MATHEMATICS COGNITIVE PROCESSES – GRADE 10

To respond correctly to provincial examination items students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. The first process, *knowing,* covers the facts, procedures, and concepts students need to know, while the second, *applying,* focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The third process, *reasoning*, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts and multi-step problems.

Knowing

Facility in using mathematics, or reasoning about mathematical situations, depends on mathematical knowledge and familiarity with mathematical concepts. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she has understood, the greater the potential for engaging a wide range of problem-solving situations and for developing mathematical understanding. Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. Facts encompass the factual knowledge that provides the basic language of mathematics, and the essential mathematical facts and properties that form the foundation for mathematical thought. Procedures form a bridge between more basic knowledge and the use of mathematics for solving routine problems, especially those encountered by many people in their daily lives. In essence a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Knowledge of concepts enables students to make connections between elements of knowledge that, at best, would otherwise be retained as isolated facts. It allows them to make extensions beyond their existing knowledge, judge the validity of mathematical statements and methods, and create mathematical representations.

This cognitive process covers the following behaviours:

1. **Recall** Recall definitions, terminology, number properties, geometric properties and notation.
2. **Recognize** Recognize mathematical objects, shapes, numbers and expressions. Recognize mathematical entities that are mathematically equivalent.
3. **Compute** Carry out procedures for $+$, $-$, $\times$, $\div$, or a combination of these with rational numbers, radicals, powers and polynomials. Approximate numbers to estimate computations. Carry out routine algebraic procedures. Compute %, factorize, and add hours in a time chart.
4. **Retrieve** Retrieve information from graphs, tables or other sources; read simple scales.
5. **Measure** Use measuring instruments; use units of measurement appropriately; estimate measures; convert units (imperial $\leftrightarrow$ SI) in one dimension; and express total time worked in decimal form and in hours and minutes.
6. **Classify/Order** Classify/group objects, shapes, numbers and expressions according to common properties; make correct decisions about class membership; and order numbers and objects by attributes.

Applying

Problem solving is a central aim, and often means, of teaching school mathematics, and hence this and supporting skills (e.g., select, represent, model) feature prominently in the process of *applying* knowledge and conceptual understanding. In items aligned with this process, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations and solve problems. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations is fundamental to success in the subject.

The problem settings are more routine than those aligned with the reasoning process. The routine problems will typically have been standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems will have been in words that set the problem situation in a quasi-real context. Though they range in difficulty, each of these types of "textbook" problems is expected to be sufficiently familiar to students that they will essentially involve selecting and applying learned procedures. Problems may be set in real-life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations or geometric figures. Therefore, problem solving is included not only in the *applying* process, with emphasis on the more familiar and routine tasks but also in the *reasoning*, process.

This cognitive process covers the following behaviours:

1. **Select** Select an efficient/appropriate operation, method or strategy for solving problems.
2. **Represent** Display mathematical information and data in diagrams, tables, charts, or graphs, and select equivalent representations for a given mathematical entity or relationship.
3. **Model** Generate an appropriate model, such as an equation or diagram for solving a routine problem.
4. **Implement** Follow and execute a set of mathematical instructions. Given specifications, draw figures and shapes.
5. **Convert** Convert units (imperial $\leftrightarrow$ SI) in two dimensions.
6. **Solve Routine Problems** Solve routine problems (i.e., problems similar to those students are likely to have encountered in class).
7. **Analyze Solution(s) to Routine Problems** Analyze solutions to routine problems to select the best one; identify errors in a solution to a routine problem.

Reasoning

Reasoning mathematically involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems. Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned.

Non-routine problems may be purely mathematical or may have real life settings. Both types of items involve transfer of knowledge and skills to new situations; interactions among reasoning skills are usually a feature. Problems requiring reasoning may do so in different ways, because of the novelty of the context or the complexity of the situation or because any solution to the problem must involve several steps, perhaps drawing on knowledge and understanding from different areas of mathematics.

Even though of the many behaviours listed within the reasoning process are those that may be drawn on in thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

This cognitive process covers the following behaviours:

1. **Analyze** Determine and describe or use relationships between variables or objects in mathematical situations; decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; visualize transformations of three-dimensional figure; and make valid inferences from given information.
2. **Generalize** Extend the process to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms.
3. **Synthesize/Integrate** Combine mathematical procedures to establish results, and combine results to produce a further result. Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas.
4. **Justify** Provide a justification for the truth or falsity of a statement by reference to mathematical results or properties.
5. **Solve Non-routine Problems** Solve problems set in mathematical or real life contexts where students are unlikely to have encountered closely similar items, and apply mathematical procedures in unfamiliar or complex contexts. Use geometric properties to solve non-routine problems.

Adapted from "Mathematics Cognitive Domain". *TIMSS 2007 Mathematics Framework* [http://timss.bc.edu/timss2007/PDF/T07_AF_chapter1.pdf]