be targeted in the unit
- identified the prerequisite knowledge and skills students needed to achieve the targeted outcomes included in this unit, and planned for review
- looked for ways to connect students' learning with other desirable outcomes, including those associated with students' attitudes
- looked for ways to encourage students' understanding of the ways in which practical problems are solved mathematically
- planned a variety of integrated instructional and assessment activities to help students achieve identified outcomes
- determined criteria with which to evaluate students' learning for marking and reporting purposes. Whenever appropriate, the teacher involved the students in setting evaluation criteria.
- developed with students a rubric that would be used to mark the unit project and its presentation. The development of

the rubric's criteria as done as early as possible and with the purpose of promoting the students understanding of the work that they were expected to accomplish.

THE UNIT

The purpose of the unit is to provide students with opportunities to use and assign meaning to vector and scalar quantities relating to real-world situations; use vector analysis to determine resultant vectors in situations involving both 2 and 3 dimensions; and use resultant vectors as solutions to practical problems.

To set the scene and review use of triangle trigonometry and parallel line rules, the teacher asked students to consider the development of a flight or watercraft simulator program that used velocities as integral parts of the game parameters. Examples might include a game that would help pilots or sailors estimate the effect of wind and/or current on the path of a plane or ship. The teacher pointed out the need to use formulas that would eventually appear in the computer program. Students noted that while they would not actually create the game program in this class, computing science students might wish to develop their work further.

Peer Assessment — Students were encouraged to visually check the vector analysis of other's work with a view to confirming both the vector representations of the problem and the solutions. In as much as possible, students were asked to confirm magnitude and directional appropriateness using head to tail and parallelogram rationale. In simple terms, students were encouraged to check that their peer' s drawings correctly represented the situations they were resolving.

Assessment Strategies — The teacher assessed the student's development of labeling and describing vector and scalar quantities. The teacher viewed the student's drawings and gave feedback on the appropriateness of their representations.

Developing Vector Analysis Skills

- The teacher had students "walk" their way through vector directions in such a way that they gained an understanding of what the term's magnitude and direction meant in a physical location sense.
- As students considered the effects of wind and current on the velocity of a boat, the teacher encouraged students to express the components as vectors.
- The teacher carefully made the point that a vector can be moved in a drawing as long as its magnitude and direction did not change. Students were shown that moving vectors allowed them to use simple trigonometry to find resultant vectors
- Students brainstormed possible navigation scenarios that required simple vector analysis.
- Small groups of students were assigned the task of writing descriptions of problems that required vector analysis to solve and would be part of the development of the game introduced earlier in unit. The groups then provided solutions to their problems. Groups then checked each other's problem sets and solutions.
- The teacher then selected problems that exemplified well constructed vector analysis situations and had students show they understood the fundamentals of vectors and their analysis by completing the questions. In as much as possible, the teacher pointed out alternative ways of finding solutions (e.g., scale diagrams or triangle solvers) to vector problems and

then discussed the strengths and weaknesses of various strategies.

- Students applied their knowledge of vectors to navigational problems to develop formulas that might appear in the programming of a flight simulator or instructional interactive program. The teacher modeled simple examples such as finding the location of an aircraft flying horizontally on a heading of 285° that is being pushed by a wind from 195° as measured clockwise from north. Parameters such as an indicated speed of 300 km/h, wind airspeed given as 90 km/h (constant) and an elapsed time of 1 hour and 15 minutes were used to show the students the affect of the wind on the planes final location.
- Simple 3-D constructs were established by having students resolve situations such as a aircraft flying at 80 m/s due north and climbing at an angle of 20° to the horizontal. Students were asked to consider the effect of a horizontal wind blowing from west to east at a speed of 30 m/s on the aircraft's track as expressed as a vector.

Teacher Review — As the students grew in their understanding of using vectors to solve navigational problems, the teacher pointed out other uses of vector analysis including forces as expressed in newton's specifically as applied to structural designs such as trusses. In as much as possible students were asked to apply their learning to new situations.

Peer, Small Group and Teacher Assessment — Students were asked at various developmental stages to report their concepts in a small group setting. Members of the group were asked to provide feedback to the reporter.

The teacher created an environment that encouraged students to participate fully in the development of sample simulations and scenarios for the flight simulator model. The demonstration of student's ability to meet the prescribed outcomes, in individual terms, was encouraged by reporting on each student's ability to construct vector diagrams and use appropriate strategies to resolve them. Students were also required to effectively communicate the meaning of the resultant vectors imbedded in the solutions to most if not all analyses that were required of them.

Using Performance Activities in Instruction

To help students understand various ways of resolving vector problems using technology such as triangle solvers, each student was asked to report on the pro and cons of using various strategies and tools.

DEFINING THE CRITERIA

Mathematical Thinking

To what extent did students

- Demonstrate an understanding vector representations of quantities
- Use vector analysis to solve problems
- Use technology to solve problems
- Use appropriate terminology in to describe problems and solutions

CASE STUDY FOR APPLICATIONS OF MATH 12

The following questions are based on the predicted effects of wind on a small plane heading due north at 150 mph. The planes altitude is 8000 feet.

1. A. Draw a scale “head to tail” vector representation of the effect of a 60 mph wind from the west on the plane above. Indicate scale of drawing and show resultant vector.
   B. Find the effective ground velocity of the plane as exactly as possible.

2. Assume the plane slows to 120 mph and descends to 5000 feet at the request of air traffic control. The plane then turns to head due east and maintains a steady speed.
   A. Draw a side view vector diagram that would show the effect of the plane flying through a down draft where the air is dropping at 30 mph. Find the value of the resultant vector. How many degrees from level flight is the planes projected path?

3. Assume the plane is redirected to head due south. It is now flying at 5000 feet at 120 mph. The plane then enters turbulent air. The air mass the plane is travelling through is moving up at 30 mph and is moving west at 60 mph.
   A. Find the resultant vector that represents the turbulent air.
   B. Show the vector that shows the planes resultant velocity whilst passing through the turbulent air. Indicate magnitude.

UNIT TEST

Students were given a carefully constructed test that initially required them to show an understanding of vector representations, terminology and the solution of simple resultant forms. Students were then required to show how formulas could be developed to find resultant vectors for relatively simple 2-D and 3-D situations. In as much as possible the initial modeling of the situations was related to the work the students had done on the navigation simulations while the final modeling required students to analyze new situations such as those involving forces on bridge components, for example.

SELF ASSESSMENT

Students’ assessment of the simulator project was based on understandings of how to find and apply resultant vector solutions. Students commented on their ability the way in which they used technology to solve vector problems.

▼ Sample 4: Essentials of Mathematics 10

Topic: *Wages, Salaries, and Expenses*

Prescribed Learning Outcomes:

Problem Solving

It is expected that students will:

- solve problems that involve a specific content area
- solve problems that involve more than one content area
- solve problems that involve mathematics within other disciplines
- analyze problems and identify the significant elements
- demonstrate the ability to work individually and cooperatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

Wages, Salaries and Expenses

It is expected that students will:

- calculate hours worked and gross pay
- calculate net income using deduction tables with different pay periods
- calculate changes in income
- develop a budget that matches predicted income

Unit Focus

The overall goal of the unit was to familiarize students with the use of budgets to more effectively manage income. Assessment required students to demonstrate their ability to read and comprehend problems whose components contained income calculations, deduction estimation, changes in income, and budget creation.

Planning the Unit

To develop the unit, the teacher:

- determined the overall goals of the unit
- identified the related IRP outcomes to be targeted in the unit
- identified the prerequisite knowledge and skills students needed to achieve the targeted outcomes included in this unit, and planned for review
- looked for ways to connect students' learning with other desirable outcomes, including those associated with students' attitudes
- looked for ways to encourage students' understanding of the ways in which practical problems are solved mathematically
- planned a variety of integrated instructional and assessment activities to help students achieve identified outcomes
- determined criteria with which to evaluate students' learning for marking and reporting purposes. Whenever appropriate, the teacher involved the students in setting evaluation criteria.
- developed with students a rubric that would be used to mark the unit project and its presentation. The development of the rubric's criteria as done as early as possible and with the purpose of promoting the students understanding of the work that they were expected to accomplish.

The Unit

Recalling, Reviewing and Extending Relevant Concepts

- To review and evaluate students' understanding of calculations of hours worked, the teacher presented a 24-hour clock scenario and had students fill out time cards indicating the times they arrived and left work. The teacher then had students calculate the hours worked per shift. The teacher gradually introduced time sheets involving split shifts and overtime to the students. The teacher then added the concept of calculating gross pay using a calculator.

Peer Assessment — Students were encouraged to check a partner's calculations of both time spent at work and calculations of gross pay.

Assessment Strategies — the teacher assessed the student's development of completing time sheets and finding gross pay. The teacher viewed the student's work and gave feedback on the appropriateness of their representations.

Creating a Budget

- The teacher had students research jobs that they might expect to have upon completion of high school. Students found the hourly pay associated with one particular job of their choice. As the students considered the job opportunities available to them, the teacher encouraged students to consider the type of lifestyle they would expect to have given their career choice.
- The teacher then had students assume a typical week and asked them to calculate their gross income for that week. Using deduction tables provided by the teacher, the student then calculated take home pay.
- The students were asked to consider typical weekly expenses. Using budget tables provided to them, the students estimated expenses such as rent, utilities, food, telephone, and transportation costs. The total estimated weekly expenses were calculated and checked with a partner.
- With the teacher's guidance, the students then reconciled their calculated take home pay with their estimated expenses and produced an acceptable budget.
- The teacher asked students to create a new budget based on a 10% increase in income that resulted from a student taking extra training or education.

Teacher Review — As student understanding of ways in which income could be managed grew, initiating retirement or education savings plans was discussed. In as much as possible students were asked to apply their learning to new a variety of practical financial problems.

Peer, Small Group, and Teacher Assessment — Students were asked at various developmental stages to report their concepts in a small group setting. Members of the group were asked to provide feedback to the reporter.

The teacher created an environment that encouraged students to participate fully in the development of individual budgets, goal setting and planning.

The demonstration of student's abilities to meet the prescribed learning outcomes was encouraged by reporting on the student's individual ability to use appropriate strategies to estimate and produce realistic and workable budgets. Students were also required to effectively communicate how they arrived at their estimates and what their goals were as well as how their goals affected their budgets.

Using Performance Activities in Instruction

To help students understand various ways of creating viable budgets, students were asked to experiment with making changes to a budget and seeing what effect that had on the complete picture. For example, students were asked to find the effect of buying a car on their ability to save. Other students were asked to find the effect of working overtime on their budgets. Students were asked to comment on the effect of a windfall such as a cash gift or income tax refund on their budget. Some students were able to use spreadsheets to more effectively manage their data and consider changes.

DEFINING THE CRITERIA

Mathematical Thinking

To what extend do students:

- demonstrate an ability to fill in time sheets and find gross pay.
- use deduction tables to find net pay
- calculate the net effect of pay rate changes
- create rationalized budgets

Unit Test

Students were given a carefully constructed test that initially required them to show an understanding of time sheets, and how to find net and gross pay. Students were then required to show how a time sheet could be used to estimate average weekly take home pay, given access to deduction tables. Students were given a list of expenses for an individual whose time sheet was provided. Students were then assessed on their ability to create a workable budget for that individual.

Self Assessment

Students' assessment of their understanding of the budgeting process included a written explanation of how they would go about creating a budget, given an hourly pay rate and access to typical housing costs and other expenses.

SAMPLE 5: ESSENTIALS OF MATHEMATICS 11

Topic: *Owning and Operating a Vehicle*

Prescribed Learning Outcomes:

Problem Solving

It is expected that students will:

- analyse problems and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original approaches
  - analyse keywords
- determine that their solutions are correct and reasonable
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

Owning and Operating a Vehicle

It is expected that students will:

- solve problems involving the acquisition and operation of a vehicle including
  - renting
  - leasing
  - buying
  - licensing
  - insuring
  - operating (e.g., fuel and oil)
  - maintaining (e.g., reparis, tune-ups)

In addition to these outcomes, the teacher assessed students' attitudes, group work, and communication skills.

UNIT FOCUS

The teacher's overall goal for this unit was to help students develop mathematical tools to calculate and analyse the costs associated with acquiring and operating a vehicle. To introduce and reinforce new concepts, the teacher provided concrete examples and guided practice, and prompted discussions of real-world applications. The teacher designed activities to assess students' application of concepts, mathematical thinking, and problem-solving and communication skills.

PLANNING THE UNIT

To develop the unit, the teacher:

- identified the prescribed learning outcomes for this unit
- examined the specific prerequisite knowledge and skills needed to achieve these outcomes
- determined which prerequisites were already in place and which should be reviewed
- looked for ways to connect students' learning to other desirable learning outcomes, including those associated with problem solving, communication skills, and attitudes
- identified criteria to evaluate students' learning
- designated assessment that was an integral part of the instructional process

THE UNIT

Leasing and Buying Vehicles

- Students were given advertisements taken from newspapers and magazines that displayed the costs of leasing vehicles. Working as a class, students identified monthly costs, fixed costs (e.g., down-payments, security payments, levies, taxes) and special costs (e.g., mileage costs)
- The teacher presented students with different scenarios for leasing vehicles (e.g., length of leases, driving over the allowable mileage limit, paying different down-payments) and asked the class to calculate the costs of leasing in each scenario.
- The class then found advertisements for vehicles that could be appealing to younger consumers. The teacher asked students to:
  - present the cost of leasing the vehicle for a set amount of time
  - find a cost of leasing that could be used to compare to other forms of transportation
  - determine how much of a personal budget leasing a vehicle would consume
- The teacher used the same process to develop students' understanding of purchasing a vehicle. The teacher and students investigated how the purchase price of a vehicle is not actually the amount of money paid for a vehicle, unless it was paid for in cash.
- Students worked with spreadsheets and programmable calculator lists to discover that monthly payments consisted of finance charges and reductions to the principle amount of the loan. To help the students' understanding, the teacher assigned students to use spreadsheet templates to calculate the interest paid in paying off a loan.
- When students had more of an understanding of the nature of loans, the teacher guided them to discover how the length of loan, the size of the monthly payments and the interest rates affected the total paid on a loan
- A loan manager from a credit institution was invited in to discuss the advantages and disadvantages of leasing and purchasing, and then worked together to compile pertinent points.
- Teacher guided students in a discussion on a costs and benefits of extended warranty.

Licensing and Insuring

- The teacher, using pamphlets supplied by an insurance broker, worked with students to help them:
  - learn basic insurance terminology (e.g., basic coverage, liability, third-party, no-fault, comprehensive, safe driver discount)
  - calculate basic and optional insurance costs
- When students had a firm understanding of insurance costs, the teacher guided them to discover how different vehicles had different insurance requirements and costs. The teacher had students research why some vehicles had higher insurance and license costs, and how these costs could affect purchasing decisions.
- Students worked with the teacher to calculate safe-driving discounts, and then worked independently on a number of application problems based on years of safe driving.

Operating and Maintaining

- Students were given examples of real-life scenarios of operating a vehicle (e.g., 24,000 km of driving in one year). Working as a class, students estimated the costs of operating a vehicle (e.g., gas and oil), and converted the cost to cost per kilometre.
- Using supplied material on maintenance intervals, the class worked to develop a chart of the costs of operating the vehicle based on:
  - tire life and replacement costs
  - tune-ups and oil changes
  - insurance and license costs

Applying Concepts in Other Curriculum Areas

- The teacher demonstrated to the class how personal loans and mortgages were calculated and applied in similar fashion to automobile purchase loans. The students then used spreadsheet templates to calculate various mortgages.

DEFINING THE CRITERIA

Mathematical Thinking

To what extend do students:

- demonstrate an understanding in the difference of monthly costs and fixed costs in a lease
- demonstrate facility in solving loans (e.g., time, total paid, interest costs) using templates or appropriate technology
- demonstrate facility in solving application problems involving insurance requirements and safe-driver discounts
- demonstrate facility in estimating operating costs per year or per kilometre

Attitudes

To what extend do students:

- approach problem situations with confidence
- show a willingness to persevere in solving difficult problems
- show flexibility in using available resources (e.g., calculators, Internet, outside expertise, help from other students or the teacher)
- contribute to class discussions and problem solving sessions

Group Skills

To what extent do students:

- communicate ideas clearly and efficiently
- work with other students in whole-class and small-group discussions to build on ideas and reinforce understanding
- present logical arguments to support their conclusions
- listen to the ideas of other students and respond appropriately

ASSESSING AND EVALUATING PERFORMANCE

Observing and Questioning

The teacher assessed students' understanding, attitudes, and group skills informally throughout the unit.

The teacher:
- observed students as they participated in class and small-group activities, taking note of behaviour that indicated whether they were achieving the criteria established for the unit
- asked questions to assess students' understanding of the central concepts
- moved through the class, observing students' work, encouraging their efforts, and listening to comments as they worked in small groups. (The teacher

knew students were involved in the unit when they made comments such as "Maybe we should try ..." or "I wonder what would happen if ...")

Journal Entries

The teacher had students paste in their mathematical workbooks examples of leasing and purchasing vehicles. Students used these examples to create problems for other students within small groups to solve. (Each group had to be able to solve the challenges it designed.) The teacher reviewed the work created by each group and evaluated the complexity and appropriateness of the problems as an indication of students' understanding of the concepts included in the unit.

Lease or Purchase Project

Students chose a vehicle from an advertisement that featured a lease and purchase plan. The students calculated the total cost of leasing and buying including all hidden costs and taxes. Students worked on deciding whether leasing or purchasing was preferable, and were encouraged to express their answers in cents per kilometre.

A loan manager from a credit institution was invited to the class to show students how to complete a loan application, as well as discuss the implications of leasing, buying new and buying used.

Students and the teacher perused the finished projects, and worked together to find discrepancies in the projects. Students were assigned points on a scale of 1 to 5 for each of the following criteria:

Lease or Purchase Project

Criteria	Rating				
• extent to which calculations are correct and include all costs	1	2	3	4	5
• extent to which data is meaningfully displayed	1	2	3	4	5
• extent of students' contributions to discussions regarding reasons for discrepancies	1	2	3	4	5

Key: 5 Excellent
4-Good
3-Average
2-Needs improvement
1-Needs complete review

Summary Sheets

Prior to instruction, the teacher gave students summary sheets designed to provide a reference and study guide for future use. Students could use one sheet for comparing the costs of mandatory and optional insurance and licensing costs including:
- mandatory basic coverage
- third-party coverage
- collision
- comprehensive
- safe driving discount
- other features of coverage (e.g., loss of vehicle costs, medical)

Another summary sheet could contain costs of leasing and purchasing including:
- down payment
- damage and security costs
- battery and tire levies
- transportation and dealer preparation costs
- options
- provincial and federal taxes
- tune-ups and maintenance costs
- extended warranty

The teacher assigned marks on the summary sheets based on the clarity, thoroughness and accuracy of students' responses. Written feedback was provided, and students were given the opportunity to make corrections and resubmit their sheets for improved grades.

Solving Problems

The teacher used a performance scale to evaluate students' abilities to solve problems and present their conclusions.

▼ SAMPLE 6: ESSENTIALS OF MATHEMATICS 12

Topic: *Comparative Study of Career Choices*

Prescribed Learning Outcomes:

Problem Solving

It is expected that students will:

- solve problems that involve more than one content area
- analyse problems and identify the significant elements
- demonstrate the ability to work individually and cooperatively to solve problems
- clearly communicate a solution to a problem and the process used to solve it
- use appropriate technology to assist in problem solving

Life/Career Project

It is expected that students will:

- determine what factors are important in analysing careers
- describe two specific career opportunities
- identify mathematical educational requirements for two careers
- compare two careers in terms of characteristics such as salary, hours of work, training cost and time, cost of living, and benefits

UNIT FOCUS

The overall goal of the unit was to give students the tools necessary for comparing career choices. Students needed to demonstrate their ability to research, compile, interpret, and present career factors. The teacher assessed students with respect to their descriptions, calculations, budget forms, and an assortment of other documents supporting the career choices and the life style it entails.

PLANNING THE UNIT

To plan the unit, the teacher:

- identified the prescribed learning outcomes for instruction and assessment for the unit and the prerequisite knowledge and skills needed to achieve these outcomes
- determined which of these prerequisites to review
- planned a variety of activities to help students achieve the learning outcomes, including those associated with group-work skills, communication skills, and attitude
- identified criteria to use in evaluating students' learning
- designed assessment that was an integral part of the instructional process

THE UNIT

Introduction

- The unit began with a discussion on selecting a career, the motivations behind it, the reasons for the selection, and the intended goals to be reached with that career choice
- People employed in various fields (e.g., trades, industry, business, research) were invited to give a brief synopsis on what their job entails, why they chose that particular field, and how they went about getting the job.
- Individually, students were asked to list as many career factors affecting career choices as they could. The class then came together and compiled all of these factors into one complete list.
- To reinforce the importance of considering all career factors when selecting a career, students were asked to interview an important adult in their life on her or his career choices.

Review of Relevant Concepts

- The teacher provided the class with a number of case studies involving budget information. Students worked in small groups to review their skills in presenting the data using spreadsheets, graphs, and diagrams. The groups had to provide reasons for choosing a particular style of presentation.
- The teacher provided specific problems and the students practiced calculations involving wages/salary and expenses.
- The teacher reviewed how to do a budgetary analysis.

Performing Research and Presenting the Results

- Students were given access to the library and the outside world via the internet. The teacher demonstrated necessary research skills and how to summarize data collected. Students selected two careers they were interested in and researched these careers with respect to the list of career factors the class had established. The students kept a log of their research which included a detailed description of the item and the source. Students were encouraged to record any additional factors that might influence their career choices.
- For further research, students were assigned to interview a person currently holding one of their career choices either in person, by phone, by mail, or by e-mail. The teacher discussed with the class how such contact could be made and what attitude and behaviour is necessary to achieve results.
- Once the research had been completed, the students were asked to organize their results and to present them meaningfully. Students were partnered up to use their partner as a sounding board for their presentation.

Analyzing the Research

- the teacher guided the students in comparing their two researched career choices. The teacher emphasized that it is important to provide reasons for decisions made.

DEFINING THE CRITERIA

Mathematical Thinking

To what extent do students:

- demonstrate an understanding of how to prepare a budget analysis
- prepare a budget analysis
- use spreadsheets, graphs and diagrams to present information
- describe the financial factors involved in selecting a career
- compare two careers with respect to salary, hours of work, training cost and time, cost of living, and benefits
- apply their mathematical knowledge while comparing two careers
- use appropriate mathematical terminology and notation

Attitudes

To what extent do students:

- approach problem situations with confidence
- show flexibility in using available resources (e.g., calculators , spreadsheets, help from other students or the teacher)
- show interest, participate in activities, and volunteer responses
- make connections between real-life situations and mathematics.

Group Skills

To what extent do students:

- work with other students in whole-class discussions and small groups, to build on ideas and understanding
- initiate, develop, and maintain interactions within the group.

Communication Skills

To what extent do students:

- communicate ideas clearly and understandably
- listen to and make use of the ideas of other students
- present logical arguments to support their conclusions

ASSESSING AND EVALUATING STUDENT PERFORMANCE

Observing and Questioning

The teacher observed as students participated in whole-class discussions, small-group activities, and individual research, taking note of behaviours that indicated students were or were not achieving the criteria established for the unit. Students' understanding, attitudes, and group and communication skills were evaluated informally throughout the unit. The teacher reviewed students' work to note the extent to which they showed progress in researching two careers and in compiling information on established career factors.

Individual Project

Each student completed a research project on comparing two careers of their interest. The project required student to:

- provide a job description for each career
- identify the mathematical educational requirements needed for each career
- list the overall educational requirements and associated costs for each career
- perform a budgetary analysis
- describe their findings on income, salary, and health factors associated with each career
- perform a lifestyle analysis
- compare the two careers objectively using their findings and provide reasons for their conclusion
- use and cite a variety of appropriate information sources
- complete a written report, and present their findings to the class

The report was evaluated using the following assessment checklist, and the presentation was evaluated using the following holistic scale. Students received copies of the checklist and scale before marking the project and used them as guidelines in developing their presentations and written reports. At the end of the project they used the guidelines to self-assess their work. The teacher held conferences to discuss discrepancies between students' and teacher's ratings. Students were given suggestions for improvements. Students who received scores of less that 50 points on their written reports were given the opportunity to redo them. The final score for the written report was the higher of the two scores.

ASSESSMENT CHECKLIST: CAREER PROJECT

Student Name:__________________________ Points: __________/100

POINTS

10 **Job Description**
- duties
- dress code
- employment opportunities
- other:

Requirements

15 • education
- requirements
- mathematics needed
- costs
- other:

15 • budgetary analysis
- food
- shelter
- clothing
- transportation
- other:

5 • income
- job(s) held
- salary/wages
- other:

Selected Careers (for each career)

10 • salary
- wages / starting salary
- increments
- benefits
- other:

10 • health issues
- stress
- insurance
- ergonomics
- risk factors
- other:

20 • life style
- food
- clothing
- shelter
- transportation
- recreation
- family size
- investments
- retirement planning
- charitable political contributions
- budgetary analysis
- other

15 **Comparison between the two careers**
- education
- salary
- health
- life style
- other:

Career Project Presentation Rating Scale

4 - Outstanding	Information is presented clearly, logically and understandably. Examples or demonstrations were used appropriately to illustrate explanations. Findings are well organized and effectively displayed. Explanations indicate a clear understanding of the topic and the use of information to perform a comparative study. References are appropriate for the topic and indicate that the student understands where to look for information.
3 - Adequate	The presentation indicates that the student has a basic understanding of the topic and of using information to perform a comparative study. Information is understandable. Findings are organized and displayed acceptably. References are appropriate for the topic.
2 - Needs Improvement	The presentation indicates a limited understanding of either the topic or the use of information to perform a comparative study, or both. The presentation may be illogical or difficult to follow. Findings may be organized poorly or ineffectively. References may indicate that the student is not clear on finding the best sources of information.
1 - Inadequate	The presentation indicates a lack of understanding of either the topic or the use of information to perform a comparative study. The presentation is illogical and difficult to follow. Findings are poorly organized and ineffectively presented. References may be lacking or inappropriate for the topic.

Journal Entries

During the project, students kept a log of their research, findings, and the approach they took. Students had to do on-going reflections on their position towards the two careers they chose. The teacher reviewed their journal entries and evaluated the quality of their responses. The teacher also used the journal to offer further guidance or pointers to the students' projects.

▼ SAMPLE 7: PRINCIPLES OF MATHEMATICS 10

Topic: *Radicals*

Prescribed Learning Outcomes:

Problem Solving

It is expected that the student will:

- analyse problems and identify the significant elements
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly explain the solution to a problem and justify the processes used to solve it

Number (Number Concepts)

It is expected that students will:

- classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system.

Number (Number Operations)

It is expected that students will:

- communicate a set of instructions used to solve an arithmetic problem.
- perform arithmetic operations on irrational numbers, using appropriate decimal approximations.
- perform operations on irrational numbers of monomial and binomial form, using exact values.

Unit Focus

The teacher provided students with concrete examples to help them realize that the study of radicals can help them extend their understanding of irrational numbers. The teacher provided small-group instruction and guided practice to help students gain the skills necessary to do the work in the unit. In order to demonstrate their abilities to solve problems involving radicals,
and use technology in solving problems, students participated in team competitions and special projects.

PLANNING THE UNIT

To develop the unit, the teacher:

- identified the prescribed learning outcomes for this unit
- examined the specific prerequisite knowledge and skills needed to achieve these outcomes
- determined which prerequisites were already in place and which should be reviewed
- planned a variety of activities to help the students achieve the outcomes
- looked for ways to connect students' learning to other desirable learning outcomes, including those associated with attitudes, work groups, and communication skills
- identified criteria to use to evaluate students' learning
- designed assessment that was an integral part of the instructional process

THE UNIT

Review of Squares and Square Roots

- The teacher used numerical examples to ensure that all students had a firm understanding of squares and square roots (e.g., $\sqrt{25} = \pm 5, 5^2 = 25$).
- Students responded to questions like the following:
  - What is the square root of ________?
  - What is the square of _______?
  - What is_______ squared?

- The students and teacher worked together to develop a table of perfect squares to 1000 that students could keep and use through the remainder of the unit.
- Do the same with perfect cubes, fourth powers, fifth powers (see table).

Table of Perfect Squares, Cubes, etc to ....

n	1	2	3	4	5	6	7	8	9	10
n^2	1	4	9	16	25	36	49	64	81	100
n^3										
n^4										

Rules for Working with Squares and Square Roots

- The teacher asked "what if?" questions like the following to help students discover that it is possible to take the rot of only a positive real number.
  - What if I wanted to find the square root of -25?
  - Try finding $\sqrt{-25}$ using your calculator. What happens? Why?
- The teacher used numerical examples and "what if"? questions to help students develop the rule for multiplying square roots: $\sqrt{a}\sqrt{b} = \sqrt{ab}\ (a \neq 0,\ b \neq 0)$

  For example, the teacher asked "What if you want to multiply $\sqrt{100}$ times $\sqrt{9}$? $\sqrt{100}\sqrt{9} = 10 \times 3 = 30$ is the same as $\sqrt{100 \text{ x } 9} = \sqrt{900} = 30$
- A similar process was used to help students develop the rule

  $$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}\ (a \neq 0,\ b \neq 0)$$.
- Students worked as a class to practise a number of these types of problems until they demonstrated clear understanding and could easily apply the rules.

Simplifying Radicals

- The teacher guided students to discover how to use the product rule in reverse to simplify radials. For example: $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$
- The teacher made it clear that when simplifying radicals, students should start by determining if any of the squares from their table of perfect squares was a factor of the radicand. If they found a factor with a nice square root (*perfect square factor*), they were to take it out from under the square root sign.
- The teacher had students do the same with perfect cubes.
- Students did the same with simplifying radicals with cube roots, fourth roots, etc. For example, given $\sqrt[3]{54}$, the teacher asked students to find a factor of 54 in their list of perfect squares, then rewrite $\sqrt[3]{54}$ as $\sqrt[3]{27 \text{x} 2}$, and factor out the cube root of 27 to get $3\sqrt[3]{2}$. The teacher ensured that students understood that the small index 3 in the $\sqrt[3]{\ }$ symbol is attached to the root sign.
- Students who found it difficult to recognize perfect squares were instructed to factor the radicand completely and use

this prime factorization to rewrite the expression in its simplest radical form. For example: $\sqrt{150} = \sqrt{2\times3\times5\times5} = 5\sqrt{6}$

- Students used their calculators to convert their final answers to decimal form.
- Students practised simplifying radicals, compared their answers with those of their peers, and resolved any differences.

Multiplying, Dividing, Adding, and Subtracting Radicals

- At this point, students were ready to move on to multiplying radicals. To help students understand that they should work first with coefficients and then with radicals, the teacher guided them as they worked through increasingly complex examples. For example:
  $5\sqrt{3}\times7\sqrt{2} = (5\times7)(\sqrt{3}\sqrt{2}) = 35\sqrt{6}$
- The class discovered how simplifying radicals before multiplying made their work easier.
  $\sqrt{18}\times\sqrt{12} = \sqrt{216}$ is difficult to simplify.
  $3\sqrt{2}\times2\sqrt{3} = 6\sqrt{6}$ is much easier.
- Individually, students solved problems to practise multiplying radicals. They were encouraged to compare their answers with those of their peers and resolve any differences.
- From multiplication, the class moved to division and rationalizing denominators. The teacher used numerical examples of increasing complexity. The more complex examples illustrated the advantage of simplifying radicals before dividing.
- Students were told that rationalizing denominators involves both removing radicals from denominators and removing denominators from within radicals.
- Together, the class worked through an example of a problem that required removing a radical from the denominator.
  For example: $\frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$
  or
  $\frac{27\sqrt{14}}{18\sqrt{6}} = \frac{3\sqrt{7}}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{21}}{6} = \frac{\sqrt{21}}{2}$
- Students worked in small groups to solve complex problems requiring the multiplication or division of radicals, or a combination of both operations. They displayed their answers in both simplest radical and decimal form.
- In a discussion about adding and subtracting radicals, the teacher asked questions that prompted students to recall the algebraic rules for adding and subtracting like and unlike terms and to apply these rules to problems involving radicals. The class worked with the teacher through several examples of increasing difficulty and discussed the advantages of simplifying the radical before performing operations.
- Students practised adding and subtracting radicals by working independently on problems prepared by the teachers. They displayed their answers both in simplest radical and decimal forms.
- Students worked in small groups to solve application problems involving all the skills they learned in this unit.
- Throughout the unit the teacher used numerical examples such as $\sqrt{2}\sqrt{2} = 2$ to encourage students to recognize immediately that $\sqrt{x}\sqrt{x} = x$ without having to work through the problem each time.
- As an alternative approach, the teacher drew on students' proficiency with basic algebra skills to develop the rules for working with radicals. The teacher provided opportunities that helped students realize that the rules for dealing with radicals are parallel to those they used in algebra.

Working with Radicals

Operation	Algebra Skill (Collect like terms)	Radical (You may have to simplify first!)
Addition and Subtraction	$5x + 3x = 8x$	$5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$
Multiplication	$3x(2y) = 6xy$	$3\sqrt{5} \times 2\sqrt{3} = 6\sqrt{15}$
Division	$\dfrac{10x^2y}{2xy} = 5x$	$\dfrac{10\sqrt{15}}{2\sqrt{3}} = 5\sqrt{5}$

Defining the Criteria

Mathematical Thinking

To what extend do students:

- Apply the rules for simplifying radicals
- solve problems requiring the multiplication of radical
- solve problems requiring the addition of radicals
- solve problems requiring the subtraction of radicals
- solve application problems involving radicals
- demonstrate an understanding of the processes involved in performing operations with radicals by developing complex problems for other students to solve
- explain the steps involved in completing the operations and procedures addressed in the unit
- report answers both in simplest radical and decimal forms

Attitudes

To what extend do students:

- show confidence in their abilities to solve problems involving radicals
- show flexibility in dealing with challenges and in using various resources to solve problems
- express their enjoyment of learning
- show interest, participate in activities, and volunteer responses

Communication Skills

To what extent do students:

- communicate ideas clearly and understandably
- explain their reasoning to others
- help other students
- accurately use the language and symbols of mathematics
- present ideas clearly and logically
- use examples to clarify explanations or arguments

Assessing and Evaluating Students

Observation and Questioning

Students' understanding, attitudes, and communication skills were assessed informally throughout the unit. The teacher:

- observed students during class discussions and while working individually and in small groups to monitor the extent to which they were meeting the criteria
- took note of which students seemed to be having difficulty
- asked questions to further assess students' understanding of concepts and to help ensure that the pace of instruction was appropriate

- observed students at work for evidence of understanding of concepts
- evaluated the quality of questions they asked and listened to their discussion in small groups
- moved through the classroom as the students worked independently and in groups to review their work and provide individual assistance as needed.

Team Competition

Students worked in teams to develop and solve a specified number of problems. Teams took turns writing one of their problems on the board. Students in the other groups were given three minutes to solve it. (Time limits may need adjusting.) Each group that correctly solved the problem in the specified time received one point. If none of the teams solved the problem, the team that developed the problem received one point. If none of the teams solved the problem, the team that developed the problem received one point. Any team that created a problem that was not solvable using the information learned in this unit or that incorrectly solved the problem, lost one point. Play rotated between the groups until all of the problems had been presented. Each group received a mark for the activity; the mark was then assigned to each student in the group. Marks were based on the number of points the group accrued plus the teacher's 1-5 rating of the complexity of the problems developed by the group.

Project

To assess individual student's understanding of the concepts covered in the unit, the teacher directed them to prepare detailed, step-by-step directions on how to complete the operations they had discussed. Students chose two topics from each of the following categories:

- Category 1:
  - simplifying radicals
  - removing radicals from the denominator
  - converting answers from simplest radical to decimal form
- Category 2:
  - adding radicals
  - subtracting radicals
  - multiplying radicals
  - dividing radicals and solving combination problems

Students prepared detailed instructions for completing the operations they chose. Students were asked to think of the teacher as a new students who did not know how to do the operations or procedures they were describing. Students were told to use whatever resources were available in preparing their instructions and to build in examples when-ever possible. The teacher rated students' instructions on the following scale.

- Students received copies of the rating scale when they started the assignment. At the end of the project they used the scale for self-assessments. Students compared their ratings with those of the teacher during a brief conference. Finally the teacher compiled instructions for each topic and gave them to students for future reference.

Project Scale

Criteria	Rating
• explanations are clear and easy to understand	5 4 3 2 1
• ideas are logically sequenced	5 4 3 2 1
• the project demonstrates a good understanding of the topic	5 4 3 2 1
• examples support and clarify the explanation	5 4 3 2 1
• language and symbols of mathematics are used correctly	5 4 3 2 1

Key 5 Excellent
4-Good
3-Average
2-Needs improvement
1-Not Acceptable

Attitude Scale

Students completed the following rating scale. Students then summarized the class results and discussed their meaning.

Please complete the following rating scale to summarize your feelings about the activities you participated in and the things you learned in this unit. Circle the number that best describes your feelings.

Attitude Scale

Statement	Rating
• I like working with square roots.	5 4 3 2 1
• I enjoy working with radicals.	5 4 3 2 1
• Working with radicals is easy.	5 4 3 2 1
• I feel comfortable putting radicals in their simplest form.	5 4 3 2 1
• I find it easier to add and subtract radicals than to multiply and divide them.	5 4 3 2 1
• I learned enough from this unit to feel comfortable working with radicals.	5 4 3 2 1
• I enjoyed the activities included in this unit	5 4 3 2 1

Key 5 - Strongly Agree
4 - Agree
3 - No Opinion
2 - Disagree
1 - Strongly Disagree

SAMPLE 8: PRINCIPLES OF MATHEMATICS 11

Topic: *Graphing Techniques*

Prescribed Learning Outcomes:

Problem Solving

It is expected that the student will:

- demonstrate the ability to work individually and co-operatively to solve problems
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies
- use appropriate technology to assist in problem solving

Patterns and Relations (Relations and Functions)

It is expected that students will:

- connect algebraic and graphical transformations of quadratic functions, using completing the square as required.
- describe, graph and analyze polynomial and rational functions, using technology

In addition to these outcomes, the teacher assessed students' attitudes and group skills.

UNIT FOCUS

The teacher emphasized the relationships between algebraic and graphical representations of functions to help students gain insight into the interconnectedness of various branches of mathematics. Students were given opportunity to study patterns and develop the graphing skills they needed to visually represent the abstract relationships described by equations. Students were given opportunities to demonstrate higher-order thinking skills as they determined the complex equations represented by graphs and learned to use graphing calculators.

PLANNING THE UNIT

To plan the unit, the teacher:

- identified the prescribed learning outcomes for this unit
- examined the prerequisite knowledge and skills needed to achieve these outcomes interpret function notation
- determined which prerequisites were already in place and which should be reviewed
- planned a variety of activities to help students achieve the learning outcomes and to connect their learning to other desirable outcomes (e.g., group skills, attitudes)
- identified criteria to use to evaluate students' learning
- designed assessment that was an integral part of the instructional process

THE UNIT

Demonstrating Understanding of Basic Functions and Their Graphs

- Together, students and teacher created graphs of all seven basic functions.
- Games were used to reinforce students' understanding of the basic functions (square route, rational, quadratic, linear, polynomial) and their associated graphs. In this manner, students received drill and practice until the teacher judged that the associations were automatic, and that all students were familiar with the basic functions and could graph them quickly.

Changing the Basic Equations

- In order to develop transformations to the basic equations that would result in vertical translations, the teacher worked

with the class to sketch graphs of formulae such as:
$y = x^2 + 1$, $y = x^2 - 2$
comparing them with the basic graph of
$y = x^2$
The teachers used these comparisons to work with students to develop the rule.

- A similar process was used to develop students' understanding of, first, horizontal translations, then reflections, and finally dilations (expansions and compressions).
- To further develop students' understanding of transformations, the teacher asked "what if?" questions. (e.g., What if y = $\sqrt{x}$ became y = $-\sqrt{x}$?) The teacher asked increasingly complex questions to challenge students' understanding.
- Once all the transformations were identified, the teacher introduced combinations of them to the discussion, further increasing the complexity.

Reinforcing and Demonstrating Understanding

- Students worked in small groups to determine the equations and tables of values that would result from various graphs, showing translations, reflections and dilations. Students used a graphing calculator to check their results.
- The teacher set up a team competition where groups of students developed equations for another group to graph. These graphs were then exchanged with a second groups, whose task was to determine which equations were used. As a variation, groups exchanged a graphing calculator that displayed a graph, and challenged other groups to replicate the graph or determine the equations used.

DEFINING THE CRITERIA

Mathematical Thinking

To what extend do students:

- use their knowledge of the basic functions and the rules for transformations to draw the graphs associated with complex forms of the basic functions. For example: $y = -2(x - 1)^2 + 3$
- predict changes that occur in a graph given variations to the equations for the basic functions
- demonstrate the higher-level thinking necessary to reverse the activity (develop a complex equation when given its graph)
- use appropriate mathematical terminology and notation
- demonstrate the ability to input functions into a graphing calculator
- compare graphs by explaining similarities and differences with respect to their defining equations
- explain properties of graphs based on the mathematical properties of the equation (e.g., $\sqrt{\ }$ can not be negative, therefore graph has restrictions)
- matching a relation or real data to an appropriate function or graph.
- demonstrate a willingness to explore, experiment, and use alternative strategies to make predictions and test hypotheses
- show a willingness to persevere in solving difficult problems
- show flexibility in the use of available resources (e.g., calculators, sketches, graphs, help from other students or the teacher)

Group Skills

To what extent do students:

- communicate ideas clearly and understandably

- work with other students in whole-class and small-group discussions, to build on ideas and reinforce understanding
- present logical arguments to support their conclusions
- listen to the ideas of other students

ASSESSING AND EVALUATING PERFORMANCE

Observing and Questioning

The teacher assessed students' understanding, attitudes, and group skills informally throughout the unit by:

- observing students as they participated in class and small-group activities, taking note of behaviors that indicated whether they were achieving the criteria established for the unit
- asking questions to assess students' understanding of the concepts being taught
- moving through the class, observing students' work, encouraging their efforts, and listening to comments as they worked in small groups (the teacher knew students were involved in the unit when they made comments like: "Let's see what happens if..."; "Maybe we should try..."; "I wonder what would happen if...").

Students completed a summary sheet designed to help them pull together and organize their learning in this unit. Students used available resources (e.g., notes, textbooks, in peer and teacher consultations) to complete this assignment. The teacher collected and reviewed the summary sheets, had students make corrections, and advised them to keep the sheets for future reference.

The teacher used the rating scale on the next page to evaluate the summary sheets.

Summary Sheet

Criteria	Rating
• draws the basic graph associated with each form of the basic equation	1 2 3 4
• uses a knowledge of the basic functions and the rules for transformations to draw the graphs associated with complex forms of the basic functions	1 2 3 4
• identifies and plots key points or lines on the graphs	1 2 3 4
• identifies each type of transformation that occurs as a result of identified changes to the basic equations	1 2 3 4

Key: 4-Exceptional.
3-Acceptable.
2-Needs improvement.
1-Little understanding shown.

Peer Assessment

The team competitions described in this unit was used as an activity for peer assessment. Each group kept records of the results of their team challenges. The teacher reviewed these records at the end of the activity. As an indication of the depth of students' understanding, the teacher took note of the complexity of the questions that students posed to other groups.

Self-Evaluation

Students completed the following evaluation of their group skills.

Assessing Group Skills

Self-Evaluation of Group Work

Based on your participation in your group, rate yourself on each of the following. Circle the number that corresponds with how satisfied you are with your performance in each area. Put a check mark in from of areas you would like help with in the future.

Need Help?	Criteria	Rating
	• I listened to the ideas of others in my group.	1 2 3 4
	• I communicated my ideas clearly and understandably.	1 2 3 4
	• I helped to build on or summarize the ideas of other students in my group.	1 2 3 4
	• I supported my conclusions and ideas with logic la arguments.	1 2 3 4
	• I encouraged other group members.	1 2 3 4
	• I attempted to resolve conflicts that arose in my group.	1 2 3 4
	• I participated in all phases of the activity.	1 2 3 4

Key: 4-Very satisfied.
3-Satisfied.
2-Need some improvement.
1-Not satisfied.

The teacher used two sections, "Social" and "Ideas," of the reference set *Evaluating Group Skills Across Curriculum* to evaluate the way individual students contributed OT the success of their groups. Minimally acceptable was defined as scale point 2. The teacher rated the performance of each student by using information from observations made while moving among the groups, and from the students' self-evaluations.

Tool and Material Use

Group Communications Skills

Rating	Social Interaction	Ideas Development
5 • Shapes the way the group works. • Develops and extends the group's work in ideas and development.	The student is able to initiate, develop, and sustain interactions so that the group is able to work together harmoniously. The student frequently encourages the efforts of other group members, often asking them questions. The student is comfortable - but not driven - to provide leadership when needed, attempts to resolve conflicts among other group members, is able to let go of personal ideas to further group progress, and approaches the task with obvious enjoyment, fen accompanied by humour.	The student participates in all phases of the activity, although contributions vary according to relevant information or experience. The student provides constructive feedback, offers predictions and hypotheses, and poses intriguing questions. The student is able to offer clarification, elaboration, or explanations as needed, and builds upon and in some cases synthesized the ideas others offer. The student may use comparisons, analogies, examples, or humour to illustrate or emphasize a point.
4 • Social interactions comfortable and well developed. • Flexible and well developed ideas.	The student is comfortable working in a group and contributes to the social dynamics. The student may take a leadership role in organizing how the group will interact. The student takes responsibility for group processes by facilitating and extending discussions, and by persevering beyond initial solutions. The student is responsive to other group members and their ideas. The student tends to interact effectively with the group but may not have much effect on how the other group members work with each other.	The student contributes ideas, experience, and information that the group is able to use. The student may help to develop ideas by providing details, examples, reasons, and explanations. The student often makes suggestions, asks questions, or adjusts personal thinking after listening to others. The student may also rephrase, paraphrase, or pose questions as a way of challenging or building on ideas from other group members. The student is able to make relevant connections to other situations or ideas.
3 • Socially engaged. • ideas are appropriate and related to the task.	The student takes part in group discussions and follows the basic rules for working with others: taking turns, listening while others are speaking, and sometimes offering recognition or support. The student may ask for or offer required information. The student is willing to accept group decisions and may share some responsibility for how the group works.	The student contributes dome suggestions and ideas to the group. The student responds and sometimes adds to suggestions that others make, participates in brainstorming activities, shows interest in the ideas of others, and adds information. The student may not defend personal ideas, and tends to give in quickly when someone disagrees.

Rating	Social Interaction	Ideas Development
2 • Inconsistent social interaction. • Ideas often disconnected.	The student may begin to show some awareness of the responsibilities of contributing to a group, may sometimes recognize and respond to the needs and ideas of others, and may show appreciation or support. At other times the student may have difficulty taking turns or accepting suggestions from other students. The student may remain uncommitted, focusing on personal needs rather than the group's task.	The student contributes ideas that address the task, but these may be unconnected to the ideas of others. The student may offer several suggestions but appears unable to elaborate, explain, or clarify ideas. The student often related the activity to personal experiences by telling stories. The student may think out loud, judge others' ideas quickly, or drift off-task easily.
1 • Largely unaware of others' needs • Limited contribution of ideas.	The student may not understand how personal behaviour affects others, and may be disruptive, aggressive, uninvolved, or easily frustrated.	The student may remain silent throughout the activity, contribute one idea repeatedly, or recount personal experiences unrelated to the group task. The student may not acknowledge or add to the contributions of other group members, but may respond to direct questions or other prompts from adults.

The students' performances with respect to the safe and proper use of tools and materials was evaluated using the following performance scale:

Outstanding
The student makes exceptionally thoughtful choices of materials, uses tools with proficiency, is especially conscientious about the safety of self and others, and assists in the organization and maintenance of a safe, orderly work environment.

Competent
The student chooses appropriate materials and tools, uses tools correctly, uses the proper safety equipment and procedures, and exhibits personal preparedness with respect to clothes, shoes, hair, jewelry, sleeves, and so forth.

Unacceptable
The student may be able to identify common tools but is unsure what tools to use for particular tasks and materials. Materials choices may be inappropriate. The student may not use proper safety measures or may need excessive supervision in order to do so. The student may be unaware of how personal actions affect the safety of others.

▼ SAMPLE 9: PRINCIPLES OF MATHEMATICS 12

Topic: *Logarithms*

Prescribed Learning Outcomes:

Problem Solving

It is expected that the student will:

- analyse a problem and identify the significant elements
- develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following:
  - guess and check
  - look for a pattern
  - make a systematic list
  - make and use a drawing or model
  - eliminate possibilities
  - work backward
  - simplify the original problem
  - develop alternative original; approaches
  - analyse key words
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that the solutions are correct and reasonable, and clearly communicate the process used to solve the problem
- use appropriate technology to assist in problem solving

Patterns and Relations (Relations and functions)

It is expected that students will:

- model, graph, and apply exponential functions to solve problems
- change functions from exponential form to logarithmic form and vice versa
- model, graph, and apply logarithmic functions to solve problems
- explain the relationship between the laws of logarithms and the laws of exponents

In addition to these outcomes, the teacher assessed students' attitudes, and group and communication skills.

UNIT FOCUS

Real-life problems in such divers situations as investment analysis, population growth, and radioactive decay cannot easily be solved using basic algebra. This unit focuses on building students' understanding of the inverse relationship between exponential and logarithmic functions so they can find solutions to such problems. The teacher provided opportunities for students to work individually and in small groups to demonstrate their abilities to solve application problems and their knowledge about why logarithms are important and how they can be used.

PLANNING THE UNIT

To plan the unit, the teacher:

- identified the prescribed learning outcomes for instruction and assessment for the unit and the prerequisite knowledge and skills needed to achieve these outcomes.
- determined which of these prerequisites were already in place and which to review
- looked for ways to connect students' learning to other desirable learning outcomes, including those associated with group-work skills, communication skills, and attitudes
- identified the criteria to use to evaluate students' success in the unit
- designed assessment to be an integral part of the instructional process

- began the unit by reviewing exponent laws and operations with exponents with the students

THE UNIT

Understanding Base 10 Logarithms

- The unit began with a review of exponent laws and operations with exponents.
- The teacher worked with the students to develop the definition of base 10 logs. Using numeric examples to establish students' understanding, the teacher guided them to discover the pattern: $1000 = 10^?$, $100 = 10^?$, $10 = 10^?$, $1 = 10^?$

The teacher related students' discovery and understanding of the pattern to the formal definition of base 10 logs, and explained that *log* was really just another name for exponent.

- To move students toward working with base 10 logs of numbers that are not integral powers of 10, the teacher asked students to estimate, based on their knowledge of base 10 logs, what power 10 would be raised to in order to get the log of a number such as 65. The teachers asked questions to prompt their thinking (e.g., What is it between? Is it closer to 1 or 2?).
- Students were asked to find a button on their calculator that would help them find the log of 65. Together, the students and teacher worked through a number of examples, first estimating what the log might be, then using their calculators to find the actual log.
- To develop the restriction to the definition:

  log x is defined only if $x > 0$ (the log of zero and negative numbers is undefined)

  students were challenged to guess the logs of one and zero and then find them on their calculators. The teacher asked questions like:
  - Why does zero give you an error message on your calculators?
  - Can 10 raised to some power give you a negative number? Why not?
  - Can you find the log of a negative number?
- The teacher used similar strategies to develop other rules associated with logarithms, such as: $\log 10^x = x$ and $10^{\log x} = x$

Students were given practice moving between exponential and logarithmic forms using their calculators before proceeding to the next part of the unit.

Performing Operations with Logarithms

- The class discussed why it is important to be able to establish the same base for numbers. (Logs allow us to establish a similar base for numbers and to work with them more easily as a result.) the teacher asked specific questions that made use of exponent properties. This reminded students about their experience working with exponential expressions in algebra and of the exponent laws for multiplying and dividing expressions with the same base. The teacher prompted students to make a connection between their experiences with algebra and what they were learning about logs.
- The teacher used simple concrete examples, such as:
  $1000 \times 100 = 10^3 \times 10^2 = 10^5$
  as students began to experiment with the rules for multiplying logarithms students worked with increasingly complex examples, such as:
  $(750)(83) = \log a + \log b$

until they were ready to develop the formula:
$\log(ab) = \log a + \log b$

- The teacher used a similar method for division of logarithms - drawing on students' experiences working with exponents to help them develop that formula.
- A similar method was also used to develop the rule:
  $\log a^n = n \log a$
- To establish a historical perspective, the teacher showed students how log tables were used to help perform complicated numerical operations. Demonstrating with a slide rule showed how theory was applied before the advent of calculators.
- Students practiced applying what they had learned about logarithms, and the teacher emphasized the connections between logs and exponents. Students were encouraged to assess their own work by using calculators to verify answers.
- The teacher assigned a worksheet to help students reinforce their learning. Students were encouraged to exchange answers and resolve any differences. The teacher moved among the students as they worked, evaluating their understanding and providing assistance as necessary.
- The teacher provided examples to help students solve equations with common bases and to reinforce the need for a common base, such as:
  $2^{3x} = 2^7$ moving to $2^{3x} = 8^{-x+4}$
- Students were then assigned the problem $5^x = 7$ (at this point, students should recognize why having a common base is important). The teacher and students worked together to see how to solve this problem. They used these and similar examples to develop the process of taking logs of both sides of an equation to generate solutions. Students demonstrated their understanding by solving additional complex equations using their calculators.

Demonstrating Change of Base

- The class moved from base 10 to other bases. The teacher used concrete numeric examples to help students see the connections and relationships between exponents and logarithmic forms. For example:
  $\log_5 25 = 2$ is the same as $5^2 = 25$
  $\log_3 81 = 4$ is the same as $3^4 = 81$
- By increasing the complexity of the examples, the teacher helped students generalize to:
  $\log_b a = n \int \approx b^n = a$
  The teacher stressed the that base and the argument of the function (b & a) must be greater than zero, and $b \neq 1$.
- Students confirmed their understanding by working through examples that required them to move back and forth between exponential and logarithmic forms.
- The teacher conducted a brief discussion and provided examples to help students discover that the rules for base 10 apply to other bases.
- The teacher challenged students to find a log they could do on their calculators (e.g., $\log_{10} 17$ and then find a log they could not do on their calculators (e.g., $\log_5 17$). Students learned to use the change of base formula
  $$\log_b a = \frac{\log a}{\log b}$$
  to solve the second type of problem. Students demonstrated their understanding of these ideas by working individually on a number of prepared problems.

