Applications of Mathematics 12

THE PROBLEM SET

September 2002

Student Assessment and Program Evaluation Branch

ACKNOWLEDGEMENT

The Ministry of Education gratefully acknowledges the advice, assistance, and contribution of the following professionals in the development of *The Problem Set* for Applications of Mathematics 12:

Vic Keehn	Summerland Secondary School	S.D. # 67 (Okanagan Skaha)
Ken Mayson	Carson Graham Secondary School	S.D. # 44 (North Vancouver)
Ray Mickelson	Earl Marriott Secondary School	S.D. # 36 (Surrey)
Patricia O'Brien	Earl Marriott Secondary School	S.D. # 36 (Surrey)
Sam Vandermeulen	Abbotsford Senior Secondary School	S.D. # 34 (Abbotsford)

There are eight problems in *The Problem Set*. This number of problems should allow time for intense investigation of each individual problem while maintaining a variety of problem-solving experiences for the students. Effective use of these problems in a cooperative setting requires time. Teachers have found success with introducing problems at regular intervals and allowing time for students to research the ideas and consult with peers and other resource persons before discussing possible solutions.

The Problem Set is part of the **Problem Solving** organizer in Applications of Mathematics 12.

The Problem Set
September 2001

RATIONALE

Problem solving is an integral component of the Mathematics curriculum from grades 1 through 12. In Applications of Mathematics 12, the Prescribed Learning Outcome states that a student should be able to

"use a variety of methods to solve all forms of problems
(that is, real-life, practical, technical, theoretical)"

The Student Assessment and Program Evaluation Branch of the Ministry of Education develops a set of problems that reflects the intent of this learning outcome.

Through the Ministry providing a set of questions for study, both students and teachers can experience the challenge of discovering appropriate strategies that might be applied to reach a solution to a novel or unfamiliar problem.

THE NATURE OF THE PROBLEMS

It is not intended that these problems require teaching beyond the Applications of Mathematics 12 curriculum.

The problems may:

- be multi-step;
- be multi-strand;
- require a variety of problem-solving strategies;
- encourage a variety of methods of solution;
- be drawn from any topics in the Applications of Mathematics curriculum from grades 1 through 12;
- require the use of appropriate technology to assist in the problem solving process.

The problems are intended to:

- be worked, under the guidance of a teacher, by all students registered in Applications of Mathematics 12;
- provide both students and teachers with a variety of challenging problems;
- encourage students and teachers to explore mathematical problems and discover appropriate strategies;
- create an atmosphere in which students and teachers realize that, acting in concert, they can bring all their individual resources to the solution of mathematical problems;
- encourage "what if" discussions by having students and teachers modify the parameters of the original problem.

ASSESSMENT

An integral part of the study of mathematics is proficiency in solving problems and this skill should be included in the assessment of a student's progress in Applications of Mathematics 12. As well, provincial examinations each year will include a part (or parts, or facsimile) of several of the problems from *The Problem Set*.

NOTES TO THE TEACHER

Teachers are expected to integrate these problems into the Applications of Mathematics 12 program. Some of the problems could be introduced at the beginning of the course, while other problems may be presented at appropriate points in the discussion of particular topics. Teachers are encouraged to pass previous problem sets to mathematics classes in other grades, where appropriate, and to use them as a source of additional problems for Applications of Mathematics 12.

Students, in general, will find the problems in this year’s set challenging. It is essential that teachers play the role of facilitators with students in solving such problems. Small group work and cooperation within that group should be encouraged. Perhaps working with these problems will give students a glimpse of the excitement and ingenuity that are involved in creative thinking. Certainly students should understand that problems need not be solved in one sitting. Often it may be necessary to “play around” with a concept, put it down, and then pick it up a few days later. Students should be encouraged to try some of the problem-solving strategies to which they have been exposed in their mathematical education. It is intended that this problem-solving strand will provide students with some real-life skills and attitudes that could be transferred to their jobs in future years. As teachers, we would like our students to be thinking, reasoning human beings, rather than simply formula manipulators. Our students should be capable of relating difficult concepts and be able to acquire the confidence necessary to analyze, explore, and form conjectures when approaching any problem.

In order to act as facilitators, it is important that the teachers not become prescriptive in their approach to these problems, but allow the students leeway to solve problems in their own unique ways. It may be very desirable, though, **that teachers ask questions which may provide additional insight into the problem**. The intent is not to lead the students to a specific method of solving the problem, but to provide an entry in some cases, or to provide a context in which to consider what is being asked. Students should be encouraged to look beyond the problem at hand in order to discover a variety of techniques for arriving at a solution.