- The teacher worked with the class to develop the graphs of exponential functions. Using graphing techniques and the students' knowledge of inverses, the class developed a number of graphs of exponential and logarithmic functions. Here are some examples:
  $y = 3^{x+1} - 2$
  $y = \log_{1/2}(x-1) + 3$

Applying Knowledge of Logarithms

To encourage students to apply their knowledge of logarithms to real-life situations, the teacher asked them to brainstorm subjects which interested them (e.g., money, earthquakes). The teacher demonstrated how to use logs and exponents to solve problems related to their interests. Students worked in small groups to solve a number of application problems and then challenged each other with problems.

DEFINING THE CRITERIA

Mathematical Thinking

to what extend do students:

- demonstrate an understanding of the definition of base 10 logs
- explain the for working with logarithms
- describe why logarithms are important and how they can be used
- describe the relationship between exponents and logarithms
- apply the rules for multiplying and dividing logs to solve simple and complex problems
- use the rules for logarithms to solve problems involving exponential and logarithmic expressions
- find the logs of numbers with bases other than 10
- use appropriate mathematical terminology and notation
- use their calculators to solve problems involving logarithms
- apply their knowledge of logarithms to real-life problems

Attitudes

To what extent do students:

- approach problem situations with confidence
- show a willingness to persevere in solving difficult problems
- show flexibility in using available resources (e.g., calculators, textbooks, help from other students or the teacher)

Group Skills

To what extent do students:

- work with other students in whole-class discussions and small groups, to build on ideas and understanding
- initiate, develop, and maintain interactions within the group
- help other students develop understanding

Communication Skills

To what extend do students:

- communicate ideas clearly and understandably
- listen to and make use of the ideas of other students

ASSESSING AND EVALUATING STUDENT PERFORMANCE

Observation and Questioning

Students' understanding, attitudes, and group and communication skills were evaluated informally throughout the unit.

The pace of the unit was determined in part by the speed with which students seemed to grasp the concepts.

The teacher:

- observed students as they participated in whole-class and small-group activities looking for evidence that they understood the concepts, built on ideas of others, initiated, developed, and maintained interactions within the group, and assisted others in developing understanding
- reviewed students' work to note the extent to which they met the criteria for mathematical thinking
- made note of behaviours that students were or were not achieving the criteria established for the unit (e.g., using
- calculators to solve problems involving logarithms)
- used questions to assess students' understanding of central concepts, the level of confidence they displayed when problem solving, and their willingness to persevere when solving difficult problems
- checked students' work to note the extent to which they used appropriate mathematical terminology and notation and applied their knowledge of logarithms to real-life problems

Individual Projects

Each student completed a research project on an issue or current event of personal interest. The project required students to:

- use what they learned about exponents and logarithms
- describe the topic and why it was of interest
- describe how the use of exponents and logarithms related to their topic
- look for data and display their findings in a meaningful way
- explain what their finding meant, why they were relevant, and why they were organized as they were
- use and cite a variety of appropriate information sources
- complete a written report, and present their findings to the class

Presentations and reports were evaluated separately using the following holistic scale. Students received copies of the scale before starting the project and used it to rate their presentations and written reports. The teacher held conferences to discuss discrepancies between students' and teacher's ratings. Students were given suggestions for improvements. Students who received scores of one or two on their written reports were given the opportunity to redo them. The final score for the written report was the higher of the two scores.
Project Rating Scale

Project Rating Scale

4 - Outstanding	Information is presented clearly, logically, and understandably. Examples or demonstrations are used appropriately to illustrate explanations. Findings are well organized and effectively displayed. Explanations indicate a clear understanding of the topic and the use of logarithms and exponents. References are appropriate for the topic and indicate that the student understands where to look for information.
3 - Adequate	The presentation indicates that the student has a basic understanding of the topic and of exponents and logarithms. Information is understandable. Findings are organized and displayed acceptably. References are appropriate for the Topic.
2 - Needs Improvement	The presentation indicates a limited understanding of either the topic or the use of logarithms and exponents, or both. The presentation may be illogical or difficult to follow. Findings may be organized poorly or ineffectively. References may indicate that the student is not clear on finding the best sources of information.
1 - Inadequate	The presentation indicates a lack of understanding of either the topic or the use of logarithms and exponents. The presentation is illogical and difficult to follow. Findings are poorly organized and ineffectively presented. References may be lacking or inappropriate for the topic.

Problem-Solving Observation Sheet

As students worked individually and in small groups to solve problems relating to logarithms, the teacher moved through the class observing and asking questions to probe for understanding. "The Problem-Solving Class Observation Sheet" included in the publication *Evaluating Problem Solving Across Curriculum* was used to evaluate students' problem-solving skills. The teacher used only those parts of the checklist that were appropriate to the classroom activities. Students received a summary of the information compiled on the sheet to give them feedback concerning their problem-solving skills and techniques.

Take-Home Test

Students were given a take-home test that contained similar problems to those studied during the unit. As part of the test, students were required to explain the rules they had learned for working with logarithms and use the rules to solve numeric examples that illustrated their explanations.

The last page of the test included a self-evaluation sheet that asked the following questions:

- What things in this unit did your find easy?
- What things did you find difficult?
- Are there any areas in the unit that you need more help with? If so, what are they?
- Would you be interested in after-school peer tutoring for help in these areas?
- Would you be willing to tutor another student after school in the areas you feel most comfortable with?

The teacher used the results of the take-home test and students' responses to the self-evaluation to pair students for after-school peer tutoring. Students were required to correct errors on their take-home tests and resubmit them for a second evaluations.

▼ SAMPLE 10: CALCULUS 12

Topic: *Limits*

Prescribed Learning Outcomes:

Problem Solving

It is expected that students will:

- analyse a problem and identify the significant elements
- demonstrate the ability to work individually and co-operatively to solve problems
- determine that their solutions are correct and reasonable
- clearly communicate a solution to problem and justify the process used to solve it
- use appropriate technology to assist in problem solving

Functions, Graphs and Limits (Limits)

It is expected that students will:

- demonstrate an understanding of the concept of limit and notation used in expressing the limit of a function $f(x)$ as x approaches a: $\lim_{x \to a} f(x)$
- evaluate the limit of a function:
  - analytically
  - graphically
  - numerically
- distinguish between the limit of a function as x approaches a and the value of the function at $x = a$
- demonstrate an understanding of the concept of one-sided limits and evaluate one-sided limits
- determine limits that result in infinity (infinite limits)
- evaluate limits of functions as x approaches infinity (limits at infinity)
- determine vertical and horizontal asymptotes of a function, using limits
- determine whether a function is continuous at $x = a$

In addition to these outcomes, the teacher assessed outcomes related to the historical development of calculus, students' attitudes, and group and communication skills.

UNIT FOCUS

The concept of a limit plays a central role in students future understanding of the importance and usefulness calculus. It is this understanding that enables students to effectively interpret the results of a variety of problem solutions using calculus.

PLANNING THE UNIT

To plan the unit, the teacher:

- identified the prescribed learning outcomes for instruction and assessment for the unit and the prerequisite knowledge and skills needed to achieve these outcomes.
- determined which of these prerequisites were already in place and which to review
- looked for ways to connect students' learning to other desirable learning outcomes, including those associated with group-work skills, communication skills, and attitudes
- identified the criteria to use to evaluate students' success in the unit
- designed assessment to be an integral part of the instructional process
- began the unit by reviewing and discussing with students characteristics of functions that they were familiar with (exponential, logarithmic, sinusoidal, etc.)

THE UNIT

Introducing the Concept of Limit

- The unit began with a review of the characteristics of familiar functions and their graphs.
- The teacher worked with the students to develop the concept of limit using an overhead graphing utility to demonstrate graphically the relationship between a variety of functions and the limits of the function at a number of different points.
- Students used graphing calculators to perform "what if" investigations to help them determine graphically the effect on the limits of different types of functions if the characteristic equation of the function were altered.

One-sided Limits

- Students were encouraged to explore the concept of one-sided limits by comparing functions with one-sided limits to those that had different left-hand and right-hand limits.

Infinite Limits and Limits at Infinity

- The teacher provided examples illustrating how to calculate limits of functions that result in infinity and limits of functions as x approaches infinity.
- Guided practice reinforced students' understanding of the difference between infinite limits and limits at infinity.

Asymptotes of a Function

- The teacher then guided class discussion from infinite limits and limits at infinity to vertical and horizontal asymptotes of a function
- Students were given practice determining the vertical and horizontal asymptotes of different functions, using limits.

Continuous Functions

- The teacher began by demonstrating that a continuous function is one which can be graphed over each interval of its domain with one continuous motion of the pen.
- This concept was expanded to the use of a graphing calculator where graphs of functions that were not continuous at $x = a$ had "holes" were no pixels were represented.
- Students were then asked to use limits to describe this phenomenon. The teacher reinforced that a function is continuous at a point $x = a$ if $\lim\limits_{x \to a} f(x) = f(a)$

Applying Knowledge of Limits

To encourage students to apply their knowledge of limits to solve some of the historical problems that the teacher had introduced at the beginning of the unit (i.e., tangent line problem and the area problem). Students worked in small groups to develop possible solutions to the problems.

DEFINING THE CRITERIA

Mathematical Thinking

to what extend do students:

- demonstrate an understanding of the concept of limit
- can evaluate the limit of a function in a variety of ways and show the connection between the methods (analytical, graphical, numerical)
- describe why limits are important and how they can be used
- describe the difference between infinite limits and limits at infinity

- use limits to determine asymptotes of a function and to determine whether the function is continuous at a given point.

Attitudes

To what extent do students:

- approach problem situations with confidence
- show a willingness to persevere in solving difficult problems
- show flexibility in using available resources (e.g., graphing calculators, textbooks, help from other students or the teacher)

Group Skills

To what extent do students:

- work with other students in whole-class discussions and small groups, to build on ideas and understanding
- initiate, develop, and maintain interactions within the group
- help other students develop understanding

Communication Skills

To what extend do students:

- communicate ideas clearly and understandably
- listen to and make use of the ideas of other students

ASSESSING AND EVALUATING STUDENT PERFORMANCE

Observation and Questioning

Students' understanding, attitudes, and group and communication skills were evaluated informally throughout the unit. The pace of the unit was determined in part by the speed with which students seemed to grasp the concepts.

The teacher:

- observed students as they participated in whole-class and small-group activities looking for evidence that they understood the concepts, built on ideas of others, initiated, developed, and maintained interactions within the group, and assisted others in developing understanding
- reviewed students' work to note the extent to which they met the criteria for mathematical thinking
- made note of behaviours that students were or were not achieving the criteria established for the unit (e.g., using graphing calculators to determine limits when appropriate)
- used questions to assess students' understanding of central concepts, the level of confidence they displayed when problem solving, and their willingness to persevere when solving difficult problems
- checked students' work to note the extent to which they used appropriate limit terminology and notation.

Individual Projects

Each student completed a research project on an topic or mathematician of personal interest. The project required students to:

- use what they learned about limits
- describe the topic and why it was of interest
- describe how the use of limits related to their topic
- explain what their finding meant, why they were relevant, and why they were organized as they were
- use and cite a variety of appropriate information sources
- complete a written report, and present their findings to the class

Presentations and reports were evaluated separately using the following holistic scale. Students received copies of the scale before starting the project and used it to rate their presentations and written reports. The teacher held conferences to discuss discrepancies between students' and teacher's ratings. Students were given suggestions for improvements. Students who received scores of one or two on their written reports were given the opportunity to redo them. The final score for the written report was the higher of the two scores.

Project Rating Scale

4 - Outstanding	Information is presented clearly, logically, and understandably. Examples or demonstrations are used appropriately to illustrate explanations. Findings are well organized and effectively displayed. Explanations indicate a clear understanding of the topic and the use of limits in a variety of contexts (e.g., to determine asymptotes of a function). References are appropriate for the topic and indicate that the student understands where to look for information.
3 - Adequate	The presentation indicates that the student has a basic understanding of the topic and of limits. Information is understandable. Findings are organized and displayed acceptably. References are appropriate for the Topic.
2 - Needs Improvement	The presentation indicates a limited understanding of either the topic or the use of limits, or both. The presentation may be illogical or difficult to follow. Findings may be organized poorly or ineffectively. References may indicate that the student is not clear on finding the best sources of information.
1 - Inadequate	The presentation indicates a lack of understanding of either the topic or the use of limits. The presentation is illogical and difficult to follow. Findings are poorly organized and ineffectively presented. References may be lacking or inappropriate for the topic.

Test/Team Competition

To evaluate students' understanding of the concepts developed in this unit, the teacher designed a team competition based on completing a set of items on a study sheet. Students worked in groups of four and used all available resources to answer each of the items. The groups were given opportunities to drill each other. When all members of the group felt comfortable with the information, they were asked to take a test individually that contained items similar to those on the study sheet. The teacher collected the tests, scored them, and gave each student a grade. Groups worked together to correct their tests and returned their corrections for additional points. The winning group had the highest average individual test scores and the highest average scores on their corrections.

Take-Home Test

Students were given a take-home test that contained similar problems to those studied during the unit. As part of the test, students were required to explain the rules they had learned for working with limits and use the rules to solve examples that illustrated their explanations.

The last page of the test included a self-evaluation sheet that asked the following questions:

- What things in this unit did your find easy?
- What things did you find difficult?
- Are there any areas in the unit that you need more help with? If so, what are they?
- Would you be interested in after-school peer tutoring for help in these areas?
- Would you be willing to tutor another student after school in the areas you feel most comfortable with?

The teacher used the results of the take-home test and students' responses to the self-evaluation to pair students for after-school peer tutoring. Students were required to correct errors on their take-home tests and resubmit them for a second evaluations.

Appendix D

Assessment Practices

Teachers should base their assessment and evaluation of student perfor mance on a wide variety of methods and tools, including observation, student self-assessments, daily practice assignments, quizzes, work samples, pencil-and-paper tests, holistic rating scales, projects, oral and written reports, performance reviews, and portfolio assessments. Using a variety of assessment methods can help teachers to compile comprehensive profiles of student learning. The Assessment Handbook Series—*Performance Assessment*, *Portfolio Assessment*, *Student Self-Assessment*, *Student-Centred Conferences*, and various reference sets—provide useful, detailed information about a range of appropriate assessment practices. The rest of this appendix provides guidance in creating classroom tests.

CONSTRUCTING CLASSROOM TESTS

There are two types of evaluations that take place in a classroom, each with its own distinct purpose.

- Formative evaluations are ongoing assessments used to guide learning rather than draw final conclusions.
- Summative evaluations refer to evaluative procedures (e.g., tests, reports, projects) usually conducted at the end of major units, to assess performance against predetermined criteria. These normally represent a substantial portion of final grades. The classroom tests described in this section fall under the category of summative evaluations.

All tests should be developed using the principles of criterion-referenced evaluation. In criterion-referenced evaluation, student achievement is interpreted in relation to previously defined levels of achievement, rather than relative to the achievement of other students. Questions on a criterion-referenced test should be representative of a clearly defined domain of prescribed learning outcomes. In this way, scores more accurately represent the student's present status with respect to those outcomes.

A test should measure what it intends to measure. For example, if a test requires a reading level far above the abilities of many of the students taking the test, test results would measure differences in reading levels rather than differences in subject knowledge.

STEPS IN CLASSROOM TEST CONSTRUCTION

The following suggests the key points to consider when developing classroom tests.

Steps in Classroom Test Construction	Points for Consideration
Plan the Test	• Begin the planning process well in advance. • Identify the prescribed learning outcomes to be tested. Learning outcomes provide a framework for the development of criteria. • Create a table of specifications that covers the learning outcomes and cognitive levels (knowledge, understanding, higher mental processes). • Balance the table of specifications to reflect the topics and cognitive levels in the curriculum.
Write Test Items	• Word questions clearly. (e.g., Use "determine a value for *x*" rather than "find *x*.") • Define the answer format so students understand the form their answers must take. • Don't repeat questions on one outcome. • When possible, design questions that connect various topics within the curriculum area and between various curricula. • Create questions that require various forms of answers (e.g., explaining, comparing, illustrating, graphing, calculating, solving, justifying). • Categorize every question according to the criteria. • Don't phrase distractors in multiple-choice questions so that they point to the answer. • Review questions for appropriate vocabulary and targetted reading levels. • Ask a colleague to answer the questions to identify possible marking problems and time requirements, and to provide feedback.
Format the Test	• Use easier questions at the beginning of the test to establish students' confidence. • Group similar items. • Organize items on the page so they are easy to read and provide adequate space for responses. • Develop test instructions that are clear and unambiguous.

Develop a Scoring Key	• Mark for processes as well as correct answers. • Allow for possible alternative solutions or different forms of answers (e.g., format, notation, detail). • Consider a range of scoring methods (e.g., holistic, analytic).
Prepare Students	• Establish test criteria with students. • Help students brainstorm topics that are likely to be covered. • Discuss how to approach the test (budget time and the weight given test results in the final grade). • Give students sufficient time to prepare for the test. • Review terminology used on the test (e.g., evaluate, simplify).
Administer the Test	• Allow time for all, or nearly all, students to finish. • Ensure a test location free of distractions. • Ensure all necessary supplies are available.
Score the Tests	• Test the effectiveness of the key on a few sample tests. Adjust the key as necessary. Note exemplar papers. • Mark tests at one sitting (or one question at one sitting) to ensure consistency. • Return marked tests promptly, and review them with the students to help them improve their understanding of the concepts involved.
Table of Specifications	A unit test in mathematics must measure the skills or concepts taught in the unit. A table of specifications can help the teacher plan the amount of emphasis to give each skill or concept. A table of specifications is a chart showing the content categories and cognitive levels to be tested. The percentage weighting of test items within any row or column is determined by the amount of time devoted to the content categories and their degree of difficulty.

Table of Specifications

Unit # ________ Variables and Equations Principles of Mathematics 11

Content	**Knowledge** • recall • conventions • classification • notation	**Understanding** • application of theories, ideas, principles, or methods to a new situation	**Higher Mental Processes** • analysis • synthesis • evaluation	**Percentage of Total**
Problem Solving		2 questions	4 questions	24%
Algebraic Skills	3 questions	5 questions	2 questions	40%
Use of Technology	2 questions	2 questions		16%
Mathematical Reasoning			5 questions	20%
% of Total	20%	36%	44%	100%

Appendix E

Acknowledgments

Many people contributed their expertise to this document. The project coordinator was Bruce McAskill of the Curriculum Branch, working with ministry personnel and our partners in education. We would like to thank all who participated in this process.

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Appendix F

Glossary

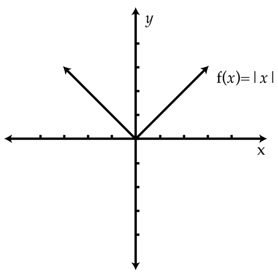
ABOUT APPENDIX F

This appendix provides an illustrated glossary of terms used in this Integrated Resource Package.

The terms and definitions are intended to be used by readers unfamiliar with mathematical terminology. For a more complete definition of each term, refer to a mathematical dictionary such as the *Nelson Canadian School Mathematics Dictionary* (ISBN 17-604800-6).

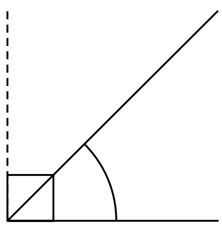
A

absolute value of a number
How far the number is from 0. Example: the absolute values of -4.2 and of 4.2 are each 4.2



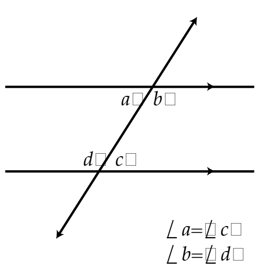
absolute value function
The function f defined by $f(x)=|x|$, where $|x|$ denotes the absolute value of x.

accuracy
A measure of how far an estimate is from the true value.

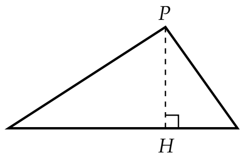


acute angle
An angle whose measure is between 0° and 90°.

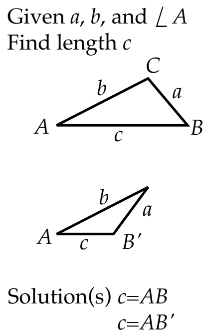
algorithm
A mechanical method for solving a certain type of problem, often a method in which one kind of step is repeated a number of times.



alternate interior angles
In the diagram to the left, the angles labelled *a* and *c* are alternate interior angles, as are the angles *b* and *d*.



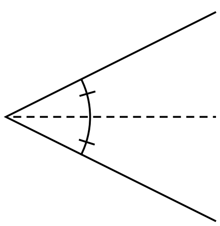
altitude of a triangle
A line segment PH, where P is a vertex of the triangle, H lies on the line through the other two vertices, and PH is perpendicular to that line.



ambiguous case
Two sides of a triangle and the angle opposite one of them are specified, and we want to calculate the remaining angles or side. There may be no solution, exactly one, or exactly two.

amplitude (of a periodic curve)
The maximum displacement from a reference level in either a positive or negative direction. That reference level is often chosen halfway between the biggest and smallest values taken on by the curve.

analytic geometry (coordinate geometry)
An approach to geometry in which position is indicated by using coordinates, lines, curves, and other objects are represented by equations, and algebraic techniques are used to solve geometric problems.



angle bisector
A line that divides an angle into two equal parts.

antiderivative
If $f(x)$ is the derivative of $F(x)$, then $F(x)$ is an *antiderivative* of $f(x)$. *Indefinite integral* means the same thing.

antidifferentiation
The process of finding antiderivatives.

arc
A connected segment of a circle or curve.

arc sine (of x)
The angle (in radians) between $-\pi/2$ and $\pi/2$ whose sine is x. Notation: $\sin^{-1} x$ or $\arcsin x$.

arc tangent
The angle (in radians) between $-\pi/2$ and $\pi/2$ whose tangent is x. Notation: $\tan^{-1} x$ or $\arctan x$.

arithmetic operation
Addition, subtraction, multiplication, and division.

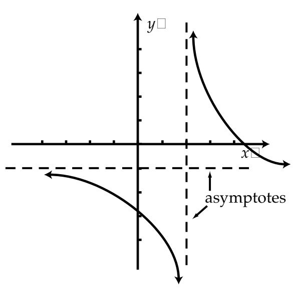
arithmetic sequence (arithmetic progression)
A sequence in which any term is obtained from the preceding term by adding a fixed amount, the *common difference*.

If a is the first term and d the common difference, then the sequence is $a, a + d, a + 2d, a + 3d, \ldots$. The n-th term t_n is given by the formula $t_n = a + (n - 1)d$.

arithmetic series
The sum S_n of the first n terms of an arithmetic sequence. If the sequence has first term a and common difference d, then

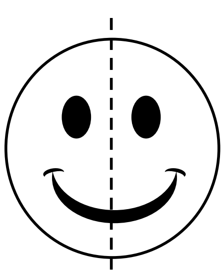
$$S_n = \frac{1}{2}n[2a + (n - 1)d] = \frac{1}{2}n\,(a + l),$$

where l is $a + (n - 1)d$, the "last" term.



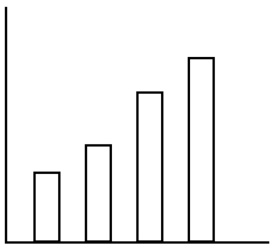
asymptote (to a curve)
A line l such that the distance from points P on the curve to l approaches zero as the distance of P from an origin increases without bound as P travels on a certain path along the curve.

average velocity
The net change in position of a moving object divided by the elapsed time.



axis of symmetry (of a geometric figure)
A line such that for any point P of the figure, the mirror image of P in the line is also in the figure.

B



bar graph
A graph using parallel bars (vertical or horizontal) which are proportional in length to the data they represent.

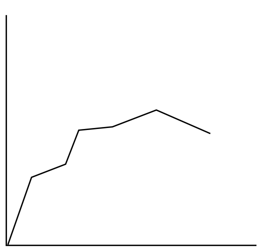
base
In the expression s^t, the number or expression s is called the *base*, and t is the *exponent*. In the expression $\log_a u$, the base is a.

binomial
The sum of two monomials.

binomial distribution
The probabilities associated with the number of "successes" when an experiment is repeated independently a fixed number of times. For example, the number of times a "six" is obtained when a fair die is tossed 100 times has a binomial distribution.

Binomial Theorem
A rule for expanding expressions of the form $(x+y)^n$.

bisect
to divide into two equal parts.



broken-line graph
A graph using line segments to join the plotted points to represent data.

C

Cartesian (rectangular) coordinate system
A coordinate system in which the position of a point is specified by using its signed distances from two perpendicular reference lines (axes).

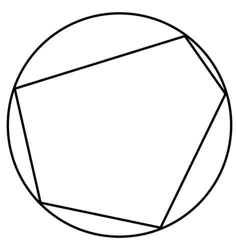
central angle
An angle determined by two radii of a given circle; equivalently, an angle whose vertex is at the center of the circle.

chain rule
A rule for differentiating composite functions.

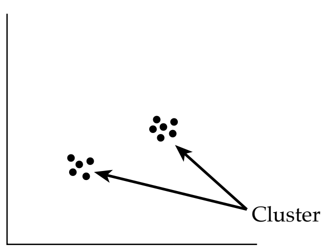
If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$.

chord
The line segment that joins two points on a curve, usually a circle.

circumference
The boundary of a closed curve, such as a circle; also, the measure (length) of that boundary. Please see *perimeter*.



circumscribed
The polygon P is *circumscribed* about the circle C if P is inside C and the edges of P are tangent to C. The circle C is *circumscribed* about the polygon Q if Q is inside C and the vertices of Q are on the boundary of C. The notion can be extended to other figures, and to three dimensions.



cluster
A collection of closely grouped data points.

coefficient
A numerical or constant multiplier in an algebraic expression. The coefficient of x^2 in $4x^2 - 2axy$ is 4, and the coefficient of xy is $-2a$.

collinear
Lying on the same line.

combination
A set of objects chosen from another set, with no attention paid to the order in which the objects are listed (see also *permutation*). The number of possible combinations of r objects selected from a set of n distinct objects is ${}_nC_r$, pronounced "n choose r."

common factor (CF)
A number that is a factor of two or more numbers. For example, 3 is a common factor of 6 and 12. *Common divisor* means the same thing. The term is also used with polynomials. For example, $x - 1$ is a common factor of $x^2 - x$ and $x^2 - 2x + 1$.

common fraction
A number written as $\frac{a}{b}$, where the numerator a and the denominator b are integers, and b is not zero. Examples:

$$\frac{4}{5}; \quad \frac{-13}{6}; \quad \frac{3}{1}.$$

compass
An instrument for drawing circles or arcs of circles.

complementary angles
Two angles that add up to a right angle.

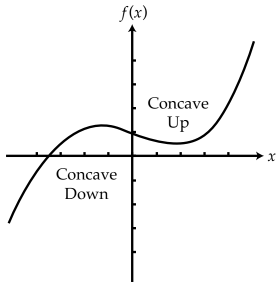
completing the square
Rewriting the quadratic polynomial $ax^2 + bx + c$ in the form $a(x - p)^2 + q$, perhaps to solve the equation $ax^2 + bx + c = 0$.

complex fraction
A fraction in which the numerator or the denominator, or both, contain fractions.

composite function
A function $h(x)$ obtained from two functions f and g by using the rule $h(x) = f(g(x))$ (first do g to x, then do f to the result).

composite number
An integer greater than 1 that is not prime, such as 9 or 14.

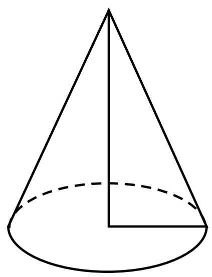
compound interest
The interest that accumulates over a given period when each successive interest payment is added to the principal in order to calculate the next interest payment.



concave down (or downward)
The function $f(x)$ is concave down on an interval if the graph of $y = f(x)$ lies below its tangent lines on that interval.

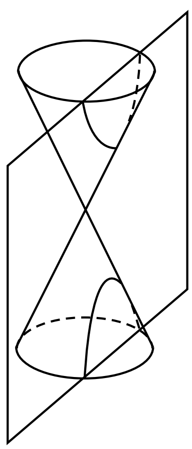
concave up (or upward)
The function f(x) is concave up on an interval if the graph of $y = f(x)$ lies above its tangent lines on that interval.

conditional probability
The probability of an event *given* that another event has occurred. The (conditional) probability that someone earns more than $200,000 a year, given that the person plays in the NHL, is different from the probability that a randomly chosen person earns more than $200,000.



cone (right circular)
The three-dimensional object generated by rotating a right triangle about one of its legs.

confidence interval(s)
An interval that is believed, with a preassigned degree of confidence, to include the particular value of some parameter being estimated.



congruent
Having identical shape and size.

conic section
A curve formed by intersecting a plane and the surface of a double cone. Apart from degenerate cases, the conic sections are the ellipses, the parabola, and the hyperbolas.

conjecture
A mathematical assertion that is believed, at least by some, to be true, but has not been proved.

constant
A fixed quantity or numerical value.

continuous data
Data that can, in principle, take on any real value in some interval. For example, the "exact" height of a randomly chosen individual, or the exact length of life of a U-235 atom can be modelled by a continuous distribution.

continuous function
Informally, a function $f(x)$ is *continuous* at a if $f(x)$ does not make a sharp jump at a. More formally, $f(x)$ is continuous at a if $f(x)$ approaches $f(a)$ as x approaches a.

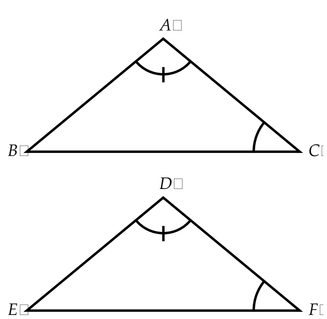
contrapositive
The contrapositive of "Whenever A is true, B must be true." is "Whenever B is false, then A must be false." Any assertion is logically equivalent to its contrapositive, so one strategy for proving an assertion is to prove its contrapositive.

converse (of a theorem)
The assertion obtained by interchanging the premise and the conclusion. If the theorem is "Whenever A happens, B must happen," then its converse is "Whenever B happens, A must happen." The converse of a theorem need not be true.

coordinate geometry
Please see *analytic* geometry.

coordinates
Numbers that uniquely identify the position of a point relative to a coordinate system.

correlation coefficient
A number (between -1 and 1) that measures the degree to which a collection of data points lies on a line.



corresponding angles and corresponding sides
Angles or sides that have the same relative position in geometric figures.

cosecant (of x)
This is $\frac{1}{\sin x}$. Notation: $\csc x$.

cosine law (law of cosines)
A formula used for solving triangles in plane geometry.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

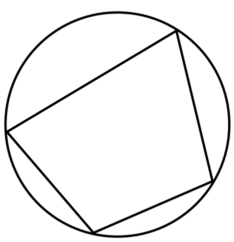
cosine (function)
See *primary trigonomic functions*.

cotangent (of x)
This is $\frac{1}{\tan x}$. Notation: $\cot x$.

coterminal angles
Angles that are rotations between the same two lines, termed the initial and terminal arms. For example:

$$20°, -340°, 380° \text{ are coterminal angles}$$

critical number (of a function)
A number where the function is defined, and where the derivative of the function is equal to 0 or doesn't exist.



cyclic (inscribed) quadrilateral
A quadrilateral whose vertices all lie on a circle.

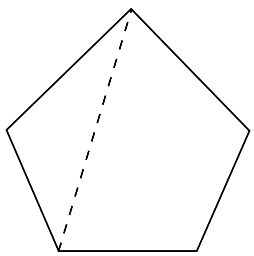
D

decimal fraction
In principle, a fraction a/b where a is an integer and b is a power of 10. For example, 1/4=25/100, so 1/4 can be expressed as a decimal fraction, usually written as 0.25.

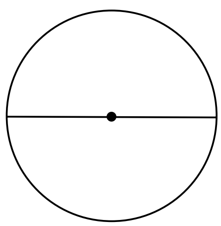
decreasing function
The function $f(x)$ is decreasing on an interval if for any numbers s and t in that interval, if t is greater than s then $f(t)$ is less than $f(s)$.

deductive reasoning
A process by which a conclusion is reached from certain assumptions by the use of logic alone.

degree
The highest power or sum of powers that occurs in any term of a given polynomial or polynomial equation. For example, $6x + 17$ has degree 1, and $2 + x^3 + 7x$ has degree 3, as does $2 + 6x + 7y + xy^2$.



diagonal
A line segment that joins two non-adjacent vertices in a polygon or polyhedron.



diameter
A line segment that joins two points on a circle or sphere and passes through the center. All diameters of a circle or sphere have the same length. That common length is called *the* diameter.

difference of squares
An expression of the form $A^2 - B^2$, where A and B are numbers, polynomials, or perhaps other mathematical expressions. We can factor $A^2 - B^2$ as $(A + B)(A - B)$.

differentiable
A function is *differentiable* at $x = a$ if under extremely high magnification, the graph of the function looks almost like a straight line near a. Most familiar functions are differentiable everywhere that they are defined.

differential equation
An equation that involves only two variable quantities, say x and y, and the first derivative, or higher derivatives, of y with respect to x.

Example: $3y^2 \dfrac{dy}{dx} = e^x$.

differentiate; differentiation
To find the derivative; the process of finding derivatives.

direct variation
The quantity Q varies directly with x if $Q = ax$ for some constant a. This can be contrasted with *inverse* variation, in which $Q = \frac{a}{x}$ for some a.

discrete data
Data arising from situations in which the possible outcomes lie in a finite or infinite sequence.

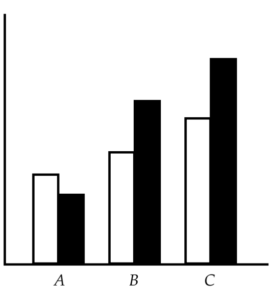
discriminant
The *discriminant* of the quadratic polynomial $ax^2 + bx + c$ (or of the equation $ax^2 + bx + c = 0$) is $b^2 - 4ac$.

displacement
Position, as measured from some reference point.

distance formula
The formula used in coordinate geometry to find the distance between two points. If A has coordinates (x_1, y_1) and B has coordinates (x_2, y_2), then the distance from A to B is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

domain (of a function)
The set of numbers where the function is defined. For example, if $f(x) = \frac{\sqrt{x - 2}}{x - 5}$, then the domain of $f(x)$ consists of all real numbers greater than or equal to 2, except for the number 5.



double bar graph
A bar graph that uses bars to represent two sets of data visually.

E

edge
The straight line segment that is formed where two faces of a polyhedron meet.

ellipse
A closed curve obtained by intersecting the surface of a cone with a plane. Please see *conic section*.

equation
A statement that two mathematical expressions are equal, such as:

$$3x + y = 7$$

equidistant
Having equal distances from some specified object, point, or line.

estimate
v. To approximate a quantity, perhaps only roughly.
n. The result of estimating. Also, an approximation, based on sampling, to some number associated with a population, such as the average age.

Euclidean geometry
Geometry based on the definitions and axioms set out in Euclid's Elements.

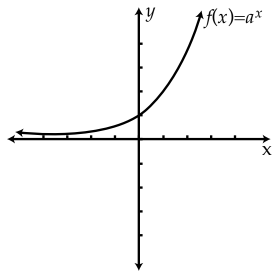
event
A subset of the sample space of all possible outcomes of an experiment.

experimental probability
An estimate of the probability of an event obtained by repeating an experiment many times. If the event occurred in k of the n experiments, it has experimental probability k/n.

exponent
The number that indicates the power to which the base is raised. For example:

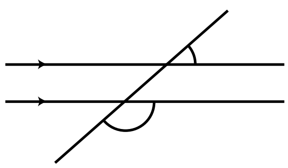
3^4: exponent is 4

exponential decay
A quantity undergoes exponential decay if its rate of decrease at any time is proportional to its size at the time. Exponential decay models well the decay of radioactive substances.



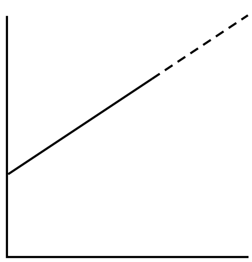
exponential function
An exponential function is a function of the form $f(x) = a^x$, where $a > 0$ and the variable x occurs as the exponent. The exponential function is the function $f(x)=e^x$, where e is a mathematical constant roughly equal to 2.7182818284.

exponential growth
A quantity undergoes exponential growth if its rate of increase at any time is proportional to its size at the time. Exponential growth models well the growth of a population of bacteria under ideal conditions.



exterior angles on the same side of the transversal
A transversal of two parallel lines forms two supplementary exterior angles.

extraneous root
A spurious root obtained by manipulating an equation. For example, if we square both sides of $1 - x = \sqrt{x - 1}$ and simplify, we obtain $(x - 1)(x - 2)=0$, that is, $x = 1$ or $x = 2$. Since 2 is not a root of the original equation, it is sometimes called an extraneous root.



extrapolate
Estimate the value of a function at a point from values at places on one side of the point only.

extreme values
The highest and lowest numbers in a set.

F

face
One of the plane surfaces of a polyhedron.

factor

n. *A factor* of a number n is a number (usually taken to be positive) that divides n exactly. For example, the factors of 18 are 1, 2, 3, 6, 9, and 18. Similarly, a factor of a polynomial $P(x)$ is a polynomial that divides $P(x)$ exactly. Thus x and x - 1 are two of the factors of x^3 - x.
v. *To factor* a number or polynomial is to express it as a product of basic terms. For example, $x^3 - x$ factors as $x(x - 1)(x + 1)$.

Factor Theorem

If $P(x)$ is a polynomial, and a is a root of the equation $P(x) = 0$, then $x - a$ is a factor of $P(x)$.

factor(s)

Numbers multiplied to produce a specific product. For example:

> $2 \times 3 \times 3 = 18$: factors are 2 and 3; $(x - 2)$ and $(x + 1)$ are factors of $x^2 + x - 2$

first-hand data

Data collected by an individual directly from observations or measurements.

flip

Another word for *reflection*.

fractal

Very roughly, a set of such a jagged and fractured appearance that small parts of it, however magnified, have the same complexity as the whole.

frequency diagram

A diagram used to record the number of times various events occurred.

function

A rule that produces, for any element x of a certain set A, an object $f(x)$. The set A is the *domain* of the function; the set of values taken on by $f(x)$ is the *range* of the function.

More formally, a function is a collection of ordered pairs (x, y) such that the second entry y is completely determined by the first entry x.

function notation
If a quantity y is completely determined by a quantity x, then y is called a *function* of x. For example, the area of a circle of general radius x might be denoted by $A(x)$ (pronounced "A of x.) In this case, $A(x) = \pi x^2$.

fundamental counting principle
If an event can happen in x different ways, and for *each* of these ways a second event can happen in y different ways, then the two events can happen in $x \times y$ different ways.

G

general polynomial equation
An equation of the form

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0.$$

general term (of a sequence)
If n is unspecified, a_n is called the *general term* of the sequence a_1, a_2, a_3, . . . Sometimes there is an explicit formula for a_n in terms of n.

geometric sequence (progression)
A sequence in which each term except the first is a fixed multiple of the preceding term. If the first term is a, and each term is r times the previous one, (r is the *common ratio*) then the general term t_n is given by $t_n = ar^{n-1}$.

geometric series
The sum S_n of the first n terms of a geometric sequence. If a is the first term, r the common ratio, where $r \neq 1$, then

$$S_n = \frac{a(1 - r^n)}{1 - r}\ .$$

See also *infinite geometric series*.

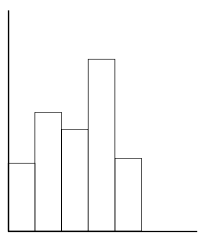
greatest common factor (GCF)
The largest positive integer that divides two or more given numbers. For example, the GCF of 12 and 18 is 6. The GCF is also called the greatest common divisor (GCD).

H

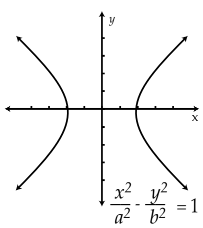
Heron's formula
A formula for the area of a triangle in terms of its sides. The area of a triangle with sides of length a, b, and c is equal to

$$\sqrt{s(s - a)(s - b)(s - c)}, \quad \text{where} \quad s = \frac{a + b + c}{2}.$$

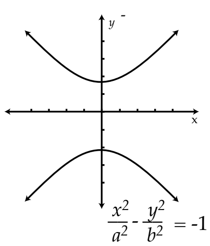
higher derivatives
The derivative of the derivative of $f(x)$, the derivative of the derivative of the derivative of $f(x)$, and so on.



histogram
A bar graph showing the frequency in each class using class intervals of the same length.



hyperbola
A curve with two branches where a plane and a circular conical surface meet. Please see *conic section*.



hypotenuse
The side opposite the right angle in a right triangle.

hypothesis
A statement or condition from which consequences are derived.

I

identity
A statement that two mathematical expressions are equal for all values of their variables.

if-then proposition
A mathematical statement that asserts that *if* certain conditions hold, *then* certain other conditions hold.

Imperial measure
The system of units (foot, pound, and so on) for measuring length, mass, and so on that was once the legal standard in Great Britain.

implicit function
A function y of x defined by a formula of shape $H(x, y) = 0$. For example, $y_3 - x^2 + 1 = 0$ defines y implicitly as a function of x. In this case, y is given *explicitly* by $y = (x^2 - 1)^{1/3}$. But often (example: $H(x, y) = y^7 + (x^2 + 1)y - 1$), it is not possible to give an explicit formula for y.

improper fraction
A *proper fraction* is a fraction whose numerator is less in absolute value than its denominator. An improper fraction is a fraction that is not a proper fraction.

increasing function
The function $f(x)$ is increasing on an interval if for any numbers s and t in that interval, if t is greater than s then $f(t)$ is greater than $f(s)$.

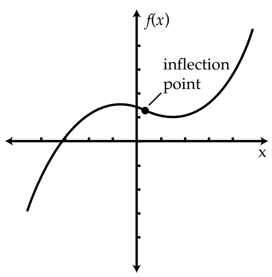
indefinite integral
Another word for *antiderivative*.

independent events
Two events are *independent* if whether or not one of them occurs has no effect on the probability that the other occurs.

inductive reasoning
A form of reasoning in which the truth of an assertion in some particular cases is used to leap to the (tentative) conclusion that the assertion is true in general.

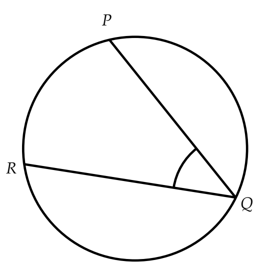
inequality
A mathematical statement that one quantity is greater than or less than the other. The statement $s > t$ means that s is greater than t, while $s < t$ means that s is less than t.

infinite geometric series
The "sum" $a + ar + ar^2 + \ldots + ar^{n-1} + \ldots$ of all of the terms of a geometric sequence. If $|r| < 1$, then this sum is equal to $\frac{a}{1-r}$.



inflection point
A point on a curve that separates a part of the curve that is concave up from one that is concave down.

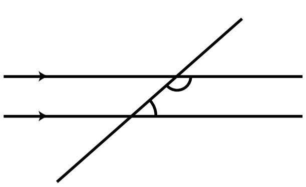
initial value problem
A function is described by specifying a differential equation that it satisfies, together with the value of the function at some "initial" point; the problem is to find the function.



inscribed angle
The angle PQR, where P, Q, and R are three points on a curve, in most cases a circle.

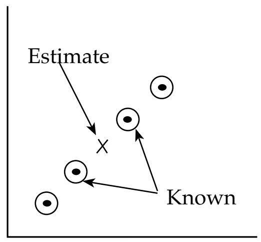
instantaneous velocity (at a particular time)
The exact rate at which the position is changing at that time.

integer
One of 0, 1, -1, 2, -2, 3, -3, 4, -4, and so on.



interior angles on the same side of the transversal
The transversal of two parallel lines forms interior supplementary angles.

integration
In part, the process of finding antiderivatives.



interpolate
Estimate the value of a function at a point from values of the function at places on both sides of the point.

intersection
The point or points where two curves meet.

interval
The set of all real numbers between two given numbers, which may or may not be included. The set of all real numbers from a given point on, or up to a given point, is also an interval, as is the set of all real numbers.

inverse (of a function)
The function $g(x)$ is the inverse of the function $f(x)$ if $f(g(x)) = x$ and $g(f(x)) = x$ for all x, or more informally if each function "undoes" what the other did.

inverse operations
Operations that counteract each other. For example, addition and subtraction are inverse operations.

inverse trigonometric functions
Inverses of the six basic trigonometric functions. For the two most commonly used, please see *arc sine* and *arc tan*.

irrational number
A number that cannot be expressed as a quotient of two integers. For example, $\sqrt{2}$, π, and e are irrational numbers.

irregular
Lacking in symmetry or pattern.

isosceles triangle
A triangle that has two or more equal sides. Occasionally defined as a triangle that has *exactly* two equal sides.

L

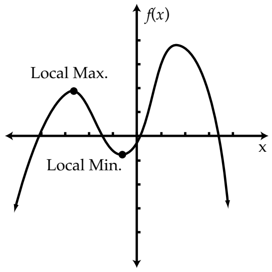
least squares
A criterion used to find the line of best fit, namely that the sum of the squares of the differences between "predicted values" and actual values should be as small as possible.

limit
The *limit* of $f(x)$ as x approaches a (notation: $\lim_{x \to a} f(x)$) is the number that $f(x)$ tends to as x moves closer and closer to a. There may not be such a number. For example, if x is measured in radians, $\lim_{x \to 0} \frac{\sin x}{x} = 1$, but $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

line of best fit
For a collection of points in the plane obtained from an experiment, a line that comes in some sense closest to the points. Please see *least squares*.

linear function
A function f given by a formula of the type $f(x) = ax + b$, where a and b are specific numbers.

linear programming
Finding the largest or smallest value taken on by a given function $a_1 x_1 + a_2 x_2 + \ldots + a_n x_n$ (the *objective function*) given that $x_1, x_2, \ldots, x_n$ satisfy certain *linear constraints*. The constraints are inequalities of the form $b_1 x_1 + b_2 x_2 + \ldots + b_n \geq c$. Many applied problems, such as designing the cheapest animal feed that meets given nutritional goals, can be formulated as linear programming problems.

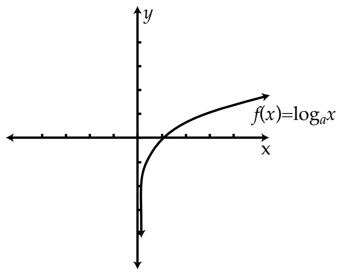


local maximum
The function $f(x)$ is said to reach a local maximum at $x = a$ if there is a neighborhood of a such that $f(x) \leq f(a)$ for any x in the neighborhood; informally, $(a, f(a))$ is at the top of a hill.

local minimum
The function $f(x)$ is said to reach a local minimum at $x = a$ if there is a neighborhood of a such that $f(x) \geq f(a)$ for any x in the neighborhood; informally, $(a, f(a))$ is at the bottom of a valley.

logarithmic differentiation
The process of differentiating a product/quotient of functions by finding the logarithm and then differentiating. For example, let $y = (1 + x)^2/(1 + 3x)$. Then

$$\ln y = 2\ln(1 + x) - \ln(1 + 3x) \text{ and } \frac{1}{y}\frac{dy}{dx} = \frac{2}{1+x} - \frac{3}{1+3x}.$$



logarithmic function
Let a be positive and not equal to 1. The logarithm of x to the base a is the number u such that $a^u = x$, and is denoted by $\log_a x$. Any function of the form $f(x) = \log_a x$ is called a logarithmic function.

lowest common multiple (LCM)
The smallest positive integer that is a multiple of two or more given positive integers. For example, the LCM of 3, 4, and 6 is 12. The LCM is often called the *least* common multiple.

M

matrix
A rectangular array of numbers. For example:

$$\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

2×2 matrix $\quad$ 3×1 matrix

maximum point (or value)
The greatest value of a function.

mean (of a sequence of numerical data)
A measure of the average value, obtained by adding up the terms of the sequence and dividing by the number of items.

median (of a sequence of numerical data)
The "middle value" when the data are arranged in order of size. If there is an even number of data, then the average of the two middle values. For example, the median of 5, 3, 7.4, 5, 8, is 5, while the median of 5, 7.4, 5, 8 is 6.2.

median (of a triangle)
The line segment that joins a vertex of the triangle to the midpoint of the opposite side.

minimum point (or value)
The lowest value of a function.

mixed number
A number that is expressed as the sum of a whole number and a fraction. For example:

$$3\tfrac{2}{5}$$

mode
The value that occurs most often in a sequence of data.

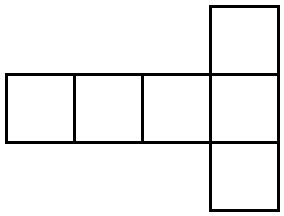
monomial
An algebraic expression that is a product of variables and constants. Examples: $6x^2$, 1, $\left(\frac{3}{4}\right) x^2y$,

multiple (of an integer)
The result obtained when the given integer is multiplied by some integer. Equivalently, an integer that has the given integer as a factor. (Often negative integers are not allowed.)

N

natural logarithm
Logarithm to the base *e*, where e is a fundamental mathematical constant roughly equal to 2.7182818284. The natural logarithm of x is usually written $\ln x$.

natural number (counting number)
One if the numbers 1, 2, 3, 4, . . . *Positive integer* means the same thing.

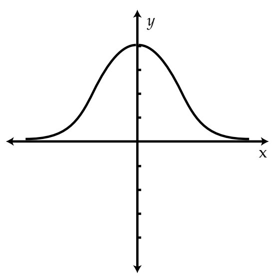


net
A flat diagram consisting of plane faces arranged so that it may be folded to form a solid.

Newton's Law of Cooling
The assertion that if a warm object is placed in a cool room, its temperature decreases at a rate proportional to the difference in temperature between the object and its surroundings.

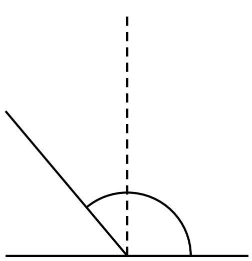
Newton's Method
An often highly efficient iterative method for approximating the roots of $f(x) = 0$. If r_n is the current estimate, then the next estimate is the x-intercept of the tangent line to $y = f(x)$ at $x = r_n$.

non-differentiable (function)
A function $f(x)$ is non-differentiable at $x = a$ if it does not have a derivative there. Example: if $f(x) = |x|$ then $f(x)$ is not differentiable at $x = 0$, basically because the curve $y=|x|$ has a sharp kink there.



normal distribution curve
The standard normal distribution curve has equation $y = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$.
The general normal is obtained by shifting the standard normal to the left or right, and/or rescaling. These curves are sometimes called *bell-shaped curves*. They figure importantly in probability, statistics, and signal processing.

O



obtuse angle
An angle whose measure is between 90° and 180°.

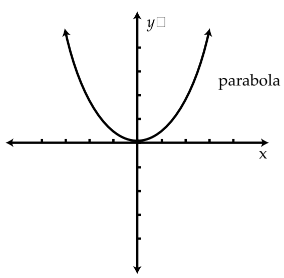
one-sided limit
Sometimes $f(x)$ exhibits different behaviour depending on whether x approaches a from the right (through values of x greater than a) or from the left. For example, let $f(x) = 1/(1+2^{1/x})$. As x approaches 0 from the right, $f(x)$ approaches 0 (notation: $\lim_{x\to 0^+} f(x) = 0$,) while $f(x)$ approaches 1 as x approaches 0 from the left.

optimization problem
A problem, often of an applied nature, in which we need to find the largest or smallest possible value of a quantity; also called a *max/min problem*.

ordered pair
A sequence of length 2. Ordered pairs (x, y) of real numbers are used to indicate the x and y coordinates of a point in the plane.

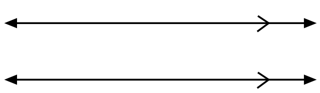
ordinal number
A number designating the place occupied by an item in an ordered sequence (e.g., first, second, and third).

origin
The point in a coordinate system at the intersection of the axes.

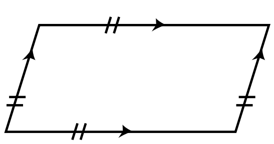


P

parabola
The intersection of a conical surface and a plane parallel to a line on the surface.



parallel lines
Two lines in the *plane* are *parallel* if they do not meet. In three-dimensional space, two lines are parallel if they do not meet *and* there is a plane that contains them both. Alternately, in the plane or in space, two lines are parallel if they stay a constant distance apart.



parallelogram
A quadrilateral such that pairs of opposite sides are parallel.

percentage
In a problem such as "Find 15% of 400." the number 400 is sometimes called the *base*, 15% or 0.15 is called the *rate*, and the answer 60 is sometimes called the *percentage*.

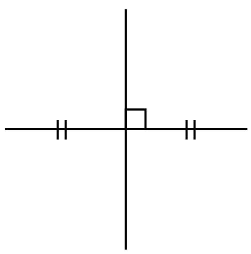
percent error
The relative error expressed in parts per hundred. Let A be an estimate of a quantity whose true value is T. Then A - T is the *error*, and $(A - T)/T$ is the *relative error*.

percentile
The k-th *percentile* of a sequence of numerical data is the number x such that k percent of the data points are less than or equal to x. (Often x is not precisely determined, particularly if the data set is not large.)

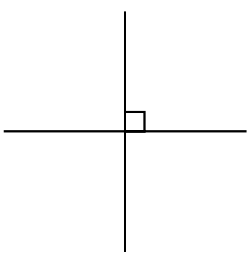
perimeter
The length of the boundary of a closed figure.

period
The interval taken to make one complete oscillation or cycle.

permutation
An ordered arrangement of objects. The number of ways of producing a permutation of r (distinct) objects from a collection of n objects is ${}_nP_r$, where ${}_nP_r = n(n - 1)(n - 2) \ldots (n - r+1)$.

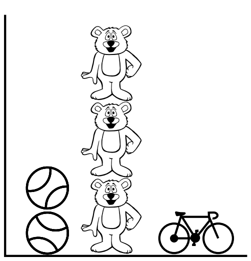


perpendicular bisector
A line that intersects a line segment at a right angle and divides the line segment into two equal parts.

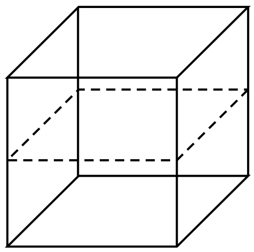


perpendicular line
Two lines that intersect at a right angle.

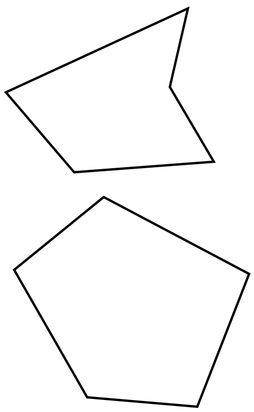
phase shift
A horizontal translation of a periodic function. For example, the function cos $2(x-\pi/3)$ is cos $2x$ with a phase shift of $\pi/3$.



pictograph
A graph that uses pictures or symbols to represent similar data.



plane of symmetry
A 2-D flat surface that cuts through a 3-D object, forming two parts which are mirror images.



polygon
A closed curve formed by line segments that do not intersect other than at the vertices.

polygonal region
A part of a plane that has a polygon as boundary.

polyhedron
A solid bounded by plane polygonal regions.

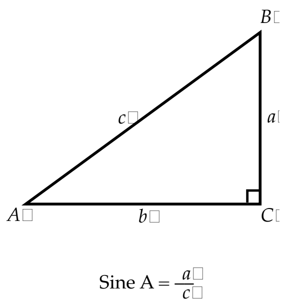
polynomial
A mathematical expression that is a sum of monomials. Examples:

$$4x^3 - 3x - \frac{1}{2};\ \ \pi x^2 + 2\pi xy;\ \ xyz.$$

population
The items, actual or theoretical, from which a sample is drawn.

power
A power of q is any term of the form q^k. Often but not always, k is taken to be a positive integer.

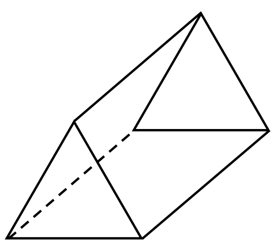
precision
A measure of the estimated degree of repeatability of a measurement, often described by a phrase such as "correct to two decimal places."



primary trigonometric functions
sine A= $a/c = opp/hyp$
cosine A= $b/c = adj/hyp$
tangent A= $a/b = opp/adj$
Functions of angles defined, for an acute angle, as ratios of sides in a right triangle.

prime
A positive integer that is divisible by exactly two positive integers, namely 1 and itself. The first few primes are 2, 3, 5, 7, 11, 13.

prime factorization (of a positive integer)
The given integer expressed as a product of primes. For example, $2 \times 5 \times 3 \times 2$ is a prime factorization of 60. Usually the primes are listed in increasing order. The standard prime factorization of 60 is $2^2 \times 3 \times 5$.



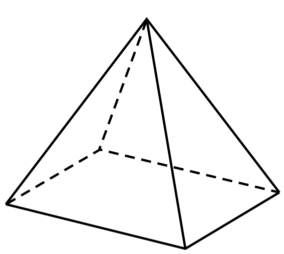
prism
A solid with two parallel and congruent bases in the shape of polygons; the other faces are parallelograms.

probability (of an event)
A number between 0 and 1 that measures the likelihood that the event will occur. The probability of the event A is often denoted by $\Pr(A)$.

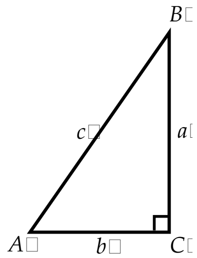
product
The *product* of two or more objects (numbers, functions, etc.) is the result of multiplying these objects together.

product rule
The rule for finding the derivative of a product of two functions.

$$\text{If } p(x) = f(x)g(x) \quad \text{then } p'(x) = f(x)g'(x) + g(x)f'(x).$$

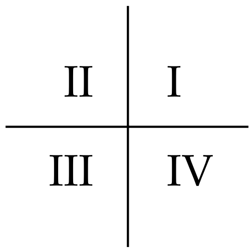


pyramid
A polyhedron one of whose faces is an arbitrary polygon (called the *base*) and whose remaining faces are triangles with a common vertex called the *apex*).



Pythagorean theorem
In a right-angled triangle, the sum of the squares of the lengths of the sides containing the right angle is equal to the square of the hypotenuse ($a^2 + b^2 = c^2$).

Q

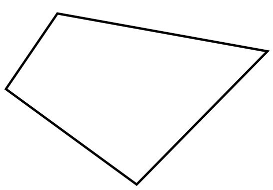


quadrant
One of the four regions that the plane is divided into by two perpendicular lines. When these lines are the usual coordinate axes, the quadrants are called the first quadrant, the second quadrant, and so on as in the diagram.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic formula
A formula used to determine the roots of a quadratic equation.

quadratic function
A function of the form $f(x) = \mathrm{a}x^2 + bx + c$, where $a \neq 0$. The graph of such a function is a parabola.



quadrilateral
A polygon with four sides.

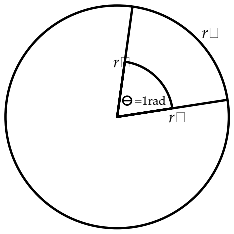
quartile
The 25th percentile is the first quartile, the 50th percentile is the second quartile (or median), and the 75th percentile is the third quartile. Please see *percentile*.

quotient
The result of dividing one object (number, function) by another.

quotient rule
The rule for differentiating the quotient of two functions.

$$\text{If } q(x) = \frac{f(x)}{g(x)} \quad \text{then} \quad q'(x) = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{(g(x))^2}.$$

R



radian
Equal to the central angle subtended by an arc of unit length at the centre of a circle of unit radius.

radical
The square root, or cube root, and so on of a quantity. For example, the cube root of the quantity Q is the quantity R such that R^3 (the cube of R) is equal to Q. The square root of Q is written $\sqrt{Q}$ ($\sqrt{\ }$ is the *radical sign*.) The cube root of Q is written $\sqrt[3]{Q}$.

radius
A line segment that joins the center of a circle or sphere to a point on the boundary. All radii of a circle or sphere have the same length. That common length is called *the* radius.

range
A measure of variability of a sequence of data, defined to be the difference between the extremes in the sequence. For example, if the data are 27, 22, 27, 20, 35, and 34, then the range is 15.

range (of a function)
The set of values taken on by a function. Please see *function*.

rank ordering
Ordering (of a sample) according to the value of some statistical characteristic.

rate
A comparison of two measurements with different units. For example, the speed of an object measured in kilometres per hour.

rate of change (of a function at a point)
How fast the function is changing. If $f(x)$ is the function, its rate of change with respect to x at $x = a$ is the derivative of $f(x)$ at $x = a$.

ratio
Another word for quotient. Also, an indication of the relative size of two quantities. We say that P and Q are in the ratio $a{:}b$ if the "size'' of A divided by the size of B is a/b.

rational expression
The quotient of two polynomials.

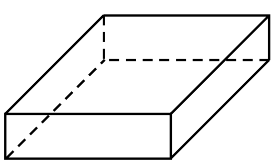
rational number
A number that can be expressed as a/b, where a and b are integers.

rationalize the denominator
Transform a quotient P/Q where the denominator Q involves radicals into an equivalent expression with the denominator free of radicals. For example

$$\frac{4}{4-\sqrt{7}} = \frac{(4)(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})} = \frac{4(4+\sqrt{7})}{9}.$$

real number
An indicator of location on a line with respect to an origin; a quantity represented by an arbitrary decimal expansion.

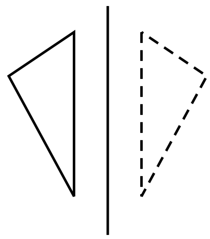
reciprocal
The number or expression produced by dividing 1 by a given number or expression.



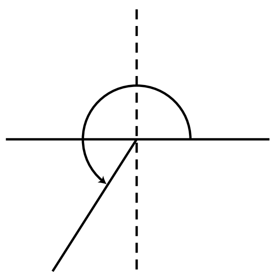
rectangular prism
A prism whose bases are congruent rectangles.

recursive definition (of a sequence)
A way of defining a sequence by possibly specifying some terms directly, and giving an algorithm by which any term can be obtained from its predecessors. For example, the Fibonacci sequence is defined recursively by the rules $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

reference angle
The acute angle between the ray line and the x-axis. For example, the reference angles of a 165˚and of a 195˚ angle are each 15˚ angles.



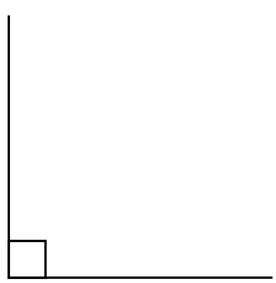
reflection (in a line)
The transformation that takes any 2-dimensional object to the object that is symmetrical to it with respect to the line, that is, to its mirror image in the line. *Flip* means the same thing. In three-dimensions, we can define analogously *reflection in a plane*.



relative maximum, minimum
Please see *local maximum, local minimum*.

Remainder Theorem
If we divide the polynomial $P(x)$ by $x - a$, the remainder is equal to $P(a)$.

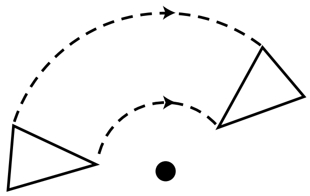
repeating decimal
A decimal expansion that has a block of digits that ultimately cycles forever. For example, 23/22 has the decimal expansion 1.0454545 . . ., with the block 45 ultimately cycling forever. A *terminating* decimal like 0.25 is usually viewed as being a repeating decimal, indeed in two ways: as 0.25000 . . . and 0.24999 . . .



resultant
The sum of two or more vectors.

right angle
An angle whose measure is 90°.

root of an equation (in one variable)
If the equation has the form $F(x) = G(x)$, a root of the equation is a number a such that $F(a) = G(a)$.



rotation (in the plane)
A transformation in which an object is turned through some angle about a point. An analogous notion can be defined in three dimensions; there the turn is about a line.

rounding
A process to follow when making an approximation to a given number by using fewer significant figures.

S

sample
A selection from a population.

sample space
The set of all possible outcomes of an experiment.

scalar
A number. Usually used in contexts where there are also vectors around, or functions. Examples of usage: "the length of a vector is a scalar;" "-3 sin x is a scalar multiple of sin x."

scatter plot
If each item in a sample yields two measurements, such as the height x and weight y of the individual chosen, the point with coordinates (x,y) is plotted. If this is repeated for all members of the sample, the resulting collection of points is a scatter plot.

secant (of x)
This is $\frac{1}{\cos x}$. Notation: sec x.

secant line
A line that passes through two points on a curve.

second derivative
The second derivative of $f(x)$ is the derivative of the derivative of $f(x)$. Two common notations: $f''(x)$ and $\frac{d^2 f}{dx^2}$.

second derivative test
Suppose that $f'(a) = 0$. The second derivative test gives a way of checking whether at $x = a$ the function $f(x)$ reaches a local minimum or a local maximum.

second-hand data
Data not collected directly by the researcher. For example, encyclopedia.

self-similar
Having the same general appearance under all magnifications.

semicircle
A half-circle; any diameter cuts a circle into two semicircles.

sequence
A finite ordered list $t_1, t_2, t_3, \ldots, t_n$ of terms (finite sequence) or a list $t_1, t_2, \ldots, t_n, \ldots$ that goes on forever (an infinite sequence.)

series
Any sum $t_1 + t_2 + \ldots + t_n$ of the first n terms of a sequence. The "sum" $t_1 + t_2 + \ldots + t_n + \ldots$ of all the terms of an infinite sequence is an *infinite series*. The concept of limit is required to define the sum of infinitely many terms.

SI measure
Abbreviation for Système International d'Unités—International System of Units—kilogram, second, ampere, kelvin, candela, mole, radian, and so on.

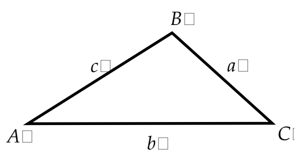
side (of a polygon)
Any of the line segments that make up the boundary of the polygon.

sigma notation
The use of the sign $\sum$ (Greek capital sigma) to denote sum. For example

$$\sum_{i=1}^{5} a_i \; a_1 + a_2 + a_3 + a_4 + a_5.$$

simple interest
Interest computed only on the original principal of a loan or bank deposit.

sine (function)
Please see *primary trigonometric functions.*

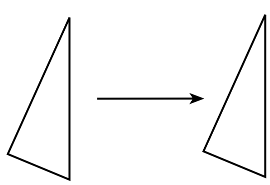


sine law (law of sines)
A formula used for solving triangles in plane trigonometry.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

skeleton
A representation of the edges of a polyhedron.

skip counting
Counting by multiples of a number. For example, 2, 4, 6, 8.

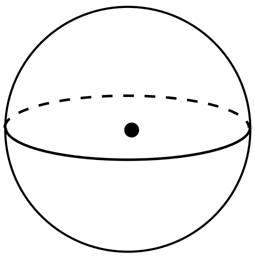


slide
A transformation of a figure by moving it up/down and/or left/right without any rotation. The word is a synonym for the more standard mathematical term *translation*.

slope
The slope of a (non-vertical) line is a measure of how fast the line is climbing. It can be defined as the change in the y-coordinate of a point on the line when the x-coordinate is increased by 1. If a curve has a (non-vertical) tangent line at the point, the slope of the curve at the point is defined to be the slope of that tangent line.

slope-intercept form (of the equation of a line)
An equation of the form $y = mx + b$. All lines in the plane except for vertical lines can be written in this form. The number m is the slope and b is the y-intercept.

solution (of a differential equation)
A function that satisfies the differential equation. For example, for any constant C, the function given by $y = (x^2 + C)^{1/3}$ is a solution of the differential equation $3y^2 \frac{dy}{dx} = 2x$.

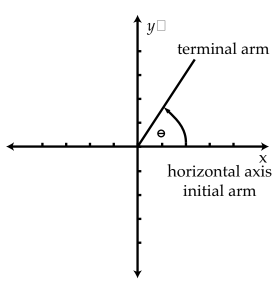


sphere
A solid whose surface is all points equidistant from a centre point.

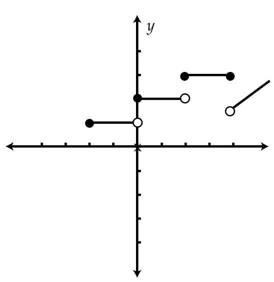
square root
The square root of x is the non-negative number that when multiplied by itself produces x. For example 5 is the square root of 25. In general the square root of x^2 is $|x|$, the absolute value of x.

standard deviation
Sample standard deviation is the square root of the sample variance. Population standard deviation is the square root of the population variance.

standard form
The usual form of an equation. For example, the standard form of the equation of a circle is $(x - a)^2 + (y - b)^2 = r^2$, because it reveals geometrically important features, the center and the radius.



standard position (angle in)
The initial arm of the angle is the positive horizontal axis (x-axis.) Counterclockwise rotation gives a positive angle.



step function
A function whose graph is flat except at a finite number of points, where it takes a sudden jump.

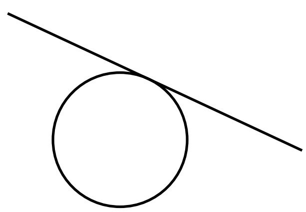
supplementary angles
Two angles whose sum is 180°.

symmetrical (has symmetry)
A geometrical figure is symmetrical if there is a rotation reflection, or combination of these that takes the figure to itself but moves some points. For example, a square has symmetry because it is taken to itself by a rotation about its center through 90˚.

system of equations
A set of equations. A *solution* of the system is an assignment of values to the variables such that all of the equations are (simultaneously) satisfied. For example, $x = 1$, $y = 2$, $z = -3$ is a solution of the system $x + y + z = 0$, $x - y - 4x = 11$.

T

tangent (function)
Please see *primary trigonometric functions.*



tangent (to a curve)
A line is tangent to a curve at the point P if under very high magnification, the line is nearly indistinguishable from the curve at points close to P. A tangent line to a circle can be thought of as a line that meets the circle at only one point.

tangent line approximation
If P is a point on a curve, then close to P the curve can be approximated by the tangent line at P. In symbols, if x is close to a, then $f(x)$ is very closely approximated by $f(a) + (x - a)f'(a)$.

tangram
A square cut into seven shapes: two large triangles, one medium triangle, two small triangles, one square, and one parallelogram.

term
Part of an algebraic expression. For example, x^3 and $5x$ are terms of the polynomial $x^3 + 3x^2 + 5x - 1$.

terminating decimal
A decimal expansion that (ultimately) ends. Example: 3.73.

tesselation
A covering of a surface (usually the entire plane) without overlap or bare spots, by copies of a given geometric figure or of a finite number of given geometric figures. The word comes from *tessala*, the Latin word for a small tile.

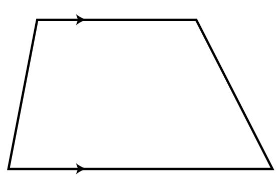
theoretical probability (of an event)
A numerical measure of the likelihood that the event will occur, based on a probability model. If, as can happen with dice or coins, an experiment has only a finite number n of possible outcomes, all equally likely, and in k of these the event occurs, then the theoretical probability of the event is k/n.

tolerance (interval)
The set of numbers that are considered acceptable as the dimension of an item. Example: a manufacturer's tolerance interval for the weight of a "400 gram'' box of cereal might be from 395 grams to 420 grams.

transformation
A change in the position of an object, and/or a change in size, and related changes. Also, a change in the form of a mathematical expression.

translation
Please see *slide.*

transversal
A line that intersects two or more lines at different points.



trapezoid
A quadrilateral that has two parallel sides. Some definitions require that the remaining two sides not be parallel.

tree diagram
A pictorial way of representing the outcomes of an experiment that involves more than one step.

trigonometry
The branch of mathematics concerned with the properties and applications of the trigonometric functions, in particular their use in "solving" triangles, in surveying, in the study of periodic phenomena, and so on.

trinomial
A polynomial that has three terms. For example:

$$ax^2 + bx + c$$

turn
Please see *rotation*.

U

unbiased
A sampling procedure for estimating a population parameter (like the proportion of BC teenagers who smoke) is unbiased if on average it should yield the correct value. At a more informal level, a polling procedure is unbiased if proper randomization procedures are used to select the sample, the wording of the questions is neutral, and so on.

unit circle
A circle of radius 1.

unit vector
A vector of length 1.

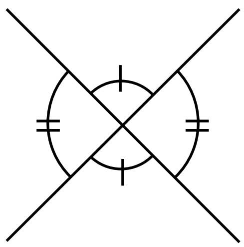
V

variable
A mathematical entity that can stand for any of the members of a given set.

variance
Sample variance is a measure of the variability of a sample, based on the sum of the squared deviations of the data values about the mean. *Population variance* is a theoretical measure of the variability of a population.

vector
A directed line segment (arrow) used to describe a quantity that has direction as well as magnitude.

vertex (pl. vertices)
In a polygon, a point of intersection of two sides. In a polyhedron, a vertex of a face.



vertically opposite angles
Opposite (and equal) angles resulting from the intersection of two lines.

W

whole number
One of the counting numbers 0, 1, 2, 3, 4, and so on; a non-negative integer.

X

***x*-intercept(s)**
The point(s) at which a curve meets the x-axis (horizontal axis).

Y

***y*-intercept(s)**
The point(s) at which a curve meets the y-axis (vertical axis).

Z

***z*-score**
If x is the numerical value of one observation in a sample, the z-score of x is $(x - \bar{x})/s$, where $\bar{x}$ is the sample mean and s is the sample standard deviation. The z-score measures how far x is from the mean.

zero (root) of $f(x)$
Any number a such that $f(a)=0$.

Appendix G

Illustrative Examples

Appendix G Introduction

This appendix contains a series of examples designed to help teachers understand the prescribed learning outcomes for Applications of Mathematics 10 to 12, Essentials of Mathematics 10 to 12, Principles of Mathematics 10 to 12, and Calculus 12.

The prescribed learning outcomes have been paired with examples that illustrate the types of activities an average student should be able to complete for each course.

- With the exception of the Problem Solving organizer, all prescribed learning outcomes for earlier courses are listed.
- In some cases an illustrative example may encompass more than one learning outcome; conversely, some outcomes have more than one example.

Please note that these illustrative examples are not intended to be used for assessment of student performance.

Examples of multi-strand and interdiciplinary problems that most students should be able to solve are indicated with an asterisk (*).

Appendix G

Illustrative Examples

Applications of Mathematics 10

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

NUMBER *(Number Concepts)*

It is expected that students will analyse the numerical data in a table for trends, patterns and interrelationships.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data)

c

Price	GST	PST	Total
$120.00	$8.40	$12.84	$141.24
$275.00	$19.25	$29.43	$323.68

a) What is the rate of GST?

b) What could be the rate of PST?

c) What could be the rule for calculating PST?

d) What is the total GST paid on the two items in the table?

e) What is the total PST paid on the two items in the table?

c National Hockey League (NHL)
Western Conference: February 1, 1996

	W	L	T	Points
Detroit	35	9	4	74
Colorado	26	14	9	61
Chicago	25	15	11	61
Toronto	22	19	9	53
St. Louis	21	20	8	50
Winnipeg	21	24	4	46
Vancouver	17	20	12	46
Los Angeles	17	22	11	45
Calgary	18	23	9	45
Edmonton	18	25	6	42
Anaheim	17	27	5	39
Dallas	14	24	10	38
San Jose	11	35	4	26

What happens to the NHL standings if wins are worth three points and ties are worth one point?

NUMBER *(Number Concepts)*

It is expected that students will analyse the numerical data in a table for trends, patterns and interrelationships.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data)

c *The following table provides data on the repayment of a $100,000 farm loan. The farmer has negotiated for one annual payment to be made each year after harvest and for the right to make an extra payment, if the harvest is good. Use the table to answer the questions.

Year	Opening Balance	Interest Rate (%)	Interest Charged	Regular Payment	Extra Payment	Closing Balance
1	$100 000.00	8	$8000.00	$14 902.95		$93 097.05
2	$ 93 097.05	8	$7447.76	$14 902.95		$85 641.87
3	$ 85 641.87	8	$6851.35	$14 902.95		$77 590.27
4	$ 77 590.27	8	$6207.22	$14 902.95		$68 894.54
5	$ 68 894.54	8	$5511.56	$14 902.95		$59 503.15
6	$ 59 503.15	8	$4760.25	$14 902.95		$49 360.46
7	$ 49 360.46	8	$3948.84	$14 902.95		$38 406.34
8	$ 38 406.34	8	$3072.51	$14 902.95		$26 575.90
9	$ 26 575.90	8	$2126.07	$14 902.95		$13 799.03
10	$ 13 799.03	8	$1103.92	$14 902.95		$ 0.00

a) What is the period of the loan?

b) What is the amount of the annual payment?

c) How much of the annual payment at the end of Year 5 went toward the opening balance? Show how to determine the answer in two different ways.

d) Create an algebraic expression to find the answer in c).

e) If the interest rate went up to 11% in Year 10, how much would be owing at the end of Year 10?

f) What extra payment at the end of Year 4 would pay the loan off at the end of Year 8?

NUMBER *(Number Operations)*

It is expected that students will use basic arithmetic operations on real numbers to solve problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • communicate a set of instructions used to solve an arithmetic problem	c Write a set of instructions that will allow another student to find: a) 1 1 2 4 3 b) 9 3 4 4 3 3 5 c) the reciprocal of a square root of a number, using a scientific calculator d) a 5% commission on a sale of $40,200
• perform arithmetic operations on irrational numbers, using appropriate decimal approximations	c Mahal indicates that $\sqrt{2}$ 1 $\sqrt{8}$ has an approximate value of 3.16. Use estimates to show whether Mahal's answer is reasonable, and use a calculator to verify the accuracy of Mahal's answer. c Find a decimal approximation of $\frac{3}{\sqrt{5}-\sqrt{2}}$ to three decimal places. c Arrange the following in order of value from least to greatest: $7, 2\sqrt{13}, 3\sqrt{6}, 4\sqrt{5}, 5\sqrt{2}$. Use decimal approximations. c Find the length of the base and the height of an equilateral triangle of area 24 cm^2.

NUMBER *(Number Operations)*

It is expected that students will describe and apply arithmetic operations on tables to solve problems, using technology as required.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- create and modify tables from both recursive and nonrecursive situations

c

Price	GST	PST	Total
$120.00	$8.40	$12.84	$141.24
$275.00	$19.25	$29.43	$323.68

a) Modify the table to allow for a PST of 6.5% of the price before taxes.

b) In another province, where PST is charged at a different rate, the total price is $138 What is the rate of PST for that province?

c *In 1993, sales of a particular video game doubled every month. The game was released in May 1993 with sales of 32 000 for May. Prepare a table to illustrate the 1993 monthly sales figures. How many video games were sold in December 1993? Identify the assumptions you made when determining the solution.
In 1994, the demand for the video game peaked. Starting in January 1994, and every month thereafter, sales were cut to one quarter of what they were in the previous month. How many video games were sold in April 1994? If April 1994 was the last month of sales, how many video games were sold over the entire twelve months?

NUMBER *(Number Operations)*

It is expected that students will describe and apply arithmetic operations on tables to solve problems, using technology as required.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- use and modify a spreadsheet template to model recursive situations

c Modify the given template for a 10-year, $85,000 farm mortgage with fixed annual payments, to allow for a change in interest rate.

Year	Opening Balance	Interest Rate (%)	Interest Charged	Regular Payment	Closing Balance
1	$ 85 000.00	8	$6800.00	$12 667.51	$ 79 132.49
2	$ 79 132.49	8	$6330.60	$12 667.51	$ 72 795.59
3	$ 72 795.59	8	$5823.65	$12 667.51	$ 65 951.73
4	$ 65 951.73	8	$5276.14	$12 667.51	$ 58 560.36
5	$ 58 560.36	8	$4684.83	$12 667.51	$ 50 577.68
6	$ 50 577.68	8	$4046.21	$12 667.51	$ 41 956.39
7	$ 41 956.39	8	$3356.51	$12 667.51	$ 32 645.39
8	$ 32 645.39	8	$2611.63	$12 667.51	$ 22 589.52
9	$ 22 589.52	8	$1807.16	$12 667.51	$11 729.17
10	$ 11 729.17	8	$ 938.33	$12 667.51	$ 0.00

a) What alternatives are open to the farmer, if the interest rate increases?

b) What alternatives are open to the farmer, if the interest rate decreases?

c Modify the template in the previous illustrative example to reflect a 25-year home mortgage with monthly payments that gives the customer the option of making an annual extra payment of $1,500 at the end of any year. Interest is charged monthly.

NUMBER *(Number Operations)*

It is expected that students will describe and apply arithmetic operations on tables to solve problems, using technology as required.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- solve problems involving combinations of tables, using:
  - addition or subtraction of two tables
  - multiplication of a table by a real number
  - spreadsheet functions and templates

c *The following is an income and expenses report for a business for the year ending December 31.

	Year 1	Year 2	Year 3	Year 4	Year 5
Sales					
Laundry	$135 000	$148 000	$150 000	$148 000	$140 000
Dry Cleaning	45 000	47 000	48 000	45 000	45 000
Repairs and Sundry	10 000	11 000	11 000	10 000	9 000
Total Sales	$190 000	$206 000	$209 000	$203 000	$194 000
Operating Expenses					
Salaries and Wages	$ 94 000	$ 99 000	$101 000	$101 000	$ 96 000
Operating Supplies	22 000	24 000	25 000	24 000	23 000
Repairs and Misc.	4 000	5 000	6 000	8 000	5 000
Accounting and Legal	2 000	2 000	2 000	2 000	2 000
Advertising	2 000	2 000	2 000	2 000	2 000
Sundry	4 000	5 000	5 000	4 500	4 000
Total Operating Expenses	$128 000	$137 000	$141 000	$141 500	$132 000
Profit Before Overhead	$ 62 000	$ 69 000	$ 68 000	$ 61 500	$ 62 000
Overhead Expenses					
Rent	$ 12 000	$ 14 000	$ 16 000	$ 18 000	$ 18 000
Utilities	6 000	7 000	8 000	9 000	10 000
Insurance	3 000	3 000	3 000	3 000	3 000
Taxes and Licenses	3 000	3 000	4 000	4 000	5 000
Depreciation – Equip.	10 000	8 000	7 000	6 000	5 000
Total Overhead Exp.	$ 34 000	$ 35 000	$ 38 000	$ 40 000	$ 41 000
Profit Before Tax	$ 28 000	$ 34 000	$ 30 000	$ 21 500	$ 21 000
Income Tax	$ 7 000	$ 8 500	$ 7 500	$ 5 375	$ 5 250
Net Profit	$ 21 000	$ 25 500	$ 22 500	$ 16 125	$ 15 750

Enter the data above onto a spreadsheet template provided to students, and modify in such a way to permit the following:

a) Calculate the dollar change in total sales, total operating expenses and total overhead expenses, between each year in the table.

b) Which is the greatest dollar change?

c) Calculate the percentage change in total sales, total operating expenses and total overhead expenses, between each year in the table.

d) Which is the greatest percentage change?

NUMBER *(Number Operations)*

It is expected that students will describe and apply arithmetic operations on tables to solve problems, using technology as required.

Prescribed Learning Outcomes	Illustrative Examples
	e) Determine the percentage change for each item for each year. f) Predict the figures for each type of income and expense for Year 6, and predict the net profit for Year 6. g) Prepare a line graph showing the annual sales, operating expenses and overhead expenses for the five year period. Use the graph to determine which item has the greatest rate of increase, and which item has the greatest rate of decrease. h) For the five year period, use a line of best fit procedure (spreadsheet software or graphing calculator) to determine equations of lines of best fit for total sales, total operating expenses and total overhead expenses. Use these equations to predict the values in Year 6. From these values, predict the net profit in Year 6. i) Calculate the net profit as a percentage of sales for each of the five years. In which year did the net profit represent the highest proportion of sales? j) Derive a formula relating total sales, total operating expenses, total overhead expenses, income tax and net profit.

NUMBER *(Number Operations)*

It is expected that students will describe and apply arithmetic operations on tables to solve problems, using technology as required.

Prescribed Learning Outcomes	Illustrative Examples

c A banker needs to provide clients with information on foreign exchange. Use the foreign exchange chart provided, or a current chart from a newspaper, to answer the following questions.

a) Calculate the cost in Canadian dollars of a refrigerator that costs $850 US.

b) Calculate the cost in US dollars of an outboard motor selling in Canada for $1,200.

c) Hans receives a cheque for 100 Swiss francs from his uncle in Berne. How many Dutch guilders would he get for this cheque? How many Canadian dollars?

d) Elsa is going on a holiday to Venezuela. She is told that she will have to pay $3.48 US for every 100 bolivars. How many bolivars will she get for $500 Canadian?

February 1, 1996

Exchange - Cross Rates

	Canada dollar	US dollar	British pound	German mark	Japanese yen
Canada dollar	—	1.3743	2.0762	0.9227	0.012850
US dollar	0.7276	—	1.5107	0.6714	0.009350
British pound	0.4816	0.6619	—	0.4444	0.006189
German mark	1.0838	1.4894	2.2501	—	0.013927
Japanese yen	77.82	106.95	161.57	71.81	—
Swiss franc	0.8821	1.2122	1.8313	0.8139	0.011335
French franc	3.7230	5.1165	7.7297	3.4352	0.047841
Dutch guilder	1.2134	1.6676	2.5194	1.1196	0.015593
Italian lira	1156.07	1588.79	2400.23	1066.71	14.855491

Exchange - Cross Rates

	Swiss franc	French franc	Dutch guilder	Italian lira
Canada dollar	1.1337	0.2686	0.8241	0.000865
US dollar	0.8249	0.1954	0.5997	0.000629
British pound	0.5460	0.1294	0.3969	0.000417
German mark	1.2287	0.2911	0.8931	0.000937
Japanese yen	88.23	20.90	64.13	0.067315
Swiss franc	—	0.2369	0.7269	0.000763
French franc	4.2208	—	3.0681	0.003220
Dutch guilder	1.3757	0.3259	—	0.001050
Italian lira	1310.64	310.52	952.72	—

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- plot linear and nonlinear data, using appropriate scales

c The mass of a beaker is recorded when the beaker contains varying volumes of ethanol. The results of the experiment are recorded in the table below.

Volume of Ethanol (mL)	Mass of Beaker and Liquid (g)
0	90
50	129
100	168
150	207
200	246

Measurements may be assumed correct to the nearest mL and to the nearest g.

Plot this data on a scatterplot, use appropriate scales, and answer the following questions.

a) Assuming that this pattern continues, determine the mass of the beaker and liquid when 250 mL of ethanol is present.

b) When a volume of 200 mL of ethanol is in the beaker, determine the mass of the ethanol alone.

c) The density of a liquid is defined as the mass of 1 mL of the liquid. Determine the density of the ethanol.

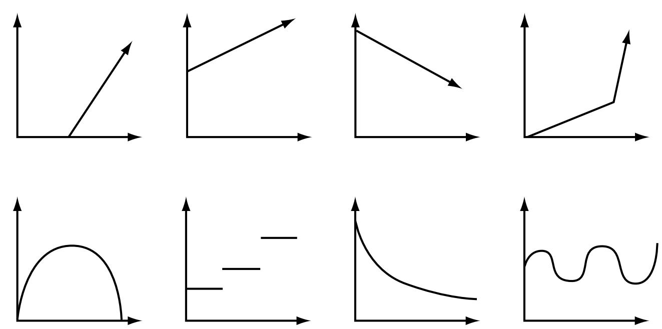
c Nannook's Pizza uses the following price structure.

Diameter (inches)	Cost ($)
8	6.50
10	10.20
12	14.65
14	19.90
16	26.00

Plot this data on a scatterplot, use appropriate scales, and describe the pattern.

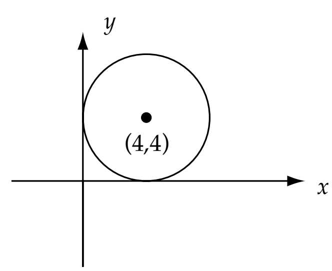
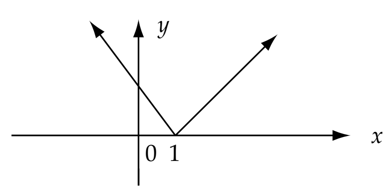
PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • represent data, using function models	c *Sketch graphs to illustrate the following situations. If sufficient information is given, represent the situation by a suitable equation. Sketch and, if possible, represent by an equation: a) the area of a circle as a function of its radius b) the cost of mailing a letter as a function of the mass of the letter c) the cost of renting a car for one day as a function of the kilometres driven d) the population of Canada as a function of the year e) the length of daylight as a function of the date. c For each of the following graphs, describe a practical situation that could be represented by the graph. In describing the situation, state the meanings of any intercepts, slopes, maxima and/or minima. 
• use a graphing tool to draw the graph of a function from its equation	c Graph the function $y = x + 1$, using a graphing tool. c Graph the function $y = x^2 + 100$, using a graphing tool. Explain the process used, so that the graph appears on the screen.

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • describe a function in terms of: - ordered pairs - a rule, in word or equation form - a graph	c Describe the parking charges at a parkade in terms of ordered pairs, a rule and a graph.
• use function notation to evaluate and represent functions	c If $f(x) = x^2 - 5x + 3$, find $f(2)$. What is the ordered pair describing the point on the graph having a y-coordinate of $f(2)$? c If $f(x) = 3x^2 - 6x + 5$, find $f(\sqrt{3})$, $f(2x)$ and $f(3t + 2)$.
• determine the domain and range of a relation from its graph	c If the coordinate axes touch the circle, what is the domain and range of the circle shown in the graph to the right?  c Determine, from its graph shown below, the domain and range of the function $y = \lvert x - 1 \rvert$ 

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples
• determine the following characteristics of the graph of a linear function, given its equation: - intercepts - slope - domain - range	c *A tanker truck drives on a weigh scale and then is filled with crude oil. The mass M, measured in kilograms, of the truck and the volume V, measured in barrels, of crude oil are related by the formula: $M = 14\,000 + 180\,V;\;\; V \# 500$ a) Draw the graph with V on the horizontal axis and M on the vertical axis. b) The tank has a maximum capacity of 500 barrels. What is the mass of the truck when it contains 500 barrels of oil? c) What is the mass of the empty truck? Where is this value found on the graph? d) Find the slope, and give an interpretation for it. e) Give the domain for this problem. f) Express the range in words. c Graph each of the following equations; and indicate intercepts, slope, domain and range. a) $y = 2x;\;\; x = (0, 1, 2, 3, 4, 5, 6)$ b) $y = -\frac{1}{3}x;\;\; x =$ a real number c) $y = 3$ d) $x = 3$ e) $y = \frac{1}{3}x + 5;\;\; x =$ a real number f) $y = mx + b;\;\; x =$ a real number

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent data, using function models.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • use partial variation and arithmetic sequences as applications of linear functions	See below

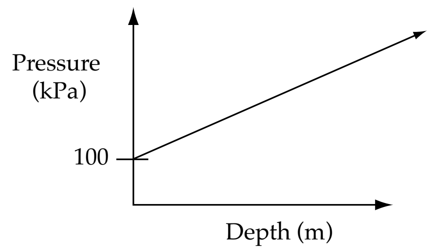
Prescribed Learning Outcomes

It is expected that students will:

- use partial variation and arithmetic sequences as applications of linear functions

Illustrative Examples

- A hydrologist studied the relationship between the pressure on an object and its depth of submersion in a liquid. The following graph was sketched.



  Draw conclusions based upon the sketch.

- Simple interest varies directly with the amount borrowed.

  a) If the interest is \$5 for \$100 borrowed, what would the interest be for \$325 borrowed?

  b) Graph the relation, and write the equation of the graph.

- *A jet ski rental operation at Lake Okanagan charges a fixed insurance premium, plus an hourly rate. The total cost for two hours is \$50 and for five hours is \$110.

  a) Graph the relation.

  b) Determine the fixed insurance premium and the hourly rate to rent the jet ski.

- With new equipment coming on line, a soft drink manufacturer has been increasing its production each day according to the following table. Assume a maximum daily output of 25 000 cans.

Day	1	2	3	4
Units	4000	4200	4400	4600

  a) Graph the relation. Hint: this is a discrete case.

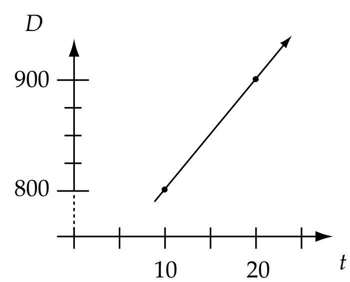
  b) On what day will they be able to produce 20 000 cans, if this trend continues?

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent data, using function models.

Prescribed Learning Outcomes	Illustrative Examples

- Given the distance–time graph shown, answer the following questions.
  - a) If $D = 850$, what is t?
  - b) If $t = 25$, what is D?
  - c) If $D = 1500$, what is t?
  - d) Write the equation of the function.
  - e) Verify the accuracy of your estimates in a), b), and c), using the equation of the function.



- Given the data in the table, predict the fuel consumption for the following engines:
  - a) 2.5 L
  - b) 5.0 L.

Engine Size (L)	Consumption (L/100 km)
2.2	6.4
3.0	7.5
3.8	8.1
4.1	8.6

- *A video game operator gives all her change in quarters. From a $20 bill, she gives 56 quarters change for a $6 purchase. She gives 8 quarters change from a $20 bill for an $18 purchase.
  - a) Graph the number of quarters given as change N on the vertical axis and the amount of the purchase P on the horizontal axis. Assume that a $20 bill was given.
  - b) What is the domain and range of the function?
  - c) How does the graph change, if a $10 bill is used?

SHAPE AND SPACE (*Measurement*)

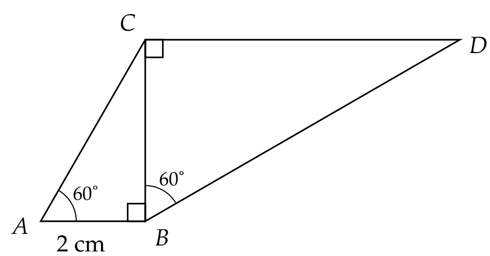
It is expected that students will demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • calculate the volume and surface area of a sphere, using formulas that are provided	c Calculate the volume and surface area of a beach ball of radius 15 cm. c *A hot air balloon has a spherical shape and a diameter of 4 m. If 30 additional cubic metres of air are pumped into the balloon, what will be the new values for the diameter, volume and surface area?
• determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects	c The area of a region in a plane is 10 cm^2. How many ways can each of the dimensions of this region be multiplied to increase the area by 20 cm^2? c A model train is built to a scale of 1:50. If the length of the model engine is 20 cm and the area of sheet metal used to cover the outside surface of the model is 180 cm^2, what is the actual length of the engine and the actual area of the sheeting used to cover the engine? If the volume displaced by the model engine is 126 cm^3, what is the volume displaced by the real engine, in m^3? c *It is improbable that a giant human, 6 m in height (three or four times normal human height), could exist. Which biological systems are most likely to break down? Explain your answer.

SHAPE AND SPACE *(Measurement)*

It is expected that students will solve problems involving triangles, including those found in 2-D and 3-D applications.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems involving two right triangles	c *From the top of a 100 m fire tower, a fire ranger observes two fires, one at an angle of depression of 5° and the other at an angle of depression of 2°. Assuming that the fires and the tower are in a straight line, determine the distance between the fires for the following: a) when the fires are on the same side of the tower b) when the fires are on opposite sides of the tower. c The triangles *ABC* and *BCD* have right angles at *B* and *C* respectively. Calculate the length of side *CD*, and state the ratio of length *BD* to length *AC*.



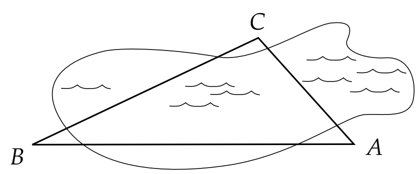
Prescribed Learning Outcomes	Illustrative Examples
• extend the concepts of sine and cosine for angles through to 180°	c Find sin 130°. c Use a calculator to find multiple solutions for angle *A*, if sin *A* = sin 130°. Use trial and error to find as many solutions as possible. Summarize the pattern found in the solutions. c Find the value(s) for /*A* (0° # *A* # 180°) when sin *A* = $^1/_2$. c Find the value(s) for *A* (0° # *A* # 180°) when cos *A* = $^1/_2$. c Find the value(s) for /*A* (0° # *A* # 180°) when cos *A* = –$^1/_2$.

SHAPE AND SPACE *(Measurement)*

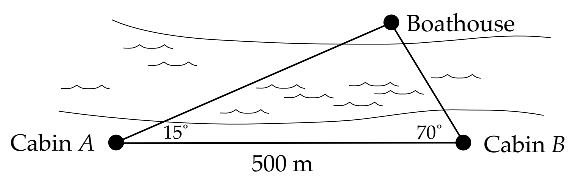
It is expected that students will solve problems involving triangles, including those found in 2-D and 3-D applications.

Prescribed Learning Outcomes	Illustrative Examples
• apply the sine and cosine laws, excluding the ambiguous case, to solve problems	

- An electric transmission line is planned to go directly over a pond. The power line will be supported by posts at points *A* and *B*. A surveyor measures the distance from *B* to *C* as 580 m, the distance from *A* to *C* as 337 m and $\angle BCA$ as 105.34°. What is the distance from post *A* to post *B*?



- Two cabins are located 500 m apart on the same side of a river. Across the river from the two cabins is a boathouse. This situation is illustrated in the diagram below. Use the measurements to find the width of the river.



- A farmer has a field in the shape of a triangle. From one corner, it is 530 m to the second corner and 750 m to the third corner. The angle between the lines of sight to the second and to the third corners is 53°. Find the perimeter and area of the field.

- *A sailboat leaves the dock at Gibson's Landing on a bearing of S57°W. After sailing for 8 km, the ship tacks and travels S31°E for 5 km.

  a) How far is the sailboat from Gibson's Landing?

  b) What direction would it have to sail to return to the dock at Gibson's Landing?

  Marshall P. Bye et al. Holt, Rinehart and Winston of Canada, Limited. By permission of CanCopy Agreement, 1998.

- *Canada's highest waterfall is Della Falls on Vancouver Island. An observer standing at the same level as the base of the falls views the top of the falls at an angle of elevation of 58°. When the observer moves 31 m closer to the base of the falls, the angle of elevation increases to 61°. Find the height of Della Falls.

SHAPE AND SPACE (*Measurement*)

It is expected that students will use measuring devices to make estimates and to perform calculations in solving problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes	c Find a rule that relates hectares to acres. Is there a rule of thumb that can be used for estimates? Estimate the area of a plot of land shown in a plan, using both units of measurement. c Use a micrometer to measure the thickness of 10 sheets of paper. Use the results of this measurement to determine the thickness of one sheet of paper. c Use a micrometer to measure the thickness of a human hair. c Calculate the area of a flat rectangular surface measuring 21 m by 14 m. Give the answer in cm^2, m^2 and dm^2. c Estimate the volume of a water bed bladder having a depth of 300 mm, a width of 1.8 m and a length of 210 cm. c *Given a cylindrical pipe of known length, choose appropriate measuring devices to find the internal and external diameters of the pipe. Find the volume of metal in the pipe. Explain your measurement and calculation procedures. c Measure the internal dimensions of a rectangular container, and calculate its volume in cm^3. Find its volume, in litres or in millilitres, using a calibrated cylinder. c Use a vernier calliper to measure the inside diameter of a piece of PVC pipe. c Measure the angle between two faces of a pyramid to the nearest degree. c Measure the angle of a bevel to the nearest tenth of a degree, using a vernier bevel protractor.

SHAPE AND SPACE *(Measurement)*

It is expected that students will use measuring devices to make estimates and to perform calculations in solving problems.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- analyse the limitations of measuring instruments and measurement strategies

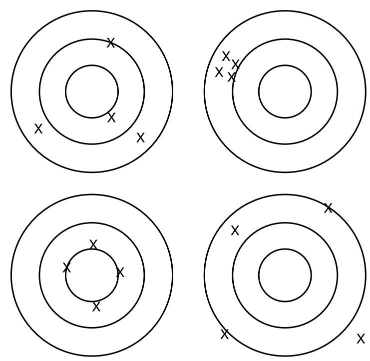
c Which ruler is most precise?

a) a ruler divided into tenths of an inch

b) a ruler divided into eighths of an inch

c) a ruler divided into millimetres

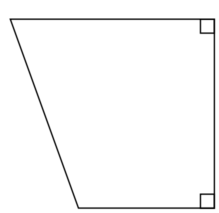
c Of the four diagrams revealing shots on a target, which best represents accuracy and precision?



- solve problems involving length, area, volume, time, mass and rates derived from these

c *A room is 16 feet long, 12 feet wide and 8 feet high. The walls and ceiling are to be painted. There are two doors in the room, each 6 feet 6 inches high and 30 inches wide. There are two windows, each 2 feet by 4 feet. Information on the paint can states that you should allow 3.79 L for every 38 m^2 of smooth surface. Two coats of paint are needed. How many cans of paint are needed, if each can contains 3.79 L? If a painter is able to paint 3 m^2 in 10 minutes, how long will it take to paint the room?

c A person buys a property that is irregularly shaped. See scale drawing below.



1 cm : 10 m

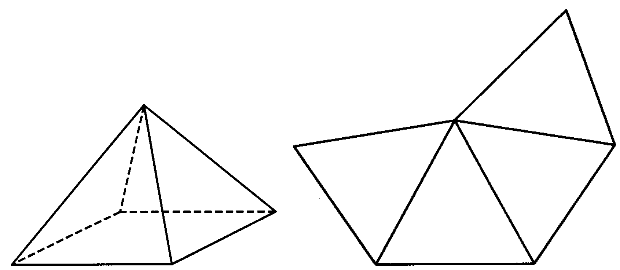
What is the total area, in m^2, of the lot?

c *A car has a highway fuel consumption of 34 miles per Imperial gallon. What is this in litres per 100 kilometres? Explain the conversion strategy used.

SHAPE AND SPACE *(Measurement)*

It is expected that students will use measuring devices to make estimates and to perform calculations in solving problems.

Prescribed Learning Outcomes	Illustrative Examples
	c A sheet metal worker must fabricate a pyramidal cap for a square column. The base of the cap is 1.5 m by 1.5 m and the height is 1.2 m. Determine the area of material required.



c *A building contractor is to provide wheel chair access to a new building. A space of 10 m by 10 m is available, on the west side of the entrance stairs, for a ramp. Municipal building codes specify that wheel chair ramps must have a minimum width of 1.5 m and a maximum slope of 10°. The vertical rise needed is 2 m. Construction costs for ramps of this kind average $300 per linear metre.

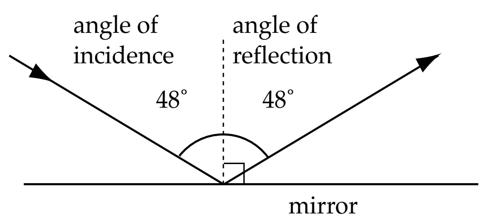
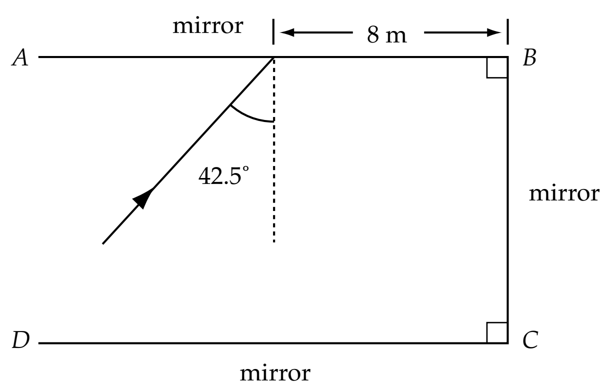
a) Design a ramp to meet the above specifications.

b) Make a plan or drawing of the proposed ramp showing the measurements, including the slopes, of the various parts.

c) Give an estimate of the cost of construction.

SHAPE AND SPACE (*Measurement*)

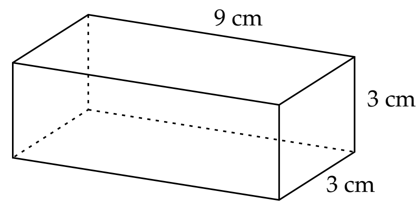
It is expected that students will use measuring devices to make estimates and to perform calculations in solving problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • interpret drawings, and use the information to solve problems	c *The law of reflection states that when a ray of light is reflected at a surface, the angle of reflection is equal to the angle of incidence. Therefore, if light hits a mirror at an angle of incidence of 48°, the angle of reflection will also be 48°.  The following diagram of the interior of a hall of mirrors shows a ray of light hitting mirror *AB* at a point 8 m from *B* and at an angle of incidence of 42.5°. Using the law of reflection, and either trigonometric relationships or scale drawings, find the angle of reflection from mirror *CD* and the distance from *C* at which the ray will hit mirror *CD,* if mirror *BC* is 12 m long. 

SHAPE AND SPACE *(Measurement)*

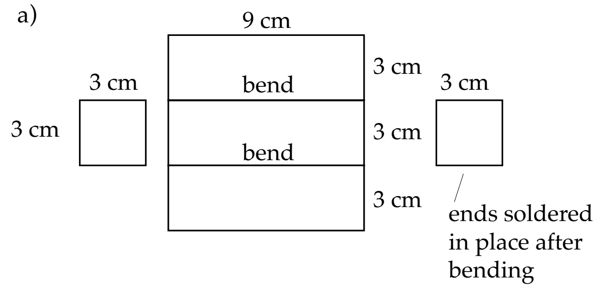
It is expected that students will use measuring devices to make estimates and to perform calculations in solving problems.

Prescribed Learning Outcomes	Illustrative Examples
	c A silver box, with dimensions as outlined below, is made from sheet metal.

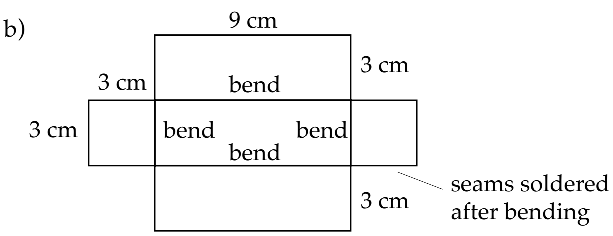


Two methods of construction are shown.

a)



b)



The material cost is \$2.50/cm², and soldering costs \$0.70/cm. For each method of construction, calculate the cost for the box.

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will solve coordinate geometry problems involving lines and line segments.

Prescribed Learning Outcomes	Illustrative Examples

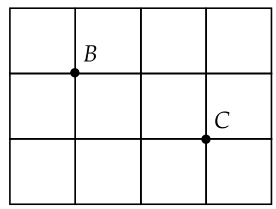
Prescribed Learning Outcomes

It is expected that students will:

- Solve problems involving distances between points in the coordinate plane.

Illustrative Examples

- Bob and Christine want to meet; see map below. Each block has dimensions of 120 m by 120 m. Assuming the roads are of negligible width, how far does Bob have to travel to get to Christine? Find two separate answers, one for a path along the roads and one for a direct path.



- Plot the points (-4, -2) and (1, 5) on the coordinate plane. Describe two different ways to calculate the distance between the two points.
- Generate a method of determining the distance between any two points in the coordinate plane without having to plot the points. Justify your method.
- Program a calculator or computer to accept, as input, the coordinates of two points and to give, as output, the distance between the two points. Document the program so that someone else can use it without assistance.

Prescribed Learning Outcomes

- solve problems involving midpoints of line segments

Illustrative Examples

- Explain to a partner the meaning of the midpoint of the line segment joining two points without using the word midpoint.
- *On a map with numerical coordinates in kilometres, the village of Sundown is at (6.3, 2.9), while the town of Sunup is at (4.7,   13.2). It was decided to construct a water main on the direct line joining Sunup with Sundown. Each community was responsible for the cost of construction from the community to the midpoint. Find the coordinates of the midpoint and Sundown's costs, if Sundown spent $63,475 per kilometre for construction. Determine alternative methods that could be used to solve the problem.

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will solve coordinate geometry problems involving lines and line segments.

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving rise, run and slope of line segments	c If the slope of a line is 6 and the line passes through the points (2, 5) and (1, k), what is the value of k? c If two points on a line are (4, 3) and (6, 4), find one other point on the line. Use a graphing utility to demonstrate the reasonableness of your answer.
• determine the equation of a line, given information that uniquely determines the line	c Use a graphing device to examine changes in the graph of $y = mx + b$ as the values of m and b are changed. Use the results to explain why the equation $y = mx + b$ is called the slope and y-intercept form of a linear equation. c Write a clear explanation of the nature of the following lines: $x = a$, $y = b$, $x = y$. c Manipulate the standard form of a straight line ($Ax + By + C = 0$) into the slope and y-intercept form of the same line. Determine rules that connect A, B and C to the slope (m) and to the intercepts. c Find the equation of a line passing through the points (-1, 3) and (4, 2). c Given the graph of an oblique line, determine an equation for the line. c *A spring with no masses attached is 25.2 cm long. For each 1-g mass attached to the spring, the spring's length increases by 4 mm. Graph this scenario, label the axes, and find an equation for the graph.
• solve problems using slopes of: - parallel lines - perpendicular lines	c *Graphically examine the slopes of various lines, all of which are perpendicular to the line y 5 $\frac{2}{3}x$1 2. Describe the slopes, and make a rule for finding the slope of a perpendicular to a given line. c Two perpendicular lines intersect on the x-axis. The equation of one of the lines is $y = 2x - 6$. Find the equation of the second line.