Teachers are encouraged to use a variety of strategies in the classroom in the assessment of problem solving. Some of these may include:

- having students present solutions to the class individually, in pairs, or in small groups.
- awarding marks for **complete** written solutions for problems. Teachers should reinforce the importance of style of presentation and reward students for showing multiple methods of solving problems, when possible. It is hoped that presentations will be clear, complete, and well developed. Asking for a plan or an outline of the solution path and refining the plan as work progresses may help students in solving the problem.
- having questions on examinations or quizzes which test a part of the problem or demand similar thought processes.
- asking students to keep a journal describing their progress in dealing with the problem (including ideas and attempts which were not successful).

PROBLEM 1

Finance companies sometimes offer a type of loan known as a discount loan where the interest is calculated and deducted from the amount requested before the funds are advanced.

a) A construction company is advanced funds from a finance company as a discount loan. The company borrows money for two years at a rate of 8.5% compounded quarterly. If it must pay back $22 500 at the end of the two-year period, and if this amount includes both the principal and the interest, how much does the construction company actually receive at the start of the two-year period?

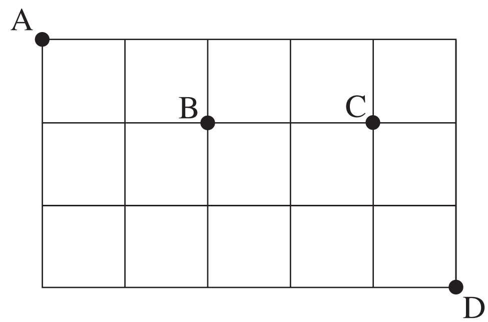
b) The company arranges a second loan, also a discount loan, payable in three years at a rate of 8.25% compounded monthly. The amount payable in three years on this second loan is $25 000 (again this amount includes principal and interest). What is the total amount of funds advanced on the two loans?

c) After advancing these funds, the finance company offers to consolidate the two loans into one discount loan payable in 30 months at a rate of 8.35% compounded semi-annually. What is the total amount the construction company must pay out on the consolidated discount loan at the end of 30 months?

d) How much will the construction company save by consolidating the loans?

PROBLEM 2

The diagram below shows a partial map of a certain city park, with walking paths located on the grid lines.



A tourist on a walking tour of the park **starts** at point A and randomly selects a path to point D, walking along the grid lines only to the south and east.

a) What is the probability the tourist passes through point B?

b) What is the probability the tourist passes through point C?

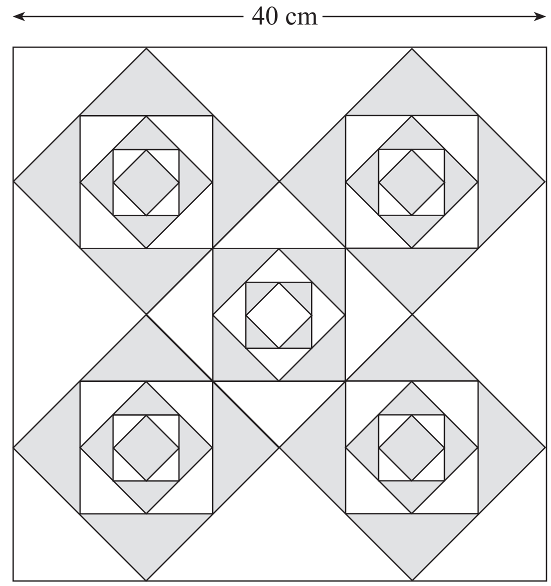
c) What is the probability the tourist passes through point B **and** point C?

d) What is the probability the tourist passes through point B **or** point C?

e) What is the probability the tourist chooses a path that misses both point B and point C?

PROBLEM 3

An art project requires students to create a fractal pattern poster on a $40 \text{ cm} \times 40 \text{ cm}$ sheet of paper. One student created her pattern by constructing squares as shown in the diagram below. Alternate squares were shaded as shown. Assume this process continues forever.



a) What is the total area of all squares on the poster?

b) What is the total length of all lines drawn?

c) What fraction of the total fractal poster will be shaded?