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will implement and analyse sampling procedures, and draw appropriate inferences from the data collected.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population	c A toothpaste company advertises that three out of four dentists prefer their product. Analyse this statement for its completeness and its accuracy in terms of population, sample, possible sampling technique, validity, and bias. c A school cafeteria wants to introduce a new dessert. Describe how a survey could be conducted to decide which of three choices should be the new dessert. c *To predict a winner in a federal election, a magazine compiled a list of about 200 000 names from sources such as telephone books, lists of automobile owners, club membership lists and its own subscription lists. The magazine mailed a questionnaire to everybody on the list, and 4000 returned it. The 4000 responses became the sample. Discuss the potential sources of bias.
• defend or oppose inferences and generalizations about populations, based on data from samples	c *To determine a preference for spending $50 in either a clothing store, an electronics shop or a restaurant, customers were surveyed one Saturday morning at the mall. Fifty-nine percent preferred spending in a clothing store, 32% in an electronics shop and 9% in a restaurant. What generalizations can be made from these results? Does the sample adequately represent the population to be surveyed? Design a more reliable sampling method to obtain this information, and include details of the questionnaires used and the method of selecting the sample. c Search through various forms of media to find examples of generalizations that have been made about populations, based on data from samples. Do you agree or disagree with the generalizations? Explain why.

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will apply line-fitting and correlation techniques to analyse experimental results.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- determine the equation of a line of best fit, using:
  - estimate of slope and one point
  - least squares method with technology

c Below are the heights, in metres, and masses, in kilograms, of 13 students.

Student	Height (m)	Mass(kg)
a	1.50	51
b	1.51	56
c	1.52	54
d	1.54	58
e	1.56	56
f	1.58	62
g	1.60	91
h	1.61	65
i	1.64	66
j	1.65	70
k	1.66	71
l	1.70	74
m	1.72	74

Plot the data and determine lines of best fit, using:
a) estimation
b) least squares method and a computing tool

Calculate the slope and intercept of each of the lines, and compare the results.

c *Use the following data to answer questions a – d.

Oil changes per year	3	5	2	3	1	4	6	4	3	2	0	10	7
Cost of repairs	$300	300	500	400	700	400	100	250	450	650	600	0	150

a) Use graphing technology to prepare a scatterplot. Draw a line of best fit.

b) From the line of best fit, make predictions of the repair cost with eight oil changes and with 14 oil changes.

c) How reliable are these predictions?

d) Beyond what point are the predictions unreliable?

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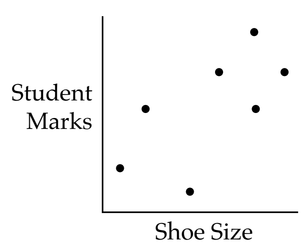
STATISTICS AND PROBABILITY *(Data Analysis)*

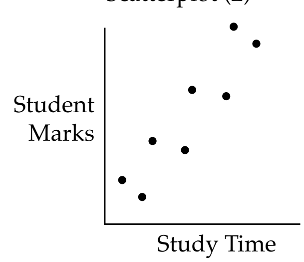
It is expected that students will apply line-fitting and correlation techniques to analyse experimental results.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • use technological devices to determine the correlation coefficient *r*	c Measure the height of each person in a class and the distance, from fingertip to fingertip, of their outstretched arms. a) Record this data as a set of ordered pairs, with height as the first element and fingertip to fingertip distance as the second. b) Plot the data on a coordinate system. c) By examining the data, predict a value for the correlation coefficient *r*. d) Using a calculating tool, determine the correlation coefficient *r* for this data.
• interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots	c *What do the following scatterplots and corresponding *r*-values represent?

Scatterplot (1)



Scatterplot (2)



Scatterplot (1) is the plot of student marks on their last test against their shoe size. The value for *r* was calculated to be 0.2. Scatterplot (2) is the plot of student marks on their last test against the time spent studying. The value for *r* was calculated to be 0.8. Describe the relationship between the values of *r* and the shape of the scatterplots.

Appendix G

Illustrative Examples

Applications of Mathematics 11

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

NUMBER *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples
• solve consumer problems, including: - wages earned in various situations - property taxation - exchange rates - unit prices	► Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus quota, and graduated commission. ► Jane has a choice of two restaurants at which to work. Mario's pays \$8/h, and tips average \$24 daily. Teppan's pays \$5.50/h, and tips average \$35 daily. If Jane works 30 hours weekly, spread over four days, how much would she earn at each restaurant? ► Identify and calculate various payroll deductions, including income tax, CPP, UI, medical benefits, union and professional dues and life insurance premiums. ► Estimate, calculate and compare gross and net pay for various wage or salary earners in your community. ► The Ningart property has a market value of \$105,000. The assessed values in the area are 60% of market values. The tax rate is 32.3 mills of assessed value. What is the Ningarts' monthly tax payment? ► The exchange rate on a given day in the United States is 28% and in Canada 38.8%. Explain why this is possible. ► A Canadian traveller goes from Switzerland to Germany. She knows that one Swiss franc is equivalent to \$1.26 Canadian (including exchange cost) and that one German mark is \$0.97 Canadian (including exchange cost). How many German marks does she get for 100 Swiss francs? ► Which provides better value for tomato soup, \$0.69 for 284 mL or \$1.79 for 907 mL?

NUMBER *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- reconcile financial statements including:
  - cheque books with bank statements
  - cash register tallies with daily receipts

Illustrative Examples

► The following petty cash transactions occurred during the first week of March.

March 4	$100 cheque was received to establish the fund.
March 5	Bought $12.50 worth of postage stamps.
March 5	Spent $10 to have something delivered by taxi.
March 6	Spent $6.50 for lunch.
March 7	Paid a courier service $25 for deliveries.
March 7	Bought flowers for opening day, $28.
March 8	Replenished the fund by $25.
March 9	Postage stamps purchased for $21.50.

Determine if a final balance of $20 is correct. If not, provide an explanation for the difference, and indicate possible ways to correct the problem.

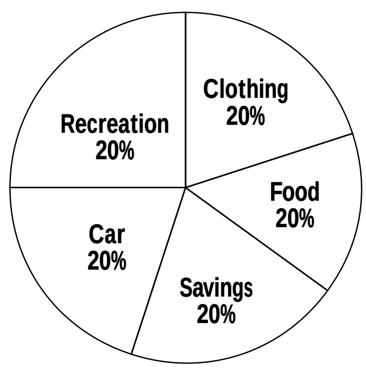
► Complete the table below to determine the cost of credit for using a department store charge account for the period shown. Monthly credit charges are 1.4% of the balance due.

Month	Previous Balance -	Payment Made +	Purchases Charged ⇒	Balance Due +	Credit Charges ⇒	New Balance
February	$314.65	$100.00	$193.75		$5.72	$414.12
March		$150.00	$ 59.60			
April		$140.00	$421.83			$618.62
May	$618.62	$200.00	$ 39.65			
June		$250.00	$ 58.11			
July		$150.00	$ 77.21			
August	$206.68	$120.00	$163.09		$3.50	$253.27

NUMBER *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples
• solve budget problems, using graphs and tables to communicate solutions	► *Research and calculate the cost of operating a vehicle for a year. Decide how to classify each cost, how to collect the data and how to display the results. ► As a project, prepare a monthly budget for one of the following: a) the family b) an assumed persona; e.g., a celebrity c) a school d) a vacation e) a fishing/hunting/shopping trip f) a municipality ► The diagram shows Julie's monthly budget of $1200. She wants to move to her own apartment that costs $450 per month. Construct a new budget that will include her rent. Explain the choices and changes that Julie could make. Julie Barnes' Monthly Budget Total = $1200



NUMBER *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples
• solve investment and credit problems involving simple and compound interest	► Determine the effective annual interest rate on a loan of \$1,000 at 10% per year, compounded quarterly. ► Calculate the compound amount, after one year, of a deposit of \$1000. Assume the current nominal annual interest when the interest is compounded: a) annually b) monthly c) daily ► *A bank offers an interest rate of 8% per year, compounded annually. A second bank offers an interest rate of 8% per year, compounded quarterly. If \$2000 were deposited, for ten years, in each bank, how much more income would be gained in the second bank than in the first? ► Calculate the interest paid on various forms of credit, including: a) credit cards b) loans c) mortgages ► A loan of \$5,000 carries an interest rate of 9% per year, compounded monthly. Adele makes a payment of \$350 every month. Use a spreadsheet to determine how much she still owes after making 12 payments.

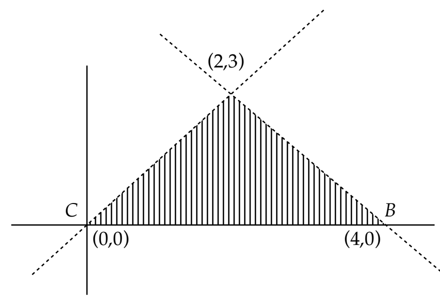
PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will represent and analyse situations that involve expressions, equations and inequalities.

Prescribed Learning Outcomes	Illustrative Examples
• graph linear inequalities, in two variables	► Solve, algebraically and graphically, for x: $2y + 5 > 3x - 1$. ► A target is described in terms of coordinates (x, y), where x and y are measured in metres. All of the following are true: • $x \leq 6$ • $y \geq 7$ • (x, y) is in the first quadrant • $x + y \leq 10$ What is the shape and the area of the target?
• solve systems of linear equations, in two variables: - algebraically (elimination and substitution) - graphically	► Solve this system of equations, using the elimination method: $x + 2y = 10$ $2x + 3y = 14$ ► Solve this system of equations, using the substitution method: $3x + 4y = 15$ $x - y = 5$ ► *A principal of $42,000 is invested partly at 7% and partly at 9.5%. If the interest is $3,700, how much is invested at each interest rate? ► Plot the graphs of $2x + 3y = 11$ and $2x - 3y = 17$. What is their point of intersection?
• solve nonlinear equations, using a graphing tool	► Using a graphing tool, solve $x^2 + 6x - 11 = 0$. ► *Solve $x^3 + x = 30$ graphically, using two different methods. Which method gives solutions that are freer from rounding errors and other inaccuracies? ► Where does the line $y = 4x + 5$ cut the curve $y = 2^x$? Use a graphing tool to find the points of intersection.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will use linear programming to solve optimization problems.

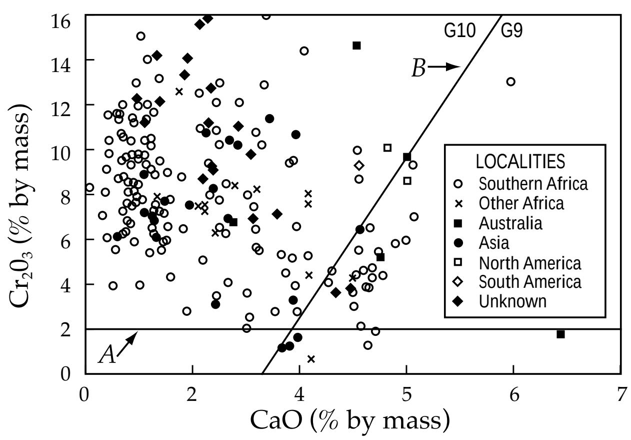
Prescribed Learning Outcomes	Illustrative Examples
• solve systems of linear inequalities in two variables using graphing technology	► Graph the solution to the following system of inequalities: $3x - y > 4$ $2x + y \le 6$ ► Given the following diagram, provide the system of inequalities whose solution is the interior of ΔABC. 
• design and solve linear and nonlinear systems, in two variables, to model problem situations	► A farmer has chickens and turkeys. He has fewer than 100 birds. He sells chickens for $10 each and turkeys for $30 each, and he earns more than $1500. Represent the situation graphically, and shade the region containing possible solutions. ► *A desktop publisher has to design formats for rectangular data tables and uses graphing grids as a design tool. Shade the region on the grid that represents the possible dimensions of rectangles in which the length is less than twice the width, the perimeter is at most 48 cm, and the area is at least 32 cm^2.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will use linear programming to solve optimization problems.

Prescribed Learning Outcomes	Illustrative Examples
	► *Diamond prospecting is done by testing the garnets found in rocks called kimberlites for the percent content of Cr_2O_3 and CaO. The following graph shows the Cr_2O_3 to CaO ratio for diamond-bearing rocks worldwide. Diamonds occur 85% of the time with garnets classed as G10. This G10 area is bounded by the function lines *A* and *B*. a) Define the system of linear inequalities that determines the G10 area. b) Which of the following samples would indicate that further prospecting is warranted?

Garnet Sample No.	Garnet Mass (g)	Cr_2O_3 mass (g)	CaO mass (g)
1	16.1	1.71	1.35
2	8.7	0.094	0.72
3	4.2	0.35	0.051
4	12.0	1.80	0.61



Prescribed Learning Outcomes	Illustrative Examples
• apply linear programming to find optimal solutions to decision-making problems	► *An agricultural club has a 10 ha plot of land available for a market garden project. It has selected corn and potatoes to plant and has $4000 for the project. The corn will cost $300/ha to grow and will generate $375/ha gross income. The potatoes will cost $500/ha to grow and will generate $650/ha gross income. a) Construct the function that describes the revenue from the project. b) Construct the inequalities that describe the restrictions. c) Plot this system of inequalities. d) Identify the feasible solutions. e) Determine the optimal solution.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will use linear programming to solve optimization problems.

Prescribed Learning Outcomes	Illustrative Examples
	► *A manufacturing company originally has three employees. The company directive is to hire additional persons to build widgets. Widgets can only be built by teams of 2 people. Eight teams can produce 500 widgets and 10 teams can produce 600 widgets. It is assumed that a linear relation exists between the number of teams and the number of widgets produced. The plant has the capacity to produce 1000 widgets. The Department of Health limits the total number of employees in the building to 15, due to the air quality problem. Write a presentation to the board of directors explaining how to optimize production. ► Find the maximum and minimum values of the quantity C, where $C = 2x - 5y$, given the constraints: $x \geq 0$ $y \geq 0$ $x \leq 12$ $y \leq x + 8$ $x + 2y \leq 28$ $3x + y \leq 39$

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse quadratic, polynomial, and rational functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• determine the following characteristics of the graph of a quadratic function: - vertex - domain and range - axis of symmetry - intercepts	► Given the graph of any quadratic function, determine the following: a) vertex b) domain c) range d) axis of symmetry e) intercepts ► Use technology to graph $f(x) = x^2 - 6x + 4$ and to determine the vertex, domain, range, axis of symmetry and intercepts. ► *One model concerning the rate of population growth of Earth has the annual rate of increase varying jointly as the population and the unused carrying capacity of Earth. The equation of the model is: $y = 0.001x(21 - x)$, where y = the rate of increase in population (in billions per year), and x = the present population (in billions). a) Plot this model of growth. b) The present population of Earth is 5.8 billion. What is the annual increase in population at present? c) What is the population when the rate of increase in population is at its greatest? d) What is the population when the rate of increase is zero? e) What is the projected maximum population that Earth can accommodate, according to this model?

SHAPE AND SPACE *(Measurement)*

It is expected that students will demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Prescribed Learning Outcomes

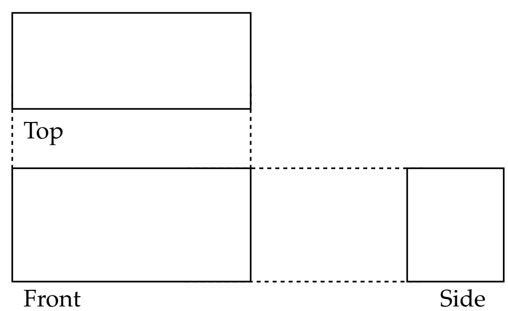
- enlarge or reduce a dimensioned object, according to a specified scale

Illustrative Examples

► A classroom has dimensions of nine metres by eight metres. Produce a scale drawing of the classroom to a scale of 1:50.

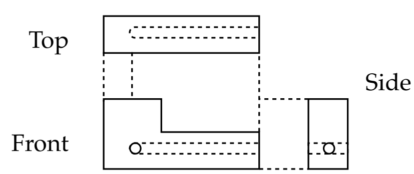
► Using surveyor's chains, tapes or other linear measuring devices, measure a chosen plot of land, and calculate its area. Make a scale drawing, using the same measurement system for the drawing as was used with the measurement instruments.

► From the scale drawing below, construct an actual sized model of the box.



Scale = 1:3

► To better visualize an object, architects often build clay models. Use molding clay or building blocks to build a model of the object that is shown in the plan below.
Scale = 2:3



SHAPE AND SPACE *(Measurement)*

It is expected that students will use measuring devices to make estimates and to perform calculations in solving problems.

Prescribed Learning Outcomes	Illustrative Examples

- calculate maximum and minimum values, using tolerances, for lengths, areas and volumes

► The diagrams represent the top and side views of a drawer handle. If the tolerance specifications are as shown below, determine the maximum and minimum dimensions for the distance between the two centres.

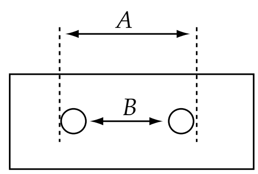


Figure 1: Top View

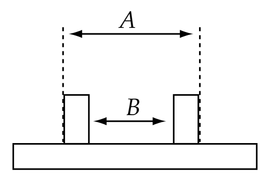


Figure 2: Side View

$A = 10.50 \pm 0.02$ cm
$B = 8.20 \pm 0.04$ cm

► To carry a high electric current to an LRT car, a wire must have a cross-sectional area of 45 ± 2 mm^2. What are the maximum and minimum diameters allowed for this wire?

► Steel ball bearings have a diameter of 0.80 ± 0.02 cm. Find the volume of one ball bearing, in cm^3, with the tolerance included. What is the maximum number of such ball bearings that can be made from 1000 cm^3 of steel?

- solve problems involving percentage error when input variables are expressed with percentage errors

► A rectangular table was measured to be 420 cm long and 170 cm wide. The length was measured with an error of 1.5% and the width with an error of 2%. Calculate the maximum and minimum possible areas, and estimate the percentage error in the calculated area.

► *An experiment is done to find the density of a ball bearing. The mass is measured to be 473 g, with a percentage error of 4%. The diameter is measured to be 5.1 cm ± 2%.

a) Calculate the density of the ball bearing, showing its percentage error.

b) Which is more effective in reducing percentage error: using a new balance that gives a mass of 473 g ± 1.5%, or using a new calliper that gives a diameter of 5.1 cm ± 1%? Justify your answer with appropriate calculations.

SHAPE AND SPACE *(Measurement)*

It is expected that students will use measuring devices to make estimates and to perform calculations in solving problems.

Prescribed Learning Outcomes	Illustrative Examples
• design an appropriate measuring process or device to solve a problem	► Design and construct a measuring device; e.g., a planimeter with a horizontal vernier scale and cardboard wheel, graduated accordingly. Apply the constructed instrument to find, according to scale, the areas of large, irregular shapes. ► *To calculate the loss of wheat after a hailstorm, a farmer counts the total number of wheat heads and the number of broken wheat heads in a small area, calculates the proportion of broken heads in the sample and extrapolates this proportion to the entire field. Explain the process used to gather the data, and explain how the estimate of loss is determined.

SHAPE AND SPACE *(Measurement)*

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

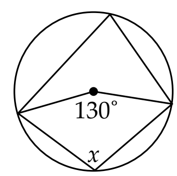
Prescribed Learning Outcomes

- use technology with dynamic geometry software, or measurement to confirm and apply the following properties:
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle, from any external point, are congruent
  - the sum of the interior angles of an n-sided polygon is equal to $(2n - 4)$ right angles

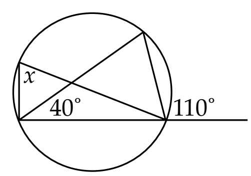
Illustrative Examples

► A plate, with a diameter of 20 cm, is placed on a square place mat, with no overhang. Calculate the length of the diagonal of the square.

► Determine the measure of angle x.

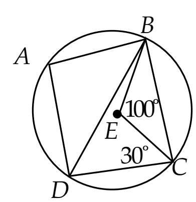


► Determine the measure of angle x.



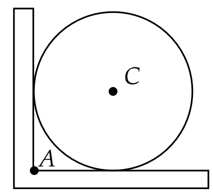
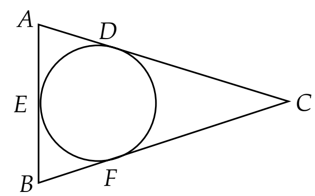
► Draw a semicircle with diameter AB. Draw an angle, ACB, with C being any point on the semicircle. What is the measure of angle ACB? Repeat for two other points C', and C'', on the semicircle. What pattern emerges?

► Determine the measure of $\angle ECB$, $\angle BDC$, $\angle BAD$ and $\angle DBE$, where E is the centre of the circle.



SHAPE AND SPACE *(Measurement)*

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

Prescribed Learning Outcomes	Illustrative Examples
	► How far from the inside corner of the shelf, *A*, is the centre *C* of the plate, if the plate has a diameter of 20 cm?  ► The perimeter of the isosceles triangle *ABC*, with $AC = BC$, is 54 cm. If $AD = 5$ cm, and *D*, *E* and *F* are points of tangency, find the length of *BC*. 

SHAPE AND SPACE *(2-D Shapes and 3-D Objects)*

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

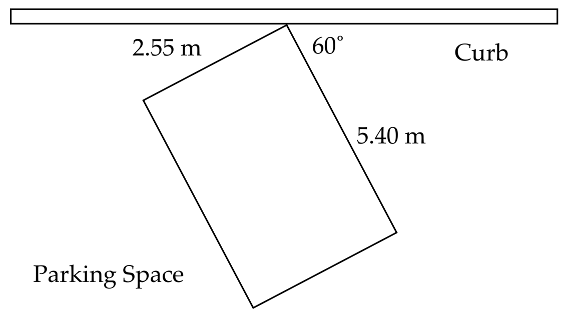
Prescribed Learning Outcomes	Illustrative Examples
• use properties of circles and polygons to solve design and layout problems	► The pattern on a piece of vinyl flooring consists of a square and four equilateral triangles. Each equilateral triangle has as its base one side of the square. Circles are inscribed in each triangle and in the square. a) Start with a square of side length 6 cm. Draw the design, full size. b) Determine the ratio of the area of the small circle to the area of the large circle. ► A standard sheet of paper is 22 cm by 28 cm. The margins are 3 cm on the left, on the right and at the top. The bottom margin is 4 cm. A project summary consists of one table that is 10 cm by 6 cm, three tables that are 8 cm by 5 cm each and 50 cm^2 of text that can be arranged in any shape(s). a) Prepare a possible layout, assuming that the tables can be oriented with their long sides parallel to any edge of the paper. b) Prepare a possible layout, assuming that the long side of any table must be parallel to the top edge of the paper. c) What is the maximum area of text that can be included with the four tables, if each table must have at least 1 cm margins? ► *A school has 325 students, all of whom have pictures to be put in the yearbook. The yearbook pages are 9.5 inches by 12 inches. The inside margins are 1.5 inches, the outside margins are 1 inch, the top margin is 1.2 inches, and the bottom margin is 1.5 inches. Each photograph is 53 mm by 35 mm. The minimum space between sides of pictures is 0.5 inches and between the bottom of one picture and the top of the next is 0.9 inches. a) How many photographs can be put on a single page? b) If the number of pages used must be divisible by 8, design a layout so that all 325 photographs can be included, without having any blank pages.

SHAPE AND SPACE (*2-D Shapes and 3-D Objects*)

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

Prescribed Learning Outcomes	Illustrative Examples

► *An average automobile requires an angle parking space with dimensions of 2.55 m wide and 5.40 m long. If spaces are being calculated for parallel parking, each automobile will require an additional length of 1.20 m as manoeuvring room. A small town's main street currently uses 60˚ angle parking.



The town council has contracted you to provide information for town planning decisions regarding parking capacity.

a. Develop a formula for the number of spaces *N* for a given curb length *L* for 60˚ angle parking.
b. Two years later, increased traffic along the main street makes angle parking unsafe. The town council wants to know how many spaces *N* they will have for a given curb length *L*, if they switch to parallel parking.

The town's main street is 200 m long. If the town council wants to retain the same parking capacity as before, how many additional spaces will have to be developed away from the main street in order to offset the spaces lost by the switch to parallel parking?

► A cylindrical can is 12 cm high and 6 cm in diameter. The can is closed, top and bottom. It is cut from a rectangular sheet of metal, and then the pieces are sealed together to form the can.

a) Determine the smallest rectangle that can be used to make one can.
b) What percentage of the metal is wasted in part a)?
c) If seams require 2 mm of extra metal per join, what are the new dimensions of the smallest rectangle?

STATISTICS AND PROBABILITY *(Data Analysis)*

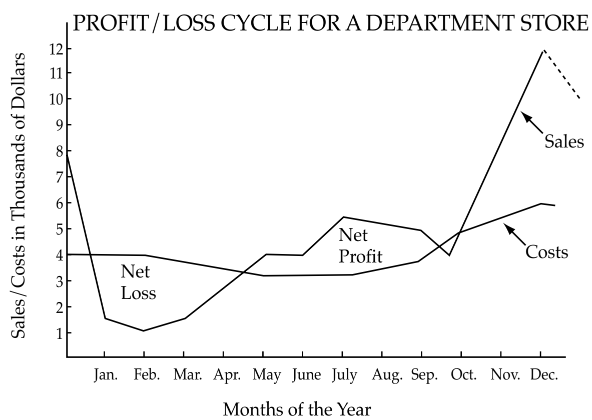
It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples

- extract information from given graphs of discrete or continuous data, using:
  - time series
  - continuous data
  - contour lines

► Sometimes points representing discrete data are joined, even though specific values for intermediate points may not be available. Give examples where such a practice is acceptable and other examples where it is not.

► *A department store may experience "peaks" and "troughs" in its revenue (sales). Christmas season and summer holidays are the two strongest periods. January to April can be the weakest period. If net profits are greater than net losses over the year, the business can stay in operation.



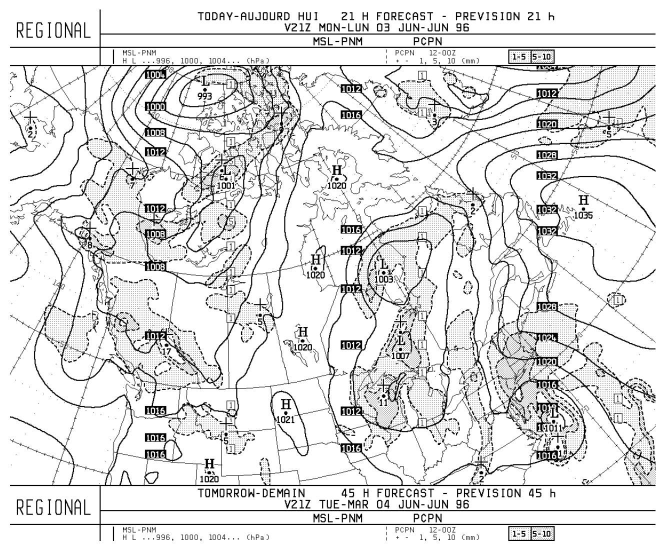
a) During periods of net loss, what might the business do for finances?
b) Over which of the two curves, Sales or Costs, does the business have the most managerial control?
c) Discuss the net profit for May.

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples

► *The map below shows the atmospheric pressure, measured in hectopascals, forecast at various weather stations for June 3, 1996. A current Environment Canada map can be found on the Internet at: http://www.cmc.ec.gc.ca/cmc/html/analysis.html



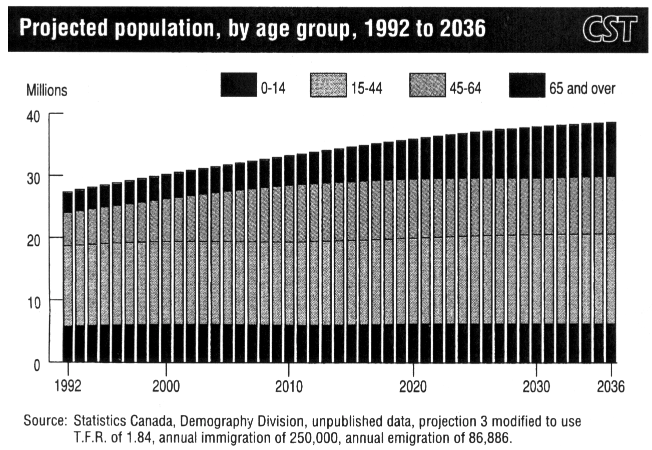
From Environment Canada, on line, June 2, 1996, with permission.

a) Using a current map, estimate the forecasted atmospheric pressure at your location.
b) What is the lowest pressure recorded in Canada for the date on your map?
c) What is the highest pressure recorded in Canada for the date on your map?
d) Shaded areas show where rain is falling. What connection is there between atmospheric pressure and rainfall?

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples
• draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data	► The bar graph below shows the projected Canadian population, by age group, for the period from 1992 to 2036.



Reproduced by authority of the Minister of Industry, 1998, Statistics Canada, *Canadian Social Trends*, Catalogue 11–008E, Number 29 Summer 1993, p. 6.

a) What year is Canada's population expected to reach 30 million?

b) Describe the rate of increase of Canada's population, both overall and by age group.

c) Estimate the median age of the Canadian population in 1992 and in 2036.

d) Estimate when Canada's population will reach 40 million.

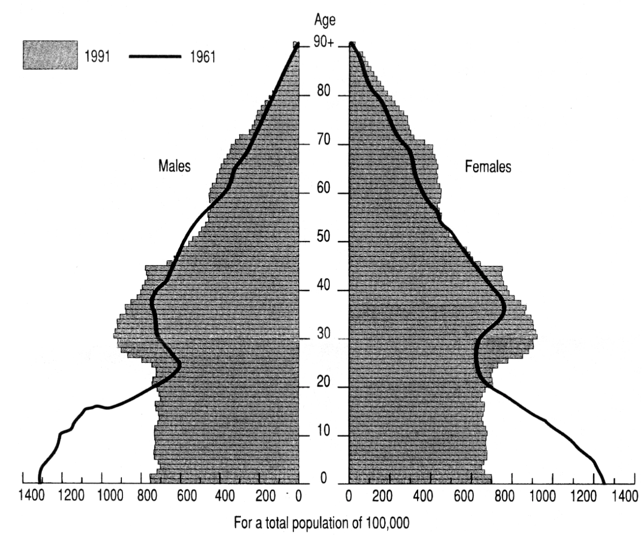
STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples

► *The population pyramids shown below are for Canada for 1961 and 1991. Separate data are shown for males and females.

Population distribution, by age and sex, 1961 and 1991



Source: Statistics Canada, Demography Division.

Reproduced by authority of the Minister of Industry, 1998, Statistics Canada, *Canadian Social Trends*, Catalogue 11–008E, Number 29 Summer 1993, p. 6.

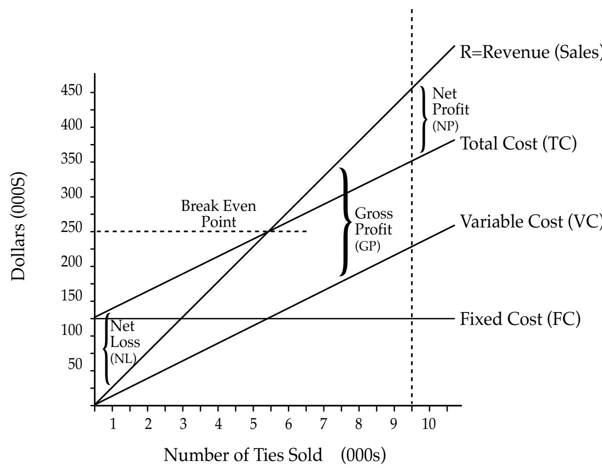
a) What is the approximate ratio of male births to female births? Has this ratio changed from 1961 to 1991? Describe any change, and make a hypothesis for the change.
b) The baby boom was a period of time that was characterized by a greater number of births than in the years before or after. What evidence is there for a baby boom, and what were the years of the baby boom?
c) The birth rate was low during the years of the Depression (1931–39) and World War II (1939–45). Where is there evidence for this?
d) The shapes of the population pyramids, especially the 1961 pyramid, show a marked lack of symmetry between the data for males and the data for females. Identify where the lack of symmetry is greatest, and make hypotheses for the lack of symmetry. How could these hypotheses be tested?

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples
	► A small store in a shopping mall sells neckties for $50 each. The ties cost the merchant $25 each. Yearly operating expenses, such as wages, rent, utilities and insurance, are $125,000.

BREAK EVEN ANALYSIS



VC + FC = TC, R – VC = GP, GP – FC = NP,
R – TC = NP (or NL)

If the store sold 100 ties, the sales (R) would not pay for the expenses; therefore, the store would be losing money. At $250 000 in sales, the store's sales just cover all the cost of the goods sold (VC) and expenses (FC). Therefore, the store just breaks even. If the store sells 9000 ties in a year:

a) What is the net profit?
b) What is the gross profit?
c) What is the fixed cost?

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples
• design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation	► Collect an example from a newspaper or magazine in which a graph has been presented in a potentially deceptive manner. Identify the source from which the graph was taken. Explain briefly the ways in which the graph might have been deceptively presented and then show ways the data might be presented more fairly or in a less distorted fashion. Include the graph with the project, and cite its source. Excerpted and adapted with permission from *Data Analysis and Statistics* (*Curriculum and Evaluation* Addenda Series, Grades 9–12), copyright 1992 by the National Council of Teachers of Mathematics. All rights reserved. ► *Using data for 10-year intervals, as provided in Canada's Population table on the following page, starting in 1921 and ending in 1991, design an honest presentation of the data that can be included in different term papers dealing with each of the following topics: a) the increase in Canada's population b) the westward shift of Canada's population c) the population of Saskatchewan d) the dominant position of Ontario and Quebec within Canada. Explain your choice of data selection and data presentation.

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples

Canada's Population[1] (Thousands)

	Nfld.	PEI	NS	NB	Que.	Ont.	Man.
1921		88.6	523.8	387.9	2,360.5	2,933.7	610.1
1931		88.0	512.8	408.2	2,874.7	3,431.7	700.1
1941		95.0	578.0	457.4	3,331.9	3,787.7	729.7
1951	361.4	98.4	642.6	515.7	4,055.7	4,597.6	776.5
1956	415.1	99.3	694.7	554.6	4,628.4	5,404.9	850.0
1961	457.9	104.6	737.0	597.9	5,259.2	6,236.1	921.7
1966	493.4	108.5	756.0	616.8	5,780.8	6,960.9	963.1
1971	522.1	111.6	789.0	634.6	6,027.8	7,703.1	988.2
1976	557.7	118.2	828.6	677.3	6,234.5	8,264.5	1,021.5
1981	567.7	122.5	847.4	696.4	6,438.2	8,624.7	1,026.2
1986	568.3	126.6	873.2	710.4	6,540.2	9,113.0	1,071.2
1987[2]	568.1	127.3	878.0	712.3	6,592.6	9,265.0	1,079.0
1988[2]	568.8	128.5	881.9	714.3	6,640.8	9,431.1	1,084.1
1989[2]	571.1	129.9	888.3	717.8	6,698.2	9,589.6	1,086.3
1990[2]	572.7	130.7	895.1	722.6	6,768.2	9,749.6	1,089.0
1991[2]	575.7	131.2	901.0	727.6	6,847.4	9,917.3	1,094.4
1992[3]	577.5	130.5	906.3	729.3	6,925.2	10,098.6	1,096.8

	Sask.	Alta.	BC	YT	NWT	Canada
1921	757.5	588.5	524.6	4.1	8.1	8,787.4
1931	921.8	731.6	694.3	4.2	9.3	10,376.7
1941	896.0	796.2	817.8	5.0	12.0	11,506.7
1951	831.7	939.5	1,165.2	9.1	16.0	14,009.4
1956	880.7	1,123.1	1,398.5	12.2	19.3	16,080.8
1961	952.2	1,332.0	1,629.1	14.6	23.0	18,265.3
1966	955.4	1,463.2	1,873.7	14.4	28.7	20,014.9
1971	926.2	1,627.9	2,184.6	18.4	34.8	21,568.3
1976	921.3	1,838.0	2,466.6	21.8	42.6	22,992.6
1981	968.3	2,237.3	2,744.2	23.2	45.7	24,341.7
1986	1,010.2	2,375.1	2,889.0	23.5	52.2	25,353.0
1987[2]	1,015.8	2,377.7	2,925.0	24.5	52.0	25,617.3
1988[2]	1,013.5	2,388.7	2,980.2	25.2	52.2	25,909.2
1989[2]	1,006.7	2,425.9	3,048.3	25.5	52.9	26,240.3
1990[2]	997.1	2,473.1	3,132.5	26.0	53.9	26,610.4
1991[2]	994.2	2,521.6	3,212.1	26.7	55.2	27,004.4
1992[3]	993.2	2,562.7	3,297.6	27.9	56.5	27,402.1

1. As of June 1.
2. Final postcensal estimates.
3. Updated postcensal estimates.

Sources
Employment and Immigration Canada
Statistics Canada

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will analyse graphs or charts of given situations to derive specific information.

Prescribed Learning Outcomes	Illustrative Examples
• collect experimental data and use best-fit exponential and quadratic functions, to make predictions and solve problems	► Plot the world population on the vertical axis and the date on the horizontal axis. Use the graph to predict the date when the population reached 4 billion and to predict the present population of the world.

Date	Population
1650	500 000 000
1850	1 100 000 000
1930	2 000 000 000
1950	2 500 000 000
1970	3 600 000 000
1988	5 100 000 000

Appendix G

Illustrative Examples

Applications of Mathematics 12

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

NUMBER *(Number Operations)*

It is expected that students will describe and apply operations on matrices to solve problems, using technology as required

Prescribed Learning Outcomes	Illustrative Examples

- model and solve problems, including those solved previously, using technology to perform matrix operations of addition, subtraction and scalar multiplication as required

c CFL Western Division Standings

Against Eastern teams

	W	*T*	*L*	*Points*
BC	*1*	*0*	*2*	*2*
Calgary	*2*	*0*	*0*	*4*
Saskatchewan	*1*	*0*	*1*	*2*
Edmonton	*1*	*0*	*2*	*2*

Against Western teams

	W	*T*	*L*	*Points*
BC	*4*	*0*	*0*	*8*
Calgary	*3*	*0*	*2*	*6*
Saskatchewan	*1*	*0*	*4*	*2*
Edmonton	*1*	*0*	*3*	*2*

a) Create a combined table using matrix addition.
b) What condition is necessary for there to be a matrix sum $A + B$, given original matrices A and B?

c *A store sells items that are tax free, items that have a 7% GST charge on the base price, and items that have both a 7% GST and a 9% PST charge on the base price. A weekend's sales, before tax, is represented in the table:

Sales by Tax Category	Saturday Sales ($)	Sunday Sales ($)
Tax free	500	700
GST only	1250	400
GST and PST	800	700

a) Determine each amount of tax (GST and PST) collected by the store on this weekend, using any method.
b) Model and solve the problem, using appropriate matrices derived from the table given.
c) How would you change your matrix model for a new GST rate of 5% and a new PST rate of 12%?

NUMBER *(Number Operations)*

It is expected that students will describe and apply operations on matrices to solve problems, using technology as required

Prescribed Learning Outcomes	Illustrative Examples
	c An Artic expedition outfitter has the following price list for tours of varying lengths and starting points. The prices are quoted in Canadian dollars.

a) Research to find a table that compares the Canadian dollar to other currencies.
b) Write a row matrix that represents the cost, in US dollars, of the four tours from a Toronto starting point.
c) Write a matrix, in Japanese yen, for the entire price list.

Starting Point	Length of Tour and Cost			
	4 Days	7 Days	10 Days	15 Days
Yellowknife	$ 740	$1145	$1550	$2225
Edmonton	$1640	$2045	$2450	$3125
Toronto	$2240	$2645	$3050	$3725
Vancouver	$1940	$2345	$2750	$3425

• model and solve consumer and network problems, using technology to perform matrix multiplication, as required

c *Singh's Grocery sells several different kinds of breakfast cereal, each at a different price:
- cereal A is 2.65/bx.
- cereal B is 3.73/bx.
- cereal C is 3.15/bx.
- cereal D is 2.99/bx.

On Wednesday, they sold the following:
- 5 boxes of cereal A
- 8 boxes of cereal B
- 7 boxes of cereal C
- 10 boxes of cereal D

a) What is Wednesday's total revenues? Model and solve this problem, using an appropriate row matrix, and an appropriate column matrix, then use matrix multiplication to solve.
b) Can the total revenue be calculated, if six boxes of cereal E (price unknown) were sold? Justify your response.

c Sales of economy cars were 200 in 1998 and fell by 4% in 1999. Sales of midsize cars were 300 in 1998 and rose by 10% in 1999. Sales of luxury cars were 40 in 1998 and rose by 3% in 1999.
a) Represent the 1998 and 1999 sales by appropriate column matrices.
b) Represent the changes from 1998 to 1999 by an appropriate matrix multiplication.

NUMBER *(Number Operations)*

It is expected that students will describe and apply operations on matrices to solve problems, using technology as required

Prescribed Learning Outcomes	Illustrative Examples

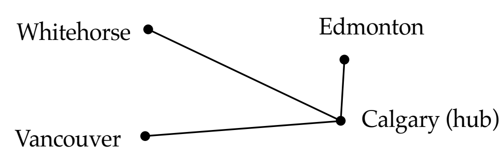
c *Soccer administrators have been experimenting with using league standings to discourage tie games, especially those in which no goals are scored. The traditional scheme of 2 points for a win and 1 point for any tie has been replaced by a scheme that awards 3 points for a win and 1 point for all ties. Two already proposed schemes are 3 points for a win, 1 point for ties that have goals scored and 0 points for ties with no goals; and a scheme with 5 points for a win, 3 points for a tie with goals scored and 0 points for a tie with no goals. In a local soccer league the top four team records after 42 games are:

	Wins	Ties with Goals	Ties with no Goals	Losses
Tigers	30	2	8	2
Irish	24	9	2	7
Colts	25	7	0	10
Jets	26	1	10	5

a) Determine the points for each team, using the traditional scheme.
b) Model this solution, using matrix multiplication.
c) Using matrices, model and determine the points for each team, using new scheme. Repeat for the other two proposed schemes.
d) Which of the alternative scoring systems can make the Irish second in the standings?
e) Which of the alternative scoring systems can make the Colts second in the standings?
f) Which of the alternative scoring systems can make the Jets second in the standings?
g) Design a system that would drop the Tigers out of first place.

NUMBER *(Number Operations)*

It is expected that students will describe and apply operations on matrices to solve problems, using technology as required

Prescribed Learning Outcomes	Illustrative Examples
	c A laundry soap is sold in 6 L and 10 L packages. Market research shows that 40% of the users of the 6 L size switch to the 10 L size for their next purchase, and 20% of the users of the 10 L size switch to the 6 L size for their next purchase. a) If the original market share was 60% for 6 L and 40% for 10 L, what is the market share for each size in the next round of purchases? b) What is the market share for each size for the third round of purchases? c *Air freight is transported among several cities in Western Canada, using Calgary as the hub. This means that all air freight either originates in Calgary or is transported through Calgary. a) Set up a network matrix A for the following cities, using 1 to represent a direct freight transport between two cities and 0 to indicate there is no direct freight transport between two cities. Cities in the Network  b) The matrix A^2 can represent the network matrix for those routes with exactly one stop. Evaluate the matrix A^2. c) Explain why there is one element equal to 3, nine elements equal to 1, and six elements equal to 0 in the matrix A^2.

NUMBER *(Number Operations)*

It is expected that students will design or use a spreadsheet to make and justify financial decisions.

Prescribed Learning Outcomes	Illustrative Examples
• design a financial spreadsheet template to allow users to input their own variables	c *For the following invoice, develop a spreadsheet that calculates the totals and that requires the operator to input a minimum number of entries.

ACME AUTO PARTS

Customer Inquiries

Item No.	Auto Parts	Quantity	Unit Price	Total	Labour	
1	Brake pads	1	26.34	26.34	O/H Front Brakes (0.9 hrs. @ 53.00/hr.)	47.70
2	Wheel seals	2	5.25	10.50	Machined and Replace Rotor (Flat Rate)	10.10
3	Rotor	1	30.16	30.16		
					Total Labour	57.70
					Total Parts	67.70
			Total Parts	67.00	PST on Parts (8%)	5.36
					GST (7%)	8.73
					TOTAL	**138.79**

NUMBER *(Number Operations)*

It is expected that students will design or use a spreadsheet to make and justify financial decisions.

Prescribed Learning Outcomes	Illustrative Examples
• analyse the costs and benefits of renting or buying an increasing asset such as land or property, under various circumstances	c *The Wong family is faced with a move and has the choice of buying a home for $145,000 with a $25,000 down payment, or renting a similar house for $975 per month. Four options are available. 1. Buy the house with a 20-year mortgage and continue investing at the same rate after the mortgage is paid. 2. Buy the house with a 30-year mortgage. 3. Rent a house and invest the $25,000. 4. Rent a house and invest both the $25,000 and the difference each month between the rent and the mortgage payment. Set up analysis spreadsheets that include the following inputs: a) mortgage interest rate, taking 8.5% as a starting value b) taxation rate, taking 1.5% of market value as a starting value c) annual rent increase, taking 5% per annum as a starting value d) annual increase in house value, taking 4% per annum as a starting value e) investment return, taking 7.0% as a starting value Try different scenarios, varying from 1 year to 30 years. Summarize circumstances in which buying makes sense, and summarize circumstances when renting makes sense.
• analyse the costs and benefits of leasing or buying a decreasing asset, such as a vehicle or computer, under various circumstances	c *A car lease runs for 36 months at $305 per month, with a down payment of $1,105, a lease-end value of $7,105 and an interest rate of 11.6%. Maintenance is the purchaser's responsibility. Set up a spreadsheet to include the monthly values of the opening balance, interest paid, lease payment and closing balance. Use the spreadsheet to answer the following questions. a) What part of the $305 is used to pay the interest on the $7,105? b) What total price is being charged for the car? c) What is the change in the monthly lease payment, if the lease-end value is reduced to $5,700? d) What is the monthly payment for a straight purchase over 36 months with a 20% down payment? e) What is the annual percentage depreciation rate assumed with the $7,105 lease-end value?

NUMBER *(Number Operations)*

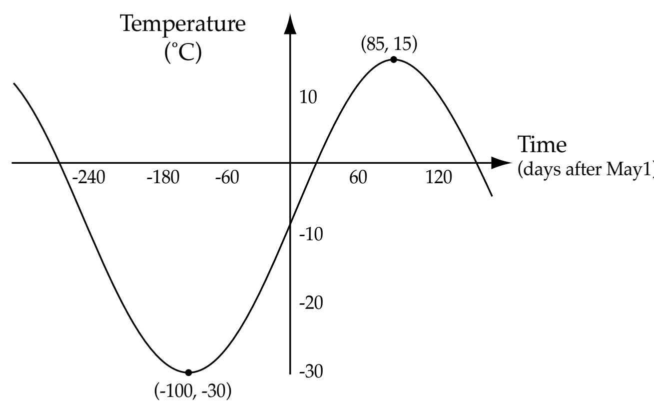
It is expected that students will design or use a spreadsheet to make and justify financial decisions.

Prescribed Learning Outcomes	Illustrative Examples
• analyse an investment portfolio applying such concepts as interest rate, rate of return and total return	c The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years. a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates: - 4% per annum, compounded annually - 8% per annum, compounded annually - 24% per annum, compounded annually b) What general conclusion can be drawn as to the accuracy of the rule of 72 calculations? c Annette makes several investments in her RRSP. She invests \$5,000 in a GIC paying 6%, compounded semi-annually, when she is 25 years old. At the age of 30, she places \$12,000 in a GIC paying 8 $^1/_2$ %, compounded annually. On her 35th birthday, she invests \$8,000 in a GIC paying 7%, compound semi-annually. a) What will her GICs be worth when she is 65, if she keeps them invested at the rates given until her 65th birthday? b) What is the average annual rate of return over the last 30 years of the investment? c An investment of \$10,000 is held from December 31, 1995 to December 31, 1998. It contains 75% Canadian stocks that track the Toronto Stock Exchange (TSE) 300 composite index, and 25% invested in guaranteed investment certificates (GICs) that pay 6.5% compounded annually. On December 31, 1995 the TSE 300 composite index was at 4714. On December 31, 1998, the TSE 300 composite index was at 6486. a) What was the value of the total portfolio on December 31, 1998? b) What average annual rate of return does this represent?

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse cyclic, recursive, and fractal patterns.

Prescribed Learning Outcomes	Illustrative Examples
• describe periodic events, including those represented by sinusoidal curves, using the terms amplitude, period, maximum and minimum values, vertical and horizontal shift	c A temperature-time graph was drawn for a northern Saskatchewan town, using long-term average data for each date of a calendar year. The variable plotted on the horizontal axis is the calendar date, with May 1 as zero and the unit of measure being days. The variable plotted on the vertical axis is the temperature in degrees Celsius. The graph is dramn below. Estimate the: a) amplitude b) period c) average maximun and minimum values d) the average temperature for the year e) average date for the maximum temperature f) average date for the minimum temperature

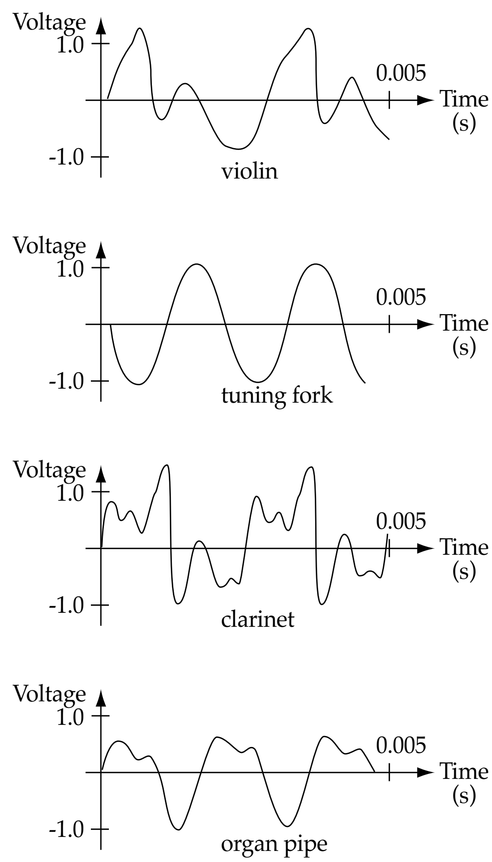


PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse cyclic, recursive, and fractal patterns.

Prescribed Learning Outcomes	Illustrative Examples

c *The following are graphs showing the patterns produced on an oscilloscope when four different musical instruments are played.



From *Fundmentals of Physics* by Martindale et al. Reprinted by permission of ITP Nelson Canada.

For each instrument:
a) estimate the amplitude
b) estimate the period
c) sketch an approximate form for the graph, if the instrument is played louder
d) sketch an approximate form for the graph, if the instrument is used to play a higher note

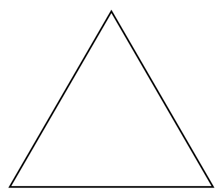
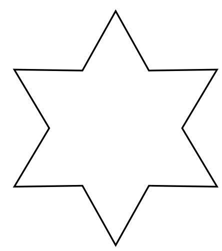
PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse cyclic, recursive, and fractal patterns.

Prescribed Learning Outcomes	Illustrative Examples
• collect sinusoidal data; graph the data using technology, and represent the data with a best fit equation of the form: - $y = a \sin (bx + c) + d$	c Research the sunrise time, each day, for a period of one year, and graph it. From your graph, determine the time of sunrise for March 12. c Collect data from real-world situations, such as: a) hours of daylight b) low tide and high tide c) average low and average high temperatures on different dates of the year Plot the data, and determine an approximate equation for the data in the form of: $y = a \sin (bx + c) + d$.
• use best fit sinusoidal equations, and their associated graphs, to make predictions (interpolation, extrapolation)	c *Sketch a graph, and build an equation to represent the following situation. From Environment Canada, find the average daily maximum temperatures on the 1st and 15th day of each month for one year, in a city of your choice. Use a sinusoidal equation to predict: a) the highest daily average maximum temperature b) the lowest daily average maximum temperature c) what time of year will the highest temperature will occur d) what time of year will the lowest temperature will occur

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse cyclic, recursive, and fractal patterns.

Prescribed Learning Outcomes	Illustrative Examples
• use technology to generate and graph sequences that model real-life phenomena	c *A mild infection spreads through a school. On any given day 10% of the uninfected population becomes infected, and 20% of the infected population is cured. Once cured, there is zero chance of further infection. On the first day of school, 100 students are infected, 900 students are uninfected, and zero students are cured. a) Track the epidemic for 30 days. b) Graph numbers of infected, uninfected, and cured as a function of time for 30 days.
• use technology to construct a fractal pattern by repeatedly applying a procedure to a geometric figure	c The following example is the Koch snowflake curve. Construct an equilateral triangle (Fig. 1). Trisect each side, construct an equilateral triangle on each middle third, and delete the middle third (Fig. 2).  Fig. 1  Fig. 2 For each segment in Fig. 2, repeat the above. c Construct your own fractal pattern.
• use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns	c *For the previous illustrative example, predict the perimeter of the fifth pattern.

PATTERNS AND RELATIONS *(Patterns)*

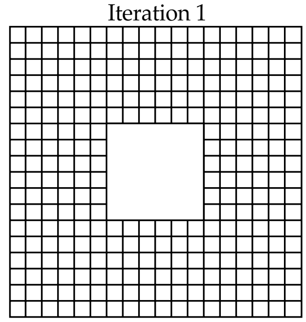
It is expected that students will generate and analyse cyclic, recursive, and fractal patterns.

Prescribed Learning Outcomes	Illustrative Examples

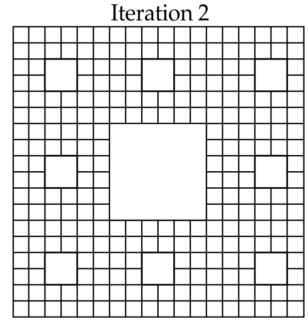
c *Fractal Carpet

A fractal can be generated by a pattern of iteration. This fractal design is called the Sierpinski carpet after the mathematician who invented it in 1916. The general rule is to start with a square and take a square out. Look at the first iteration and describe the rule that was used to determine the size of the square that was removed. Now compare the first two iterations and describe the rule that was used to construct the second from the first. Apply the rule you have stated to construct the third iteration in the space provided.

Iteration 1



Iteration 2



Now examine the third iteration you have constructed, and record the length of the side of the new squares you drew. Compare this length to the lengths of the sides of the previous squares. Write the lengths of the sides of all the squares in descending order. If you construct the fourth iteration, what will the lengths of the sides of the squares need to be? Now look at the first iteration again. What is the area of the square that was removed? What is the area of each individual square that was removed in the next two iterations? Write these areas in descending order. What is the area of each individual square to be removed in the fourth iteration?

Challenge: Find the perimeter of all the squares in the third iteration. Find the area of the figure that remains once all the squares are removed in the third iteration.

PATTERNS AND RELATIONS *(Patterns)*

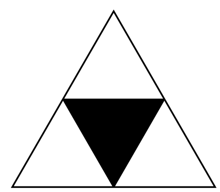
It is expected that students will generate and analyse cyclic, recursive, and fractal patterns.

Prescribed Learning Outcomes	Illustrative Examples

c *The Sierpinski triangle can be created by using dilations and isometries. You may begin with an arbitrary triangle. An equilateral triangle is used for the procedures described below.

a) Draw an equilateral triangle.
b) Reduce the triangle by a factor of $\frac{1}{2}$. Make three copies of the reduced triangle.
c) Place the three reduced similar triangles on the original, one at each vertex.
d) Eliminate the remaining portion of the original triangle by blackening it.

Your work should result in the figure shown here.



Answer the following questions:

a) Let the area of the original triangle be 1 area unit. What area remains? What area has been removed?
b) Let the side of the original triangle be 1 length unit. What is the perimeter of the figure with the dark region removed?

Repeat steps a) through d) of the original procedure for each of the triangular regions remaining in the figure shown. Sketch the result of your work.

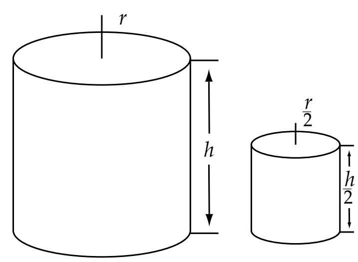
Answer the following questions:

a) What is the area of the remaining triangular region?
b) What is the perimeter of the new "holey" triangular region?
c) What would the next iteration of the procedure look like? Make a sketch.
d) Write an expression for the area of the Sierpinski triangle after carrying out the procedure *n* times.
e) Write an expression for the perimeter of the Sierpinski triangle after carrying out the procedure *n* times.
f) How would your expressions differ, if you began with a triangle other than an equilateral triangle?

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse cyclic, recursive, and fractal patterns.

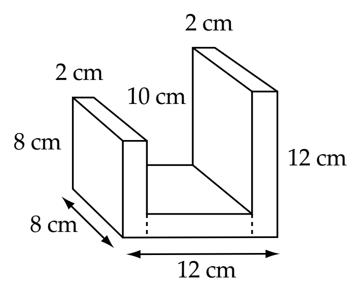
Prescribed Learning Outcomes	Illustrative Examples
	c Construct a cylinder with the dimensions: $r = 10$ cm, $h = 20$ cm. A second figure is constructed by halving the previous radius and height. A third is constructed by halving the second and so on. a) Predict the lateral surface area and the volume of the sixth pattern. b) Write an expression for the lateral surface area after carrying out the procedure n times. c) Write an expression for the volume after carrying out the procedure n times.



SHAPE AND SPACE *(Measurement)*

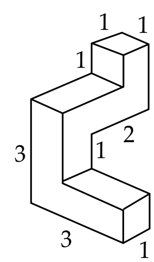
It is expected that students will analyse objects, shapes, and processes to solve cost and design problems.

Prescribed Learning Outcomes	Illustrative Examples
• use dimensions and unit prices to solve problems involving perimeter, area and volume	c Determine the volume of the plastic book end shown below.



If the book end is constructed using an injection mold, find the materials cost, if the plastic ingredients cost 6¢/cm².

c *The following diagram shows a structure for enclosing a cable system. All the angles are right angles and the lengths are in metres.



A special latex coating is applied to all outside surfaces of the structure. What is the cost of the latex coating, if it costs 28¢/cm²?

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving estimation and costing for objects, shapes or processes when a design is given	c A swimming pool is 50 m by 21 m. The deep end is 4.0 m deep and extends out 12 m. The shallow end is 1.2 m deep and extends out 12 m. There is a uniform slope connecting the deep and shallow ends. a) Draw scale diagrams showing the top view and the side view of the pool. b) Calculate the cost of filling it with water at \$2.00/m³. c) Waterproofing of the underwater surfaces costs \$17/m². Determine the cost of waterproofing.

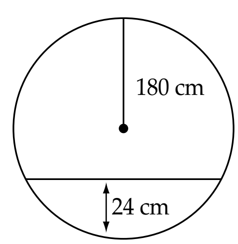
SHAPE AND SPACE *(Measurement)*

It is expected that students will analyse objects, shapes, and processes to solve cost and design problems.

Prescribed Learning Outcomes	Illustrative Examples
	c *To satisfy the building code, an auditorium has to have 1200 m^2 of washroom space. In a washroom for males, the average space needed is 1.9 m^2 per user and the average usage time is 97 s. In a washroom for females, the average space needed is 2.4 m^2 per user and the average usage time is 145 s. Determine the required washroom space: a) on the basis of equal areas for males and females b) on the basis of equal users per hour for males and females
• design an object, shape, layout or process within a specified budget	c *Tin plate for making cylindrical cans comes in sheets that are 240 cm by 160 cm and costs $3.20 per sheet. Cans are 6 cm in diameter and 11 cm high, and they have 3 seals each. Seals cost 0.8¢ each to make. One sheet of tin plate is used for making pieces for ends, and two sheets are used for making pieces for sides. a) How many ends and how many sides can be made from the three sheets of tin plate? b) How many cans can be made from the three sheets, and what is the cost per can? c) Is there another way of making more cans from the three sheets, or the same number of cans from less tin plate? d) How much money is saved doing it the second way? c To produce a voters' list for a city riding, a sum of $1.70 per voter is allocated. Four methods of enumerating are possible: For a riding of 40 000 potential voters, find the maximum number of voters who can be enumerated within the budget and the minimum budget needed to be sure of enumerating 98% of the potential voters.

SHAPE AND SPACE *(Measurement)*

It is expected that students will analyse objects, shapes, and processes to solve cost and design problems.

Prescribed Learning Outcomes	Illustrative Examples
	c A window cleaner has been asked by the owner of a large office tower to submit a quotation for cleaning the windows of the building. The window cleaner has the following information: a) there are 24 floors b) there are 14 windows per side on each floor c) there are 4 sides to the building From experience, the window cleaner knows that the transfer time between windows on the same floor and same side of the building is 60 seconds. The transfer time between sides of the building is 120 seconds and between floors is 30 seconds. The time to clean one window is 120 seconds. The window cleaner has a base charge of $120. The maximum period of time he works at one stretch is 3 hours, then he takes a 30 minute rest. In addition to his rate of $25/hour, he wants to make 25% profit from the job for reinvestment in his business. What would be the best quote?
• use simplified models to estimate the solutions to complex measurement problems	c Obtain a Mercator projection map and a polar projection map of Canada. Use a transparent overlay grid to estimate the area of the Yukon by counting squares. a) Which type of map gives the most accurate estimate of the area? Explain the sources of error when the estimates are compared to the exact area given in a reference source. b) Estimate the area by splitting it into rectangles and triangles, and compare this method to the previous one you used. Which method is most accurate? Which type of map gives the most reliable estimate for the area of the Yukon? Where are the main sources of error in the estimate? c *A water tank is a sphere of diameter 3.6 m. a) Estimate the volume of water in the tank, if the depth of water is 24 cm. b) Describe the estimation process you used. 

SHAPE AND SPACE *(3-D objects and 2-D shapes)*

It is expected that students will solve problems involving polygons and vectors, including both 3-D and 2-D applications.

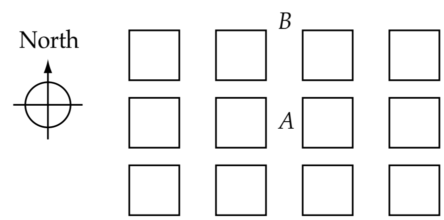
Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- use appropriate terminology to describe:
  - vector (i.e., magnitude, direction)
  - scalar quantities (i.e., magnitude)

Illustrative Examples

c A car at location *A* is travelling toward location *B*. The speed is constant at 50 km/h.



a) Describe the speed in terms of a scalar.
b) Describe the velocity in terms of a vector.

Prescribed Learning Outcomes

- assign meaning to the multiplication of a vector by a scalar

Illustrative Examples

c The vector $\vec{a}$ represents a velocity of 40 km/h east. Make a scale drawing of each of the following vectors:
a) $3\vec{a}$
b) $7\vec{a}$
c) $-3\vec{a}$
d) $1.6\vec{a} + 4\vec{a}$

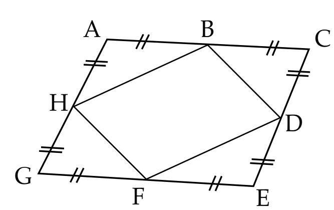
What velocities do each of these vectors represent?

Prescribed Learning Outcomes

- determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods

Illustrative Examples

c

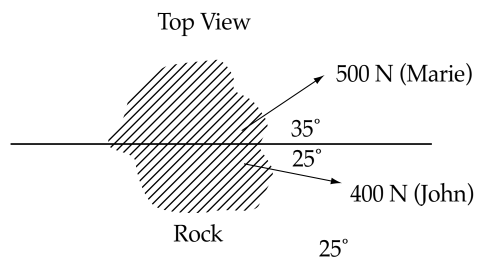


Using the above diagram of a rhombus *ACEG*, determine the vector addition of each of the following:
a) $\overline{AH} + \overline{HG}$
b) $\overline{GF} + \overline{BC}$
c) $\overline{GF} + \overline{CB}$
d) $\overline{FD} + \overline{DE}$.

c *A ski jumper encounters a horizontal friction of 85 N backward, a vertical weight of 750 N downward and an air resistance of 340 N upward. Draw the vector addition of these forces, and use the drawing to find the magnitude and direction of the resultant force.

SHAPE AND SPACE *(3-D objects and 2-D shapes)*

It is expected that students will solve problems involving polygons and vectors, including both 3-D and 2-D applications.

Prescribed Learning Outcomes	Illustrative Examples
	c A boat is traveling across a river with a forward velocity of 14 m/s, and there is a current of 3 m/s down the river. How fast is the boat traveling? c *John and Marie are using two ropes held horizontally to pull a rock on a horizontal plane.  a) Draw a vector diagram to estimate the magnitude and direction of the resultant applied force. b) Verify the estimate by a calculation. c) If the rock does not move, what is the magnitude and direction of the force of friction?
• model and solve problems in 2-D and simple 3-D, using vector diagrams and technology	c Model, by drawing a diagram, Jack's jogging route, if he jogs north at 15 km/h for 30 minutes and then turns east and jogs at 12 km/h for 20 minutes. How far has he jogged in total? How far is he from his starting point? In what direction does he need to go to return to the start by the shortest path? c *An aircraft flying horizontally on a heading of 285º is pushed by a wind from 195º. Angles are measured clockwise from north. The indicated air speed of the aircraft is 300 km/h. The wind is constant at 90 km/h. After 1 hour and 15 minutes of flight, what will be the aircraft's change in location? c *An aircraft flies at 80 m/s due north and climbs at an angle of 20º with the horizontal. A horizontal wind blows from west to east at 30 m/s. a) What is the magnitude of the velocity of the aircraft? b) What is the angle that the aircraft's track makes with the horizontal?

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will use normal and binomial probability distributions to solve problems involving uncertainty.

Prescribed Learning Outcomes	Illustrative Examples
• find the population standard deviation of a data set or a probability distribution, using technology	c Measure the height of each student in a class, and calculate the mean and standard deviation. c *A company uses an automated packaging device to produce 50-g bags of Karmel Korn. The machine needs frequent checking to see if it is actually putting 50 g in each bag. The following are the masses, in grams, of thirty bags of Karmel Korn.

54	50	47	50	51	50
53	50	47	51	50	51
52	49	46	52	50	49
52	48	48	53	49	49
51	48	49	52	49	50

a) Calculate the mean and standard deviation of this data.
b) What problems will be encountered, if the standard deviation gets too high?

Dottori et al., *Foundations of Mathematics 11*, p. 392. Adapted with permission.

Prescribed Learning Outcomes	Illustrative Examples
• use z-scores and the normal distribution to solve problems	c The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 1.5 mL. a) Calculate the z-score for a can with a volume of 355 mL. b) What percentage of production will consist of cans having content volumes between 350 mL and 355 mL? c) What percentage of production will consist of cans having content volumes less than 355 mL? d) If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected in a run of 50,000? c *For entry into the Canadian Armed Forces, the standards for height used to be set at 158 cm to 194 cm for males, and 152 cm to 184 cm for females. Use the concept of z-score to test if these two height standards are equivalent. Assume that heights for male and females are normally distributed with means of 176 cm and 163 cm and standard deviations of 8 cm and 7 cm respectively.

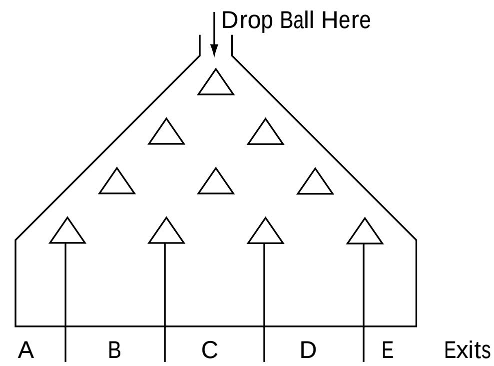
STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will use normal and binomial probability distributions to solve problems involving uncertainty.

Prescribed Learning Outcomes	Illustrative Examples
	c *A sample of 122 people gives a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find: a) the expected number of people with temperatures above 37.0°C b) the expected number of people with temperatures below 36.0°C Also, estimate the range of temperatures contained within the sample. c In the general population, the IQ scores of individuals is normally distributed with a mean of 100 and a standard deviation of 10. If a large group of people is tested: a) What proportion of this group is expected to have IQs between 100 and 120? b) What is the probability that an individual in the group has an IQ greater than 120?
• use the normal approximation to the binomial distribution to solve problems involving probability calculations for large samples (where $npq > 10$)	c Based on past experience, a car salesperson will complete sales to 10% of the customers that enter the showroom. If the salesperson has 200 customers in the next month, establish a symmetric 95% confidence interval for the number of completed sales for the month. c *From a sample of 250, pollsters estimate that the number of decided voters in favour of a particular provincial law is 64%, and the number opposed is 36%. Discuss how the percentage margin of error changes as the sample size changes. c A true/false test has 40 questions. Assume all questions are answered by guessing. Use a normal distribution to estimate the probability of guessing 25 or more questions correctly.

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations, and combinations.

Prescribed Learning Outcomes	Illustrative Examples
• solve pathway problems, interpreting and applying any constraints	c Given the following "pinball" situation, what is the probability of the ball reaching each of the exits?  What assumptions are made in the solution?
• use the fundamental counting principle to determine the number of different ways to perform multistep operations	c Joe has three different shirts, two different pairs of pants and five different pairs of shoes. List all possible outfits in such a way as to ensure that all have been counted and none have been counted twice. How many possible outfits are there? Use the fundamental counting principle to determine the number of outfits there should be. Do your answers match? c *An airline pilot reported that in seven days she spent one day in Winnipeg, one day in Regina, two days in Edmonton and three days in Yellowknife. How many different itineraries are possible? What difference would it make if the first day and the last day were spent in Yellowknife?

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will model the probability of a compound event, and solve problems based on the combining of simpler probabilites.

Prescribed Learning Outcomes	Illustrative Examples
• construct a sample space for two or three events	c List the sample space for rolling a 6-sided die and flipping a coin. c *Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15 inclusive. A train is scheduled to arrive between 07:11 and 07:17 inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the point (6, 14). Times are expressed in whole minutes. a) How many points are there in this sample space? b) How many points have the bus and the train arriving at the same time? c) How many points have the bus arriving after the train? d) What is the probability of the bus arriving after the train?
• classify events as independent or dependent	c Classify the following events as independent or dependent: a) tossing a head in a coin toss and rolling a 6 on a die b) drawing an ace for the first card and another ace for the second, if the experiment is carried out without replacement c) drawing a king for the first card and a queen for the second, if the experiment is carried out with replacement c *Sixty percent of young drivers take driver training, and 25% of young drivers have an accident in their first year of driving. Statistics show that 10% of those who do take driver training have an accident in their first year. Are taking driver training and having an accident in the first year independent events? Support your answer with the appropriate mathematical calculations.
• solve problems, using the probabilities of mutually exclusive and complementary events	c If the probability of winning a game is $\frac{1}{31}$, what is the probability of losing the game? c *A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot?

Appendix G

Illustrative Examples

Essentials of Mathematics 10

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

PERSONAL BANKING

It is expected that students will describe consumer banking services, including types of accounts and their uses.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- name and describe various types of commonly used consumer bank accounts
- complete various banking forms
- describe the use of a bank card for automated teller machines (ATMs) and debit payments
- identify different types of bank service charges and their relative costs

► There are errors in the way the following cheque is written. Can you identified the errors?

JOHN DOE
ANY STREET
CITY, PROVINCE
POSTAL CODE

December 26 19 *97*

Pay to the order of *A. Baron* $ *12*

twelve DOLLARS

ANY BANK
YOUR TOWN
PROVINCE

John Doe

:26167...001: 1234...567...

Solution:
The errors are

- in the currency area — write $12 as 12. 00
- place line before and after written monetary amount— ______________ twelve______________xx/100

► Terry Atwal is depositing the following items into her checking account 22-763-4 today. Currency: 20 toonies; five $10 bills; 20 quarters; four dimes; six pennies; three cheques of $14.67, $53.26, and $5.64. Filled in a deposit slip for her.

____
ANY STREET
CITY, PROVINCE
POSTAL CODE

____________ 19 ___

____
DEPOSITED BY

ANY CREDIT UNION
YOUR TOWN
PROVINCE

ACCT NO. __________

CURRENCY		
COIN		
CHEQUES OR		
COUPONS		
TOTAL		
LESS Received in Cash		
Please sign in presence of Teller		
AMOUNT OF DEPOSIT $		

PERSONAL BANKING

It is expected that students will describe consumer banking services, including types of accounts and their uses.

Prescribed Learning Outcomes	Illustrative Examples
• reconcile financial statements, such as cheque books	► Complete the following cheque register. Balance: Sept. 1, $767.34 Cheques: #26 Sept. 4 to Sports Wear for $50.76; #27 Sept. 4 to Value Foods for $54.90; #28 Sept 10 to K. Turner for $200.00; #29 Sept. 21 to Holm's Rental Agency for $535.00 ; #30 Sept. 23 to CD Music for $26.92; Oct. 2 #31 to Fitness Physio for $31. 00 Deposits: Sept. 5 $535.00; Oct. 2 $250.00

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT	✓	DEPOSIT AMOUNT	DEDUCT CHEQUES ADD DEPOSITS	BALANCE FORWARD
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➜	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➜	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➜	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➜	
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		FOR				BALANCE ➜	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➜	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➜	
		TO				CHEQUE – DEPOSIT +	
		FOR				BALANCE ➜	

PERSONAL BANKING

It is expected that students will describe consumer banking services, including types of accounts and their uses.

Prescribed Learning Outcomes	Illustrative Examples
	► *David Hartley had a balance of $231.18 in his account. On September 30, he wrote cheque #344 for $198.00 to Sears. On the same day, he received his pay for $488.90. October 1, he wrote a cheque #345 for $385 to Lanes Rental Group for his apartment. What is his balance? ► *Complete a reconciliation statement form for this account

ACCU CREDIT					DATE			
				BALANCE FORWARD	20	08	350	00
DESCRIPTION	DEBITS		CREDITS		DAY	MO.	BALANCE	
DEPOSIT			452	51	21	08	802	51
CHEQUE #98	102	90			25	08	699	61
CHEQUE #99	141	12					558	49
CHEQUE #100	24	88			27	08	533	61
CHEQUE #101	56	70					476	91
DEPOSIT			215	00			691	91
DEPOSIT			280	00	30	08	971	91
CHEQUE #102	125	45					846	46
SERVICE CHARGE	8	75			31	08	837	71

PERSONAL BANKING

It is expected that students will describe consumer banking services, including types of accounts and their uses.

Prescribed Learning Outcomes	Illustrative Examples

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES ADD DEPOSITS	BALANCE FORWARD	
									350	00
Aug. 21		TO *Deposit*				452	51	CHEQUE – DEPOSIT +	452	51
		FOR						BALANCE ➜	802	51
25	98	TO *Esso*	102	90				CHEQUE – DEPOSIT +	102	90
		FOR						BALANCE ➜	699	61
25	99	TO *Tires*	141	12				CHEQUE – DEPOSIT +	141	12
		FOR						BALANCE ➜	558	49
27	100	TO *Telephone*	24	88				CHEQUE – DEPOSIT +	24	88
		FOR						BALANCE ➜	533	61
27	101	TO *Hydro*	56	70				CHEQUE – DEPOSIT +	56	70
		FOR						BALANCE ➜	476	91
27		TO *Deposit*				215	00	CHEQUE – DEPOSIT +	215	00
		FOR						BALANCE ➜	691	91
30		TO *Deposit*				280	00	CHEQUE – DEPOSIT +	280	00
		FOR						BALANCE ➜	971	91
Sept. 1	102	TO *Pete's Shack*	125	45				CHEQUE – DEPOSIT +	125	45
		FOR						BALANCE ➜	846	46
3	103	TO *Insurance*	211	11				CHEQUE – DEPOSIT +	211	11
		FOR						BALANCE ➜	635	35
6		TO *Deposit*				2000	00	CHEQUE – DEPOSIT +	2000	00
		FOR						BALANCE ➜	2635	35
7		TO *Sears*	854	00				CHEQUE – DEPOSIT +	854	00
		FOR						BALANCE ➜	1781	35
7		TO *Gas*	57	10				CHEQUE – DEPOSIT +	57	10
		FOR						BALANCE ➜	1724	25
8		TO *Eaton's*	146	58				CHEQUE – DEPOSIT +	146	58
		FOR						BALANCE ➜	1577	67

PERSONAL BANKING

It is expected that students will describe consumer banking services, including types of accounts and their uses.

Prescribed Learning Outcomes	Illustrative Examples

STATEMENT OF RECONCILIATION

Bank Statement	Chequebook Record
FINAL BALANCE shown on this bank statement	FINAL BALANCE shown in chequebook record
ADD DEPOSITS made since statement date	SUBTRACT all withdrawals which are not shown in chequebook record
Subtotal ________________ Subtract outstanding cheques	
FINAL BALANCE: ________________________	FINAL BALANCE: ________________________

WAGES, SALARIES, AND EXPENSES

It is expected that students will solve problems involving wages, salaries, and expenses.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • calculate hours worked and gross pay	► Ilia works at a restaurant. She makes \$6.75 per hour. The hours she worked last week are summarized in the chart below: (see chart below) a) Calculate her hours worked. b) Calculate her gross pay
• calculate net income using deduction tables (focus on weekly) with different pay periods	► Paul Wieler works a 40-hour week and receives \$7.75 per hour. Time and a half is paid for all hours over 40 in a given week. Last week he worked 44 hours. Find his net pay if the only deductions are CPP, EI, and Income Tax (Code 0).
• calculate changes in income	► Suppose that you were earning \$13.55 an hour. After three months, you received a raise of 10%. What is your new hourly rate?
• develop a budget that matches predicted income	► *Susan predicts she will earn \$12.00 per hour working 40 hours per week construction full time. Develop a monthly budget that matches her income. Consider both deductions and expenses. Consider all types of spending. Explain your reasoning. ► Complete the table for Jane, who earns \$1500 for the weeks ending January 15 and 31.

Mon	Tue	Wed	Thu	Fri	Sat	Sun
3.5 h	–	–	4.0 h	6 h	7.5 h	3.5 h

	Payroll Ending____	Payroll Ending____
Gross Pay		
EI Premium		
CPP Premium		
Taxable Income		
Net Income		

SPREADSHEETS

It is expected that students will design and use a spreadsheet to make and justify decisions.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • create a spreadsheet using different formatting options	► What is the difference between a row and a column? ► What are the three types of information that can go into a cell? Explain in your words what each is. ► Write the specific steps to: - format a number to two decimal places - format a number to show dollars and cents (currency) - change a column's width. - change cell values - add/delete rows/columns
• use a spreadsheet template to solve problems	► (spreadsheet below)

	A	B	C	D
1	Length			
2	Width			
3				
4	Perimeter			
5	Area			
6				

Use the spreadsheet of a sod-laying company to calculate the areas and perimeters of these custom orders

	length	width	area	perimeter	diagonal length
a)	12	9			
b)	18	5			
c)	22	13			
d)	23.5	17.32			

SPREADSHEETS

It is expected that students will design and use a spreadsheet to make and justify decisions.

Prescribed Learning Outcomes	Illustrative Examples
• create a spreadsheet using formulas and functions	► Create a new spreadsheet to record a teacher's marks. The term mark is the average of the two tests. The final mark is the exam mark times 40% plus the term mark times 60%.

	A	B	C	D	E	F	G
1	*Your Name*	Name	test 1	test 2	term mark	exam	final
2		Bob	70	75		65	
3		Susan	85	82		75	
4		Jeff	75	70		65	
5		Randi	96	89		82	
6		Carrie	56	65		51	
7		Scott	87	78		72	

► *Questions for any Spreadsheet
a) Write a formula to add the values and cells C12 and C13.
b) Write a formula to find a difference between cells D10 and D9.
c) Write a formula to multiply A10, A11 and A12 together.
d) Write a formula to add cells B6 to B10.

• use a spreadsheet to answer "what-if" questions

► *

	A	B
1	initial amount	$5000.00
2	rate	0.05
3	years	3
4		
5	final value	$5788.13
6		

Roger has $5000. He would like to invest it to eventually have $10 000. He thought that investing at 5% for three years would work. As you can see, he was wrong. Find the right combination of time and rate, given $5000, to achieve the amount of $10 000. Remember that 5% is expressed as 0.05. You are also not allowed to use an interest rate greater than 15%. Can you find more that one combination?

• identify where spreadsheets could be effectively used

► List at least two possible types of problem situations that could be solved using spreadsheets. Explain your choices.

RATE, RATIO AND PROPORTION

It is expected that students will apply the concepts of rate, ratio, and proportion to solve problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • use the concept of unit rate to determine the best buy on a consumer item and justify the decision	► Bruce is considering buying orange juice. Which is the best buy, 355 mL. for 99 cents or 500 mL for $1.19?
• solve problems on the application of sales tax in Canada	► Calculate the sales tax paid for jeans priced at $44.99 when PST is 7%.
• describe a variety of sales promotion techniques and their financial implications for the consumer	► Two computers have similar features. One is priced at $2100 and the other is priced at $2400 but is on sale for 15% off. Which costs less?
• solve rate, ratio, and proportion problems involving length, area, volume, time, mass, and rates derived from these	► *Find the volume of a box that is 2 cm. x 5 cm. x 1 cm. What happens to the volume of the box if each dimension is doubled? If a box has a volume of 100 cm^3, what would the volume be if each of the dimensions were doubled? ► Provide students with a recipe that contains fractions. Ask what the quantities would be if the recipe were doubled and/or tripled. ► A logging truck can hold 44 000 kg. An average log has a mass of 1100 kg. How many logs would fit on the truck?

TRIGONOMETRY

It is expected that students will demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- apply ratio and proportion in similar triangles.

► The length of a shadow cast by a tree is 10.5 m. At the same time, the length of a shadow cast by a 45 cm tomato plant is 75 cm. How tall is the tree?

► A 6.0 m ladder rests against the side of a house. Martin is standing right under the ladder. He is 1.70 m tall and his head touches 2.0 m up the ladder. How high does the ladder reach up the wall of the house?

► *The side of a triangular roof of a house is 6 m long and the roof is 2.5 m tall. The model of the house constructed by the architect has a roof that is 6 cm long and its side measures 4 cm. Draw a sketch of both the roof of the house and the model and fill in the known measurements. Hint: Which triangle should you consider in the roof when solving this problem: the whole roof or half the roof?
- How long is the actual roof of the house?
- How tall is the roof of the model house?

► Use the data tables and diagrams to complete the tasks outlined below.

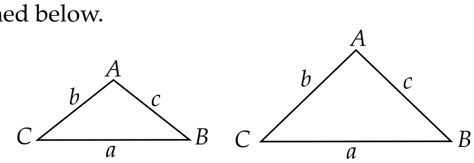


Diagram 1 Diagram 2

Part I
- Measure and record the sizes of all angles on the triangle worksheet.
- Use the table below (Table 1) to record the angles and ratios accurate to the unit digit:

Table 1

angle A (degrees)	
angle B (degrees)	
angle C (degrees)	
angle A’ (degrees)	
angle B’ (degrees)	
angle C’ (degrees)	
angle A / angle A’	
angle B / angle B’	
angle C / angle C’	

- What do you notice about the ratios of angles of the triangles?

TRIGONOMETRY

It is expected that students will demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles.

Prescribed Learning Outcomes	Illustrative Examples

Part II

- Label the sides of each triangle. For example, the side opposite of angle A is called a, the side opposite of angle A' is called a', etc..
- For each triangle, measure and record the lengths of all sides on the triangle and record this in a table like the one below (Table 2).

Table 2

Length of side a (cm)	
Length of side b (cm)	
Length of side c (cm)	
Length of side a' (cm)	
Length of side b' (cm)	
Length of side c' (cm)	
a/a'	
b/b'	
c/c'	
a/b	
a'/b'	
a/c	
a'/c'	
b/c	
b'/c'	

- Calculate and record all nine ratios of sides accurate to one decimal place. Use the values you found in the previous section. For example, triangle ABC has ratios a/b, b/a, a/c, c/a, b/c, and c/b.
- What do you notice about the lengths of sides ratios between pairs of triangles?

TRIGONOMETRY

It is expected that students will demonstrate an understanding of ratio and proportion and apply these concepts in solving triangles.

Prescribed Learning Outcomes	Illustrative Examples

- use the trigonometric ratios sine, cosine and tangent in finding the sides or angles in right triangles

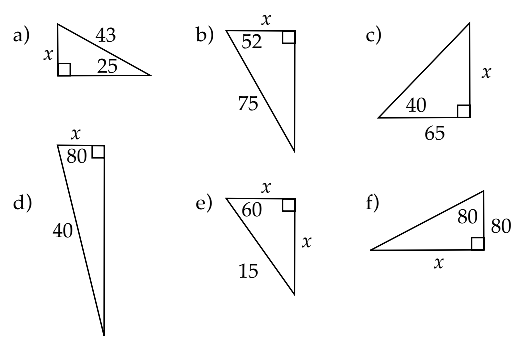
► Use the data tables from the previous illustrative example to answer the following:

- Set your calculator to degree mode. Use your findings from the previous exercise to fill in the following table for the right triangle ABC.

sine A		sine B	
cosine A		cosine B	
tangent A		tangent B	

- Compare the table entries to the lengths of sides ratios you found in the previous exercise. What do you notice?
- Repeat the exercise with triangle A'B'C' from the previous worksheet.

► State whether you would use the sine, the cosine or the tangent ratio to find x for each figure.



► If a grasshopper jumps a vertical distance of 1.5 cm and a horizontal distance of 8.0 cm., at what angle did it rise from the ground?

► A 6 m ladder is leaning against the wall of a house. The angle between the ladder and the ground is 68°. Give your answers accurate to two decimal places.
- How far away is the ladder from the wall?
- How high up the house does the ladder reach?

► *An airplane is flying at an altitude of 10 km. You spot the airplane at an angle of 55°. Give your answers to the nearest kilometre.
- How far away is the airplane directly from you?
- How far away is the airplane on the ground from you?
- Are these measurements exact? What did you assume about solving this problem? Hint: Is the Earth flat? What do you see with?

GEOMETRY PROJECT

It is expected that students will complete a project which includes a 2-D plan and a 3-D model of some physical structure.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

It is expected that students will:

- measure lengths using both SI (Metric) and Imperial units
- estimate measurements of objects in SI and Imperial systems including:
  - length
  - area
  - volume
  - mass

Illustrative Examples

► Use a tape measure to find the width of your desk or worktable. Measure it to the nearest $^1/_4$ inch and also to the nearest centimeter.

► Identify the appropriate unit of measurement in the SI and Imperial systems.

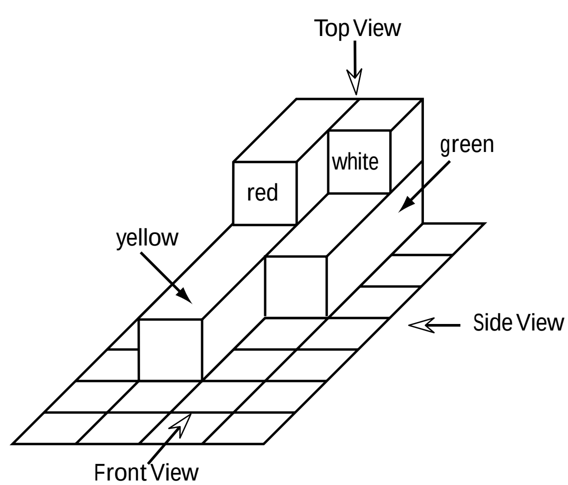
Item	Metric - SI	Imperial
a. distance from Cranbrook Abbotsford		
b. length of a pen		
c. area of a soccer field		
d. volume of a swimming pool		
e. height of Mt.Washington		
f. mass of a compact car		

Prescribed Learning Outcomes

- draw top, front, and side views for both 3-dimensional rod / block objects and their sketches

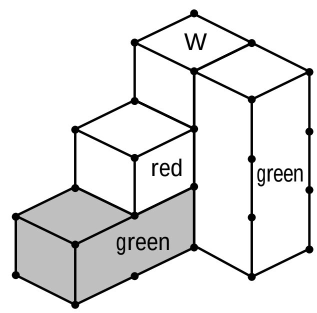
Illustrative Examples

► Given the 3-D rod design shown below, draw the 2-D top, front, and side views.



GEOMETRY PROJECT

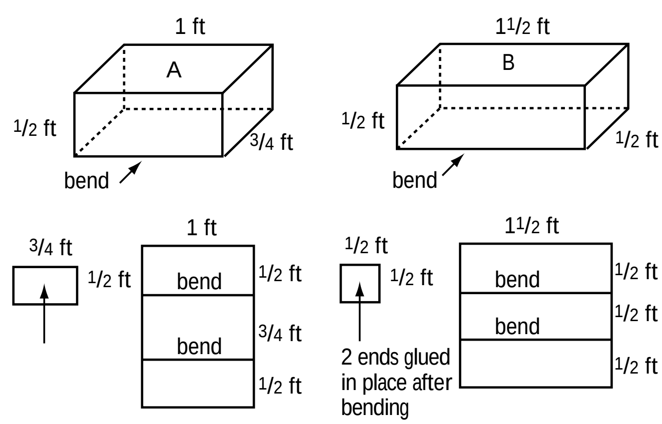
It is expected that students will complete a project which includes a 2-D plan and a 3-D model of some physical structure.

Prescribed Learning Outcomes	Illustrative Examples
• sketch and build 3-D designs using isometric dot paper	► Remove the shaded Cuisenaire rod from the figure and draw the new figure on isometric dot paper. 
• determine the relationships among linear scale factors, areas, surface areas, and volumes of similar figures and objects	► Predict what happens to the area of a 4 in. x 5 in. square if 1 inch is added to each dimension. ► Predict what happens to the area of a 5 in. x 5 in. quilting square if 1 inch is added to each dimension. Verify your prediction.
• enlarge or reduce a dimensioned object according to a specified scale	► Using graph paper draw a $^1/_2$ scale version of the cover of your math textbook.
• solve problems involving linear dimensions, area, and volume	► Find the total volume of six shoeboxes whose individual dimensions are 15 cm x 15 cm x 30 cm. ► A toy box has dimensions 120 cm x 80 cm x 60 cm. A new toy box has half the length, half he width and half the height. a) Calculate the dimensions of the new toy box. b) Calculate the volume of the new toy box. c) Is the new volume half the old volume? Explain why or why not.

GEOMETRY PROJECT

It is expected that students will complete a project which includes a 2-D plan and a 3-D model of some physical structure.

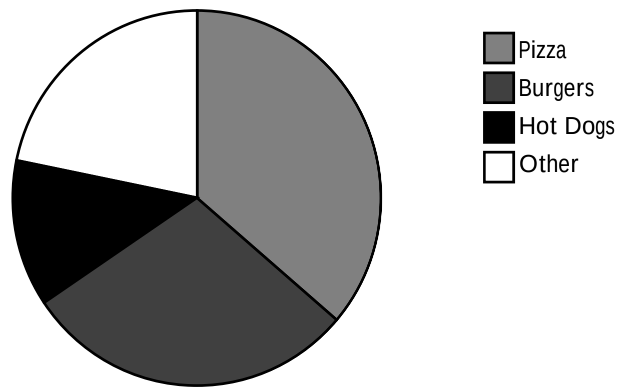
Prescribed Learning Outcomes	Illustrative Examples
• interpret drawings and use the information to solve problems	► *Use the drawings below to answer the following questions: - A box manufacturing company produces two open boxes, as shown below. Are the volumes identical? Explain your reasoning. - If the material cost is $4.50/ft² and glue costs $0.70/ft, which box is cheaper to make? How much do you save? - If a company manufactures 1000 of the cheaper boxes in a production run, how much money is saved?
• complete a project that includes a 2-D plan and 3 -D model of some physical structure	► *Prepare a 2-D plan and construct a 3-D model of a building such as a garden shed or a doghouse.



PROBABILITY AND SAMPLING

It is expected that students will implement and analyse sampling procedures, and draw appropriate inferences from the data collected.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • read and interpret graphs	► The graph below shows the results of a survey of 60 students. What do you think the survey is about? About how many students are in each category?



PROBABILITY AND SAMPLING

It is expected that students will develop and implement a plan for the collection, display, and analysis of data using technology as required.

Prescribed Learning Outcomes

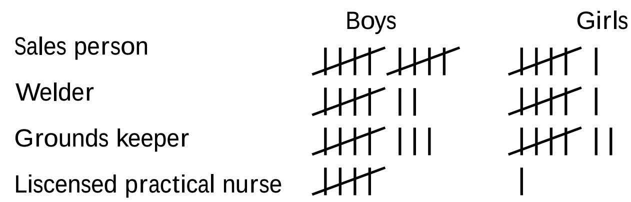
- discuss how collected data are affected by the nature of the sample, the method of collection, the sample size, and biases

Illustrative Examples

► Carmen designed and handed out 100 questionnaires to middle-year students in her school. One question she asked was this:

What do you want to be? Choose one.
o Sales person o Welder
o Grounds keeper o Liscensed pratical nurse

50 questionnaires were returned. Here are the results.



Carmen reached the conclusion that most students will work in sales.

Do you agree with each of the following?
What else might Carmen have done?
a) the wording of Carmen's question
b) the method of gathering data
c) the sample she chose to survey
d) the conclusion she reached

Prescribed Learning Outcomes

- describe issues to be considered when collecting data (e.g., appropriate language, ethics, cost, privacy, cultural sensitivity)

Illustrative Examples

► Explain what would be the most appropriate methods for collecting data

For each of these questions:
a) Does smoking cause lung cancer?
b) Does pet ownership enhance the quality of life for senior citizens?

Identify potential ethical problems, the need for sensitivity to personal and cultural beliefs, and cost when designing questions and collecting data.

PROBABILITY AND SAMPLING

It is expected that students will develop and implement a plan for the collection, display, and analysis of data using technology as required.

Prescribed Learning Outcomes	Illustrative Examples
• select, defend and use appropriate methods of collecting data: - designing and using questionnaires - interviews - experiments - research	► As a member of your school's student council, you wish to determine the number of students who will attend a school dance. Explain how you could collect this information. ► A city council is trying to determine if a major road in your district needs to be widened. Explain a procedure that could be used to help city council make this decision. ► Bill surveyed people entering a theatre and asked them if they would support building a new sports arena in the community. Discuss this method of collecting data in terms of its appropriateness.
• determine and use measures of central tendency to support decisions	► Find the mean, median, and mode for the following sets of test scores: a) 42, 62, 68, 73, 75, 75, 86, 89, 92 b) 10, 20, 20, 30, 50, 70 ► Do a survey of your class on favourite car colour, and determine the mode. ► Explain why each of the following people might select the mean, median, or mode in a set of data. a) a store owner deciding what sizes of shoes to order b) someone moving to a new city and looking at house costs c) reporting the average score on a test
• use sample data to make predictions and decisions	► Collect class data on shoe size. Use this data to predict the number of pairs of size 11 men's shoes found in a sample of 1000 grade 10 boys. If you owned a shoe store, what decision would you make concerning the purchase of men's shoes?

PROBABILITY AND SAMPLING

It is expected that students will develop and implement a plan for the collection, display, and analysis of data using technology as required.

Prescribed Learning Outcomes	Illustrative Examples
• use suitable graph types to display data (by hand or using technology)	► Research examples of statistics used in a variety of situations (e.g., population statistics, professional sport, earthquake prediction, weather, loan risk). Report your findings to the class using appropriate graphs to display data. ► Annual precipitation in the various regions of Canada is shown below. Coastal regions 100 - 140 cm Ontario and Quebec 65 - 90 cm Prairie region 40 - 55 cm Northlands 15 - 40 cm Display the above data using a suitable graph. Breifly describe why you choose the graph that you did. ► The following data were collected with a telephone survey of 1000 people. Each was asked to name his or her favourite spectator sport. (see table below) Construct a circle graph to display this data.
• critique ways in which statistical information and conclusions are presented by the media and other sources	► *Collect data from a newspaper, magazine, radio, television, or the Internet. a) How were samples for the data selected? Why do you think they were selected that way? Are they biased? b) Were the data collection methods appropriate for the data and the issue? c) How would you do it differently? Why? d) Are the data presented clearly and honestly? e) Do the conclusions follow logically for the data? f) What questions are left unanswered? Is this deliberate?

Favourite Sport	Number	As a Per Cent	Degrees
Hockey	450	45.0%	162
Football	240		
Baseball	120		
Soccer	58		
Volleyball	24		
Other	108		
Total	1000		

Appendix G

Illustrative Examples

Essentials of Mathematics 11

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

RELATIONS AND FORMULAS

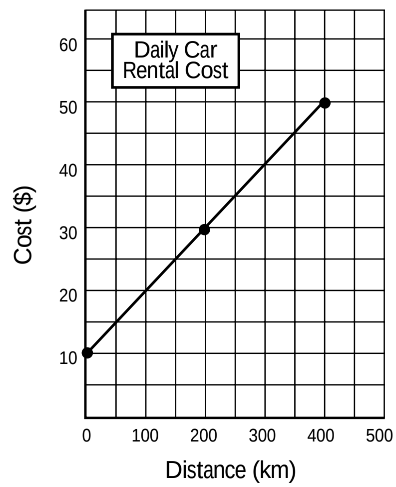
It is expected that students will represent and interpret relations in a variety of contexts.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • express a linear relation of the form $y = mx$ - in words - as a formula - with a table of values - as a graph	c *George rents a car for one day. The cost to rent a car is calculated at \$0.15/km plus a fixed charge of \$30.00 per day. Express this linear relation: - in words - as a formula - with a table of values - as a graph Describe the differences in the various forms of expressing the linear function. c Lorraine begins a new part-time job for her dad at the family restaurant. Her first week is a probationary week in which she earns \$7.50 per hour. The pay she earns each day depends upon how many hours she works. Express this linear relation in: - words - as a formula - with a table of values - as a graph
• interpolate and extrapolate values from the graph of a linear relation	c Using the previous illustrative example, determine: - what Lorraine's pay would be if she worked 2 ½ hours - what Lorraine's pay would be if she worked 5 hours
• determine the slope of a linear relation and describe it in words and interpret its meaning in a problem context	c Using Lorraine's daily pay graph, determine the slope of the graph and explain what this means.

Relations and Formulas

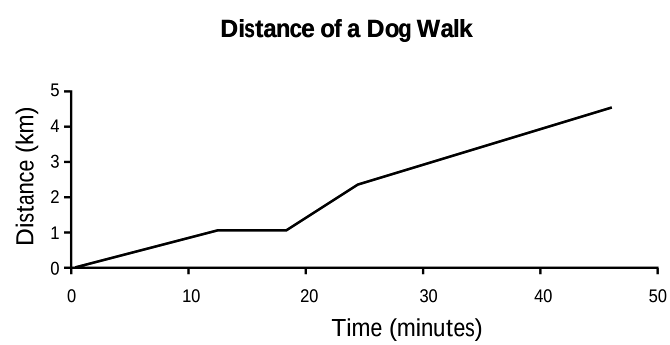
It is expected that students will represent and interpret relations in a variety of contexts.

Prescribed Learning Outcomes	Illustrative Examples
• interpret the graph of a relation and describe it in words	c The cost associated with renting a car for a day from Company A is shown on the graph. The cost depends on the number of kilometres driven plus a daily charge.



a) What does the cost depend on?
b) Determine the slope of the line.
c) What does this slope represent?
d) What is the daily charge for a rental?
e) Write the formula that describes this problem (using words and using letters).

c The following sketch represents a person walking a dog. Write a story to correspond to the graph.



RELATIONS AND FORMULAS

It is expected that students will represent and interpret relations in a variety of contexts.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • construct a graph of a relation from its description in words.	c *Construct a graph that matches each of the following statements: a) A car speeds up, maintains a constant speed, and then slows down. b) A woman climbs a hill at a steady pace and then runs down the hill. c) A person walks at a steady pace, then jogs at a steady pace, then stops to do push-ups.
• evaluate formulas	c Find the area of a trapezoid with bases of 8 cm and 12 cm, and a height of 7 cm. A = 8, b = 12, h = 7 $A = \left(\frac{a+b}{2}\right)h$ c To start a summer yard work program, a student took out a \$3000 loan for 3 months. The student paid \$90 interest on the loan. Find the monthly interest rate, using the formula: $r = \frac{I}{pt}$

INCOME AND DEBT

It is expected that students will demonstrate an awareness of selected forms of personal income and debt.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

It is expected that students will:

- solve problems involving performance-based income, including commission sales, piecework and salary plus commission

Illustrative Examples

- Weili is an insurance salesperson. He receives a 30% commission on the first year's premium of each life insurance policy he sells. Weili sells three life insurance policies this week. If the premiums for the first year are $350, $400, and $440, what was Weili's gross pay this week?
- Weili decides to work for another company that will give him a weekly salary of $350 and a commission rate of 6% for any goods sold over $3000. If he sold goods worth $5688 this week, what is his gross pay?
- *Evan works for a machine company that pays him 1% on the first $5000 sold, 2% on the next $15,000 sold and 3% on anything over $20,000. What would his gross pay be if he sold $25,000?
- Mary works for a packaging company, assembling condiment packages for a fast-food industry. She earns $0.08 for each package. Calculate her earnings for the week. The calculations for Monday have been done for you.

Day	# of Packages	Daily Earnings
Monday	760	760 x 0.08 = $60.80
Tuesday	690	
Wednesday	792	
Thursday	420	
Friday	608	
Saturday	201	
	Weekly Earnings	

- Jace Manufacturing Co. makes pallets. Their workers get $1.20 per pallet. If a worker assembles 100 pallets in a day, what would be the gross pay in a five-day week?

INCOME AND DEBT

It is expected that students will demonstrate an awareness of selected forms of personal income and debt.

Prescribed Learning Outcomes	Illustrative Examples
• use simple and compound interest calculations to solve problems	c May Hill owes $200 on her credit card. She misses a payment. She must pay interest for 28 days. If the yearly interest rate is 18%, how much interest would she pay for 28 days? c What is the compound interest earned on a deposit of $1000 after two years if it is compounded semi-annually at a rate of 6%? c Review the following information about a $20,000 loan to be repaid at the rate shown over a period of 9 years. Explain how you could use a spreadsheet to find the closing balance after 8 years.

Year	Opening Balance	Interest Rate %	Interest Charged	Annual Payment	Closing Balance
1	$20 000.00	5	$1 000.00	$3 000.00	$18 000.00
2	$18 000.00	5	$ 900.00	$3 000.00	$15 900.00
3	$15 900.00	5	$ 795.00	$3 000.00	$13 695.00

Prescribed Learning Outcomes	Illustrative Examples
• solve consumer problems involving – credit cards – exchange rates – personal loans	c A purchase of $220 was made on June 14. Nothing was paid on the first due date of July 20. Calculate the daily interest charge for the purchase if the rate is 18.6% per year. The next due date is August 20. c Amy would like to buy a computer. She has found one that she likes for $2400 plus taxes. She doesn't have the money right now so she decides to take out a personal fixed rate loan. a) How much will she pay per month if she takes the loan out for two years? b) How much interest will she pay?

INCOME AND DEBT

It is expected that students will demonstrate an awareness of selected forms of personal income and debt.

Prescribed Learning Outcomes	Illustrative Examples

- A banker needs to provide clients with information on foreign exchange. Use the foreign exchange chart provided, or a current chart from a newspaper, to answer the following questions.
  a) Calculate the cost in Canadian dollars of a refrigerator that costs $850 US.
  b) Calculate the cost in US dollars of an outboard motor selling in Canada for $1200.
  c) Hans receives a cheque for 100 Swiss francs from his uncle in Berne. How many Dutch guilders would he get for this cheque? How many Canadian dollars?

Exchange - Cross Rates

	Canada dollar	US dollar	British pound	German mark	Japanese yen
Canada dollar	—	1.3743	2.0762	0.9227	0.012850
US dollar	0.7276	—	1.5107	0.6714	0.009350
British pound	0.4816	0.6619	—	0.4444	0.006189
German mark	1.0838	1.4894	2.2501	—	0.013927
Japanese yen	77.82	106.95	161.57	71.81	—
Swiss franc	0.8821	1.2122	1.8313	0.8139	0.011335
French franc	3.7230	5.1165	7.7297	3.4352	0.047841
Dutch guilder	1.2134	1.6676	2.5194	1.1196	0.015593
Italian lira	1156.07	1588.79	2400.23	1066.71	14.855491

Exchange - Cross Rates

	Swiss franc	French franc	Dutch guilder	Italian lira
Canada dollar	1.1337	0.2686	0.8241	0.000865
US dollar	0.8249	0.1954	0.5997	0.000629
British pound	0.5460	0.1294	0.3969	0.000417
German mark	1.2287	0.2911	0.8931	0.000937
Japanese yen	88.23	20.90	64.13	0.067315
Swiss franc	—	0.2369	0.7269	0.000763
French franc	4.2208	—	3.0681	0.003220
Dutch guilder	1.3757	0.3259	—	0.001050
Italian lira	1310.64	310.52	952.72	—

INCOME AND DEBT

It is expected that students will demonstrate an awareness of selected forms of personal income and debt.

Prescribed Learning Outcomes	Illustrative Examples

c

	Card 1	Card 2
Balance Owing	$750.00	$750.00
Interest Rate Per Year	15%	18%
Annual Card Fee	$36.00	$0.00

Use the information from the chart above to answer the following questions.

a) Find the interest charged on both credit cards for the month, using the formula I = p x r x t (Interest = Principle x Interest Rate x Time).

b) Including the card fee, which card is cheaper to use to make a cash advance of $750.00 that is paid back after one month.

c *Fill in the missing information

Credit Card Statement Account #123456

09/07	Previous Balance Owing	$125.00	
	Payment Thank you	100.00	
11/07	Ralph's Tire Shop	36.99	
20/07	The Shoe Shop	27.99	cr
27/07	Super Sports	24.87	
30/07	Disco Jim's	79.99	

Maximum Credit	Balance on Last Statement	Interest Charged	Total Purchases This Statement	Total Payment and Credits This Statement	New Balance
$500.00	$125.00	$2.30			
Cue Date: August 22			Minimum Payment 10% of Balance		
			Credit Remaining		

*From the statement, identify

a) credit limit

b) daily interest rate

c) if previous bill was paid

d) total expenditures

DATA ANALYSIS AND INTERPRETATION

It is expected that students will analyse data with a focus on the validity of its presentation and the inferences made.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- display and analyse data on a line plot

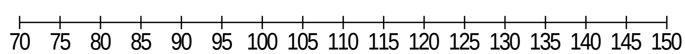
c The following table lists the year's top 10 box-office movies (as of April 17, 1997).

Movie Title	Gross Earnings	Time in Theatres
101 Dalmations	$135,848,836	19 weeks
Jerry Maguire	$148,244,457	17 weeks
Liar, Liar	$120,015,240	3 weeks
Michael	$93,767,309	15 weeks
Ransom	$136,448,821	22 weeks
Scream	$88,826,827	16 weeks
Space Jam	$90,384,232	17 weeks
Star Trek: First Contact	$92,017,585	20 weeks
Star Wars (Reissue)	$137,138,234	10 weeks
The English Patient	$73,040,322	21 weeks

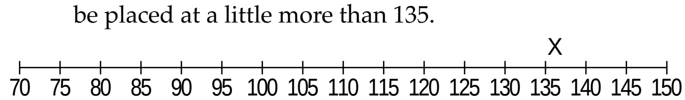
In your notebook, construct a line plot for these data following these instructions:

1. Draw a horizontal line with a rule.

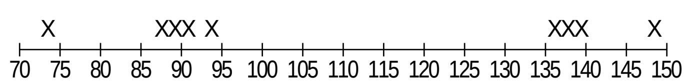
2. Now put a scale below the line using your ruler. To do this find the smallest and the largest gross earning values ($73,040,322 and $148,244,457). A suitable scale might run from $70 million to $150 million, with an interval of 5.



3. Now plot each movie's earnings by placing an X above the line at the appropriate location. For example, 101 Dalmations earned $135,848,836. Therefore, an X would be placed at a little more than 135.



4. Continue until you have plotted each movies gross earnings.



DATA ANALYSIS AND INTERPRETATION

It is expected that students will analyse data with a focus on the validity of its presentation and the inferences made.

Prescribed Learning Outcomes	Illustrative Examples

- manipulate the presentation of data to stress a particular point of view

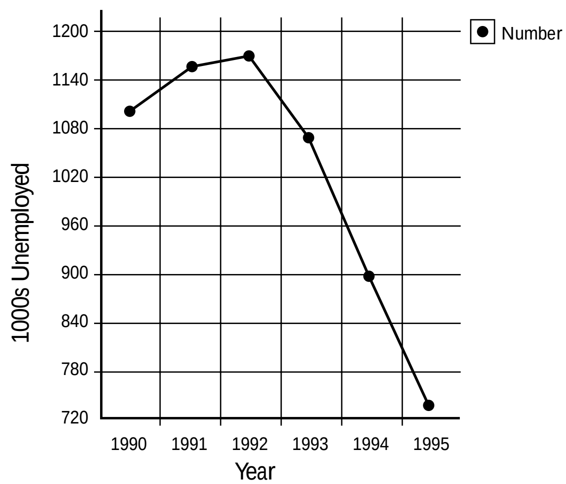
c *Assume that the following data represent the number of Canadians who were unemployed during the years 1990 - 1995.