PROBLEM 4

Airlines routinely sell more tickets than available seats (overbooking) because they know a certain percentage of customers who buy tickets do not show up. Nite-Flite Airlines has data that indicates 5% of people buying tickets are no-shows; consequently, the airline overbooks its 300-seat airplanes by selling 310 seats. After 310 sales, potential passengers are placed on a "stand-by" list.

a) What is the probability that Nite-Flite Airlines has to deal with the situation of more passengers than seats?

b) If you and a friend are the first two passengers on the "stand-by" list, what is the probability that you both get seats?

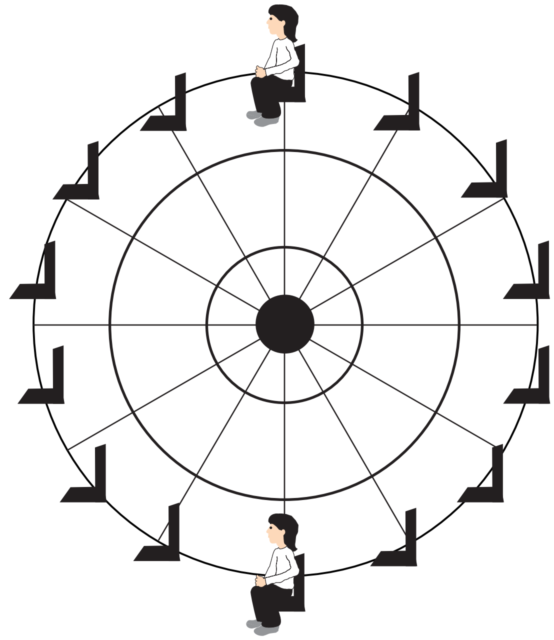
c) If you are the fourth name on the stand-by list, what is the probability that you will get a seat on the flight?

Note: The intent of this problem is to encourage students to explore the statistical capabilities of the graphing calculator. The problem can also be solved by using z-score tables.

PROBLEM 5

The following table shows the height of a seat on a Ferris wheel above the ground after it has reached its normal rotating speed.

Time (seconds)	Height (metres)
20	19.9
25	18.4
30	11.4
35	4.0
40	1.8
45	6.5
50	14.6
55	19.8
60	18.3
65	11.2



By using the sine regression function on a graphing calculator, determine:

a) how far above the ground the seat is at its highest point.

b) the number of complete revolutions the Ferris wheel makes in six minutes at normal rotation speed.

c) the radius of the Ferris wheel.

d) how far above the ground the seat is at its lowest point.

e) the probability that a seat will be a least 15 m above the ground if the Ferris wheel loses power and stops.

PROBLEM 6

A matrix is made up of i rows and j columns. Each entry a_{ij} is generated by the formula:

$$a_{ij} = f(i) \times g(j) \quad \text{where} \quad f(i) = 2i - 3 \quad \text{and} \quad g(j) = 3j + 2$$

Find:

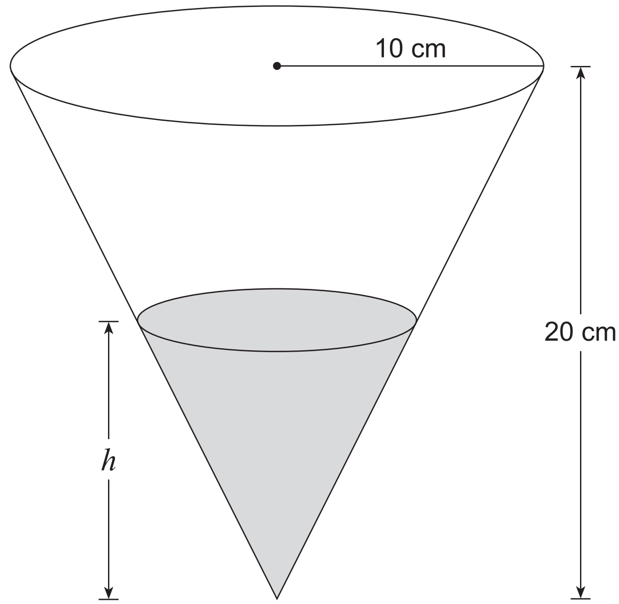
a) the entry in the 6th row and the 8th column of a 10×10 matrix.

b) the sum of all the entries in a 3×3 matrix.

c) the sum of all the entries in a 6×6 matrix.