Unemployed Canadians (1000s)

Year	1990	1991	1992	1993	1994	1995
Number	1105	1151	1164	1060	897	738

A federal employment agency wishes to demonstrate that the government's policies and practices have caused a dramatic decrease in unemployment. The agency has produced this graph:

Canadian Unemployment



a) What is your overall impression of this graph?
b) How was the graph constructed to give this impression?
c) Assume that you are a member of the political opposition party. You feel that the government's policies and practices have not caused a significant decrease in unemployment. As part of your statement to the press, you wish to include a graph. Using the data from the chart, construct a graph that will support your party's point of view.

MEASUREMENT TECHNOLOGY

It is expected that students will determine measurements in Systeme International (SI) and Imperial systems using different measuring devices.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

It is expected that students will:

- select and use appropriate measuring devices and measurement units (in Imperial or SI units)

Illustrative Examples

- Use calipers to measure:
  - the inside diameter of a piece of pipe
  - the depth of a cylinder
  - the internal and external diameters of a cylinder
- Use a micrometer to measure:
  - the thickness of a human hair
  - the thickness of one sheet of paper
- Using a metre stick, ruler, tape measure, or any other suitable device, measure the following rectangular shaped items in your classroom. For each item, give the length and width. Estimate your answers first.

Item	Estimate SI	Actual SI	Estimate Imperial	Actual Imperial
a) desktop				
b) textbook				
c) classroom				
d) window				
e) door				
f) teacher's desk				

MEASUREMENT TECHNOLOGY

It is expected that students will determine measurements in Systeme International (SI) and Imperial systems using different measuring devices.

Prescribed Learning Outcomes	Illustrative Examples

- perform basic conversions within and between the Imperial and SI systems using technology as appropriate

c

Imperial/Metric Conversion Table

Capacity	
1 Canadian Gallon	4.546 litres
1 litre	0.220 Canadian Gallons
Volume	
1 cubic inch	16.387 cubic centimetres
1 cubic centimetre	0.061 cubic inches
1 cubic yard	0.765 cubic metres
1 cubic metre	1.308 cubic yards
Length	
1 inch	2.540 centimetres
1 centimetre	0.394 inches
1 foot	0.305 metres
1 metre	3.281 feet
1 yard	0.914 metres
1 metre	1.094 yards
1 mile	1.609 kilometres
1 kilometre	.621 miles
Area	
1 square inch	6.452 square centimetres
1 square centimetre	0.155 square inches
1 square foot	0.093 square metres
1 square metre	10.764 square feet
1 square yard	0.836 square metres
1 square metre	1.196 square yards
1 acre	0.405 hectares
1 hectare	2.471 acres
Mass	
1 ounce	28.350 grams
1 gram	0.035 ounces
1 pound	0.454 kilograms
1 kilogram	2.205 pounds

Convert the following measures into the units (within the same system) as indicated:

a) 3 m = ____ cm
b) 25 mm = ____ cm
c) 0.65 m = ____ mm
d) 800 g = ____ kg
e) 5 ft. = ____ in.
f) 3 yd. = ____ ft.
g) 27 in. = ____ ft. ____ in.
h) 1 mi. = ____ yd.

MEASUREMENT TECHNOLOGY

It is expected that students will determine measurements in Systeme International (SI) and Imperial systems using different measuring devices.

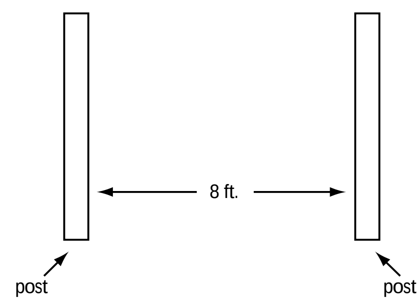
Prescribed Learning Outcomes

It is expected that students will:

- use measurement strategies to solve problems

Illustrative Examples

- How many paper clips could be made from 1000 metres of wire? How many kilometres of wire are required for a company to manufacture one million paper clips?
- A room measures 12 ft. 6 in. x 15 ft. and has a ceiling height of 116 ft.
  a) Ignoring any openings such as doors or windows, find the total square footage of wall area to be painted.
  b) If the ceiling area is to be tiled, how many tiles measuring 12 ft. x 12 ft. will it take to finish the ceiling tile?
- *Frank is building a fence with 8 ft. sections. Assuming there is to be no space between the boards, how many 1 x 6 fence boards should he order for each section? (Investigate the actual dimensions of a 1 x 6 fence board.)



Frank's fence has a total of 12 sections. How many boards should he order, if he includes 3% extra to allow for unusable lumber?

OWNING AND OPERATING A VEHICLE

It is expected that students will analyse the cost of acquiring and operating a vehicle.

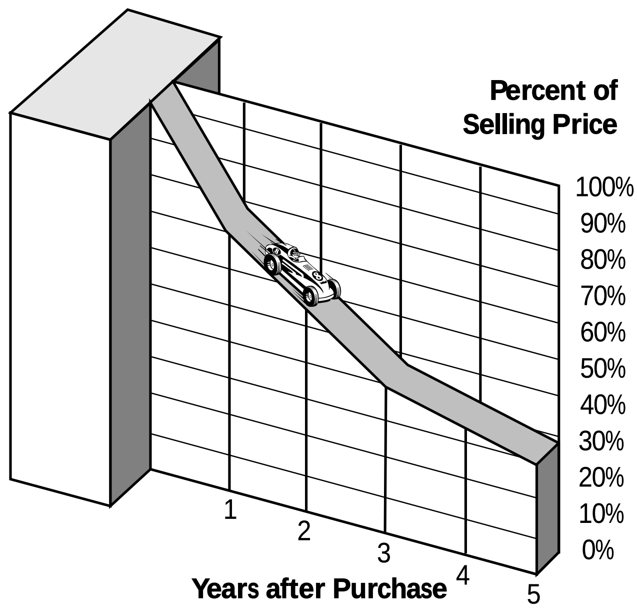
Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems involving the acquisition and operation of a vehicle including: - renting - leasing - buying - licensing - insuring - operating (e.g., fuel and oil) - maintaining (e.g., repairs, tune-ups).	• Dez would like to buy a Camaro Z28 Sports Coupe. The base price is $32,000. He adds option package #8 to it for an extra cost of $1585, plus he wants an automatic transmission for an additional $695. Freight on the car is $655. The dealership will give him a trade-in allowance of $5000 on his old car. What is his total price? • Identify some of the advantages and disadvantages of leasing a vehicle. • *The following describes a 1999 sports utility vehicle. The cost of the vehicle is $34,000 plus taxes (freight is included in this price). The monthly lease payment is $349 plus taxes for a lease term of 36 months. For leasing, a down payment of $3850 is required. As well, a refundable security deposit of $500 and the first month's payment must be made when the lease is signed. a) Calculate the total monthly payment. b) Calculate the total lease payment (total for three years), c) Calculate the residual value at the end of the lease if the residual value rate for this type of vehicle is 75% after three years. d) At the end of the three-year lease, you have the option of returning the vehicle or purchasing it for its residual value. If you decide to purchase the vehicle, what is the total cost of purchasing the vehicle (this includes the cost of the lease). • A car requires 52 litres of gasoline. If the cost of gasoline is $0.629 per litre, calculate the cost to fill the tank. • Jane took her vehicle to a car dealer for servicing. The oil and the oil filter were changed, and new wiper blades were installed. She also asked them to check over the motor as she was planning to go on a long trip. The costs were as follows: four litres of oil at $2.05 per litre, one oil filter at $5.60 and two wiper blades at $9.75 per pair. The time required for servicing was 0.6 hours. The shop rate for labour was $59 per hour. How much is this servicing going to cost Jane?

OWNING AND OPERATING A VEHICLE

It is expected that students will analyse the cost of acquiring and operating a vehicle.

Prescribed Learning Outcomes	Illustrative Examples

- The trade-in value of a car decreases yearly. This loss in value is called depreciation and is illustrated in the graph below. The per cents given in the graph are based on the price of the car when it was new.



Carli et al., *Consumer and Career Mathematics*, p 274. Reprinted and adapted with permission.

a) Mark and Cathy bought a new car for $18,900 two years ago. What is the approximate trade-in value of the car, and how much has the car depreciated?
b) Find the value of the car after seven years.
c) Make a generalization about the depreciation of a new car over several years.

PERSONAL INCOME TAX

It is expected that students will prepare a simple income tax form.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • prepare an income tax form for an individual who is single, employed and without dependents	c Calculate your personal income tax, using an appropriate printed or electronic Canada Customs and Revenue Agency form. **Note:** Canada Customs and Revenue Agency will supply student activity booklets for this.

APPLICATIONS OF PROBABILITY

It is expected that students will demonstrate an awareness of the applications of probability to real world situations.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • express probabilities as ratios, fractions, decimals, percents, and in words	c After rolling a die, express the probability that a 5 will be rolled: - as a ratio - as a fraction - as a decimal - as a percent - in words
• use probability to predict the result in a given situation	c The probability of drawing a diamond from a well shuffled deck is 1 in 4. If you made 20 draws (returning each card), how many diamonds would you theoretically expect to draw?
• determine the odds for and against a particular event occurring	c What are the odds of rolling a 1 or 2 with a 6-sided die? c What are the odds against drawing a diamond from a well shuffled deck?
• compare experimental observations with theoretical predictions	c Using a die with faces numbered 1 to 6, - what is the probability of rolling: a 6? a 4? a 1? - perform an experiment with a die and compare the results
• use probabilities to calculate expected gains and losses	c *For each game decide whether you are willing to play the game and explain your decision: a) Pay $1. Toss a coin. If it shows heads, you get $2 back. b) Pay $1. Draw a card from a shuffled deck. If it is a heart, you get $5. c) Pay $2. Draw a card from a shuffled deck. If it is a jack or ace, you get $10. d) Pay $1. Roll a dice. If a 2 or 3 shows up, you get $4 back. e) Pay $1. Toss two coins. If they are both heads, you get $3 back.

APPLICATIONS OF PROBABILITY

It is expected that students will demonstrate an awareness of the applications of probability to real world situations.

Prescribed Learning Outcomes	Illustrative Examples
	c It costs $2 to play the game "Pick the Marble." In this game, a bag contains four red marbles, one black marble, and five white marbles. You randomly pick a marble from the bag. If it is red, you win $5, and if it is black, you win $10. However, if it is white, you do not win anything. Determine the expected value of the game. If you play this game 20 times, what would your expected gain or loss be?
• communicate and justify solutions to probability problems	c Sherry takes a 100-item multiple-choice examination. Each item has four possible choices. She knows 68 of the answers and guess randomly at the other 32. calculate her expected number of correct answers. a) Explain how you calculated her expected score. b) If Sherry's score was actually higher or lower that her expected score, how would you explain it to her?

BUSINESS PLAN

It is expected that students will prepare a plan to own and operate a successful business.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • prepare a business plan to own and operate a business that includes the following information to appropriate: - monthly cost of operation (e.g. inventory, rent, wages, insurance, advertising, loan payments, etc.) - hours of operation - estimated daily sales (average) - gross/net profit, - hourly earning • draw a scale floor plan of the store	c *As owner and manager of a proposed Running Shoe Store in the mall, you must first produce a business plan before a bank will loan you the money to start. a) Decide on the type of shoe to sell, and the number of pairs of each shoe to stock. b) Determine the monthly cost of operation including rent, wages including deductions, insurance, utilities and advertising. c) Design an employee schedule. d) Develop a cash flow model including estimated daily sales, gross profit, and estimated time to profitability. e) Draw a floor plan to scale. c You decide to set up a lawn-care business. Prepare a business plan to apply for a student loan.

Appendix G

Illustrative Examples

Essentials of Mathematics 12

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

PERSONAL FINANCE

It is expected that students will solve consumer problems involving insurance, mortgages, and loans.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- solve problems involving different types of insurance

c A female smoker aged 27 buys a $65,000 policy.
a) What will she pay per month?
b) How much will it be with the disability waiver?

Permanent Plus	FEMALE		
Annual Premium Per $1,000 Insurance Non-smoker			
Age	$50,000 - 99,999	$100,000+	DWC
20	1.49	1.11	0.13
21	1.49	1.12	0.13
22	1.49	1.13	0.13
23	1.49	1.15	0.14
24	1.49	1.16	0.14
25	1.49	1.18	0.14
26	1.51	1.22	0.14
27	1.52	1.27	0.15
28	1.54	1.31	0.15
29	1.55	1.36	0.16
30	1.57	1.41	0.16
31	1.63	1.48	0.17
32	1.70	1.56	0.18
33	1.77	1.64	0.19
34	1.84	1.72	0.19
35	1.91	1.81	0.20
36	2.00	1.92	0.21
37	2.10	2.03	0.22
38	2.20	2.14	0.24
39	2.31	2.27	0.25
40	2.42	2.40	0.26
41	2.59	2.57	0.28
42	2.79	2.76	0.30
43	2.97	2.95	0.32
44	3.18	3.16	0.34
45	3.41	3.39	0.36
46	3.75	3.70	0.39
47	4.12	4.03	0.42
48	4.52	4.40	0.46
49	4.97	4.80	0.50
50	5.46	5.23	0.54
51	5.89	5.67	0.59
52	6.36	6.14	0.64
53	6.86	6.65	0.69
54	7.40	7.21	0.74
55	7.99	7.91	0.80

Add $65 annually per policy for Policy Factor

Permanent Plus	FEMALE		
Annual Premium Per $1,000 Insurance REGULAR			
Age	$50,000 - 99,999	$100,000+	DWC
20	1.69	1.30	0.15
21	1.71	1.35	0.16
22	1.73	1.39	0.16
23	1.76	1.44	0.17
24	1.78	1.50	0.17
25	1.80	1.55	0.18
26	1.97	1.64	0.19
27	1.93	1.73	0.19
28	2.00	1.82	0.20
29	2.07	1.92	0.21
30	2.15	2.03	0.22
31	2.27	2.15	0.24
32	2.39	2.28	0.25
33	2.52	2.42	0.26
34	2.66	2.57	0.28
35	2.81	2.72	0.29
36	3.00	2.92	0.31
37	3.21	3.13	0.33
38	3.43	3.36	0.36
39	3.66	3.60	0.38
40	3.91	3.86	0.41
41	4.21	4.16	0.44
42	4.54	4.49	0.47
43	4.90	4.84	0.51
44	5.28	5.22	0.54
45	5.69	5.63	0.58
46	6.25	6.12	0.63
47	6.86	6.65	0.69
48	7.53	7.22	0.74
49	8.26	7.85	0.81
50	9.07	8.53	0.97
51	9.74	9.19	0.94
52	10.47	9.88	1.01
53	11.25	10.64	1.09
54	12.08	11.45	1.17
55	12.98	12.33	1.25

Add $65 annually per policy for Policy Factor

PERSONAL FINANCE

It is expected that students will solve consumer problems involving insurance, mortgages, and loans.

Prescribed Learning Outcomes	Illustrative Examples

c

Protection I - BC $200 Deductible

Homeowners Rates

Replacement Cost Contents Included

Building Coverage	Broad Pref	Broad Std	Special Pref	Special Std
100,000	277	346	319	398
105,000	295	369	339	424
110,000	311	389	358	447
115,000	327	409	376	470
120,000	345	431	397	496
125,000	362	453	416	521
130,000	377	471	434	542
135,000	396	495	455	569
140,000	411	514	473	591
145,000	428	535	492	615
150,000	444	555	511	638
155,000	462	578	531	665
160,000	478	598	550	688
165,000	495	619	569	712
170,000	511	639	588	735
175,000	526	658	605	757
180,000	542	678	623	780
185,000	558	698	642	803
190,000	573	716	659	823
195,000	589	736	677	846
200,000	607	759	696	873
Each Add'l $5000	$15.00	$19.00	$17.25	$21.85

Contents: Each additional $1000 - $3
Outbuildings: Each additional $1000 - $3
Deductible Options - Using premium shown above
$100 - Increase by $40
$500 - Decrease by 15%, maximum $200
$1,000 - Decrease by 22%, Maximum $300

c Diane and Rod have a $100,000 home. They wish to insure the house and contents for $170,000. They have a guest cottage worth $20,000. What will their home owner's insurance cost with $100 deductible? They have had no claims and wish to purchase a comprehensive package with Company A.

c Why is it a good idea to get insurance for your house or for the contents of an apartment?

PERSONAL FINANCE

It is expected that students will solve consumer problems involving insurance, mortgages, and loans.

Prescribed Learning Outcomes	Illustrative Examples
• determine the costs involved in purchasing a home, including gross debt service ratio	c *The Baileys live in Prince George and are relocating to Victoria. They purchase a house for $220,000 and hire a mover to move their personal belongings. The mover charges $1,500. They hire a lawyer to look after legalities for a fee of $800. An appraisal is done of the property at a cost of $120. A survey of the property is done for a cost of $450. The Baileys' possession date is July 7th with the first payment due on July 15th. The interest cost for the additional 8 days is $140. Property taxes are $1,750 for which the Baileys agree to pay for the six months of July to December. Before moving in the Baileys have the yard re-sod for $3,500 and replace the stove and fridge for $750 and $900 respectively. They split the costs of the appliances with the seller. Mrs. Bailey replaces the drapes in the living room for $500 and has the master bedroom painted for $350. The seller notifies the family that they prepaid the water and garbage to the end of the year at $49.50 per month. The Baileys agree to pay for July to December. The cost to hook up the phone is $65 and to activate the natural gas costs $45. They increase their insurance to $680 from $256 per year and pay for the remaining six months. Create a spreadsheet to complete the necessary calculations for finding the additional costs besides the cost of the house itself. c A person decided to buy a house worth $85,000. The down payment will be $6,000, the monthly property taxes are $120, the heating costs are $110 per month. Calculate the maximum affordable price, the monthly mortgage payment and the gross debt ratio if the bank will finance the house at 7.50% for 25 years. The gross monthly income is $2,500.

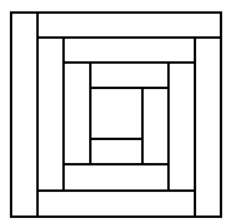
PERSONAL FINANCE

It is expected that students will solve consumer problems involving insurance, mortgages, and loans.

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving different types of mortgages	c Anne talks to her financial advisor. This is the following information she gives her advisor: - She lives in a $90,000 home on which there is an outstanding mortgage of $70,000. - She owns a $25,000 car on which she still owes $12,000. The loan was for 3 years. - She has a credit card balance of $1,575. - Anne has $30,000 in a registered pension plan and $5,000 in savings bonds. - She has $900 in her chequing account and $2,000 in a savings account Prepare a net worth statement and debt-equity ratio for Anne. c Casey and Evelyn bought a house from Mr. Jones for $165,000. They had a down payment of $35,000 and borrowed the rest from the bank at an interest rate of 6.75%. Prepare a schedule of mortgage payments for a period of nine months. The mortgage is taken over 35 years. The first mortgage payment was dated March 10. Prepare a spreadsheet or use a mortgage calculator to find the payment, interest, principle, unpaid balance, and owner's equity.

DESIGN AND MEASUREMENTS

It is expected that students will analyse objects, shapes, and processes to solve cost and design problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • analyse objects that are shown in "exploded" format	c Find an example of an exploded diagram from a hardware store, home improvement store, lumber yard, or other source. Using your exploded diagram to answer the following questions: a) Can the object in question be constructed from the drawn pieces? b) What information about the object not been included in the diagram? c) What is the purpose of providing an exploded diagam?
• draw objects in "exploded" format • solve problems involving estimation and costing for objects, shapes, or processes when a design in given	c *The following diagram is a sketch of a *log cabin* block design for quilts:  a) If you want to create a 60" x 72" twin quilt (not including a border), how many 12" x 12" blocks must you make? b) Create an exploded view of this quilt block. c) Create a diagram of the constituent parts using a scale of 1:2. d) Seams for each piece in the block are to be 1/4" wide. Bolts of material are 45" wide. What total length of material would need to be purchased to create the twin quilt? e) If material is sold only by the metre, what total length of material would need to be purchased? f) Color the design pieces using black and white. Black cloth is sold at $7.25 per metre and white cloth at $4.95 per metre. What is the cost of the quilt if these two colors are used? g) What is the cost of the material wasted?
• design an object within a specified budget	c You are given $25 to build a birdhouse. Provide a 3-D diagram and exploded view diagram of your design. Compile a materials list and cost the materials with a minimum of two suppliers. Adjust your materials to work within the $25 limit.

GOVERNMENT FINANCES

It is expected that students will demonstrate an awareness of the income and expenditures of federal, provincial, and municipal governments.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • describe government expenditures including the amounts spent on social welfare benefits, social security, education, health care, policing, armed forces, and employee wages and salaries	• What is a definition for tax? Why do we have taxes? Where do taxes come from? How are taxes distributed and who determines how they are distributed? • Research the amounts spent on various government expenditures. Using a spreadsheet make a pie chart of the various government expenditures. Change the percents that are used to show how the federal government spends its money to cents on the dollar.
• solve problems involving the calculation of selected federal taxes (e.g., GST, excise tax and duties)	• The Malden Co. imports 5000 guilders worth of flax tablecloths from the Netherlands. How much will it cost to bring the tablecloths to Canada? Assume all values are given in dollar amounts.
• calculate provincial taxes (e.g., PST, corporation capital, licenses, gasoline)	• Carr's has a payroll of $850,000 for the year. How much provincial tax is paid? • Tattlers has a corporate capital gain of $15,000,000. What will they pay in corporation capital tax?
• determine how selected municipal taxes are calculated (e.g., property tax)	• If land is assessed at $8,500 and a house is assessed at $95,000, what will the tax assessment be if the rate is 45% of the market value? • What will the mill rate be if the budget is $4,800,000 and the total assessed value is $247,000,000? • A city has a total property assessment of $375,000,000. The city prepared its budget and found that it needs $22,500,000. Find the tax rate expressed in: – cents per dollar - percent rate - mill rate

GOVERNMENT FINANCES

It is expected that students will demonstrate an awareness of the income and expenditures of federal, provincial, and municipal governments.

Prescribed Learning Outcomes	Illustrative Examples

- What is a levy? What is an educational levy and how is it used?
- A property owner receives a Statement and Demand for taxes each year. This is what it looks like. Show how the levies are calculated.

PROPERTY DESCRIPTION

Roll No.	Ward	Lot/Section	Blk/Twp	Plan	Frontage/Area
204700			12	132 384	55

Assessment		Status	Class	Total	Portion	Total Port.
Land	Building	Code	Code	Assessment	%	Assessment
$6,900	$67,900		T	$74,800	45%	$33,670

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			$33,670	19.886	$669.56
By-Law No.	**Term**	**Type**	**Frontage Levy**	**Mill Rate**	**Levy**
					0.00

EDUCATION TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	$33,670	8.019	$270.00
Provincial Education 2	$33,670		$0.00
School Division Tax	$33,670	24.278	$817.44

PROVINCIAL TAX CREDITS

Description	Credit
Manitoba Resident Homeowner Tax Assistance	$250.00

TOTAL TAXES DUE

Municipal Taxes	Education Taxes	Total Current Taxes	Provincial Credits	Net Taxes	Arrears/ Credits	Added Taxes	Total Tax Due
$669.56	$1,087.44	$1,757.00	$250.00	$1,507.00			$1,507.00

- Andy's house is assessed for $58,000. The municipal levy is 21 mills and the educational tax levy is 29 mills. There is an extra $400 that will be added for street lights, paid over the next five years. What is the total tax to be paid?

LIFE/CAREER PROJECT

It is expected that students will research career choices and perform a comparative study.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • determine what factors are important in analyzing careers • describe two specific career opportunities • identify the mathematical educational requirements for two careers • compare two careers in terms of salary, working hours, training time and cost, cost of living, and benefits	c Make a list of factors which are important to you in choosing a career. c Analyse two career choices you are interested in and look at the factors that will determine some lifestyle options. Factors that you need to consider are food, clothing, shelter, education, salary, job security, transportation, insurance, recreation, taxes, family size, and charitable political contibutions. In your descriptions of the research, provide: - a job description for each selected career - educational backgrounds or training requirements, including associated costs - salary/wage levels for each career - available employment opportunities in the field - opportunity for advancement c *From the above two careers, analyse in depth one of your choices and the lifestyle it permits. The following items should be considered in detail: - Describe the job in terms of duties, salary/wage, dress code, benefits, and employment opportunities. - Detail the educational requirements and the associated costs for the career with particular reference to the mathematical requirements. - Prepare an annual budget to determine the money needed to acquire the training. - Identify potential issues that may be relevant to the career (e.g., stress, environment, or other associated factors). - Describe a lifestyle which a career might ensure, including a budgetary analysis supporting the description. - Create a resume and application for the selected career.

INVESTMENTS

It is expected that students will demonstrate and recognize the differences among different types of financial investments.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • determine a financial plan to achieve personal goals	c List three goals you hope to achieve this year. Discuss why they are important to you and how you plan on achieving them. c List the top ten things you value most in life. Briefly describe why each value is important to you. c What are the functions of advertising and how does it influence your goals? c List your favourite and least favourite ad on TV, in magazines or newspapers. Describe why you like or dislike these ads. c List three financial goals for people at different stages of life (e.g., 20s single, 50s married). c *Complete the following worksheet for yourself. Its purpose is to help you realize the amount of income you may need later in life and the type of investments that are appropriate for your needs. a) At what age would you like to retire?____________ How old are you now?____________ b) What is the monthly income you think you will need to maintain your expected lifestyle upon retiring? (in todays dollars)____________ c) Using the figure above and assuming a 4% inflation rate, how much does this amount translate to when you expect to retire?____________(use the chart below)

Years to Retirement	Inflation factor at 4%
5	1.2
15	1.8
25	2.7
35	4.0

	d) What are your expected sources of retirement income? ____ e) What amount of investment will be required to provide an income for the retirement income above (assuming an 8% return on your investment)?______________________
• describe different investment vehicles (e.g., GICs, bonds, mutual funds, stocks, and real estate)	c What is the difference between GICs and bonds? c What is the difference between mutual funds and stocks?

INVESTMENTS

It is expected that students will demonstrate and recognize the differences among different types of financial investments.

Prescribed Learning Outcomes	Illustrative Examples
• compare and contrast different investment vehicles in terms of risk factors, rates of return, costs, and lengths of terms	c Why might someone buy a Canada Savings Bond (CSB) instead of a stock? c Why might someone choose not to purchase a 5-year term GIC?

Investment Type	Level of Risk	Yield Range in Recent Years	Liquidity	Costs to Purchase/Redeem	Inflation Protection
Savings Account	low	0.5 - 4%	high	no	no
CSBs	low	3 - 8%	high	no	no
GICs, term deposits	low	4 - 9%	varies	no	no
Gov't/Corporate Bonds	low	varies	varies	no	varies
Stocks	low to high	0 - 30%	varies	yes	yes
Mutual Funds	low to high	5 - 30%	good	yes	yes
Real Estate (income property)	varies	varies	varies	yes	varies
Collectibles	high	varies	low	yes	varies

a) Which types of investments are high risk? Low risk?
b) Which types of investments are protected against inflation?
c) Based on a comparison of level of risk to expected yield, state which investment vehicle you would use, and why.

Prescribed Learning Outcomes	Illustrative Examples
• identify reasons for investing money in RRSPs and RESPs	c What are the advantages in investing in: a) an RRSP? b) an RESP?

INVESTMENTS

It is expected that students will demonstrate and recognize the differences among different types of financial investments.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- investigate how to purchase and sell stocks

- Jon's broker advised him to invest some of his money in ABC Ltd. ABC is currently listed on a stock exchange at \$21.45 a share. How much will it cost Jon to purchase 200 shares?
- If Jon's broker charges him a fee of 2% of the value of the order, calculate the total cost to purchase the 200 shares.
- Some brokers use a commission schedule based on a per share basis, regardless of the price of the stock. Following is a fee schedule used by one broker.

Base Charge of \$40 plus:	\$0.08 per share for 1 - 499 shares
	\$0.065 per share for 500 - 999 shares
	\$0.05 per share for 1000 - 2499 shares
	\$0.04 per share for over 2500 shares

Use the schedule to answer the following:

Stock	Number of Shares	Cost per Share	Commission	Total Cost
Ashton	600	\$1.55		
Denison	3000	\$0.45		
Polyair	2300	\$3.75		
Softkey	800	\$27.27		

TAXATION

It is expected that students will demonstrate an ability to fill out an income tax form.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- fill out income tax forms for:
  - single parent with a child
  - married couple with one person working
  - married couple with children

c *Profile: Married couple with children

Tina and Brad Shoe have three children, Jimmy, Timmy, and Kimmy.

Tina works part-time at The Grocery Store. In 1998, Tina made $9,264. Tina was born April 18, 1969.

Brad is a construction worker who worked for nine months in 1998 and made $56,490. For the other three months, Brad collected employment insurance benefits. The attached T4E slip show these benefits. He paid $960 in union dues last year. In addition he contributed $4,500 to his RESP, which he will be able to claim in full.

Tina and Brad made $350worth of charitable donations last year. They also had medical expenses of $1434 and interest income of $458 on their savings account.

Complete Tina and Brad's income tax return for 1998.

T4 — STATEMENT OF REMUNERATION PAID / ÉTAT DE LA RÉMUNÉRATION PAYÉE (Revenue Canada / Revenu Canada)

- Employer's name – Nom de l'employeur: The Grocery Store
- Year / Année: 1998
- 14 Employment income – line 101 / Revenus d'emploi – ligne 101: 9264.00
- 22 Income tax deducted – line 437 / Impôt sur le revenu retenu – ligne 437: 1574.88
- 10 Province of employment / Province d'emploi: MB
- 16 Employee's CPP contributions – line 308 / Cotisations de l'employé au RPC – ligne 308: 240.86
- 24 EI insurable earnings / Gains assurables d'AE: 1204.32
- 12 Social insurance number / Numéro d'assurance sociale: 878 787 878
- 28 Exempt – Exemption CPP/QPP EI / RPC/RRQ AE
- 29 Employment Code / Code d'emploi
- 17 Employee's QPP contributions – line 308 / Cotisations de l'employé au RRQ – ligne 308
- 26 CPP/QPP pensionable earnings / Gains donnant droit à pension – RPC/RRQ
- 18 Employee's EI premiums – line 312 / Cotisations de l'employé à l'AE – ligne 312: 282.55
- 44 Union dues – line 212 / Cotisations syndicales – ligne 212
- 20 RPP contributions – line 207 / Cotisations à un RPA – ligne 207
- 46 Charitable donations – Schedule 9 / Dons de bienfaisance – Annexe 9
- 52 Pension adjustment – line 206 / Facteur d'équivalence – ligne 206
- 50 RPP or DPSP registration number / N° d'agrément d'un RPA ou d'un RPDB
- Employee's name and address – Nom et adresse de l'employé: Shoe, Tina / 20 Centre Street / Anytown, MB / R2P 0M0
- Other information (see the back) / Autres renseignements (voir au verso): Box / Case — Amount / Montant
- T4 (98)

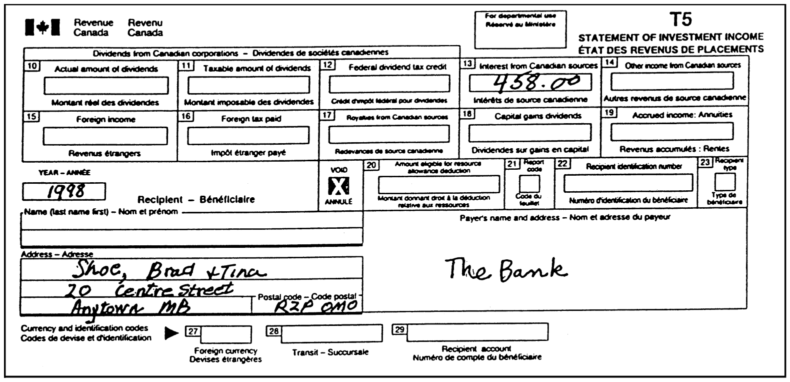
T4 — STATEMENT OF REMUNERATION PAID / ÉTAT DE LA RÉMUNÉRATION PAYÉE (Revenue Canada / Revenu Canada)

- Employer's name – Nom de l'employeur: Construction Co.
- Year / Année: 1998
- 14 Employment income – line 101 / Revenus d'emploi – ligne 101: 56490.00
- 22 Income tax deducted – line 437 / Impôt sur le revenu retenu – ligne 437: 9603.30
- 10 Province of employment / Province d'emploi: MB
- 16 Employee's CPP contributions – line 308 / Cotisations de l'employé au RPC – ligne 308: 1068.80
- 24 EI insurable earnings / Gains assurables d'AE: 7343.70
- 12 Social insurance number / Numéro d'assurance sociale: 111 222 333
- 28 Exempt – Exemption CPP/QPP EI / RPC/RRQ AE
- 29 Employment Code / Code d'emploi
- 17 Employee's QPP contributions – line 308 / Cotisations de l'employé au RRQ – ligne 308
- 26 CPP/QPP pensionable earnings / Gains donnant droit à pension – RPC/RRQ
- 18 Employee's EI premiums – line 312 / Cotisations de l'employé à l'AE – ligne 312: 1053.00
- 44 Union dues – line 212 / Cotisations syndicales – ligne 212: 960.00
- 20 RPP contributions – line 207 / Cotisations à un RPA – ligne 207
- 46 Charitable donations – Schedule 9 / Dons de bienfaisance – Annexe 9
- 52 Pension adjustment – line 206 / Facteur d'équivalence – ligne 206
- 50 RPP or DPSP registration number / N° d'agrément d'un RPA ou d'un RPDB
- Employee's name and address – Nom et adresse de l'employé: Shoe, Brad / 20 Centre Street / Anytown, MB / R2P 0M0
- Other information (see the back) / Autres renseignements (voir au verso): Box / Case — Amount / Montant
- T4 (98)

TAXATION

It is expected that students will demonstrate an ability to fill out an income tax form.

Prescribed Learning Outcomes	Illustrative Examples



Revenue Canada Revenu Canada

For departmental use
Réservé au Ministère

T5
STATEMENT OF INVESTMENT INCOME
ÉTAT DES REVENUS DE PLACEMENTS

Dividends from Canadian corporations – Dividendes de sociétés canadiennes

10 Actual amount of dividends / Montant réel des dividendes	11 Taxable amount of dividends / Montant imposable des dividendes	12 Federal dividend tax credit / Crédit d'impôt fédéral pour dividendes	13 Interest from Canadian sources / Intérêts de source canadienne	14 Other income from Canadian sources / Autres revenus de source canadienne
			458.00	
15 Foreign income / Revenus étrangers	**16 Foreign tax paid / Impôt étranger payé**	**17 Royalties from Canadian sources / Redevances de source canadienne**	**18 Capital gains dividends / Dividendes sur gains en capital**	**19 Accrued income: Annuities / Revenus accumulés : Rentes**

YEAR – ANNÉE: 1998

Recipient – Bénéficiaire

VOID / ANNULÉ: X

20 Amount eligible for resource allowance deduction / Montant donnant droit à la déduction relative aux ressources

21 Report code / Code du feuillet

22 Recipient identification number / Numéro d'identification du bénéficiaire

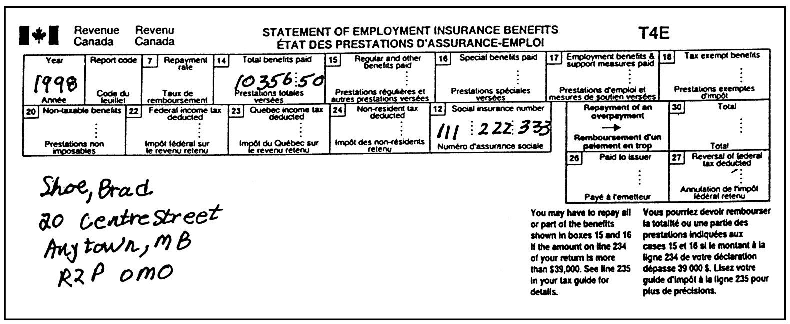
23 Recipient type / Type de bénéficiaire

Name (last name first) – Nom et prénom

Address – Adresse
Shoe, Brad + Tina
20 Centre Street
Anytown MB
Postal code – Code postal: R2P 0M0

Payer's name and address – Nom et adresse du payeur
The Bank

Currency and identification codes / Codes de devise et d'identification ▶ 27 Foreign currency / Devises étrangères 28 Transit – Succursale 29 Recipient account / Numéro de compte du bénéficiaire



Revenue Canada Revenu Canada

STATEMENT OF EMPLOYMENT INSURANCE BENEFITS
ÉTAT DES PRESTATIONS D'ASSURANCE-EMPLOI

T4E

Year / Année	Report code / Code du feuillet	7 Repayment rate / Taux de remboursement	14 Total benefits paid / Prestations totales versées	15 Regular and other benefits paid / Prestations régulières et autres prestations versées	16 Special benefits paid / Prestations spéciales versées	17 Employment benefits & support measures paid / Prestations d'emploi et mesures de soutien versées	18 Tax exempt benefits / Prestations exemptes d'impôt
1998			10356.50				

20 Non-taxable benefits / Prestations non imposables	22 Federal income tax deducted / Impôt fédéral sur le revenu retenu	23 Quebec income tax deducted / Impôt du Québec sur le revenu retenu	24 Non-resident tax deducted / Impôt des non-résidents retenu	12 Social insurance number / Numéro d'assurance sociale
				111 222 333

Repayment of an overpayment → Remboursement d'un paiement en trop

30 Total / Total

26 Paid to issuer / Payé à l'emetteur

27 Reversal of federal tax deducted / Annulation de l'impôt fédéral retenu

Shoe, Brad
20 Centre Street
Anytown, MB
R2P 0M0

You may have to repay all or part of the benefits shown in boxes 15 and 16 if the amount on line 234 of your return is more than $39,000. See line 235 in your tax guide for details.

Vous pourriez devoir rembourser la totalité ou une partie des prestations indiquées aux cases 15 et 16 si le montant à la ligne 234 de votre déclaration dépasse 39 000 $. Lisez votre guide d'impôt à la ligne 235 pour plus de précisions.

VARIATION AND FORMULAS

It is expected that students will use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • plot and analyse examples of direct variation, partial variation, and inverse variation	c The distance (d) that a car travels at 90 km/h is directly proportional to the time (t) traveled. Write an equation and find sufficient points to graph this relationship. c According to Ohm's Law, the current (A) flowing in a wire is inversely proportional to the resistance (Ω) of the wire. If the current is 5 amperes when the resistance is 24 ohms, what is the current when the resistance is 3 ohms?
• given data, graph, or a situation, recognize the variation represented	c In each of the following situations, state what kind of variation is represented. Sketch the graph. a) x | y: 0 | 0; 1 | 2; 2 | 8; 3 | 18; 4 | 32; 5 | 50 b) x | y: 1 | 360; 2 | 180; 3 | 120; 4 | 90; 5 | 72 c A private parking lot charges $1.00 to park on their lot for any amount of time up to 30 minutes. Additional time is rounded up the nearest $^1/_2$ hour and cost $0.75per half hour to a maximum of $5.50 per day. Complete a table of values in 30 minute time intervals that shows the cost of parking. What is the minimum amount of time parked needed to allow the daily maximum.

a)

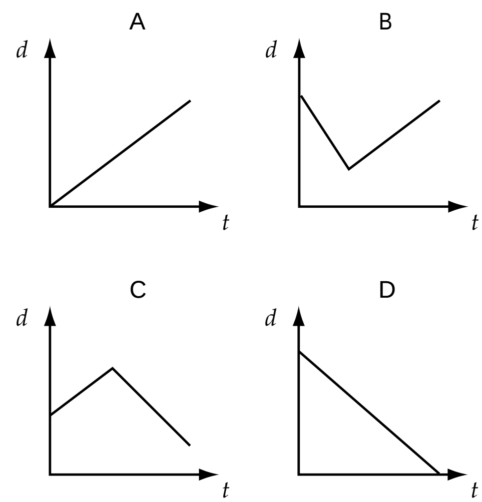
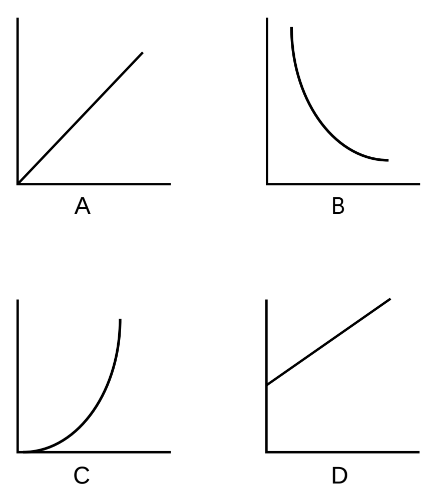
x	y
0	0
1	2
2	8
3	18
4	32
5	50

b)

x	y
1	360
2	180
3	120
4	90
5	72

VARIATION AND FORMULAS

It is expected that students will use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Illustrative Examples
	c Carla walks toward her house and then walks away from it. Which graph best represents this situation if the distance from the house is on a vertical axis and elapsed time is on the horizontal axis?  c Write a relationship that describes each of the following graphs. Be sure to label your axes. 
• use formulas to solve problems	c Consider the formula for the volume of a cylinder: $V = \pi r^2 h$ a) How is the volume affected if the height is doubled? b) How is the volume affected if the radius is doubled? c) What is the effect of halving the radius and tripling the height?

VARIATION AND FORMULAS

It is expected that students will use algebraic and graphical models to generalize patterns, make predictions, and solve problems.

Prescribed Learning Outcomes	Illustrative Examples
	c In order for a satellite to remain in orbit, the speed of the satellite can be determined by the formula $s = \sqrt{\frac{5.15x10^{12}}{d}}$, where *s* represents the speed in km/h and *d* is the distance between the satellite and the centre of the Earth in km. If the Earth has a diameter of 12,750 km, find the speed of a satellite that is 36,000 km above the Earth. c *I have just inherited some money from my grandfather. I want to spend some of it now, but also want to save some for college or university next year. a) On a one-year GIC, $5000 minimum investment, the bank pays 5.5% interest, compounded annually. How much will I have to invest in the GIC in order to have $6000 in one year's time? b) If I wait three years to go to school, I can invest the money for three years in a GIC that pays 6% interest, compounded annually. How much would I have to invest at this rate in order to have $6000 in three years?

Appendix G

Illustrative Examples

Principles of Mathematics 10

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

NUMBER *(Number Concepts)*

It is expected that students will analyse the numerical data in a table for trends, patterns, and interrelationships.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data)

Illustrative Examples

►

Price	GST	PST	Total
$120.00	$8.40	$12.84	$141.24
$275.00	$19.25	$29.43	$323.68

a) What is the rate of GST?

b) What could be the rate of PST?

c) What could be the rule for calculating PST?

d) What is the total GST paid on the two items in the table?

e) What is the total PST paid on the two items in the table?

► National Hockey League (NHL)
Western Conference: February 1, 1996

	W	L	T	Points
Detroit	35	9	4	74
Colorado	26	14	9	61
Chicago	25	15	11	61
Toronto	22	19	9	53
St. Louis	21	20	8	50
Winnipeg	21	24	4	46
Vancouver	17	20	12	46
Los Angeles	17	22	11	45
Calgary	18	23	9	45
Edmonton	18	25	6	42
Anaheim	17	27	5	39
Dallas	14	24	10	38
San Jose	11	35	4	26

What happens to the NHL standings if wins are worth three points and ties are worth one point?

NUMBER *(Number Concepts)*

It is expected that students will analyse the numerical data in a table for trends, patterns, and interrelationships.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data)

Illustrative Examples

► *The following table provides data on the repayment of a $100,000 farm loan. The farmer has negotiated for one annual payment to be made each year after harvest and for the right to make an extra payment, if the harvest is good. Use the table to answer the questions.

Year	Opening Balance	Interest Rate (%)	Interest Charged	Regular Payment	Extra Payment	Closing Balance
1	$100 000.00	8	$8000.00	$14 902.95		$93 097.05
2	$ 93 097.05	8	$7447.76	$14 902.95		$85 641.87
3	$ 85 641.87	8	$6851.35	$14 902.95		$77 590.27
4	$ 77 590.27	8	$6207.22	$14 902.95		$68 894.54
5	$ 68 894.54	8	$5511.56	$14 902.95		$59 503.15
6	$ 59 503.15	8	$4760.25	$14 902.95		$49 360.46
7	$ 49 360.46	8	$3948.84	$14 902.95		$38 406.34
8	$ 38 406.34	8	$3072.51	$14 902.95		$26 575.90
9	$ 26 575.90	8	$2126.07	$14 902.95		$13 799.03
10	$ 13 799.03	8	$1103.92	$14 902.95		$ 0.00

a) What is the period of the loan?

b) What is the amount of the annual payment?

c) How much of the annual payment at the end of Year 5 went toward the opening balance? Show how to determine the answer in two different ways.

d) Create an algebraic expression to find the answer in c).

e) If the interest rate went up to 11% in Year 10, how much would be owing at the end of Year 10?

f) What extra payment at the end of Year 4 would pay the loan off at the end of Year 8?

NUMBER *(Number Concepts)*

It is expected that students will explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Prescribed Learning Outcomes	Illustrative Examples
• classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are "nested" within the real number system	► Explain why the number 1.112111211112 . . . is irrational. ► Given a set of numbers, place them in their appropriate box in a nested Venn diagram. ► Describe, orally and in writing, whether or not a number is irrational. ► Give evidence that a particular real number, such as $\sqrt{3}$, is rational or irrational.

NUMBER *(Number Operations)*

It is expected that students will use basic arithmetic operations on real numbers to solve problems.

Prescribed Learning Outcomes	Illustrative Examples
• communicate a set of instructions used to solve an arithmetic problem	► Write a set of instructions that will allow another student to find: a) $1 + 2 \div 3$ b) $9 \times 4 \div 3 \times 5$ c) the reciprocal of the square root of a number, using a scientific calculator d) a 5% commission on a sale of \$40 200
• perform arithmetic operations on irrational numbers, using appropriate decimal approximations	► Mahal indicates that $\sqrt{2} + \sqrt{8}$ has an approximate value of 3.16. Use estimates to show whether Mahal's answer is reasonable, and use a calculator to verify the accuracy of Mahal's answer. ► Find a decimal approximation of $\frac{3}{\sqrt{5}-\sqrt{2}}$ to three decimal places. ► Arrange the following in order of value from least to greatest: $7, 2\sqrt{13}, 3\sqrt{6}, 4\sqrt{5}, 5\sqrt{2}$. Use decimal approximations. ► Evaluate $\sqrt[3]{128} + 4\left(\sqrt[3]{16}\right)$ to three decimal places. ► Find the length of the base and the height of an equilateral triangle of area 24 cm^2.

NUMBER *(Number Operations)*

It is expected that students will describe and apply arithmetic operations on tables to solve problems, using technology as required.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- create and modify tables from both recursive and nonrecursive situations

Illustrative Examples

►

Price	GST	PST	Total
$120.00	$8.40	$12.84	$141.24
$275.00	$19.25	$29.43	$323.68

a) Determine the PST Rate.

b) Modify the table to allow for a PST of 6.5% of the price.

c) If the price after both taxes is $138 and PST is charged on the $120 price, what is the rate of PST?

► *In 1993, sales of a particular video game doubled every month. The game was released in May 1993 with sales of 32 000 for MayPrepare a table to illustrate the 1993 monthly sales figures. How many video games were sold in December 1993? Identify the assumptions you made when determining the solution.

In 1994, the demand for the video game peaked. Starting in January 1994, and every month thereafter, sales were cut to one quarter of what they were in the previous month. How many video games were sold in April 1994? If April 1994 was the last month of sales, how many video games were sold over the entire twelve months?

NUMBER *(Number Operations)*

It is expected that students will describe and apply arithmetic operations on tables to solve problems, using technology as required.

Prescribed Learning Outcomes	Illustrative Examples
• use and modify a spreadsheet template to model recursive situations	► Modify the given template for a 10-year, $85,000 farm mortgage with fixed annual payments, to allow for a change in interest rate.

Year	Opening Balance	Interest Rate (%)	Interest Charged	Regular Payment	Closing Balance
1	$ 85 000.00	8	$6800.00	$12 667.51	$ 79 132.49
2	$ 79 132.49	8	$6330.60	$12 667.51	$ 72 795.59
3	$ 72 795.59	8	$5823.65	$12 667.51	$ 65 951.73
4	$ 65 951.73	8	$5276.14	$12 667.51	$ 58 560.36
5	$ 58 560.36	8	$4684.83	$12 667.51	$ 50 577.68
6	$ 50 577.68	8	$4046.21	$12 667.51	$ 41 956.39
7	$ 41 956.39	8	$3356.51	$12 667.51	$ 32 645.39
8	$ 32 645.39	8	$2611.63	$12 667.51	$ 22 589.52
9	$ 22 589.52	8	$1807.16	$12 667.51	$11 729.17
10	$ 11 729.17	8	$ 938.33	$12 667.51	$ 0.00

a) What alternatives are open to the farmer, if the interest rate increases?

b) What alternatives are open to the farmer, if the interest rate decreases?

► *Modify the template in the previous illustrative example to reflect a 25-year home mortgage with monthly payments that gives the customer the option of making an annual extra payment of $1500 at the end of any year. Interest is charged monthly.

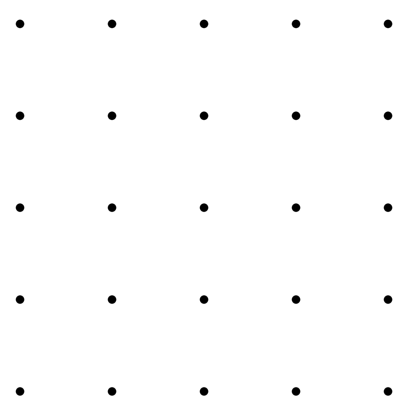
NUMBER *(Number Operations)*

It is expected that students will use exact values, arithmetic operations, and algebraic operations on real numbers to solve problems.

Prescribed Learning Outcomes	Illustrative Examples
• perform operations on irrational numbers of monomial and binomial form, using exact values	► Show that $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$. ► Find an equivalent form of $\frac{3}{\sqrt{5}-\sqrt{2}}$ that has a whole number as its denominator. ► Arrange the following in order from least to greatest: $7, 2\sqrt{13}, 3\sqrt{6}, 4\sqrt{5}, 5\sqrt{2}$. Do not use decimal approximations. ► Simplify $\sqrt[3]{128} + 4\left(\sqrt[3]{16}\right)$, giving the answer as an exact value. ► Find an equivalent of $\left(3\sqrt{5}+4\sqrt{2}\right)\left(4\sqrt{5}-3\sqrt{2}\right)$. ► *An equilateral triangle is inscribed in a circle. If the area of the circle is 36π, find the exact area of the equilateral triangle.

NUMBER *(Number Operations)*

It is expected that students will use exact values, arithmetic operations, and algebraic operations on real numbers to solve problems.

Prescribed Learning Outcomes	Illustrative Examples
• explain and apply the exponent laws for powers of numbers and for variables with rational exponents	► Simplify $\left(\frac{8}{27}\right)^{\left(-\frac{2}{3}\right)}$. ► Write the number expression $7^{\left(\frac{2}{3}\right)}$, using radicals. ► Simplify $\left(\sqrt[5]{x^3}\right)\left(\sqrt[3]{x^2}\right)$. ► Write an equivalent expression for $\sqrt[3]{\sqrt[2]{3x^5}}$, using exponents. ► Prove that $\sqrt{2}$ is an irrational number. ► *The 5×5 geoboard shown in the diagram can be used to construct squares whose areas are whole numbers. The sides of the squares can be constructed by joining dots horizontally, vertically or diagonally. What whole number areas can be constructed? Justify your answers with appropriate drawings and calculations. 

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse number patterns.

Prescribed Learning Outcomes	Illustrative Examples
• generate number patterns exhibiting arithmetic growth	► The first modern Olympiad was held in 1896. Every four years after this date the summer Olympics were held. Given such a framework, reveal what should have been the next five summer Olympic years after 1896. Explain why this pattern was never achieved. ► A salesperson receives a base salary of $12,000 per year, plus $100 for every unit sold. What is the salary, if 50 units are sold? 51 units? 52 units? ► For the arithmetic sequence 16, 23, 30, 37, . . . , find the next three terms. ► *A pile of bricks is arranged in rows. The numbers of bricks in the rows form an arithmetic sequence. There are 45 bricks in the 5th row and 33 bricks in the 11th row. a) How many bricks are in the first row? b) Write the general term for the sequence. c) What is the maximum number of rows of bricks possible?
• use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems.	► For the arithmetic sequence 7, 11, 15, 19, . . . , find the 29th term. ► Find the sum of the arithmetic series 3 + 7 + 11 + . . . + 483. ► *Mary's annual salary is on a range from $26,785 in the first year to $34,825 in the seventh year. a) If the salary range is an arithmetic sequence with seven terms, determine the raise Mary can expect each year. b) What is her salary in the fifth year? c) What is the first salary in this range that is greater than $30,000? d) What is the total amount that Mary earned in the seven years?

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse number patterns.

Prescribed Learning Outcomes	Illustrative Examples
• relate arithmetic sequences to linear functions defined over the natural numbers	► If three eggs are used for every carrot cake made in a bakery, write a function that determines the number of eggs for *n* cakes. ► To rent an ice arena, there is an initial charge for cleaning the ice, plus a rental fee for each hour or part of an hour. The rates posted on the board are: (see table below) Graph the function that models the rates posted on the board.
• generate number patterns exhibiting geometric growth	► Insert three numbers between 5 and 80, so that the five numbers form a geometric sequence. ► A store is conducting a Dutch auction. It will take 10% off the cost of an item each day. If an item originally costs $150, find its cost for each of the next five days.

Time (h)	Cost ($)
Less than 1	100
More than 1, less than 2	180
More than 2, less than 3	260
. . .	

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will generalize operations on polynomials to include rational expressions.

Prescribed Learning Outcomes	Illustrative Examples
• factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$	► Factor: a) $5x^2 + 6x - 8$ b) $6x^2 - x - 2$. ► Factor $4x^2 + 20x + 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the trinomial? ► Factor $4x^2 - 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the binomial? ► For which integral values of k can $4x^2 + kx + 3$ be factored over the set of rational numbers? ► Factor $(x + b)^2 + 6(x + b) + 8$. ► Factor $6x^4 - x^2 - 2$.
• find the product of polynomials	► Find the product and simplify: a) $(3x - 4)(2x^2 + 3x + 1)$ b) $(2x - y)^3$.
• divide a polynomial by a binomial, and express the result in the forms: - $\frac{P}{D} = Q + \frac{R}{D}$ - $P = DQ + R$ - $P(x) = D(x)Q(x) + R$	► Divide $(3x^3 + 2x^2 - 7x + 8)$ by $(x + 2)$. ► Divide $(t^2 - 3t - 10)$ by $(t - 3)$. ► Divide $(6x^3 - 2x^2 + 7x - 11)$ by $(3x^2 - 2)$. ► When the polynomial $P(t) = 4t^4 - 17t^2 - 36t - 20$ is divided by $(2t - 5)$, the remainder is -60. Express the division in the forms: a) $\frac{P(t)}{2t-5} = Q(t) + \frac{R}{2t-5}$ b) $P(t) = Q(t)(2t - 5) + R$

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will generalize operations on polynomials to include rational expressions.

Prescribed Learning Outcomes	Illustrative Examples
• determine equivalent forms of simple rational expression with polynomial numerations, and denominators that are monomials, binomials or trinomials that can be factored	► Change each rational expression to its simplest equivalent form: a) $\frac{4x^4 - 6x^3 + 2x^2 - 10x}{2x}$ b) $\frac{x^2 - 5x - 6}{x^2 - 36}$ c) $\frac{x^2 + 3x}{x^2 + x - 6}$
• determine the nonpermissible values for the variable in rational expressions	► For what value(s) of x are each of the following not defined? Explain your conclusion in each case. a) $\frac{3}{x}$ b) $\frac{-2}{x+1}$ c) $\frac{4}{3x-4}$ d) $\frac{2x+1}{x^2-4}$ e) $\frac{5x}{x^2-3x-4}$ f) $\frac{5x+y}{3x-y}$ g) $\frac{7x^2-6xy+3y^2}{4x^2-9y^2}$ h) $\frac{2}{x^3}$ i) $\frac{5}{(x^3-1)}$
• perform the operations of addition, subtraction, multiplication and division on rational expressions	► For each expression perform the indicated operations, and identify any nonpermissible values. a) $\left(\frac{1}{x}\right) + \left(\frac{3}{2x}\right)$ b) $\left(\frac{4}{x+1}\right) - \left(\frac{1}{x-2}\right)$ c) $\left(\frac{2x+1}{x-1}\right) + \left(\frac{x-1}{x^2-x-2}\right)$ d) $\left(\frac{x^2+2x+1}{x-5}\right)\left(\frac{x^2-25}{x^2+6x+5}\right)$ e) $\left(\frac{3x^2+10x+3}{x^2-9}\right) \div \left(\frac{3x+1}{x-3}\right)$

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will generalize operations on polynomials to include rational expressions.

Prescribed Learning Outcomes	Illustrative Examples
	f) $\dfrac{3}{\left(\dfrac{2}{x}\right)}$ g) $\dfrac{\left(\dfrac{2x+6}{x+1}\right)}{\left(\dfrac{x+3}{x^2+1}\right)}$ h) $\dfrac{\left(\dfrac{1}{x}+3\right)}{\left(\dfrac{1}{x}-3\right)}$
• find and verify the solutions of rational equations that reduce to linear form	► Solve for x, checking for any nonpermissible values. a) $\dfrac{2}{x} = -3$ b) $\dfrac{4}{x} + \dfrac{3}{2x} = \dfrac{11}{4}$ c) $\dfrac{5}{x-1} + \dfrac{2}{x+1} = 2$ d) $\dfrac{2x+1}{x+3} - \dfrac{x-2}{x+1} = 5$ e) $\dfrac{3}{x^2-25} + \dfrac{2}{x+5} = \dfrac{4}{x-5}$ f) $\dfrac{4}{x-5} + 6 = \dfrac{4}{x-5}$ ► *The average speed of an airplane is five times as fast as the average speed of a passenger train. To travel 400 km, the train requires four hours more than the airplane. Find the average speeds of the train and the airplane.

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- plot linear and nonlinear data, using appropriate scales

Illustrative Examples

► The mass of a beaker is recorded when the beaker contains varying volumes of ethanol. The results of the experiment are recorded in the table below.

Volume of Ethanol (mL)	Mass of Beaker and Liquid (g)
0	90
50	129
100	168
150	207
200	246

Measurements may be assumed correct to the nearest mL and to the nearest g.

Plot this data on a scatterplot, using appropriate scales, and answer the following questions.

a) Assuming that this pattern continues, determine the mass of the beaker and liquid when 250 mL of ethanol is present.

b) When a volume of 200 mL of ethanol is in the beaker, determine the mass of the ethanol alone.

c) The density of a liquid is defined as the mass of 1 mL of the liquid. Determine the density of the ethanol.

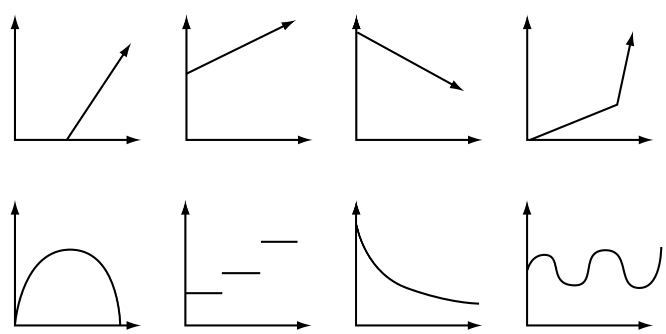
► Nannook's Pizza uses the following price structure.

Diameter (inches)	Cost ($)
8	6.50
10	10.20
12	14.65
14	19.90
16	26.00

Plot this data on a scatterplot, using appropriate scales, and describe the pattern.

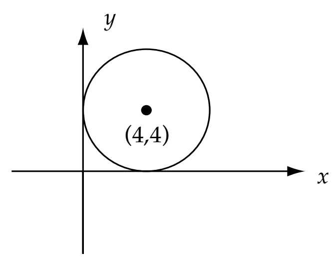
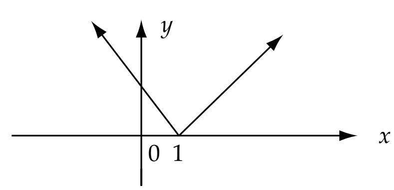
PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples
• represent data, using function models	► *Sketch graphs to illustrate the following situations. If sufficient information is given, represent the situation by a suitable equation. Sketch and, if possible, represent by an equation: a) the area of a circle as a function of its radius b) the cost of mailing a letter as a function of the mass of the letter c) the cost of renting a car for one day as a function of the kilometres driven d) the population of Canada as a function of the year e) the length of daylight as a function of the date ► For each of the following graphs, describe a practical situation that could be represented by the graph. In describing the situation, state the meanings of any intercepts, slopes, maxima and/or minima. 
• use a graphing tool to draw the graph of a function from its equation	► Graph the function $y = x + 1$, using a graphing tool. ► Graph the function $y = x^2 + 100$, using a graphing tool. Explain the process used, so that the graph appears on the screen.

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples
• describe a function in terms of: - ordered pairs - a rule, in word or equation form - a graph	► Describe the parking charges at a parkade in terms of ordered pairs, a rule, and a graph.
• use function notation to evaluate and represent functions	► $f(x) = x^2 - 5x + 3$. What are the ordered pairs describing the points on the graph having a y-coordinate of $f(2)$? ► If $f(x) = 3x^2 - 6x + 5$, find $f(\sqrt{3})$, $f(2x)$ and $f(3t + 2)$.
• determine the domain and range of a relation from its graph	► If the coordinate axes touch the circle, what is the domain and range of the circle shown in the graph to the right?  ► Determine, from its graph shown below, the domain and range of the function $y = \|x - 1\|$ 

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

Prescribed Learning Outcomes	Illustrative Examples
• determine the following characteristics of the graph of a linear function, given its equation: - intercepts - slope - domain - range	► *A tanker truck drives on a weigh scale and then is filled with crude oil. The mass M, measured in kilograms, of the truck and the volume V, measured in barrels, of crude oil are related by the formula: $M = 14\,000 + 180\,V;\ \ V \leq 500.$ a) Draw the graph with V on the horizontal axis and M on the vertical axis. b) The tank has a maximum capacity of 500 barrels. What is the mass of the truck when it contains 500 barrels of oil? c) What is the mass of the empty truck? Where is this value found on the graph? d) Find the slope, and give an interpretation for it. e) Give the domain for this problem. f) Express the range in words. ► Graph each of the following equations; and indicate intercepts, slope, domain and range. a) $y = 2x;\ \ x = (0, 1, 2, 3, 4, 5, 6)$ b) $y = -\frac{1}{3}x;\ \ x =$ a real number c) $y = 3$ d) $x = 3$ e) $y = \frac{1}{3}x + 5;\ \ x =$ a real number f) $y = mx + b;\ \ x =$ a real number

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent data, using linear function models.

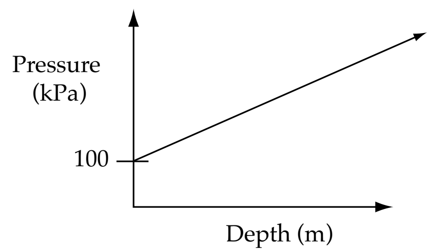
Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- use partial variation and arithmetic sequences as applications of linear functions

Illustrative Examples

► A hydrologist studied the relationship between the pressure on an object and its depth of submersion in a liquid. The following graph was sketched.



Draw conclusions based upon the sketch.

► Simple interest varies directly with the amount borrowed.

a) If the interest is $5 for $100 borrowed, what would the interest be for $325 borrowed?

b) Graph the relation, and write the equation of the graph.

► *A jet ski rental operation at Okanagan Lake charges a fixed insurance premium, plus an hourly rate. The total cost for two hours is $50 and for five hours is $110.

a) Graph the relation.

b) Determine the fixed insurance premium and the hourly rate to rent the jet ski.

► *With new equipment coming on line, a soft drink manufacturer has been increasing its production each day according to the following table. Assume a maximum daily output of 25 000 cans.

Day	1	2	3	4
Units	4000	4200	4400	4600

a) Graph the relation. Hint: this is a discrete case.

b) On what day will they be able to produce 20 000 cans, if this trend continues?

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent data, using linear function models.

Prescribed Learning Outcomes	Illustrative Examples

► Given the distance–time graph shown, answer the following questions.

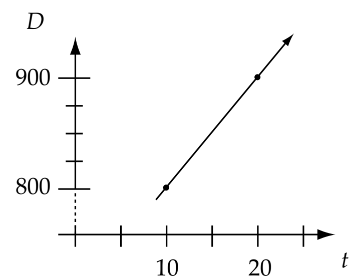
a) If $D = 850$, what is t?

b) If $t = 25$, what is D?

c) If $D = 1500$, what is t?

d) Write the equation of the function.

e) Verify the accuracy of your estimates in a), b) and c), using the equation of the function.



► Given the data in the table, predict the fuel consumption for the following engines:

a) 2.5 L

b) 5.0 L

Engine Size (L)	Consumption (L/100 km)
2.2	6.4
3.0	7.5
3.8	8.1
4.1	8.6

► *A video game operator gives all her change in quarters. From a $20 bill, she gives 56 quarters change for a $6 purchase. She gives 8 quarters change from a $20 bill for an $18 purchase.

a) Graph the number of quarters given as change N on the vertical axis and the amount of the purchase P on the horizontal axis. Assume that a $20 bill was given.

b) What is the domain and range of the function?

c) How does the graph change, if a $10 bill is used?

SHAPE AND SPACE *(Measurement)*

It is expected that students will demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Prescribed Learning Outcomes	Illustrative Examples

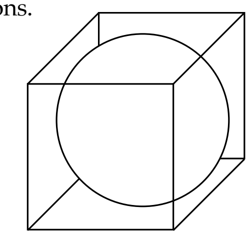
Prescribed Learning Outcomes

- calculate the volume and surface area of a sphere, using formulas that are provided

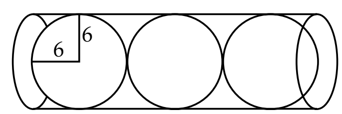
Illustrative Examples

► Calculate the volume and surface area of a beach ball of radius 15 cm.

► *A sphere fits tightly inside a cube. If the radius of the sphere is 10 cm, find the surface area of the cube. Explain and justify your calculations.



► Determine the total surface are of the container and then justify your answer.



► What is the volume of the air inside the cylinder and outside the balls?

► *A hot air balloon has a spherical shape and a diameter of 4 m. If 30 additional cubic metres of air are pumped into the balloon, what will be the new values for the diameter, volume and surface area?

► *A silo is composed of a cylinder with a hemisphere on top as the roof. What is the surface are of the silo if the height is 10m and the radius of the cylinder is 3m?

Prescribed Learning Outcomes

- determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects

Illustrative Examples

► The area of a region in a plane is 10 cm^2. By what factor must each of the dimensions of this region be multiplied to increase the area by 20 cm^2? Explain how you arrived at your answer.

► *A model train is built to a scale of 1:50. If the length of the model engine is 20 cm and the area of sheet metal used to cover the outside surface of the model is 180 cm^2, what is the actual length of the engine and the actual area of the sheeting used to cover the engine? If the volume displaced by the model engine is 126 cm^3, what is the volume displaced by the real engine, in m^3?

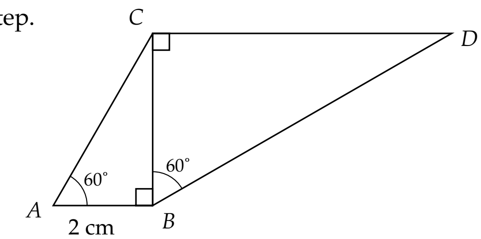
► *It is improbable that a giant human, 6 m in height (three or four times normal human height), could exist. Which biological systems are most likely to break down? Explain your answer.

SHAPE AND SPACE *(Measurement)*

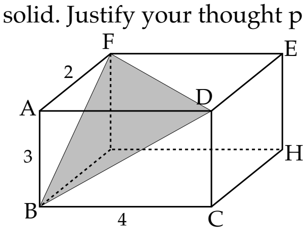
It is expected that students will solve problems involving triangles, including those found in 3-D and 2-D applications.

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving two right triangles	► *From the top of a 100 m fire tower, a fire ranger observes two fires, one at an angle of depression of 5˚ and the other at an angle of depression of 2˚. Assuming that the fires and the tower are in a straight line, determine the distance between the fires for the following: a) when the fires are on the same side of the tower b) when the fires are on opposite sides of the tower

► The triangles *ABC* and *BCD* have right angles at *B* and *C* respectively. Calculate the length of side *CD*, and state the ratio of length *BD* to length *AC*. Explain your reasoning at each step.

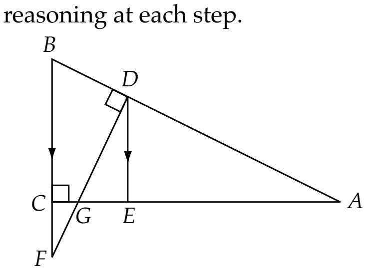


► *Find the perimeter of the shaded triangle in the rectanglular solid. Justify your thought processes.



► *Canada's highest waterfall is Della Falls on Vancouver Island. An observer standing at the same level as the base of the falls views the top of the falls at an angle of elevation of 58˚. When the observer moves 31 m closer to the base of the falls, the angle of elevation increases to 61˚. Find the height of Della Falls.

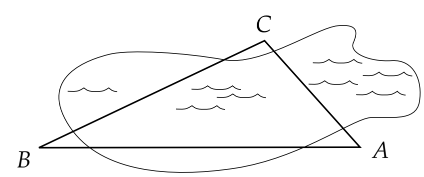
► In the figure; AD = BF, DE = DB, DE is parallel to BC and AC is perpendicular to BF, FD is perpendicular to AB. Name the triangle that is congruent to triangle ADE and justify your reasoning at each step.



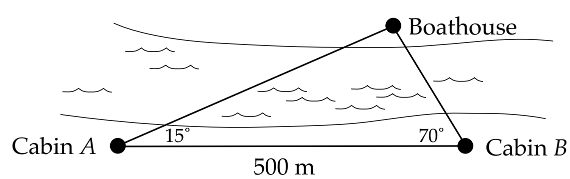
SHAPE AND SPACE *(Measurement)*

It is expected that students will solve problems involving triangles, including those found in 3-D and 2-D applications.

Prescribed Learning Outcomes	Illustrative Examples
• extend the concepts of sine and cosine for angles through to 180˚	► Find sin 130˚. ► Use a calculator to find multiple solutions for angle A, if $\sin A = \sin 130^\circ$. Use trial and error to find as many solutions as possible. Summarize the pattern found in the solutions. ► Find the value(s) for A ($0^\circ \le A \le 180^\circ$) when $\sin A = \frac{1}{2}$. ► Find the value(s) for A ($0^\circ \le A \le 180^\circ$) when $\cos A = \frac{1}{2}$. ► Find the value(s) for A ($0^\circ \le A \le 180^\circ$) when $\cos A = -\frac{1}{2}$.
• apply the sine and cosine laws, excluding the ambiguous case, to solve problems	► *An electric transmission line is planned to go directly over a pond. The power line will be supported by posts at points A and B. A surveyor measures the distance from B to C as 580 m, the distance from A to C as 337 m and BCA as 105.34˚. What is the distance from post A to post B?



► Two cabins are located 500 m apart on the same side of a river. Across the river from the two cabins is a boathouse. This situation is illustrated in the diagram below. Use the measurements to find the width of the river.



► A farmer has a field in the shape of a triangle. From one corner, it is 530 m to the second corner and 750 m to the third corner. The angle between the lines of sight to the second and to the third corners is 53˚. Find the perimeter and area of the field.

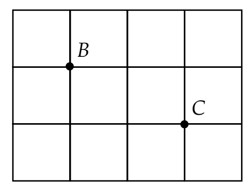
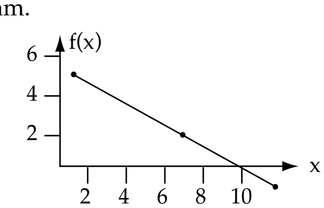
► *A sailboat leaves the dock at Gibson's Landing on a bearing of S57˚W. After sailing for 8 km, the ship tacks and travels S31˚E for 5 km.

a) How far is the sailboat from Gibson's Landing?

b) What direction would it have to sail to return to the dock at Gibson's Landing?

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will solve coordinate geometry problems involving lines and line segments.

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving distances between points in the coordinate plane	► Bob and Christine want to meet; see map below. Each block has dimensions of 120 m by 120 m. Assuming the roads are of negligible width, how far does Bob *B* have to travel to get to Christine *C*? Find two separate answers, one for a path along the roads and one for a direct path.  ► Plot the points (-4, -2) and (1, 5) on the coordinate plane. Describe two different ways to calculate the distance between the two points. ► Generate a method of determining the distance between any two points in the coordinate plane without having to plot the points. Justify your method. ► Program a calculator or computer to accept, as input, the coordinates of two points and to give, as output, the distance between the two points. Document the program so that someone else can use it without assistance.
• solve problems involving midpoints of line segments	► Explain to a partner the meaning of the midpoint of the line segment joining two points without using the word midpoint. ► *On a map with numerical coordinates in kilometres, the village of Sundown is at (6.3, 2.9), while the town of Sunup is at (4.7, 13.2). It was decided to construct a water main on the direct line joining Sunup with Sundown. Each community was responsible for the cost of construction from the community to the midpoint. Find the coordinates of the midpoint and Sundown's costs, if Sundown spent $63,475 per kilometre for construction. Determine alternative methods that could be used to solve the problem. Justify your answer. ► Given the line AB with the point A (1, 5) and midpoint M (7, 2) determine the coordinates of B. Justify your statements using a diagram. 

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will solve coordinate geometry problems involving lines and line segments.

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving rise, run and slope of line segments	► If the slope of a line is 6 ($m = 6$) and the line passes through the points (2, 5) and (1, k), what is the value of k? ► If two points on a line are (4, 3) and (6, 4), find one other point on the line. Use a graphing utility to demonstrate the reasonableness of your answer.
• determine the equation of a line, given information that uniquely determines the line	► Use a graphing device to examine changes in the graph of $y = mx + b$ as the values of m and b are changed. Use the results to explain why the equation $y = mx + b$ is called the slope and y-intercept form of a linear equation. ► Write a clear explanation of the nature of the following lines: $x = a$, $y = b$, $x = y$. ► Manipulate the standard form of a straight line ($Ax + By + C = 0$) into the slope and y-intercept form of the same line. Determine rules that connect A, B and C to the slope (m) and to the intercepts. ► Find the equation of a line passing through the points (-1, 3) and (4, 2). ► Given the graph of an oblique line, determine an equation for the line. ► *A spring with no masses attached is 25.2 cm long. For each 1-g mass attached to the spring, the spring's length increases by 4 mm. Graph this scenario, label the axes, and find an equation for the graph.
• solve problems using slopes of: - parallel lines - perpendicular lines	► Graphically examine the slopes of various lines, all of which are perpendicular to the line $y = \frac{2}{3}x + 2$. Describe the slopes, and make a rule for finding the slope of a perpendicular to a given line. ► Two perpendicular lines intersect on the x-axis. The equation of one of the lines is $y = 2x - 6$. Find the equation of the second line.

STATISTICS AND PROBABILITY *(Data Analysis)*

It is expected that students will implement and analyse sampling procedures, and draw appropriate inferences from the data collected.

Prescribed Learning Outcomes	Illustrative Examples
• choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population	► A toothpaste company advertises that three out of four dentists prefer their product. Analyse this statement for its completeness and its accuracy in terms of population, sample, possible sampling technique, validity and bias. ► A school cafeteria wants to introduce a new dessert. Describe how a survey could be conducted to decide which of three choices should be the new dessert. ► *To predict a winner in a federal election, a magazine compiled a list of about 200 000 names from sources, such as telephone books, lists of automobile owners, club membership lists and its own subscription lists. The magazine mailed a questionnaire to everybody on the list, and 4000 returned it. The 4000 responses became the sample. Discuss the potential sources of bias.
• defend or oppose inferences and generalizations about populations, based on data from samples	► *To determine a preference for spending $50 in either a clothing store, an electronics shop or a restaurant, customers were surveyed one Saturday morning at the mall. Fifty-nine precent preferred spending in a clothing store, 32% in an electronics shop and 9% in a restaurant. What generalizations can be made from these results? Does the sample adequately represent the population to be surveyed? Design a more reliable sampling method to obtain this information, and include details of the questionnaires used and the method of selecting the sample. ► Search through various forms of media to find examples of generalizations that have been made about populations, based on data from samples. Do you agree or disagree with the generalizations? Explain why.

Appendix G

Illustrative Examples

Principles of Mathematics 11

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

NUMBER *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples
• solve consumer problems, including: - wages earned in various situations - property taxation - exchange rates - unit prices	c Calculate and compare wage situations involving minimum wage rates, regular pay, overtime pay, gratuities, piecework, straight commission, salary and commission, salary plus quota and graduated commission. c Jane has a choice of two restaurants at which to work. Mario's pays \$8/h, and tips average \$24 daily. Teppan's pays \$5.50/h, and tips average \$35 daily. If Jane works 30 hours weekly, spread over four days, how much would she earn at each restaurant? c Identify and calculate various payroll deductions, including income tax, CPP, EI, medical benefits, union and professional dues and life insurance premiums. c Estimate, calculate and compare gross and net pay for various wage or salary earners in your community. c *The Ningart property has a market value of \$105 000. The assessed values in the area are 60% of market values. The tax rate is 32.3 mills of assessed value. What is the Ningarts' monthly tax payment? c The exchange rate for converting currency on a given day in the United States is 28% and in Canada 38.8%. Explain why this is possible. c Which is a better deal, 284 mL of tomato soup for \$0.69 or 907 mL for \$1.70?

NUMBER *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- reconcile financial statements including:
  - cheque books with bank statements
  - cash register tallies with daily receipts

Illustrative Examples

- *The following petty cash transactions occurred during the first week of March.

  March 4 $100 cheque was received to establish the fund.
  March 5 Bought $12.50 worth of postage stamps.
  March 5 Spent $10 to have something delivered by taxi.
  March 6 Spent $6.50 for lunch.
  March 7 Paid a courier service $25 for deliveries.
  March 7 Bought flowers for opening day, $28.
  March 8 Replenished the fund by $25.
  March 9 Postage stamps purchased for $21.50.

  Determine if a final balance of $20 is correct. If not, provide an explanation for the difference, and indicate possible ways to correct the problem.

- Complete the table below to determine the cost of credit for using a department store charge account for the period shown. Monthly credit charges are 1.4% of the balance due.

Month	Previous Balance	- Payment Made	+ Purchases Charged	⇒ Balance Due	+ Credit Charges	⇒ New Balance
February	$314.65	$100.00	$193.75		$5.72	$414.12
March		$150.00	$ 59.60			
April		$140.00	$421.83			$618.62
May	$618.62	$200.00	$ 39.65			
June		$250.00	$ 58.11			
July		$150.00	$ 77.21			
August	$206.68	$120.00	$163.09		$3.50	$253.27

NUMBER *(Number Operations)*

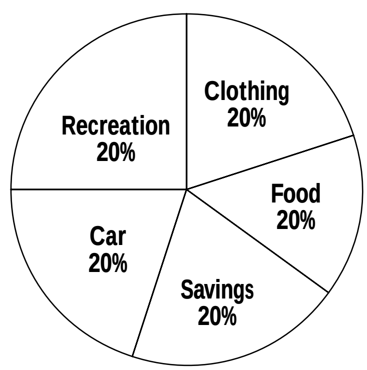
It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples
• solve budget problems, using graphs and tables to communicate solutions	c Research and calculate the cost of running a car for a year. Decide how to classify each cost, how to collect the data and how to display the results.

c *As a project, prepare a monthly budget for one of the following:
a) the family
b) an assumed persona (e.g., a celebrity)
c) a school
d) a vacation
e) a fishing/hunting/shopping trip
f) a municipality

c The diagram shows Julie's monthly budget of $1,200. She wants to move to her own apartment that costs $450 per month. Construct a new budget that will include her rent. Explain the choices and changes that Julie could make.

Julie Barnes' Monthly Budget Total = $1,200



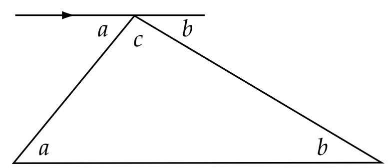
NUMBER *(Number Operations)*

It is expected that students will solve consumer problems, using arithmetic operations.

Prescribed Learning Outcomes	Illustrative Examples
• solve investment and credit problems involving simple and compound interest	c Determine the effective annual interest rate on a loan of \$1,000 at 10% per year, compounded quarterly. c Calculate the compound amount, after one year, of a deposit of \$1,000. Assume the current nominal annual interest when the interest is compounded: a) annually b) monthly c) daily c A bank offers an interest rate of 8% per year, compounded annually. A second bank offers an interest rate of 8% per year, compounded quarterly. If \$2,000 were deposited, for ten years, in each bank, how much more income would be gained in the second bank than in the first? c Calculate the interest paid on various forms of credit, including: a) credit cards b) loans c) mortgages c A loan of \$5,000 carries an interest rate of 9% per year, compounded monthly. Adele makes a payment of \$350 every month. Use a spreadsheet to determine how much she still owes after making 12 payments. c Compare two investments in an RRSP for one year with contributions starting January 1. a) \$100 is invested monthly at 10% per annum, compounded monthly. b) \$600 is invested semi-annually at 10% per annum, compounded semi-annually.

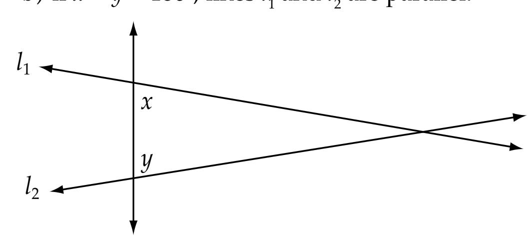
PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will apply the principles of mathematical reasoning to solve problems and to justify solutions.

Prescribed Learning Outcomes	Illustrative Examples
• differentiate between inductive and deductive reasoning	c Find, inductively, the sum of the angles of a triangle, by: a) constructing triangles and tearing the corners off b) putting the torn corners together to form a straight line. c *Show, deductively, that the sum of the measures a, b and c is 180˚, by: a) drawing a triangle b) using one side as a base and drawing a parallel line segment on the opposite vertex c) knowing that $a = a$, $b = b$, and c is included in both; $\therefore a + c + b = 180°$. 
• explain and apply connecting words, such as *and*, *or* and *not*, to solve problems	c *Each member of a sports club plays at least one of the following sports: soccer, rugby or tennis. The following information is given: a) 163 play tennis; 36 play tennis and rugby; 13 play tennis and soccer b) 6 play all three sports; 11 play soccer and rugby; 208 play rugby and tennis c) 98 play soccer and rugby. Use this information to determine the number of members in the club. c On a number line, indicate the location of the sets corresponding to the following: a) $x < 2$ or $x > 5$ b) $x < 2$ and $x > 5$ c) $x < 5$ or $x > 2$ d) $x < 5$ and not $x > 2$.

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will apply the principles of mathematical reasoning to solve problems and to justify solutions.

Prescribed Learning Outcomes	Illustrative Examples
• use examples and counterexamples to analyse conjectures	c Rajiv concluded that whenever he added two prime numbers the sum was always even. Find a counterexample to prove that Rajiv's conjecture is false. c A science text states that water boils at 100°C. Find a counterexample. c Mary used her graphing calculator to graph $y = x^x$. She found the screen to be blank for $x < 0$ and made a conjecture that x^x is undefined when $x < 0$. Find an example to show that Mary's conjecture is reasonable. Find a counterexample to show that Mary's conjecture is false.
• distinguish between an *if–then* proposition, its converse and its contrapositive	c Change the statement "Multiples of 3 are always multiples of 6" into "if–then" form, and write the converse and contrapositive of the "if–then" statement. Decide on the truth of all three propositions. c Create a true proposition whose converse and contrapositive are both true.
• prove assertions in a variety of settings, using direct and indirect reasoning	c Angle *ABC* is obtuse, and *AD* is the median of *BC*. If *AD* is not an altitude, prove that *ABC* is a scalene triangle. c Prove that the medians of a triangle cannot bisect each other. c In the diagram below, show: *a)* $x + y < 180°$ b) if $x + y = 180°$, lines l_1 and l_2 are parallel.  c Prove that the difference of squares of two odd numbers is always divisible by 4.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will represent and analyse situations that involve expressions, equations, and inequalities.

Prescribed Learning Outcomes	Illustrative Examples
• graph linear inequalities, in two variables	c Solve, algebraically and graphically, for x: $2x + 5 > 3x - 1$. c A target is described in terms of coordinates (x, y), where x and y are measured in metres. All of the following are true: • $x \leq 6$ • $y \geq 7$ • (x, y) is in the first quadrant • $x + y \leq 10$. What is the shape and the area of the target?
• solve systems of linear equations, in two variables: - algebraically (elimination and substitution) - graphically	c Solve this system of equations: a) $x + 2y = 10$ $2x + 3y = 14$ b) $3x + 4y = 15$ $x - y = 5$ c In what situations is one method of solving a system of linear equations (elimination or substitution) preferrable to another? c *A principal of $42,000 is invested partly at 7% and partly at 9.5%. If the interest is $3,700, how much is invested at each interest rate? c Plot the graphs of $2x + 3y = 11$ and $2x - 3y = 17$. What is their point of intersection?
• solve systems of linear equations, in three variables: - algebraically - with technology	c Determine the solution to the following system: $2x + y - z = 3$ $x + 2y + z = 0$ $3x - y - 2z = 11$ c The total revenue R is a quadratic function of the price p of books sold. So $R = ap^2 + bp + c$. Find the values of a, b and c, if the revenue is $6,000 at a price of $30; $6,000 at a price of $40; and $5,000 at a price of $50.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will represent and analyse situations that involve expressions, equations, and inequalities.

Prescribed Learning Outcomes	Illustrative Examples
• solve nonlinear equations, using a graphing tool	c Using a graphing tool, solve $x^2 + 6x - 11 = 0$. c Solve $x^3 + x = 30$ graphically, using two different methods. Which method gives solutions that are more free from rounding errors and other inaccuracies? c Where does the line $y = 4x + 5$ cut the curve $y = 2^x$? Use a graphing tool to find the points of intersection.
• solve nonlinear equations: - by factoring - graphically	c Solve by factoring: *a)* $x^2 - 2x = 24$ *b)* $x^3 = 1$ c) $2x^2 + 9x - 5 = 0$ d) $7x^2 + 4x - 11 = 0$ c Solve each of the above graphically. For example, $x^2 - 2x = 24$ can be solved by graphing $y = x^2 - 2x$ and $y = 24$ and using the points of intersection to determine the solution. c Solve $3x^2 + 1 = 10x - 2$ graphically in two different ways. Is there one way that gives more reliable results? Explain your procedures and the results obtained.
• use the Remainder Theorem to evaluate polynomial expressions, the Rational Zero Theorem, and the Factor Theorem to determine factors of polynomials	c The polynomial $P(x) = 4x^3 + bx^2 + cx + 11$ has a remainder of -7 when divided by $(x + 2)$ and a remainder of 14 when divided by $(x - 1)$. Find the values of b and c. c Factor $x^3 - 2x^2 - 5x + 6$.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will represent and analyse situations that involve expressions, equations, and inequalities.

Prescribed Learning Outcomes	Illustrative Examples
• determine the solution to a system of nonlinear equations, using technology as appropriate	c Find the solutions to the following system: $y = x^2$ $y = 8 - x^2$ c Graphically, find the solution set to the following system: $y = 3x + 2$ and $y = 2^x$ How do you know that the solution set is complete? c *The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. The population growth can be modelled by the equation $P_1 = 5(1.02)^n$, with the food production being modelled by $P_2 = 0.2n + 6$. The variable n is the number of years after 1987. a) When does $P_1 = P_2$? b) If $P_1 > P_2$ is true, when does this happen, and how is this inequality interpreted?

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• determine the following characteristics of the graph of a quadratic function: - vertex - domain and range - axis of symmetry - intercepts	c Given the graph of any quadratic function, determine the following: a) vertex b) domain c) range d) axis of symmetry e) intercepts c Use technology to graph $f(x) = x^2 - 6x + 4$ and to determine the vertex, domain, range, axis of symmetry and intercepts. c *One model concerning the rate of population growth of Earth has the annual rate of increase varying jointly as the population and the unused carrying capacity of Earth. The equation of the model is: $y = 0.001x(21 - x)$, where y equals the rate of increase in population (in billions per year), and x equals the present population (in billions). a) Plot this model of growth. b) The present population of Earth is 5.8 billion. What is the annual increase in population at present? c) What is the population when the rate of increase in population is at its greatest? d) What is the population when the rate of increase is zero? e) What is the projected maximum population that Earth can accommodate, according to this model?

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will examine the nature of relations with an emphasis on functions.

<table>
<tr><th>Prescribed Learning Outcomes</th><th>Illustrative Examples</th></tr>
<tr><td>

• perform operations on functions and compositions of functions

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c If $f(x) = 3x + 2$ and $g(x) = x^2$, find:
a) $3f(x)$
b) $f(x) \bullet g(x)$
c) $f(x) + g(x)$
d) $f(g(x))$
e) $f(f(x))$

c A ball thrown in the air has a velocity given by $v(t) = 49 - 9.8t$. The kinetic energy function $K(v)$ is given by $K(v) = 0.4v^2$. Express the ball's kinetic energy as a function $K(t)$ of time.

</td></tr>
<tr><td>

• determine the inverse of a function

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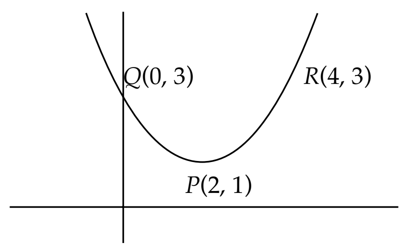
c Is the inverse of $f(x) = 2x - 5$ a function?

c Sketch the inverse of the following. Is the inverse a junction?



c Sketch the inverse of $y = x^3$

c If $f(x) = 2x - 1$ and $g(x) = \frac{x+1}{2}$, find $f(g(x))$ and $g(f(x))$, and show that the functions $f(x)$ and $g(x)$ are inverses of each other.

c Determine the domain and range for each of the functions in the two previous illustrative examples.

c *Graph the inverse of $y = \frac{x}{(x - 1)}$, and determine the equation, domain, and range of the inverse.

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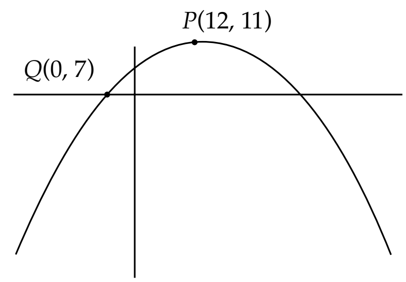
PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• connect algebraic and graphical transformations of quadratic functions, using completing the square as required	c Graph $f(x) = 2x^2 + 5x - 7$. c Give a list of events or situations that might be described by a quadratic, parabolic, shape. c Given the graph of $y = x^2$, sketch $y = -2(x - 3)^2 - 4$. c Given the graph of $y = x^2$, what is the equation for the transformed graph shown here?  c Rewrite the equation of $f(x) = 2x^2 - 12x + 13$ in the form $f(x) = a(x - p)^2 + q$, and graph the function.
• model real-world situations, using quadratic functions	c *Computer software programs are sold to students for $20 each, and 300 students are willing to buy them at that price. For every $5 increase in price, there are 30 fewer students willing to buy the software. What is the maximum revenue? c What is the maximum rectangular area that can be enclosed by 120 m of fencing, if one of the sides of the rectangle is an existing wall?

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: - factoring - the quadratic formula - graphing	c Solve $3x^2 - 5x + 2 = 0$ algebraically and by graphing the corresponding function $f(x) = 3x^2 - 5x + 2$. c *When bicycles are sold for $280 each, a cycle store can sell 80 in a season. For every $10 increase in the price, the number sold drops by 3. a) Represent the sales revenue as a quadratic function of either the number sold or the price. b) What is the number sold, and the price, if the total sales revenue is exactly $20,000? c) What is the range of prices that will give a sales revenue that exceeds $15,000? c Write a quadratic equation whose roots are $\frac{3}{2}$ and $-\frac{1}{4}$. Is this equation unique?
• determine the character of the real and non-real roots of a quadratic equation, using: - the discriminant in the quadratic formula - graphing	c If $3x^2 - mx + 2 = 0$ can be factored, what values of m are possible? c Discuss the implications of a negative discriminant when describing the zeros of a quadratic function. c Given $3x^2 - mx + 3 = 0$: a) For what value(s) of m would one root be double the other? b) For what values of m would the roots not be real? c *The profit y for publishing a book is given by the equation $y = -5x^2 + 400x - 3000$, where x is the selling price per book. a) Is it possible to set a selling price that will earn a total profit of $6,000? Explain your solution with reference to appropriate equations and graphs. b) What range of selling prices allows the publisher to make a profit on this book?

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcomes

- describe, graph and analyse polynomial and rational functions, using technology

Illustrative Examples

c Determine if each of the following examples is a rational function, a polynomial function or some other type of function, and justify your conclusion.

a) $y = x^2 - 3x + \sqrt{7}$

b) $y = (x - 5)^{-1}$

c) $y = \frac{1}{5}x^4 + 3x^3 - 12x - 0.75$

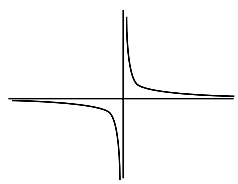
d) $y = \sqrt{7x^5} + x^2$

e) $y = 2^x - 9$

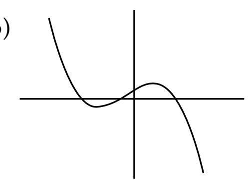
f) $y = \frac{3x - 7}{x^2 - 5x + 6}$

c Examine the following graphs. Which could be graphs of rational functions, and which could be graphs of polynomial functions?

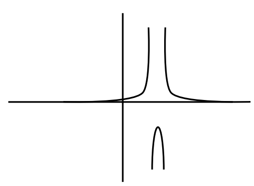
a)



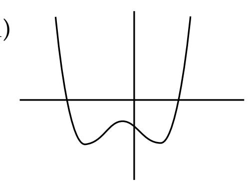
b)



c)



d)



c Graph $y = x^2(x^2 - 4)$. What is the domain and range of this function?

c Graph $y = x^2 - 1$, identify the zeros of this function, and use these to predict the asymptotes of $\frac{1}{(x^2 - 1)}$.
Then graph $y = \frac{1}{(x^2 - 1)}$, using a graphing tool. Compare the two graphs, considering domain, range, asymptotes and zeros.

c Use a graphing tool to graph $y = \frac{x^2}{(x^2 - 4)}$ and to predict the domain, range and zeros. Describe the symmetry.

c *Use technology to graph $f(x) = x^3 - 4x^2 + k$ for various values of k.

a) Estimate the values of k for which the equation $f(x) = 0$ appears to have a double root.

b) Show that $k = 0$ ensures that $f(x) = 0$ has a double root.

c) Show that $k = \frac{256}{27}$ ensures that $f(x) = 0$ has a double root.

PATTERNS AND RELATIONS *(Relations and Functions)*

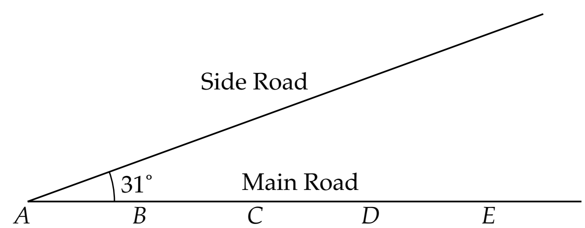
It is expected that students will represent and analyse quadratic, polynomial and rational functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• formulate and apply strategies to solve absolute value equations, radical equations, rational equations and inequalities	c Sketch $f(x) = \lvert x - 1 \rvert - 4$, and determine the values of x for which $f(x) > 0$. c Solve for x: a) $\lvert x - 1 \rvert > 7$ c) $\sqrt{(x+2)} > \frac{x}{x+2}$ b) $\sqrt{(x-1)} + \sqrt{(x+4)} = 5$ d) $\lvert x - 1 \rvert + \lvert 2x - 1 \rvert > 7$ c The point P lies on the y-axis, while points A and B are (-9, 0) and (5, 0) respectively. If $PA + PB$ is 28 units long, determine the coordinates of P.

SHAPE AND SPACE *(Measurement)*

It is expected that students will solve problems involving triangles, including those found in 3-D and 2-D applications.

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving ambiguous case triangles in 3-D and 2-D	c An 11 cm long line *AB* is drawn at an angle of 44˚ to a horizontal line *AE*. A circle with centre *B* and a radius of 9 cm is drawn, cutting the horizontal line at points *C* and *D*. Calculate the length of the chord *CD*. c The line segment of equation $y = 2.4x$, passes through *A*(0, 0) and *C*(5, 12), has a length of 13 and makes an angle of 67.3˚ with the horizontal *x*-axis. a) What points are located with *CB* = 10 and *AB* horizontal? b) Check your answer by determining the intersection points of the circle $(x - 5)^2 + (y - 12)^2 = 100$ and the line $y = 0$. c) Use a suitable diagram to explain why the answers to a) and b) are the same. c *Streetlights *A*, *B*, *C*, *D* and *E* are placed 50 m apart on the main road, as indicated on the diagram. The light from a streetlight can travel 24 m. Determine the furthest point on the side road that is lighted and the length of side road that is illuminated by both streetlight *C* and streetlight *D*.



SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will solve coordinate geometry problems involving lines and line segments, and justify the solutions.

Prescribed Learning Outcomes	Illustrative Examples
• solve problems involving distances between points and lines	c Determine the shortest distance from (3, 4) to the line $2x - 5y = 7$. c The lines $y = 3x + 1$ and $y = 3x - 9$ are parallel. Determine the vertical distance between the two lines, the horizontal distance between the two lines and the shortest distance between the two lines.
• verify and prove assertions in plane geometry, using coordinate geometry	c Given $A = (-1, 3)$, $B = (0, 5)$ and $C = (-2, 6)$: a) Verify that *ABC* is a right-angled triangle. b) Is *ABC* isosceles? Justify your assertion. c) If *M* is the midpoint of *AB* and *N* is the midpoint of *AC*, prove that *MN* is parallel to *BC*. d) Find a point *D* so that *ABCD* is a parallelogram. Prove that *ABCD* is not a rectangle. c *Use coordinate geometry to prove that: a) the diagonals of any parallelogram bisect one another in their midpoints b) if *ABC* is any triangle, with *M* as the midpoint of *AB* and *N* as the midpoint of *AC*, then *MN* is parallel to *BC* and is half its length c Use coordinate geometry to divide the line segment with end points *A*(4, 7) and *B*(-3, 8) into five congruent parts.

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

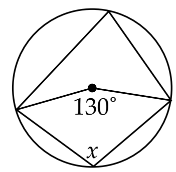
It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

Prescribed Learning Outcomes

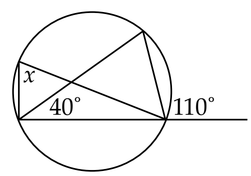
- use technology with dynamic geometry software and measurement to confirm and apply the following properties:
  - the perpendicular from the centre of a circle to a chord bisects the chord
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle, from any external point, are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is $(2n – 4)$ right angles

Illustrative Examples

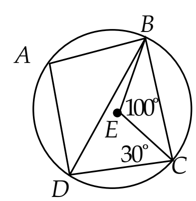
- A plate, with a diameter of 20 cm, is placed on a square place mat, with no overhang. Calculate the length of the diagonal of the square.
- Determine the measure of angle x. Justify your assertions.



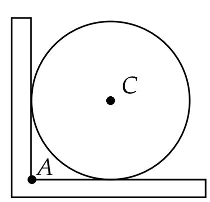
- Determine the measure of angle x. Justify your assertions.



- Draw a semicircle with diameter *AB*. Draw an angle, *ACB*, with *C* being any point on the semicircle. What is the measure of angle *ACB*? Repeat for two other points *C′*, and *C″*, on the semicircle. What pattern emerges?
- Determine the measure of /*ECB*, /*BDC*, /*BAD* and /*DBE*, where *E* is the centre of the circle.



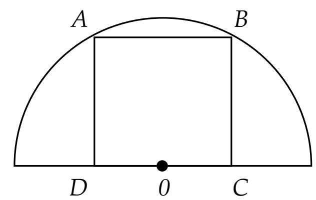
- How far from the inside corner of the shelf, *A*, is the centre *C* of the plate, if the plate has a diameter of 20 cm?



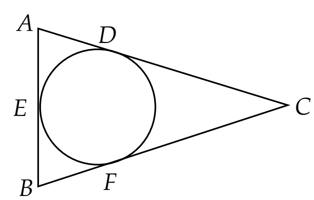
SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

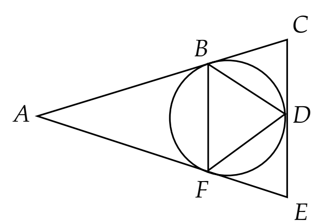
Prescribed Learning Outcomes	Illustrative Examples
	c Find the length of the side of the square if the diameter of the semicircle is 20 cm. Justify your assertions.



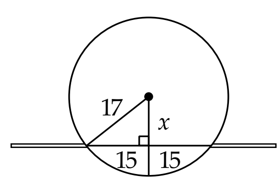
c The perimeter of the isosceles triangle *ABC*, with *AC* = *BC*, is 54 cm. If *AD* = 5 cm, and *D*, *E* and *F* are points of tangency, find the length of *BC*.



c Determine the measure of /*CAE*, if /*BDF* = 60˚ and /*FDE* = 70˚.



c A circular hole 30 cm in diameter is cut in a sheet of metal and a spherical globe 34 cm in diameter is set into the hole. How far below the top surface of the metal sheet will the sphere extend?



SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

Prescribed Learning Outcomes

- prove the following general properties, using established concepts and theorems:
  - the perpendicular bisector of a chord contains the centre of the circle
  - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle)
  - the inscribed angles subtended by the same arc are congruent
  - the angle inscribed in a semicircle is a right angle
  - the opposite angles of a cyclic quadrilateral are supplementary
  - a tangent to a circle is perpendicular to the radius at the point of tangency
  - the tangent segments to a circle from any external point are congruent
  - the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord
  - the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles.

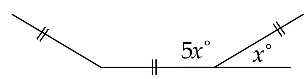
Illustrative Examples

c a) For what values of c does the line $y = c$ touch the circle $x^2 + y^2 = r^2$?
  b) Use the result from part a) to show that the tangent to a circle is perpendicular to the radius at the point of tangency.

c Show that the angle inscribed in a semicircle is a right angle.

c *The chord AB is one side of a regular polygon of n sides. The polygon is inscribed in a circle. If D is any other vertex of the polygon, prove that the magnitude of angle ADB is $\frac{180^\circ}{n}$.

c Explore the relationships between the number of sides of a regular polygon and the sum of the internal angles.



sum of interior angles
= ______ °

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

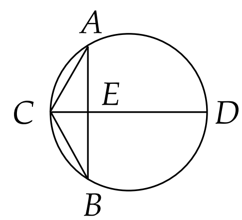
It is expected that students will develop and apply the geometric properties of circles and polygons to solve problems.

Prescribed Learning Outcomes

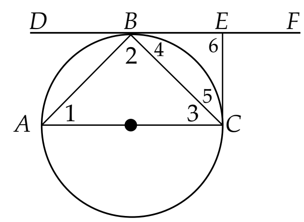
- solve problems, using a variety of circle properties, and justify the solution strategy used

Illustrative Examples

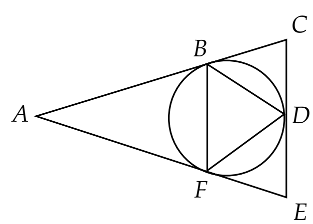
c If diameter *CD* is perpendicular to chord *AB* at *E*, prove that triangle *ABC* is isosceles.



c Show that BC bisects angle ECA, if AC is the diameter of the circle, CE is perpendicular to DF and DF is a tangent at B.



c Determine the measure of /*BAF*, if /*DEF* = 60˚ and /*BFD* = 70˚. Provide a reason for each step in the solution strategy.



c *A chain on a bicycle connects two gear wheels of diameters 9 cm and 19 cm respectively. The centres of the gear wheels are 87 cm apart. Find the minimum length of the chain.

Appendix G

Illustrative Examples

Principles of Mathematics 12

PROBLEM SOLVING

It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • solve problems that involve a specific content area • solve problems that involve more than one content area • solve problems that involve mathematics within other disciplines • analyse problems and identify the significant elements • develop specific skills in selecting and using an appropriate problem-solving strategy or combination of strategies chosen from, but not restricted to, the following: - guess and check - look for a pattern - make a systematic list - make and use a drawing or model - eliminate possibilities - work backward - simplify the original problem - develop alternative original approaches - analyse keywords • demonstrate the ability to work individually and co-operatively to solve problems • determine that their solutions are correct and reasonable • clearly communicate a solution to a problem and the process used to solve it • interpret their solutions by describing what the solution means within the context of the original problem • use appropriate technology to assist in problem solving	Illustrative Examples for Problem Solving have been indicated with an asterisk (*) throughout Appendix G.

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse exponential patterns.

Prescribed Learning Outcomes	Illustrative Examples
• derive and apply expressions to represent general terms and sums for geometric growth and to solve problems	c Determine the n^{th} term and the sum of the first n terms of the geometric sequence whose first three terms are 2, 6, and 18. c Mathematicians use sigma notation as a way to write the sum of a series. For example: $\sum_{k=1}^{5} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$. Use sigma notation to write the series $5 - 15 + 45 - \ldots + 3645$. c *Suppose that a principal of P dollars is invested at an annual interest rate r that is compounded annually. The amount A after t years is given by $A = P(1 + r)^t$. a) Find the number of years for the amount to double, if $2000 is invested at a rate of 7.5%, compounded annually. b) If the interest rate were 7.25% per annum, compounded semi-annually, how would the doubling period change? c) What would be the doubling period, if the rate were 7% per annum, compounded daily? c For the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$, find the sum of 20 terms. c *The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years. a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates: • 4% per annum, compounded annually • 8% per annum, compounded annually • 24% per annum, compounded annually. b) What general conclusion can be drawn as to the accuracy of rule of 72 calculations?

PATTERNS AND RELATIONS *(Patterns)*

It is expected that students will generate and analyse exponential patterns.

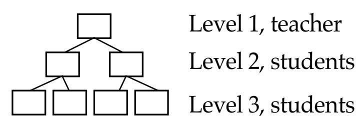
Prescribed Learning Outcomes	Illustrative Examples

Prescribed Learning Outcome:

- connect geometric sequences to exponential functions over the natural numbers

Illustrative Examples:

- The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people.
  a) Calculate the population in 1998, 2009, 2019.
  b) Calculate the population that food production could sustain in 1998, 2009, 2019.
  c) When will the population exceed the food supply?
- *The following is a school trip telephoning tree.



  a) At what level are 64 students contacted?
  b) How many are contacted at the 8th level?
  c) By the 8th level how many students, in total, have been contacted?
  d) By the nth level how many students, in total, have been contacted?
  e) If there are 300 students in total, by what level will all have been contacted?

Prescribed Learning Outcome:

- estimate values of expressions for infinite geometric processes

Illustrative Examples:

- For the infinite series $2+\frac{2}{5}+\frac{2}{25}+\ldots$, estimate the sum to four decimal places.
- An oil well produces 25 000 barrels of oil during its first month of production. If its production drops by 5% each month, estimate the total production before the well runs dry.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will solve exponential, logarithmic and trigonometric equations and identities.

Prescribed Learning Outcomes	Illustrative Examples
• solve exponential equations having bases that are powers of one another	c Solve for x: $3^{(4x-1)} = 27^{2x}$. c *A string of ones and zeros is the binary representation of a number. If this number is converted to the base-16 hexadecimal representation, it is 9 digits shorter. As well, the decimal and hexadecimal representations have the same number of digits. a) How many digits are there in the binary representation of the original number? b) Between what two decimal numbers does the original number lie?
• solve and verify exponential and logarithmic equations and identities	c Solve for x: $\log_2(x - 2) + \log_2(x) = \log_2(3)$. c Solve for x: $2 \times 3^x = 5^{(x-1)}$. c Solve for x, checking for any extraneous solutions: $\log_5(3x + 1) + \log_5(x - 3) = 3$. c *The pH of an acid is given by $pH = -\log_{10}[H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per litre. What is the hydrogen ion concentration of a weak vinegar solution of pH = 3.1? c Joe has $50,000 invested at an interest rate of 7% per annum, compounded monthly. Laura has $40,000 invested at 9.5% per annum, compounded annually. After how many years will the two investments be equal in value? c Verify the identity $\log_a\left(\frac{1}{x}\right) = -\log_a x$ for any base a and any positive value of x.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will solve exponential, logarithmic and trigonometric equations and identities.