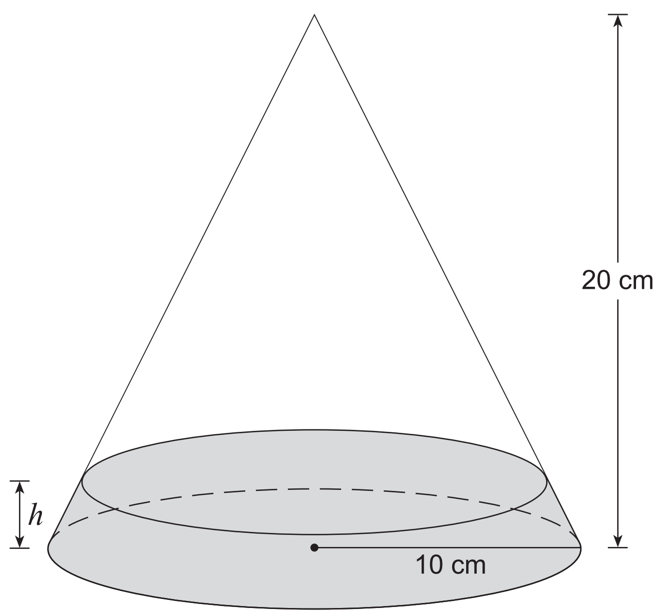
PROBLEM 7

A transparent sealed plastic cone of radius 10 cm and height 20 cm contains 300 mL of liquid.



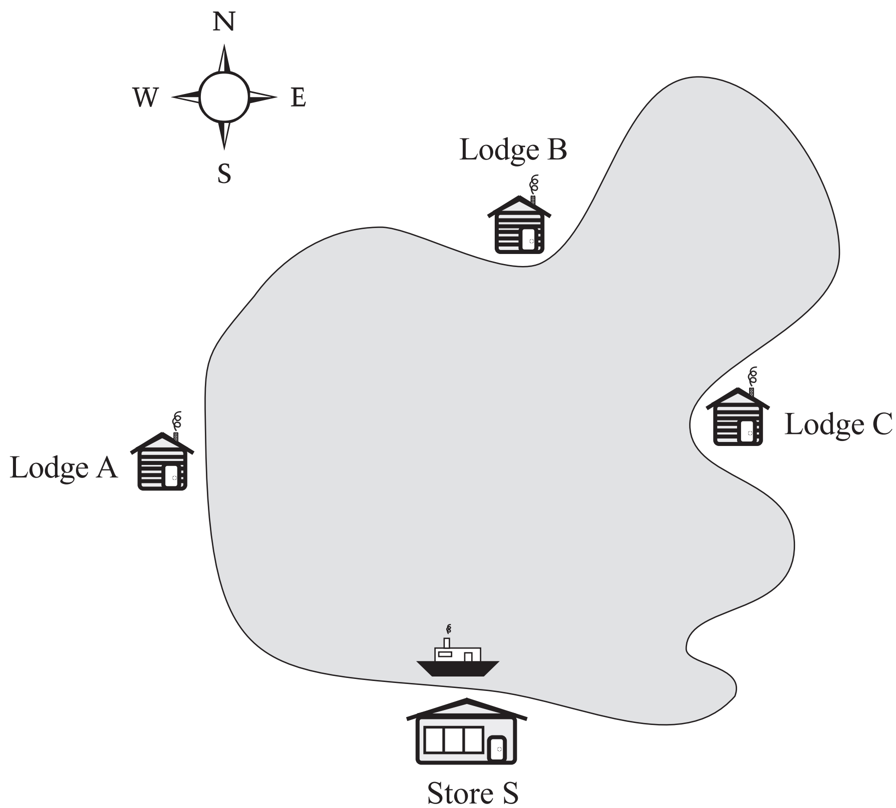
a) How high is the level of the liquid when the cone is standing on its tip as shown in the diagram above?

b) If the cone is inverted, how high is the level of the liquid?



Problem 8

The owner of a store on a remote lake supplies three wilderness lodges with groceries by boat. The lodges are situated as shown in the diagram below. (Diagram not drawn to scale.)



The owner travels from Lodge A to B to C and then back to the store to make his deliveries.

- Lodge A is 15 km from the store at a bearing of $315°$.
- Lodge B is 20 km from Lodge A at a bearing of $60°$.
- Lodge C is 10 km from Lodge B at a bearing of $120°$.
- The normal speed of the owner's boat is $30\,\text{km/h}$.
- The boat uses 18 litres of gas per hour travelling at this speed.

a) How far and at what heading is Lodge B from the store?

b) How far and at what heading is Lodge C from the store?

c) Assuming the boat has a normal operating speed of $30\,\text{km/h}$ for the entire trip and that gas costs 85.9 cents per litre, how much does the gas cost to make the trip to all the lodges and then back to the store?