Prescribed Learning Outcomes

- distinguish between degree and radian measure, and solve problems, using both

Illustrative Examples

c Draw an angle of one radian, and show how its radius and arc length are related.

c Convert the following angles to degrees: $\frac{2\pi}{3}$, 1.6 rad.

c Convert the following angles to radians expressed in terms of π: 180°, 55°.

c *In an experiment to verify the law of refraction, measurements were made of the angles of incidence and refraction. A spreadsheet was used to calculate the sines of both angles and the ratio of the two sines. The results of the spreadsheet calculation are shown in the following table.

Angle of incidence *i* (degrees)	Angle of refraction *r* (degrees)	sin *i*	sin *r*	(sin *i*) (sin *r*)
10	7	-0.544	0.657	-0.83
20	13	0.913	0.420	2.17
30	19	-0.988	0.150	-6.59
40	25	0.745	-0.132	-5.63
50	30	-0.262	-0.988	0.27
60	35	-0.305	-0.428	0.71
70	38	0.774	0.296	2.61
80	40	-0.994	0.745	-1.33

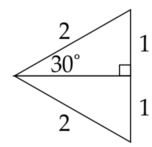
a) In the calculations of sin *i* and sin *r*, are the angles taken as being in degrees or in radians?
b) Modify the spreadsheet so that all entries reflect radian measure.
c) Modify the spreadsheet so that all entries reflect degree measure.
d) What conclusion can be drawn from either b) or c)?

Prescribed Learning Outcomes

- determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

Illustrative Examples

c Given an equilateral triangle with a side of 2 units, determine the exact trigonometric ratios of 30°.



c Find the exact values for $\sin \frac{7\pi}{6}$, $\tan \frac{2\pi}{3}$, $\cos \frac{7\pi}{4}$.

PATTERNS AND RELATIONS *(Variables and Equations)*

It is expected that students will solve exponential, logarithmic and trigonometric equations and identities.

Prescribed Learning Outcomes	Illustrative Examples
• solve first and second degree trigonometric equations over a domain of length 2π: - algebraically - graphically	c Find, algebraically and graphically, the solution to the following trigonometric equations: a) $1 + 2\cos x = 5\cos x$; $0 \le x < 2\pi$. Give solutions in decimal form. b) $\sin^2 x - \sin x = 0$; $0 \le x < 2\pi$. Give solutions as exact values. c) $\cos 4x = 0.5$; $0 \le x < 2\pi$. Give solutions as exact values.
• determine the general solutions to trigonometric equations where the domain is the set of real numbers	c Sketch the graph of $y = \sin 3x$. Use the graph to find all solutions of $\sin 3x = 0$ in the interval $0 \le x < 2\pi$. c *Use technology to graph $y = x - 2\sin x$, and use the graph to find all solutions to the equation $2\sin x = x$. Express answers to a three-decimal place accuracy. c What is the relation between the graphs of $y = \sin x$ and $y = \frac{1}{2}$ and the roots of the equation $0 = 2\sin x - 1$? c Use technology to solve $\sin 3x = \frac{1}{2}$, and then write the general solution.
• analyse trigonometric identities: - graphically - algebraically for general cases	c a) Verify that $\sin^2 x + \cos^2 x = 1$ for any real number x. b) Use this identity to show that $1 + \tan^2 x = \sec^2 x$ for all x, where $\cos x \ne 0$. c *Given the identity $\frac{\sin x}{1 - \cos x} = \frac{1 - \cos x}{\sin x}$: a) verify the identity for the particular case when $x = \frac{\pi}{3}$ b) verify for a general angle, using an algebraic approach c) verify, by graphing the left-hand side and the right-hand side of the given identity
• use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions	c Write 2 (sin 5)(cos 5) in terms of a single trigonometric function. c *Graph the function $f(x) = \frac{2\tan x}{1 + \tan^2 x}$. a) Make a conjecture for the period of the above graph. b) Simplify the expression for $f(x)$ to a single trigonometric function, and then find the period of $f(x)$. c) Compare the solutions to a) and b).

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse exponential and logarithmic functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• model, graph and apply exponential functions to solve problems	c The summertime population of gophers in a field can be modelled by the equation $P = 100(1.1)^n$, where n is measured in years. Plot the graph for a 10-summer period, and use the graph to find out how long it takes for the gopher population to double. c The half-life of sodium-24 is 14.9 hours. Suppose that a hospital buys a 40 mg sample of sodium-24. a) How much of the sample will remain after 48 hours? b) How long will it be until only 1 mg remains? c *The population of a certain country is 28 million and grows at a rate of 3% annually. Assuming the population is continuously growing, the population P, in millions, t years from now can be determined by the formula $P = 28e^{0.03t}$. a) In how many years will the population be 40 million? b) What factors could contribute to the breakdown of this model?
• change functions from exponential form to logarithmic form and vice versa	c Rewrite $y = 2^x$ as a logarithmic function. c *The ionization of pure water is shown in the equations: $[H^+]\,[OH^-] = 1.0$ ¥ 10^{-14} and $[H^+] = [OH^-]$. If the pH of any solution is defined as $pH = -\log_{10}[H^+]$, what is the pH of pure water?
• model, graph and apply logarithmic functions to solve problems	c Research the strength of earthquakes, and compare them, using the Richter scale. c *The Armenian earthquake, Richter scale 6.9, produced 3.5×10^{13} J of energy. How much energy did the Alaska earthquake, Richter scale 8.2, produce? c Graph $y = \log_{10}x$ and $y = \log_2 x$ on the same set of coordinate axes. What is the likely position of the graph of $y = \log_5 x$? c Analyse the graph of $y = \log_{10}(2x + 3)$. Identify the domain, range, asymptotes and intercepts.
• explain the relationship between the laws of logarithms and the laws of exponents	c *Explain how the exponent law $a^x \times a^y = a^{(x + y)}$ is related to the logarithmic law $\log_a(MN) = \log_a M + \log_a N$. c Use a calculator to find $\log_5 8$, and justify your procedure.

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse trigonometric functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position	c Triangle OBA has vertices $O(0, 0)$, $B(4, 0)$ and $A(4, 3)$. The unit circle, centred at (0, 0), intersects OA at point P. a) Use similar triangles to find the coordinates of point P. b) Use trigonometric ratios to find the sine and cosine of angle AOB. c) Compare your results in b) to the coordinates of point P.
• draw (using technology), sketch and analyse the graphs of sine, cosine and tangent functions, for: - amplitude, if defined - period - domain and range - asymptotes, if any - behaviour under transformations	c Using a graphing utility, graph $y = \sin x$ and $y = \cos x$ on the same set of axes. a) What relationship seems to exist between the two? b) What is the amplitude and period of each graph? c Graph $y = \tan x$ and $y = \tan 2x$. Compare the period, the domain and the range of $y = \tan x$ to those of $y = \tan 2x$. c *In the equation $y = A \sin [B(x + C)] + D$; $A = 4$, $B = 3$, $C = \frac{-3\pi}{4}$ and $D = -3$. Compare the graph of this function to the graph of $y = \sin x$ with respect to domain, range, amplitude, period, x and y intercepts, horizontal phase shift and vertical displacement.

PATTERNS AND RELATIONS *(Relations and Functions)*

It is expected that students will represent and analyse trigonometric functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
• use trigonometric functions to model and solve problems	c *For a Saskatchewan town, the latest sunrise time is on December 21, at 09:15. The earliest sunrise time is on June 21, at 03:15. Sunrise times on other dates can be predicted from a sinusoidal equation. **Note**: There is no Daylight Saving Time in Saskatchewan. a) What is the equation that describes sunrise times? b) What is the amplitude and period of the equation describing sunrise times? c) Use the equation to predict the time of sunrise on April 9. d) What is the average time of sunrise throughout the year? c The depth of water in a harbour is given by the equation $d(t) = -4.5\cos(0.16\pi t) + 13.7$, where $d(t)$ is the depth, in metres, and t is the time, in hours, after low tide. a) Sketch the graph of $d(t)$. b) What is the period of the tide, from one high tide to the next? c) A bulk carrier needs at least 14.5 m of water to dock safely. For how many hours per cycle can the bulk carrier dock safely? c *The average daily maximum temperature in Vancouver follows a sinusoidal pattern with a highest value of 23.6˚C on July 26, and a lowest value of 4.2˚C on January 26. a) Describe this variation with a sine or cosine equation. b) What is the expected maximum temperature for May 26? c) How many days will have an expected maximum of 21.0˚C or higher? d) Explain why different equations give the same answers for b) and c).

SHAPE AND SPACE *(3-D Objects and 2-D Shapes)*

It is expected that students will classify conic sections, using their shapes and equations.

Prescribed Learning Outcomes	Illustrative Examples
• classify conic sections according to shape	c Visualize the shapes generated from the intersection of a double-napped cone and a plane. For each conic section, describe the relationship between the plane, the central axis of the cone and the cone's generator.
• classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only)	c A circle with a radius of 4 units has the equation $x^2 + y^2 - 16 = 0$. What are the values of A, C and F in the general form? What is the radius of the circle $25x^2 + 25y^2 - 100 = 0$? c a) Graph the circle $x^2 + y^2 = 25$. b) Graph $Ax^2 + y^2 = 25$ where $A > 1$. c) Graph $Ax^2 + y^2 = 25$ where $0 < A < 1$. d) Graph $Ax^2 + y^2 = 25$ where $A = 0$. e) Graph $x^2 + Cy^2 = 25$ where $C > 1$. f) Graph $x^2 + Cy^2 = 25$ where $0 < C < 1$. g) Graph $x^2 + Cy^2 = 25$ where $C = 0$. h) Draw a conclusion based on the results found in b) through g). c Graph $2x^2 + y^2 - 12 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph? c Graph $4x^2 - 25y^2 - 100 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph?
• convert a given equation of a conic section from general to standard form and vice versa	c Convert to standard form: a) $x^2 + y^2 + 6x - 8y = 11$ b) $3x^2 + y^2 + 6x + 4y = 9$ c Convert to general form: a) $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{16} = 1$ b) $\frac{(x+3)^2}{25} - \frac{(y-4)^2}{16} = 1$

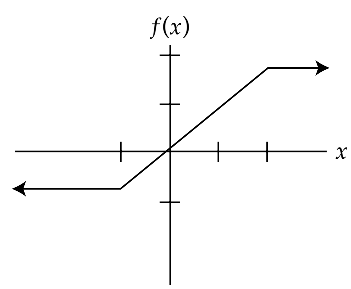
SHAPE AND SPACE (*Transformations*)

It is expected that students will perform, analyse and create transformations of functions and relations that are described by equations or graphs.

Prescribed Learning Outcomes	Illustrative Examples
• describe how various translations of functions affect graphs and their related equations: - $y = f(x - h)$ - $y - k = f(x)$	c Describe how the graph of $y = x^2$ compares to the graph of $y = x^2 - 2$. c Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $f(x) - 2$ b) $f(x - 2)$ c) $f(x - 2) + 1$
• describe how various stretches of functions (compressions and expansions) affect graphs and their related equations: - $y = af(x)$ - $y = f(kx)$	c Describe how the graph of $y = x^2$ compares to the graph of: a) $y = 2x^2$ b) $y = \frac{2}{3}x^2$ c Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $2f(x)$ b) $-2f(x)$ c) $\frac{2}{3}f(x)$ Discuss the changes. c Given the graph of $f(x) = \sin x$, sketch the graph of: a) $f(2x)$ b) $\frac{2}{3}f(x)$ c Given the graph of $f(x) = x^3$ and its image under the transformation $g(x) = 3f(x)$, find the equation describing $g(x)$.
• describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations: - $y = f(-x)$ - $y = -f(x)$ - $y = f^{1}(x)$	c Graph any function $f(x)$. Sketch the graph of: a) $-f(x)$ b) $f(-x)$ c) $f^{1}(x)$ d) $f^{1}[f(x)]$ c If $g(x)$ is the reflection of $f(x)$ in the y-axis, write the equation of $g(x)$ in terms of $f(x)$.

SHAPE AND SPACE *(Transformations)*

It is expected that students will perform, analyse and create transformations of functions and relations that are described by equations or graphs.

Prescribed Learning Outcomes	Illustrative Examples
• using the graph and/or the equation of $f(x)$, describe and sketch $\lvert f(x) \rvert$	c Given the graph of $f(x) = 2x + 1$, sketch $\lvert f(x) \rvert$. c Sketch $y = 3 \lvert \sin x \rvert$. What is the period of this function? c Sketch $f(x) = \frac{1}{\lvert x^2 - 1 \rvert}$. c *An AC generator has a voltage given by $V = 170 \cos (120\pi t)$, where V is the voltage and t the time in seconds. A simple DC rectifier has voltage output given by $V = 170 \lvert \cos (120\pi t) \rvert$. Sketch the output graphs for both devices, and describe the similarities and differences.
• using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$	c Given $f(x) = 2x + 1$, sketch the graph of $f(x)$ and of $\frac{1}{f(x)}$. What happens to the x-intercepts of $f(x)$? c Sketch the graph of $f(x) = \sin x$, and sketch $\frac{1}{\sin x}$. c Sketch $\frac{1}{f(x)}$, if $f(x)$ is shown by the accompanying sketch. 
• describe and perform single transformations and combinations of transformations on functions and relations	c Given $f(x) = x^2$, sketch the graph of $f(x)$, and sketch the graph of $-2f(x - 1) + 3$. c Determine the equation of the ellipse $x^2 + 4y^2 - 25 = 0$, after each of the following transformations: a) translated two units to the right b) translated three units down c) expanded by a factor of two along the horizontal axis d) expanded by a factor of one quarter along the vertical axis c *Given the circle $x^2 + y^2 = 1$ and its image under a translation described by the ordered pair (2, -3); a) Write the equation of the image b) If a point $P(a, b)$ is on the graph of the circle $x^2 + y^2 = 1$ and $P_1(a_1, b_1)$ is the transformed image of P, what are the coordinates of P_1 in terms of a and b?

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will use normal and binomial probability distributions to solve problems involving uncertainty.

Prescribed Learning Outcomes	Illustrative Examples
• find the standard deviation of a data set or a probability distribution, using technology	c Measure the height of each student in a class, and calculate the mean and standard deviation. c *A company uses an automated packaging device to produce 50-g bags of Karmel Korn. The machine needs frequent checking to see if it is actually putting 50 g in each bag. The following are the masses, in grams, of thirty bags of Karmel Korn. 54, 50, 47, 50, 51, 50 53, 50, 47, 51, 50, 51 52, 49, 46, 52, 50, 49 52, 48, 48, 53, 49, 49 51, 48, 49, 52, 49, 50 a) Calculate the mean and standard deviation of this data. b) What problems will be encountered, if the standard deviation gets too high? Dottori et al., *Foundations of Mathematics 11*, p. 391. Adapted with permission.
• use z-scores and the normal distribution to solve problems	c *The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 15 mL. a) Calculate the z-score for a can with a volume of 355 mL. b) What percentage of production will consist of cans having content volumes between 350 mL and 355 mL? c) What percentage of production will consist of cans having content volumes less than 355 mL? d) If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected in a run of 50 000? c For entry into the Canadian Armed Forces, the standards for height used to be set at 158 cm to 194 cm for males, and 152 cm to 184 cm for females. Use the concept of z-score to test if these two height standards are equivalent. Assume population means of 176 cm and 163 cm and standard deviations of 8 cm and 7 cm respectively.

Masses table (Karmel Korn):

54	50	47	50	51	50
53	50	47	51	50	51
52	49	46	52	50	49
52	48	48	53	49	49
51	48	49	52	49	50

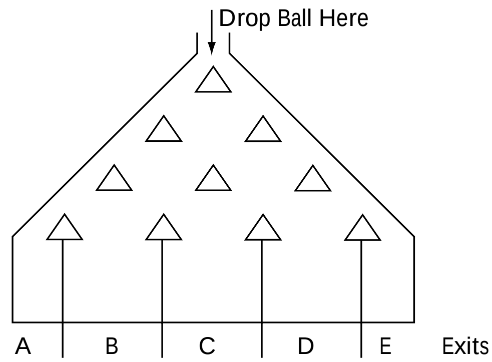
STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will use normal and binomial probability distributions to solve problems involving uncertainty.

Prescribed Learning Outcomes	Illustrative Examples
	c A sample of 122 people gives a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find: a) the expected number of people with temperatures above 37.0°C b) the expected number of people with temperatures below 36.0°C Also, estimate the range of temperatures contained within the sample. c In the general population, the IQ scores of individuals is normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested: a) What proportion of this group is expected to have IQs between 100 and 120? b) What is the probability that an individual in the group has an IQ greater than 120?
• use the normal approximation to the binomial distribution to solve problems involving probability calculations for large samples (where $npq > 10$)	c *Pollsters estimate that the number of decided voters in favour of a particular bylaw is 64%, and the number opposed is 36%. a) If the sample size is 250, find the expected mean and standard deviation of *yes* voters. b) Estimate, for this sample, the expected percentage of *yes* voters, with a symmetric 95% confidence interval used to establish the margin of error. c) If the margin of error for the percentage of *yes* voters must be less than ±1.0%, what would be the minimum sample size required? c The probability that a car salesperson will complete a sale is 0.10. a) If the salesperson has 200 customers in the next month, estimate the number of sales that she will make during that interval. b) Construct a 95% confidence interval for her monthly sales.

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations, and combinations.

Prescribed Learning Outcomes	Illustrative Examples
• solve pathway problems, interpreting and applying any constraints	c Given the following "pinball" situation, what is the probability of the ball reaching each of the exits?  What assumptions are made in the solution?
• use the fundamental counting principle to determine the number of different ways to perform multistep operations	c Joe has three different shirts, two different pairs of pants and five different pairs of shoes. List all possible outfits in such a way as to ensure that all have been counted and none have been counted twice. How many possible outfits are there? Use the fundamental counting principle to determine the number of outfits there should be. Do your answers match? c *An airline pilot reported that in seven days she spent one day in Winnipeg, one day in Regina, two days in Edmonton and three days in Yellowknife. How many different itineraries are possible? What difference would it make if the first day and the last day had to be spent in Yellowknife?

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations, and combinations.

Prescribed Learning Outcomes	Illustrative Examples
• determine the number of permutations of *n* different objects taken *r* at a time, and use this to solve problems	c List all possible permutations of the letters in the word **bold**. c Calculate the number of ways that an executive consisting of four people (president, vice-president, treasurer and secretary) can be selected from a group of 20 people. c Explain the meaning of ${}_8P_3$. Why does ${}_3P_8$ not make sense? c Develop and solve a problem where ${}_8P_3$ would be applicable. c Solve ${}_nP_2 = 30$. c On a 12-question multiple-choice test, three answers are A, three are B, three are C and three are D. How many different answer keys are possible?
• determine the number of combinations of *n* different objects taken *r* at a time, and use this to solve problems	c *From a group of five student representatives, three will be chosen to work on the dance committee. a) List all possible committees. b) Calculate ${}_5C_3$, and compare to the answer in part a). c) If the committee had to have a chairperson, would it still be a combination? Why or why not? d) How many committees of three, with a chairperson, can be chosen from a group of 10 student representatives? c Show that ${}_nC_k = {}_nC_{(n-k)}$, using two different methods. Verify the truth of this assertion for the special case with $n = 10$ and $k = 3$. c *How many diagonals are there in a regular polygon with 20 sides? What is the general formula for the number of diagonals in an *n*-sided polygon?
• solve problems, using the binomial theorem where *N* belongs to the set of natural numbers	c Expand $(x + y)^7$, using the binomial theorem. c Find the 11th term of the expansion of $(x - 2)^{13}$. c *Investigate the sample space for flipping 1 coin, 2 coins, 3 coins, 4 coins . . . , and make an organized list. Relate this organized list to Pascal's triangle and the binomial theorem. c Given a set of four elements, list the different proper and improper subsets, and organize them. How is this related to Pascal's triangle? How many subsets are there in total?

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

Prescribed Learning Outcomes	Illustrative Examples
• construct a sample space for two or three events	c List the sample space for rolling a 6-sided die and flipping a coin. c *Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15 inclusive. A train is scheduled to arrive between 07:11 and 07:17 inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the point (6, 14). Times are expressed in whole minutes. a) How many points are there in this sample space? b) How many points have the bus and the train arriving at the same time? c) How many points have the bus arriving after the train? d) What is the probability of the bus arriving after the train?
• classify events as independent or dependent	c Classify the following events as independent or dependent: a) tossing a head in a coin toss and rolling a 6 on a die b) drawing an ace for the first card and another ace for the second, if the experiment is carried out without replacement c) drawing a king for the first card and a queen for the second, if the experiment is carried out with replacement c *Sixty percent of young drivers take driver training, and 25% of young drivers have an accident in their first year of driving. Statistics show that 10% of those who do take driver training have an accident in their first year. Are taking driver training and having an accident in the first year independent events?
• solve problems, using the probabilities of mutually exclusive and complementary events	c If the probability of winning a game is $\frac{1}{31}$, what is the probability of losing the game? c *A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot?

STATISTICS AND PROBABILITY *(Chance and Uncertainty)*

It is expected that students will model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

Prescribed Learning Outcomes	Illustrative Examples
• determine the conditional probability of two events	c In a particular country, the probability of a child being a girl is 0.510. A family of five children is known to have at least two girls. What is the probability of this family having exactly four girls? c *It is known that 10% of a population has a certain disease. For a patient without the disease, a blood test for the disease gives a correct diagnosis 95% of the time. For a patient with the disease, the test gives a correct diagnosis 99% of the time. What is the probability that a person whose blood test shows the disease actually has the disease?
• solve probability problems involving permutations, and combinations and conditional probability	c Five books, each of a different colour, and including one red and one green book, are placed on a shelf. What is the probability of the red book being at one end and the green book at the other? c What is the probability of holding all four aces in a five-card hand dealt from a standard 52-card deck? c *A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot? c) What is the probability of Team A eventually winning? d) If Team B shot first, what is the probability of Team B eventually winning?

Appendix G

Illustrative Examples

Calculus 12

FUNCTIONS, GRAPHS AND LIMITS *(Functions and their Graphs)*

It is expected that students will represent and analyse inverse trigonometric, base e exponential, natural logarithmic, elementary implicit, and composite functions, using technology as appropriate.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will:	
• model and apply inverse trigonometric, base e exponential, natural logarithmic, elementary implicit, and composite functions to solve problems	c Find the area of the part of the first quadrant that is inside the circle $x^2 + y^2 = 4$ and to the left of the line $x = u$. (The inverse sine function will be useful here.) c Suppose that under continuous compounding 1 dollar grows after t years to $(1.05)^t$ dollars. Find the number r such that $(1.05)^t = e^{rt}$. (This r is called the nominal yearly interest rate.)
• draw (using technology), sketch and analyse the graphs for rational, inverse trigonometric, base e exponential, natural logarithmic, elementary implicit and composite functions, for: - domain and range - intercepts	c Sketch the graph of $\frac{x^2}{x^2 - 1}$. c Write $\sin(\tan^{-1} x)$ in a form that does not involve any trigonometric functions. c a) Let $f(x) = \ln(ex)$. Sketch the graph of $f(x)$. b) Let $g(x)$ be the inverse function of $f(x)$. Sketch the graph of $g(x)$. c) Find an explicit formula for $g(x)$. c Sketch the curve $y = \ln(e^x + 1)$.
• recognize the relationship between a base a exponential function (a>0) and the equivalent base e exponential function (convert $y = a^x$ to $y = e^{x(\ln a)}$) (Differentiation problems will also require students to demonstrate achievement of this outcome.)	c Let $f(x) = x^x$. a) Express $f(x)$ in the form $f(x) = e^{g(x)}$. b) Find the minimum value taken on by $f(x)$ on the interval $(0, \infty)$.
• determine using the appropriate method (analylitic or graaphing utility) the points where $f(x) = 0$	c [No example for this prescribed learning outcome}

FUNCTIONS, GRAPHS AND LIMITS *(Limits)*

It is expected that students will understand the concept of a limit of a function, notation used, and be able to evaluate the limit of a function.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- demonstrate an understanding of the concept of limit and notation used in expressing the limit of a function $f(x)$ as x approaches a: $\lim\limits_{x \to a} f(x)$

c * Sketch and explain what each of the following means:

a) $\lim\limits_{x\to a} f(x) = L$

b) $\lim\limits_{x\to a^-} f(x) = L$

c) $\lim\limits_{x\to a^+} f(x) = L$

d) $\lim\limits_{x\to a^+} f(x) = \infty$

e) $\lim\limits_{x\to a^-} f(x) = \infty$

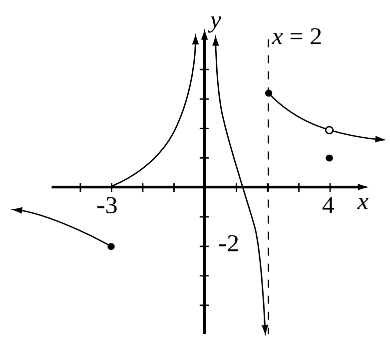
f) $\lim\limits_{x\to \infty} f(x) = L$

- evaluate the limit of a function
  - analytically
  - graphically
  - numerically
- distinguish between the limit of a function as x approaches a and the value of the function at $x = a$
- demonstrate an understanding of the concept of one-sided limits and evaluate one-sided limits
- determine limits that result in infinity (infinite limits)
- evaluate limits of functions as x approaches infinity (limits at infinity)

c Use a calculator to draw conclusions about:

a) $\lim\limits_{t\to 0} \dfrac{\sin t - t}{t^3}$

b) $\lim\limits_{x\to 0^+} x^x$



c From the given graph find the following limits. If there is no limit, explain why:

a) $\lim\limits_{x\to 2^+} f(x)$

b) $\lim\limits_{x\to 2^-} f(x)$

c) $\lim\limits_{x\to -3^+} f(x)$

d) $\lim\limits_{x\to -3^-} f(x)$

e) $\lim\limits_{x\to -3} f(x)$

FUNCTIONS, GRAPHS AND LIMITS *(Limits)*

It is expected that students will understand the concept of a limit of a function, notation used, and be able to evaluate the limit of a function.

Prescribed Learning Outcomes	Illustrative Examples
	f) $\lim_{x \to 4} f(x)$ g) $f(4)$ h) $\lim_{x \to 0} f(x)$ C Find the limits: a) $\lim_{h \to 0} \frac{(1+h)^2 - 1}{h}$ b) $\lim_{t \to 6} \frac{17}{(t-6)^2}$ c) $\lim_{h \to 0} \frac{(2+h)^{-2} - \frac{1}{4}}{h}$ d) $\lim_{v \to 2} \frac{v^2 + 2v - 8}{v^4 - 16}$ e) $\lim_{x \to 8^-} \frac{\lvert x-8 \rvert}{x-8}$ f) $\lim_{x \to 0} \frac{1 - \sqrt{1-x^2}}{x}$ g) $\lim_{x \to 0} \frac{\sin 3x}{x}$ h) $\lim_{\theta \to 0} \frac{\tan 3\theta}{\theta}$ i) $\lim_{\theta \to 0} \frac{\sin 3\theta}{\sin 5\theta}$ j) $\lim_{x \to \infty} \frac{3x^2 + 5}{4 - x^2}$ k) $\lim_{x \to \infty} x \sin \frac{1}{x}$ l) $\lim_{x \to \infty} \frac{\sqrt{x^2 + 4x}}{4x + 1}$ m) $\lim_{x \to 0^+} \frac{1}{2 + 10^{\frac{1}{x}}}$ n) $\lim_{x \to 0^-} \frac{1}{2 + 10^{\frac{1}{x}}}$

FUNCTIONS, GRAPHS AND LIMITS *(Limits)*

It is expected that students will understand the concept of a limit of a function, notation used, and be able to evaluate the limit of a function.

Prescribed Learning Outcomes	Illustrative Examples
• determine vertical and horizontal asymptotes of a function, using limits	c Find the horizontal and vertical asymptotes of each of the following curves: a) $y=\dfrac{x}{x+4}$ b) $y=\dfrac{x^2}{x^2-1}$ c) $y=\dfrac{x^2}{x^2+1}$ d) $y=\dfrac{1}{(x-1)^2}$
• determine whether a function is continuous at $x=a$	c *Given $f(x)=\begin{cases}\sqrt{-x} & ; \quad x<0\\ 3-x & ; \quad 0\le x\le 3\\ (x-3)^2 & ; \quad x>3\end{cases}$ a) find: i) $\lim\limits_{x\to 0^+} f(x)$ ii) $\lim\limits_{x\to 0^-} f(x)$ iii) $\lim\limits_{x\to 0} f(x)$ iv) $\lim\limits_{x\to 3^-} f(x)$ v) $\lim\limits_{x\to 3^+} f(x)$ vi) $\lim\limits_{x\to 3} f(x)$ b) Where is the graph discontinuous. Why? c) Sketch the graph.

THE DERIVATIVE *(Concept and Interpretations)*

It is expected that students will understand the concept of a derivative and evaluate derivatives of a function using the definition of derivative.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • describe geometrically a secant line and a tangent line for the graph of a function at $x=a$. • define and evaluate the derivative at $x = a$ as: $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ and $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ • define and calculate the derivative of a function using: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ or $f'(x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ • use alternate notation interchangeably to express derivatives (i.e., $f'(x), \frac{dy}{dx}, y'$, etc.) • compute derivatives using the definition of derivative	c Calculate the derivatives of the given function directly from the definition: a) $f(x)=x^2+3x$ b) $g(x)=\frac{x}{x-1}$ c) $H(t)=\sqrt{t+1}$ c Using the definition of derivative, find the equation of the tangent line to the curve: $y=5+4x-x^2$ at the point $(2,9)$.
• distinguish between continuity and differentiability of a function at a point • determine when a function is non-differentiable, and explain why • determine the slope of a tangent line to a curve at a given point • determine the equation of the tangent line to a curve at a given point	c Determine where $f(x)=\lvert x-3\rvert$ is non differentiable. c Find an equation of the line tangent to the given curve at the point indicated. Sketch the curve: $y=\frac{1}{x^2+1}$ at $x=-1$ c *Find all the values of a for which the tangent line to $y=\ln x$ at $x=a$ is parallel to the tangent line to $y=\tan^{-1}x$ at $x=a$.

THE DERIVATIVE *(Concept and Interpretations)*

It is expected that students will understand the concept of a derivative and evaluate derivatives of a function using the definition of derivative.

Prescribed Learning Outcomes	Illustrative Examples
• for a displacement function $s = s(t)$, calculate the average velocity over a given time interval and the instantaneous velocity at a given time • distinguish between average and instantaneous rate of change	c The position of a particle is given by $s = t^3$. Find the velocity when $t = 2$. c *The displacement in metres of a particle moving in a straight line is given by $s = 5 + 4t - t^2$ where t is measured in seconds. a) Find the average velocity from $t = 2$ to $t = 3$, $t = 2$ to $t = 2.1$, and $t = 2$ to $t = 2.01$. b) Find the instantaneous velocity when $t = 2$. c) Draw the graph of s as a function of t and draw the secant lines whose slopes are average velocity as in a). d) Draw the tangent line whose slope is the instantaneous velocity in b).

THE DERIVATIVE *(Computing Derivatives)*

It is expected that students will determine derivatives of functions using a variety of techniques.

Prescribed Learning Outcomes	Illustrative Examples

It is expected that students will:

- compute and recall the derivatives of elementary functions including:
  - $\frac{d}{dx}(x^r) = rx^{r-1}$, r e ℜ
  - $\frac{d}{dx}(e^x) = e^x$
  - $\frac{d}{dx}(\ln x) = \frac{1}{x}$
  - $\frac{d}{dx}(\cos x) = -\sin x$
  - $\frac{d}{dx}(\sin x) = \cos x$
  - $\frac{d}{dx}(\tan x) = \sec^2 x$
  - $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
  - $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- use the following derivative formulas to compute derivatives for the corresponding types of functions:
  - constant times a function: $\frac{d}{dx}cu = c\frac{du}{dx}$
  - $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$ (sum rule)
  - $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ (product rule)
  - $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (quotient rule)
  - $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$ (power rule)
- use the Chain Rule to compute the derivative of a composite function:
  $\frac{dy}{dx} = \frac{du}{dx}\frac{dy}{du}$ or $\frac{d}{dx}(F(g(x))) = g'(x)F'(g(x))$

c Calculate the derivatives of the following functions:

a) $y = 3 - 4x - 5x^2 + 6x^2$

b) $z = \frac{S^5 - S^3}{15}$

c) $f(t) = \frac{4}{2-5t}$

d) $y = (2x+3)^6$

e) $y = (x^2+9)\sqrt{x^2+3}$

f) $f(x) = \sin 2x$

g) $y(x) = \cos^2(5-4x^3)$

h) $y = \ln(3t^2+6)$

i) $y = 2e^{-x}$

j) $y = \tan(e^x)$

k) $F(x) = \log_5(3x-8)$

l) $y = 2^{x^4}$

m) $y = \sin^{-1}(\sqrt{2x})$

n) $y = \tan^{-1}(3x)$

c Let $f(x) = \ln\left(x + \sqrt{4+x^2}\right)$. Find $f'(x)$ and simplify.

c A certain function $f(x)$ has $f(1) = 4$ and $f'(1) = 5$. Let $g(x) = \frac{1}{\sqrt{2f(x)+1}}$. Find $g'(1)$.

THE DERIVATIVE *(Computing Derivatives)*

It is expected that students will determine derivatives of functions using a variety of techniques.

Prescribed Learning Outcomes	Illustrative Examples
• compute the derivative of an implicit function.	c Find $\frac{dy}{dx}$ in terms of x and y if: a) $xy = x + 2y + 1$ b) $x^3y + xy^5 = 2$ c Find the equation of the tangent line at the given point: $x^2y^3 - x^3y^2 = 12$ at $(-1,2)$ c Find y'' by implicit differentiation: $x^2 - y^2 = 1$ c The equation $x^4 + x^2y + y^4 = 27 - 6y$ defines y implicitly as a function of x near the point (2,1). Calculate y' and y'' at (2,1).
• use the technique of logarithmic differentiation.	c Let $y = (x+1)^2(2x+1)^3(3x+1)^4e^{5x}$. Use logarithmic differentiation to find $\frac{dy}{dx}$. c Using logarithmic differentiation, find a) y' if $y = x^x \; (x > 0)$ b) $\frac{dy}{dx}$ if $y = 5^{2x+1}$ c) $f'(x)$ if $f(x) = \frac{x^5\sqrt{3+x^2}}{(4+x^2)^3}$
• compute higher order derivatives	c Find y', y'', and y''' for the functions: a) $y = x^2 - \frac{1}{x}$ b) $y = \frac{x-1}{x+1}$ c Let $f(x) = \frac{1}{1+x}$. Find $f^{(7)}(x)$, the seventh derivative of $f(x)$.

APPLICATIONS OF DERIVATIVE *(Derivatives and the Graph of the Function)*

It is expected that students will use the first and second derivatives to describe the characteristic of the graph of a function.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • given the graph of $y = f(x)$: - graph $y = f'(x)$ and $y = f''(x)$ - relate the sign of the derivative on an interval to whether the function is increasing or decreasing over that interval. - relate the sign of the second derivative to the concavity of a function • determine the critical numbers and inflection points of a function. • determine the maximum and minimum values of a function and use the first and / or second derivative test(s) to justify their solutions	c For the following functions, find the critical numbers, the inflection points, and the vertical and horizontal asymptotes. Sketch the graph, and verify using a graphing calculator. a) $f(x) = \frac{x^3}{x^2 - 1}$ b) $y = \frac{3x^2}{2x^2 + 1}$ c) $f(x) = \frac{1}{x^2 - x}$ d) $f(x) = (x^2 - 1)^{\frac{2}{3}}$ c Find the critical and inflection points for $f(x) = -2xe^{-x}$ and sketch the graph of $f(x)$ for $x \geq 0$. c Sketch the graph $g(x) = x + \cos x$. Determine where the function is increasing most rapidly and least rapidly.
• use Newton's iterative formula (with technology) to find the solution of given equations, $f(x) = 0$	c Using a calculator, complete at least 5 iterations of Newton's method for $g(x) = x - \sin x + 1$ if $x_0 = -1$. c Explain why the function *f*, where $f(x) = \frac{2\ln x}{1 + x^2}$, has exactly one critical number. Use Newton's Method to find that critical number, correct to two decimal places. c Use Newton's Method to find, correct to two decimal places, the *x*-coordinate of a point at which the curve $y = \tan x$ meets the curve $y = x + 2$. Also look at the problem by using the "solve" feature of your calculator and by graphing the curves.

APPLICATIONS OF DERIVATIVE *(Derivatives and the Graph of the Function)*

It is expected that students will use the first and second derivatives to describe the characteristic of the graph of a function.

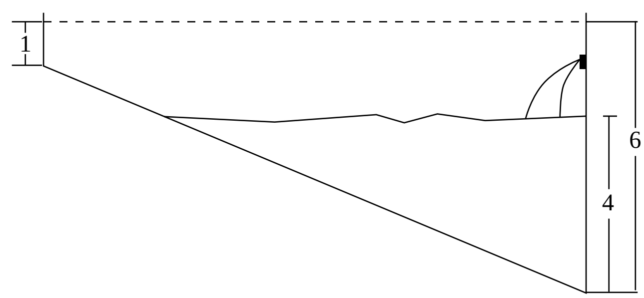
Prescribed Learning Outcomes	Illustrative Examples
• use the tangent line approximation to estimate values of a function near a point and analyse the approximation using the second derivative.	c Determine the tangent line approximation for $f(x) = \sin x$ in the neighbourhood of $x = 0$. Zoom in on both graphs and compare the results. c A certain curve has equation $e^{2y} + y = x^2$. Note that the point (1,0) lies on the curve. Use a suitable tangent line approximation to give an estimate of the y-coordinate of the point on the curve that has x-coordinate equal to 1.2. c A certain function $f(x)$ has derivative given by $f'(x) = \sqrt[3]{x^2 - 1}$. It is also known that $f(3) = 4$. a) Use the tangent line approximation to approximate $f(3.2)$. b) Use the second derivative of f to determine whether the approximation in part (a) is bigger or smaller than the true value of $f(3.2)$. c The diameter of a ball bearing is found to be 0.48 cm, with possible error ± 0.005 cm. Use the tangent line approximation to approximate: a) the largest possible error in computing the volume of the ball bearing, b) the maximum percentage error

APPLICATIONS OF DERIVATIVE *(Applied Problems)*

It is expected that students will solve applied problems from a variety of fields including the physical and biological sciences, economics, and business.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will:	
• solve problems involving distance, velocity, acceleration	c The average velocity of a particle in the time interval from 1 to t is $\frac{e^t}{e^t - 1}$ for $t > 1$. Find the displacement and velocity at time $t = 2$.
• solve related rate problems (rates of change of two or more related variables that are changing with respect to time)	c Car P (a police car) was travelling in a northerly direction along the y-axis at a steady 50 kilometres per hour, while car Q was travelling eastward along the x-axis at varying speeds. At the instant when P was 60 metres north of the origin, Q was 90 metrs east of the origin. A radar unit on P recorded that the straight-line distance between P and Q was, at that instant, increasing at the rate of 80 kilometres per hour. How fast was car Q going?

c *A swimming pool is 25 metres wide and 25 metres long. When the pool is full, the water at the shallow end is 1 metre deep, and the water at the other end is 6 metres deep. The bottom of the pool slopes at a constant angle from the shallow end to the deep end. Water is flowing into the deep end of the pool at 1 cubic meter per minute. How fast is the depth of water at the deep end increasing when the water there is 4 metres deep?



c *Boyle's Law* states that if gas is compressed but kept at constant temperature, then the pressure P and the volume V of the gas are related by the equation $PV=C$, where C is a constant that depends on temperature and the quantity of gas present. At a certain instant, the volume of gas inside a pressure chamber is 2000 cubic centimetres, the pressure is 100 kilopascals, and the pressure is increasing at 15 kilopascals per minute. How fast is the volume of the gas decreasing at this instant?

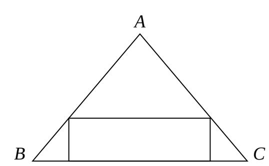
APPLICATIONS OF DERIVATIVE *(Applied Problems)*

It is expected that students will solve applied problems from a variety of fields including the physical and biological sciences, economics, and business.

Prescribed Learning Outcomes	Illustrative Examples
• solve optimization problems (applied maximum/minimum problems)	c A striped ball is thrown vertically upward, having a velocity of 12 m/s. After *t* seconds its altitude is represented by $s = 12t - 4.9t^2$. At what instant will it reach a maximum height and how high will it rise?

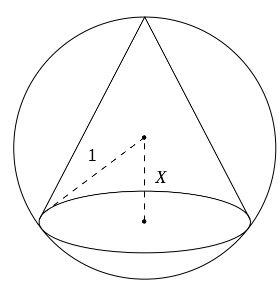
c *Construct an open-topped box with a maximum volume from a 20 cm x 20 cm piece of cardboard by cutting out four equal corners. Use more than one method. Compare your results.

c Triangle ABC has AB=AC=8, BC=10. Find the dimensions of the rectangle of maximum area that can be inscribed in ΔABC with one side on the rectangle on BC.



c A pharmaceuticals company can sell its veterinary antibiotic for a price of $450 per unit. Assume that the total cost of producing *x* units in a year is given by $C(x)$ where $C(x) = 800\,000 + 50x + 0.004x^2$. How many units per year should the company produce in order to maximize yearly profit?

c Find the volume of the cone of maximum volume that can be inscribed in a sphere of radius 1. Hint: let *X* be as shown in the diagram.



ANTIDIFFERENTIATION *(Recovering Functions from their Derivatives)*

It is expected that students will recognize antidifferentiation (indefinite integral) as the reverse of the differentiation process.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • explain the meaning of the phrase "$F(x)$ is an antiderivative (or indefinite integral) of $f(x)$" • use antiderivative notation appropriately (i.e., $\int f(x)dx$ for the antiderivative of $f(x)$) • compute the antiderivatives of linear combinations of functions whose individual antiderivatives are known including: - $\int k\,dx = kx + C$ - $\int x^r dx = \frac{x^{r+1}}{r+1} + C$ if $r \neq -1$ - $\int \frac{dx}{x} = \ln\lvert x\rvert + C$ - $\int e^x dx = e^x + C$ - $\int \sin x\,dx = -\cos x + C$ - $\int \cos x\,dx = \sin x + C$ - $\int \sec^2 x\,dx = \tan x + C$ - $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$ - $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$ • compute $\int f(ax+b)\,dx$ if $\int f(u)\,du$ is known	c Is it true that $\int 2xe^{2x} dx = xe^{2x} - \frac{e^{2x}}{2} + C$? c Is it true that $\int \sqrt{1-x^2}\,dx = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x + C$? c Is it true that $\int x\cos x\,dx = \frac{x^2}{2}\sin x + C$? c Evaluate the following indefinite integrals: a) $\int (1+3x)^5 dx$ b) $\int e^{x/2}\,dx$ c) $\int \frac{5dx}{(1+4x)^2}$ c Suppose that $\frac{d^2y}{dx^2} = \sin \pi x$. a) Find a general formula for $\frac{dy}{dx}$ as a function of x. b) Find a general formula for y as a function of x.
• create integration formulas from the known differentiation formulas	c Let a be a positive constant other than 1. Find $\int a^x dx$. c Use the fact that $\sec^2 x = 1 + \tan^2 x$ to find $\int \tan^2 x\,dx$. c Use the identity $\cos 2x = 1 - 2\sin^2 x$ to find $\int \sin^2 x\,dx$. c Find $\int x\cos 3x\,dx$ by guessing that one antiderivative is $x\sin 3x$, differentiating, and then adjusting your guess.

ANTIDIFFERENTIATION *(Recovering Functions from their Derivatives)*

It is expected that students will recognize antidifferentiation (indefinite integral) as the reverse of the differentiation process.

Prescribed Learning Outcomes	Illustrative Examples
• solve initial value problems using the concept that if $F'(x) = G'(x)$ on an interval, then $F(x)$ and $G(x)$ differ by a constant on that interval	c Suspose that $f''(t) = 3t^2$ for all t and $f(1) = 2$, $f'(1) = 5$. Find a formula for $f(t)$. c It is known that $f'(x) = e^{x/2}$ and $f(6 \ln 2) = 10$. Find a general formula for $f(x)$. c a) Verify that the function f given by $f(x) = 3e^{-x} \sin 3x - e^{-x} \cos 3x$ is an antiderivative of $10e^{-x} \cos 3x$. b) Use the result of part a) to find an antiderivative $G(x)$ of $10e^{-x} \cos 3x$ such that $G(0) = 5$.

ANTIDIFFERENTIATION *(Applications of Antidifferentiation)*

It is expected that students will use antidifferentiation to solve a variety of problems.

Prescribed Learning Outcomes	Illustrative Examples
It is expected that students will: • use antidifferentiation to solve problems about motion along a line that involve: - computing the displacement given initial position and velocity as a function of time - computing velocity and/or displacement given suitable initial conditions and acceleration as a function of time • use antidifferentiation to find the area under the curve $y = f(x)$, above the x-axis, from $x = a$ to $x = b$	c A particle is moving back and forth along the x-axis, with velocity at time t given by $v(t) = \sin(t/2)$. When $t = 0$, the particle is at the point with coordinates (4,0). Where is the particle at time $t = \pi$? c Let a be a given positive number. Find the area of the region which is under the curve with equation $y = \frac{1}{x}$, above the x-axis, and between the vertical lines $x = a$ and $x = 2a$. Simplify your answer. c Use calculus to find the area under the curve, above the x-axis, and lying between the lines $x = \frac{1}{\sqrt{3}}$ and $x = \sqrt{3}$.
• use differentiation to determine whether a given function or family of functions is a solution of a given differential equation • use correct notation and form when writing the general and particular solution for differential equations • model and solve exponential growth and decay problems using a differential equation of the form: $\frac{dy}{dt} = ky$	c a) Verify that $y = \sin 3t$ is a solution of the differential equation $y'' = -9y$. b) Find a solution of the above differential equation that is not a constant multiple of $\sin 3t$. c) Find a solution y of the differential equation such that $y(0) = 2$ and $y'(0) = 1$. c *Torricelli's Law says that if a tank has liquid in it to a depth h, and there is a hole in the bottom of the tank, then liquid leaves the tank with a speed of $\sqrt{2gh}$. Use the metre as the unit of length and measure time in seconds. Then g is about 9.81. Take in particular a cylindrical tank with base radius R, with a hole of radius r at the bottom. a) Verify that h satisfies the differential equation $\frac{dh}{dt} = \frac{r^2}{R^2}\sqrt{2gh}$ b) Verify that for any constant C, the function $h(t)$ is a solution of the above equation, where $h(t) = \frac{1}{4}\left(C + \frac{\sqrt{2g}\, r^2}{R^2} t^2\right)^2$ c) A hot water tank has a base radius of 0.3 metres and is being drained through a circular hole of radius 0.1 at the bottom of the tank. At a certain time, the depth of the water in the tank is 1.5 metres. What is the depth of the water 30 seconds later?

ANTIDIFFERENTIATION *(Applications of Antidifferentiation)*

It is expected that students will use antidifferentiation to solve a variety of problems.

Prescribed Learning Outcomes	Illustrative Examples
	c *Shortly after a person was exposed to iodine-131, the level of radioactivity in the person's thyroid was 20 times the "safe" level. Three hours later, the level had decreased to 15.5 time the safe level. How much longer must the person wait until the level of radioactivity in the thyroid reaches "safe" territory? c *A simple model for the growth of a fish of a particular species goes as follows: Let M be the maximum length that can be reached by a member of that species. At the instant when the length of an individual of that species is y, the length of the individual is growing at a rate proportional to $M - y$, that is, y satisfies the differential equation $\frac{dy}{dt} = k(M - y)$ for some constant k. a) Let $w = M - y$. Verify that w satisfies the differential equation $\frac{dw}{dt} = -kw$. b) Write down the general solution of the differential equation of part a). Then write down a general formula for y as a function of time. c) Suppose that for a certain species, the maximum achievable length M is 60 cm, and that when t is measured in years, k = 0.05. A fish of that species has current length of 10 cm. How long will it take to reach a length of 20c m?
• model and solve problems involving Newton's Law of Cooling using a differential equation of the form: $\frac{dy}{dt} = ay + b$	c *Coffee in a well-insulated cup started out at 95°, and was brought into an office that was held at a constant 20°. After 10 minutes, the temperature of the coffee was 90°. Assume that Newton's Law of Cooling applies. How much additional time must elapse until the coffee reaches 80°?

Appendix H

Additional Support for Instruction

This appendix contains detailed information about specific teaching strategies referred to in this Integrated Resource Package. This information is provided in cases where explanations of strategies and activities are too extensive to fit in the Suggested Instructional Strategies column or where the same activity is referred to several times in the IRP.

Derivation of Quadratic Formula

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

or

$$ax^2 + bx + c = 0$$

$$4a^2x^2 + 4abx + 4ac = 0$$

$$(2ax + b)^2 - b^2 + 4ac = 0$$

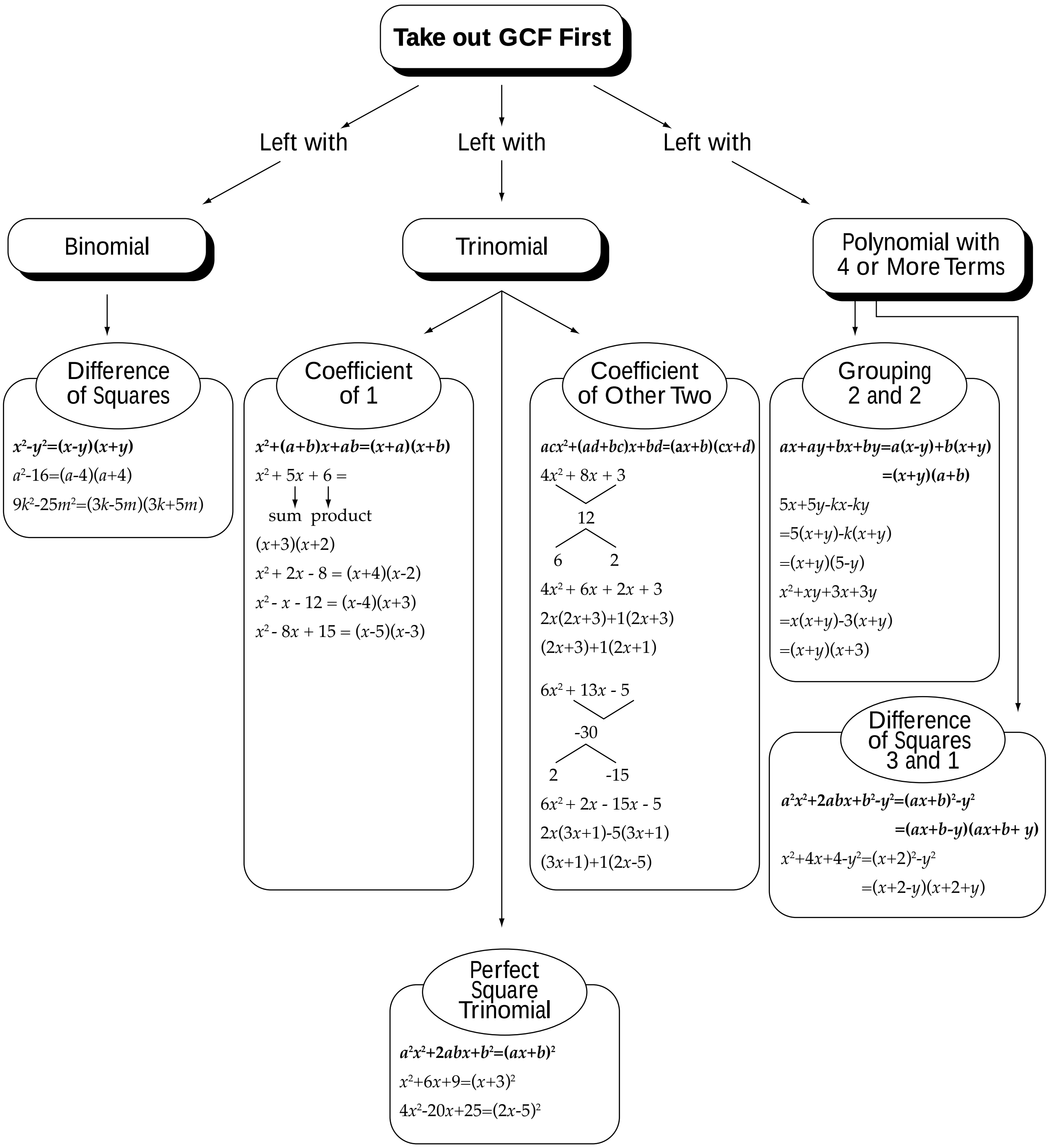
$$(2ax + b)^2 = b^2 - 4ac$$

$$2ax + b = \pm\sqrt{b^2 - 4ac}$$

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

FACTORING POLYNOMIALS

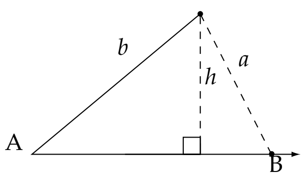


COSINE/SINE LAW MATRIX

The following chart may be useful in the presentation and development of the cosine law, sine law and the solution of oblique triangles.

In oblique triangles there are six cases

SSS	SAS	ASA
1. Cosine law to find /1	1. Cosine law to find 3rd side	1. 180˚ - (/1+/2)=/3
2. ••sine law or cosine law to find /2	2. ••use either law to find /2	2. sine law to find the other 2 sides
3. 180˚ - (/1+/2)=/3	3. 180˚ - (/1+/2)=/3	

AAS	AAS (SSA)		AAA
1. 180˚ - (/1+/2)=/3 2. sine law to find the other 2 sides	Given: /A, a, b 	Case 1: a<h ∴ no triangle Case 2: a=h ∴ 1 right triangle (/B=90˚) Case 3: h<a<b ∴ 2 triangles (1. /B<90˚)(2. /B>90˚) Case 4: b≤a ∴ 1 triangle	no unique solution

Note:
- • verify larges sides are opposite larges angles
- •• to avoid the ambiguous case with sine law, always solve for the smallest angles first